

Estimating MPI Penalty

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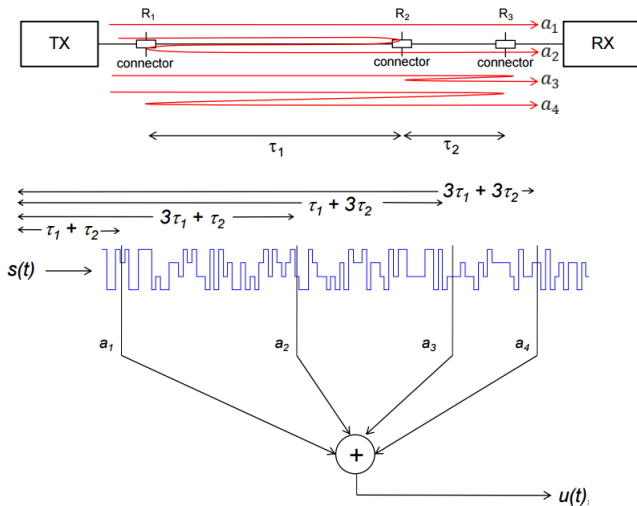
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- ▶ Optical link power penalty associated with MPI (Multi-Path Interference) is an important part of link power budget. It also helps determine maximum discrete reflectance and optical return loss of a channel.
- ▶ Although the worst-case outcome, an outage, has a low probability of occurring, it can severely impair link performance for long when it occurs.
- ▶ MPI penalty is difficult to measure experimentally.
- ▶ Comprehensive, closed-form analytical solution is also difficult.
- ▶ Here we present a combination of approximation and simulation in order to help estimate MPI penalty.

Plan of This Presentation

1. Describe Upper Bound model
2. Introduce a Discount Factor
3. Describe simulation
4. Show alignment between Discounted Upper Bound and simulation
5. Estimate a range of values of MPI penalty for various link scenarios

Upper Bound



The received signal $u(t)$ is the sum of these delayed replicas of transmitted signals. Received power is $|u(t)|^2$.

Upper Bound

- ▶ For PAM- m , amplitudes A_i , $i = 1..m$, are transmitted.
- ▶ Received signal field $u(t) = B_0 e^{j\omega t} + \sum_{k=1}^N \sqrt{R^2} B_k e^{j(\omega t + \tilde{\theta}_k)}$, where
 - ▶ B_0 is the victim amplitude; B_k are the interfering amplitudes
 - ▶ $\tilde{\theta}_k$ is a random variable in $[0, 2\pi)$. It accounts for various path lengths of interference etalons, as well as spectral width / phase noise. For a more granular treatment of $\tilde{\theta}$ that separately accounts for phase noise and path length, see reference [1].
 - ▶ N is the number of interfering terms. $N = p(p-1)/2$, where p is the number of reflectance points in a link: n number of connectors + 2 PMD reflectance points.
 - ▶ PMD reflectance is assumed equal to connector reflectance R .
- ▶ We make two worst-case assumptions:
 - ▶ $B_j = A_m$ for all $j \in [0, N]$. Victim is at highest PAM amplitude, and all interfering terms are of highest PAM amplitude.
 - ▶ $\tilde{\theta}_k = \tilde{\theta}$, i.e., it is common to all interferers

Upper Bound

- ▶ Therefore, $u(t) = A_m e^{j\omega t} (1 + NR e^{j\tilde{\theta}})$ where $NR e^{j\tilde{\theta}}$ is the interference term.
- ▶ $I(t) = |u(t)|^2 \approx A_m^2 (1 + 2NR \cos\tilde{\theta})$ where $2NR \cos\tilde{\theta}$ is the noise intensity term.
- ▶ Since $\cos\tilde{\theta}$ is bounded within $[-1, 1]$, peak-to-peak noise intensity $\leq 4NR A_m^2$.
- ▶ MPI Penalty, dB = $10 \log_{10} \left(\frac{OMA_{inner}}{OMA_{inner} - 4NR A_m^2} \right)$
- ▶ Substitute $OMA_{inner} = \frac{A_m^2 - A_1^2}{m-1}$, extinction ratio $E = \frac{A_m^2}{A_1^2}$
- ▶ MPI Penalty, dB = $10 \log_{10} \left(\frac{1}{1-x} \right)$, $x = (m-1)4NR \left(\frac{E}{E-1} \right)$
- ▶ This is an upper bound. *The reward of this conservative choice is elimination of outage risk.*

Accounting for PMD Reflectances Separately

- ▶ It is helpful to separate out reflectance values of transmitter, receiver, and connectors, because it enables us to explore various scenarios.
- ▶ For n connectors between T_x and R_x , We can count various reflections separately and add them up [4].
 - ▶ One reflection between T_x and R_x
 - ▶ n reflections between T_x and n connectors
 - ▶ n reflections between R_x and n connectors
 - ▶ $n(n-1)/2$ reflections among n connectors
- ▶ MPI Penalty, dB = $10 \log_{10}\left(\frac{1}{1-x}\right)$, $x = (m-1)4S\left(\frac{E}{E-1}\right)$,
where $S = \sqrt{R_t R_r} + n\sqrt{R_t R_c} + n\sqrt{R_r R_c} + \frac{n(n-1)}{2} R_c$
 R_c, R_t, R_r are discrete reflectances of connectors, transmitter and receiver, respectively. Table 1 lists a few examples.

MPI Penalty, Upper Bound

Extinction Ratio 4.5 dB

Table 1: MPI Penalty, Upper Bound, for 2, 4 and 6 connectors. PAM4, Ext. Ratio 4.5 dB. All values in dB. No discount factor applied ($D = 1$).

Cases	Tx	Rx	Conn	Pmpi(2)	Pmpi(4)	Pmpi(6)
Case A	26	26	26	1.43	5.24	-
Case B	20	20	26	4.04	-	-
Case C	26	26	35	0.55	1.05	1.76
Case D	35	35	35	0.16	0.40	0.78
Case E	26	26	55	0.24	0.27	0.30
Case F	26	26	45	0.31	0.42	0.55
Case G	20	26	55	0.47	0.52	0.57
Case H	20	26	45	0.58	0.75	0.95
Case I	20	26	35	0.96	1.72	2.83

MPI Penalty, Upper Bound

Extinction Ratio 5 dB

Table 2: MPI Penalty, Upper Bound, for 2, 4 and 6 connectors. PAM4, Ext. Ratio 5 dB. All values in dB. No discount factor applied ($D = 1$).

Cases	Tx	Rx	Conn	Pmpi(2)	Pmpi(4)	Pmpi(6)
Case A	26	26	26	1.33	4.70	-
Case B	20	20	26	3.68	-	-
Case C	26	26	35	0.52	0.98	1.64
Case D	35	35	35	0.15	0.38	0.73
Case E	26	26	55	0.22	0.25	0.29
Case F	26	26	45	0.29	0.40	0.51
Case G	20	26	55	0.44	0.49	0.54
Case H	20	26	45	0.55	0.71	0.89
Case I	20	26	35	0.90	1.60	2.61

MPI Penalty, Upper Bound

Extinction Ratio 6 dB

Table 3: MPI Penalty, Upper Bound, for 2, 4 and 6 connectors. PAM4, Ext. Ratio 6 dB. All values in dB. No discount factor applied ($D = 1$).

Cases	Tx	Rx	Conn	Pmpi(2)	Pmpi(4)	Pmpi(6)
Case A	26	26	26	1.20	4.01	-
Case B	20	20	26	3.20	-	-
Case C	26	26	35	0.47	0.89	1.47
Case D	35	35	35	0.13	0.34	0.66
Case E	26	26	55	0.20	0.23	0.26
Case F	26	26	45	0.26	0.36	0.47
Case G	20	26	55	0.40	0.45	0.49
Case H	20	26	45	0.49	0.64	0.80
Case I	20	26	35	0.81	1.44	2.31

Discount Factor

- ▶ We now introduce an arbitrary discount factor D , to compensate for the highly conservative nature of this upper bound – but without raising the outage risk.
- ▶ MPI Penalty, $\text{dB} = 10 \log_{10}\left(\frac{1}{1-x}\right)$, $x = D(m-1)4S\left(\frac{E}{E-1}\right)$ where $0 < D \leq 1$
- ▶ How should we determine the appropriate value of D ?
 - ▶ Precedents: Look in past IEEE link models
 - ▶ Estimation: Derive a simple approximation
 - ▶ Simulation: Perform Monte Carlo analysis
 - ▶ Measurement: Preferred but hard to get it right
 - ▶ A combination of the above, using good judgment. This presentation includes the first two.

Discount Factor: Precedents

- ▶ In the past, IEEE link models have used a similar discount factor called Reflection Noise factor [3].
- ▶ From Notes: *"Reflection noise factor of 0.6 introduced to avoid undue pessimism. The value needs further consideration."*

Table 4: Reflection Noise Factors Used in past IEEE Link Models*

File	Tab	Cell	Value
10GEPBud3_1_16a.xls	LX4_SMF	L10	0.6
	1310S	L10	0.6
	1550S40km	L10	0.6
EFM0_0_2.7.xls	1000LX10SMF	L11	0.2
	1000BX10.1490	L11	0.6
	1000PX10.1310	L11	0.2

*Binary NRZ, 2 PMD reflectances only (no connectors)

Two Components of Discount Factor

- ▶ Let's consider two discounts, using simple approximations.
- ▶ Amplitude Discount
 - ▶ At 25 GBaud, a PAM symbol occupies only 8 meters of fiber. If we assume that interfering terms are from fairly independent symbols, where each symbol has PAM amplitude from $\{0,1,2,3\}$, we can scale down the magnitude of interference.
 - ▶ Risk Scenario: A long burst of PAM 3 symbols.
- ▶ Attenuation Discount
 - ▶ We can view a link as made of multiple segments, where each segment represents a combination of connector insertion loss and fiber attenuation. Interfering terms get more attenuated than signal, as they get bounced around the link.

Amplitude Discount

- ▶ Amplitude Discount Factor

$$D_1 = \frac{1}{4} \left(\frac{1}{\sqrt{E}} + \sqrt{\frac{E+2}{3E}} + \sqrt{\frac{2E+1}{3E}} + 1 \right)$$

- ▶ See Appendix B for derivation of D_1
- ▶ MPI Penalty, dB = $10 \log_{10} \left(\frac{1}{1-x} \right)$, $x = D_1(m-1)4S \left(\frac{E}{E-1} \right)$

Table 5: Amplitude Discount Factor D_1 for PAM4

E(dB)	D_1
4	0.82
4.5	0.81
5	0.79
6	0.77
8	0.73
100	0.60

Attenuation Discount

- ▶ Attenuation Discount Factor $D_2 = \frac{\hat{S}}{S}$
- ▶ See Appendix C for derivation of D_2 , based on the assumption that channel insertion loss is evenly divided over n segments.
- ▶ MPI Penalty, dB = $10 \log_{10}(\frac{1}{1-x})$, $x = D_2(m-1)4S(\frac{E}{E-1})$
- ▶ See table on the next page for values of D_2 and how they affect overall discount.

Example Values of Discount Factor D

Table 6: Example values of Discount Factor D and MPI Penalty

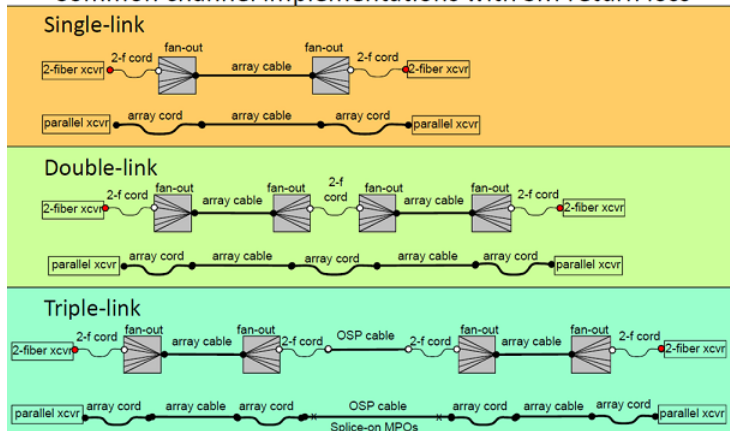
Cases	E	n	seg	Rt	Rr	Rc	ChIL	D_1	D_2	D
DR4-A	5	4	0.01	-20	-26	-35	0.0	0.79	1.00	0.79
DR4-B	5	4	0.75	-20	-26	-35	3.0	0.79	0.72	0.57
DR4-C	5	4	0.75	-20	-26	-45	3.0	0.79	0.62	0.49
FR8-A	4.5	2	2.00	-26	-26	-26	4.0	0.81	0.78	0.63
FR8-B	4.5	4	0.01	-26	-26	-35	0.0	0.81	1.00	0.81
FR8-C	4.5	4	1.00	-26	-26	-35	4.0	0.81	0.68	0.55
LR8-A	4.5	2	3.00	-26	-26	-26	6.0	0.81	0.71	0.57
LR8-B	4.5	6	0.01	-26	-26	-35	0.1	0.81	0.99	0.80
LR8-C	4.5	6	1.00	-26	-26	-35	6.0	0.81	0.60	0.48

- ▶ Notice how D_2 moves in opposite direction to ChIL, making D stay near 0.5 at max ChIL (marked in red). This suggests that we should estimate MPI penalty for $D=0.5$ and $D=0.6$.

Channel Model Diagram

See Reference [5]

Common channel implementations with SM return loss



Return Loss Legend

- 26 dB
- 26 to 35 dB
- 55 dB

1

MPI Penalty for $D=0.5$ and $D=0.6$

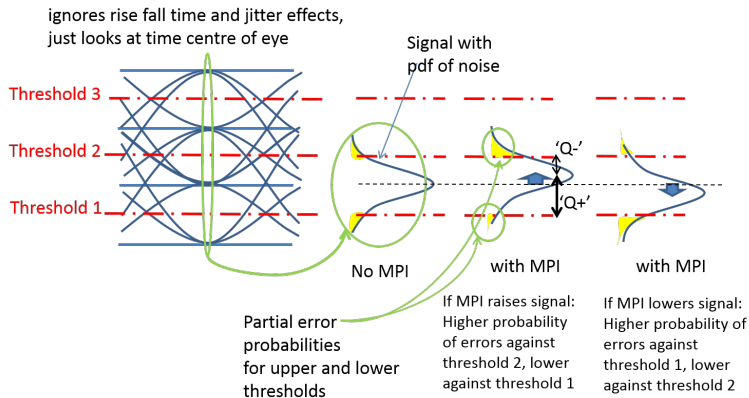
Table 7: MPI Penalty for Discount Factor $D=0.5$ and $D=0.6$. Also shown is upper bound ($D=1$), for comparison. D includes D_1 , which accounts for varying amplitudes of PAM interference terms, and D_2 , which accounts for channel insertion loss.

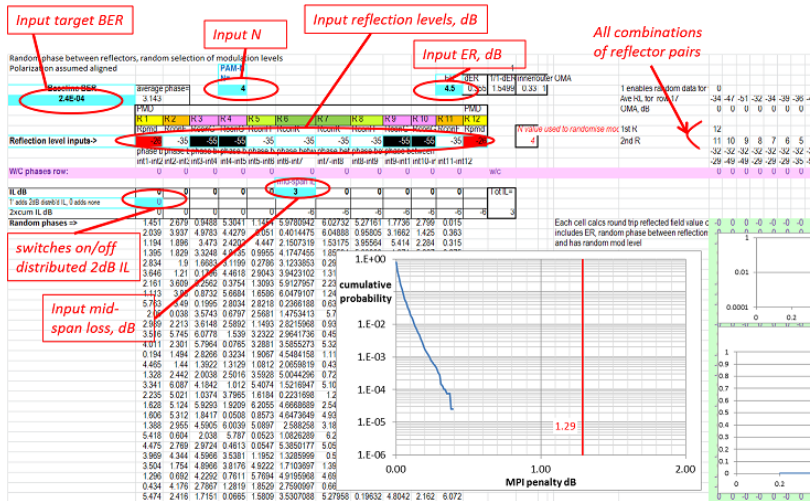
Cases	Fiber	E	n	Rt	Rr	Rc	Pmpi D=0.5	Pmpi D=0.6	Pmpi D=1.0
Single-Link	duplex	4.5	2	-26	-26	-26	0.66	0.80	1.43
	duplex	4.5	2	-26	-26	-35	0.27	0.32	0.55
	parallel	4.5	2	-20	-26	-45	0.28	0.34	0.58
Double-Link	duplex	4.5	4	-26	-26	-35	0.49	0.60	1.05
	parallel	4.5	4	-20	-26	-45	0.36	0.44	0.75
Triple-Link	duplex	4.5	6	-26	-26	-35	0.79	0.97	1.76
	parallel	4.5	6	-20	-26	-45	0.45	0.54	0.95

- ▶ Independently, a spreadsheet-based statistical model using the Monte Carlo simulation technique has been developed.
- ▶ It is capable of modeling 12 reflectance points that can be specified individually, including PMD reflectance at each end of the link.
- ▶ It is available for sharing. See reference [6]
- ▶ Filename is king_02_0116_smf.7z. It's a 7z zipped file which extracts to about 27M and then needs to be extended by duplicating the bottom row of the spreadsheet.

PAM4 Cartoon – What the Statistical Model Does

- Partial error probabilities are calculated for each signal modulation level and its adjacent thresholds, and used to derive a Q penalty due to MPI.





Link Model Cases Considered

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
S1	26	35	55	55	35	26						
S2	55	55	55	55								
S2a	35	35	35	35								
D1	26	35	55	55	35	35	55	55	35	26		
D2	55	55	55	55	55	55						
D2a	35	35	35	35	35	35						
T1	26	35	55	55	35	35	35	35	55	55	35	26
T2	55	55	55	55	55	55	55	55				
T2a	35	35	35	35	35	35	35	35				

- ▶ Same channel model diagram as shown on page 18.
- ▶ Single-Link: S1, S2 (4x55 dB), S2a (4x 35 dB)
- ▶ Double-Link: D1, D2 (6x55 dB), D2a (6x35 dB)
- ▶ Triple-Link: T1, T2 (8x55 dB), T2a (8x35 dB)
- ▶ ER 4.5 dB, 0 dB link loss

Single-Link S1

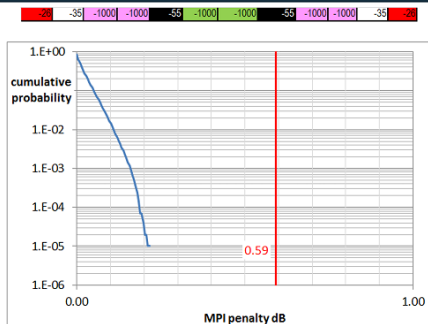


Table 8: MPI Penalty, dB, for Single-Link S1. 2 connectors at -35 dB, 2 connectors at -55 dB, PMD at -26 dB, zero insertion loss, ER 4.5 dB. Monte Carlo high confidence is defined as 99.9999%

	Worst-Case	High-Confidence
Upper Bound, $D=0.5$		0.27
Upper Bound, $D=1.0$	0.55	
Monte Carlo	0.59	0.25

Double-Link D1

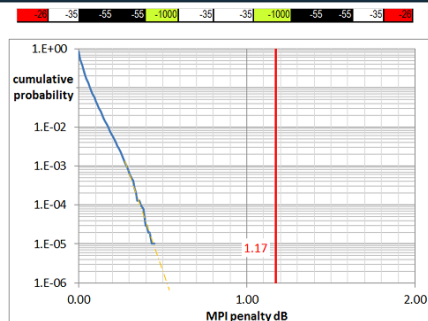


Table 9: MPI Penalty, dB, for Double-Link D1. 4 connectors at -35 dB, 4 connectors at -55 dB, PMD at -26 dB, zero insertion loss, ER 4.5 dB. Monte Carlo high confidence is defined as 99.9999%

	Worst-Case	High-Confidence
Upper Bound, D=0.5		0.49
Upper Bound, D=1.0	1.05	
Monte Carlo	1.17	0.52

Triple-Link T1

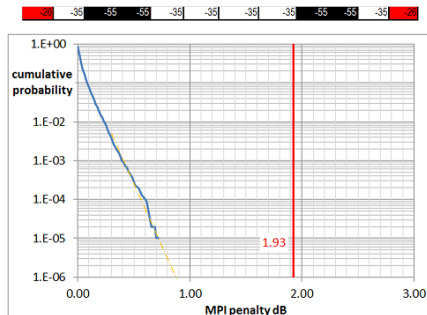
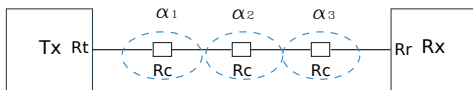


Table 10: MPI Penalty, dB, for Triple-Link T1. 6 connectors at -35 dB, 4 connectors at -55 dB, PMD at -26 dB, zero insertion loss, ER 4.5 dB. Monte Carlo high confidence is defined as 99.9999%

	Worst-Case	High-Confidence
Upper Bound, D=0.5		0.79
Upper Bound, D=1.0	1.76	
Monte Carlo	1.93	0.90

D_2 vs. Loss Location



- ▶ In Appendix C, we derive D_2 for evenly distributed loss.
- ▶ Now let's consider the case where loss is not evenly distributed. For convenience, take $n = 3$ connectors.
- ▶ By counting each reflection separately, it can be shown that:

$$\hat{S}' = \sqrt{R_t R_r}(\alpha_1 \alpha_2 \alpha_3) + \sqrt{R_t R_c}(1 + \alpha_1 + \alpha_1 \alpha_2) + \sqrt{R_r R_c}(1 + \alpha_3 + \alpha_2 \alpha_3) + R_c(2 + \alpha_2)$$
$$\text{and } D'_2 = \frac{\hat{S}'}{S}$$

- ▶ For evenly distributed load, $\alpha_1 = \alpha_2 = \alpha_3$ and $D'_2 = D_2$
- ▶ Let's consider 4 cases of loss location.

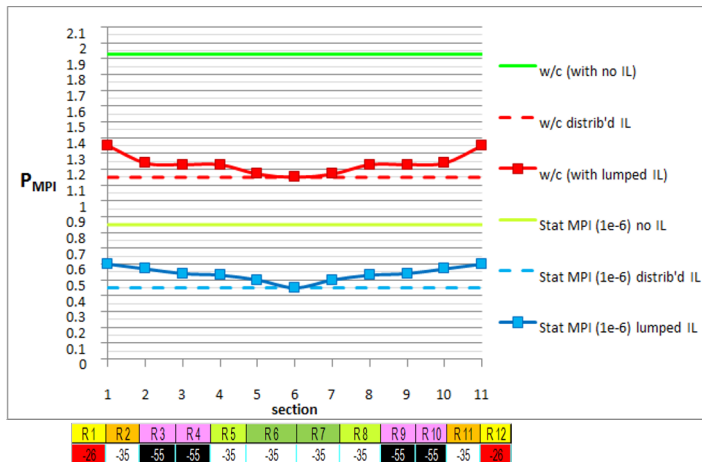
D'_2 vs. D_2 for $n = 3$

Table 11: Comparison of D_2 and D'_2 , to illustrate the effect of location of attenuation in the link. ChIL 6 dB, $n=3$, connector discrete reflectance -35 dB, PMD reflectance -26 dB.

Loss Location	α_1	α_2	α_3	D_2	D'_2
Left	0.25	1.00	1.00	0.58	0.63
Right	1.00	1.00	0.25	0.58	0.63
Middle	1.00	0.25	1.00	0.58	0.61
Distributed	0.63	0.63	0.63	0.58	0.58

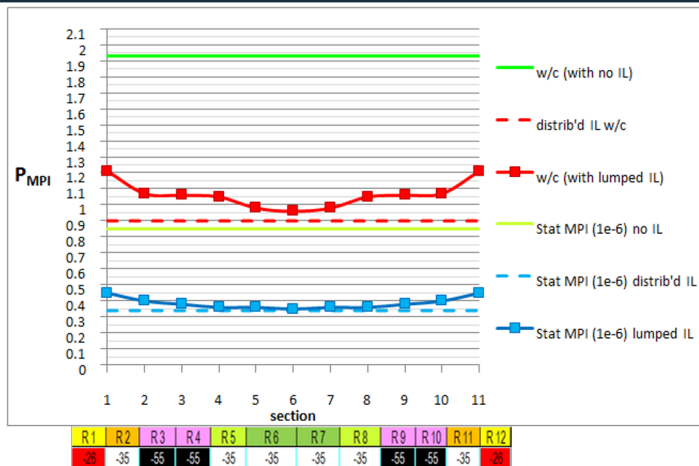
- ▶ D'_2 is about 10% greater than D_2 in the corner case of all channel loss being concentrated at either end of the link.
- ▶ It can be shown that this effect is milder for smaller channel loss and better connector reflectance.
- ▶ We now show that Monte Carlo simulations corroborate this.

Effect of Loss Location: 4 dB IL



- ▶ Solid lines: x axis denotes the link segment number where loss is localized.
- ▶ Dotted lines: Show results based on distributed insertion loss.

Effect of Loss Location: 6.3 dB IL



- ▶ For links where channel loss is in the span closest to PMD, a slightly higher allocation of MPI penalty may be necessary.
- ▶ See Appendix D for MPI Penalty plots of various cases.

Conclusion

- ▶ We presented an analytic approximation of MPI penalty using discounted upper bound. We also presented Monte Carlo simulations for various cases of interest.
- ▶ Discounted upper bound and Monte Carlo simulations represent two views of the same problem, with two different perspectives and methods. They present a range of values of MPI penalty.
 - ▶ Discounted upper bound is based on fixed PAM3 level of transmitted signal but varying interfering amplitudes, and fixed worst-case phases.
 - ▶ Monte Carlo assumes both transmitted and interfering signals have varying amplitudes and phases.
 - ▶ For zero insertion loss, a range of $D = 0.5$ (matching Monte Carlo at 99.9999% confidence) to $D = 0.8$ ($D_1 = 0.8$, $D_2 = 1$) is a good starting point of consideration for estimating MPI penalty.
 - ▶ More practically, for nonzero insertion loss, this range can be lowered to, say, $D = 0.4$ to $D = 0.6$.
 - ▶ Both methods confirm that there is some dependence on where the insertion loss is concentrated.

References

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Appendix A: Summary of Equations

$$\text{MPI Penalty, dB} = 10 \log_{10}\left(\frac{1}{1-x}\right) \quad (1)$$

$$x = D(m-1)4S\left(\frac{E}{E-1}\right) \quad (2)$$

$$S = \sqrt{R_t R_r} + n\sqrt{R_t R_c} + n\sqrt{R_r R_c} + \frac{n(n-1)}{2} R_c \quad (3)$$

$$D = D_1 D_2 \quad (4)$$

$$D_1 = \frac{1}{4} \left(\frac{1}{\sqrt{E}} + \sqrt{\frac{E+2}{3E}} + \sqrt{\frac{2E+1}{3E}} + 1 \right) \quad (5)$$

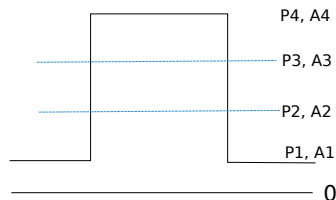
$$D_2 = \frac{\hat{S}}{S} \quad (6)$$

$$\hat{S} = \sqrt{R_t R_r} \cdot \sqrt{\alpha^{2n}} + \frac{1-\alpha^n}{1-\alpha} \cdot \left(\sqrt{R_t R_c} + \sqrt{R_c R_r} \right) + R_c \cdot \left(\frac{n}{1-\alpha} + \frac{\alpha^n - 1}{(1-\alpha)^2} \right) \quad (7)$$

α : transmission coefficient of a link segment, E: extinction ratio, m: number of PAM levels, n: number of connectors, R_c, R_t, R_r : reflectance values of connectors, transmitter and receiver, respectively.

Appendix B: Derivation of D1 (Amplitude Discount)

- ▶ For upper bound, we had assumed $B_j = A_4, \forall j$, for PAM4, in received field $u(t) = B_0 e^{j\omega t} + \sum_{k=1}^N \sqrt{R^2} B_k e^{j(\omega t + \tilde{\theta})}$
- ▶ Let's change that to $B_0 = A_4$, and $B_k, k \in [1, N]$, equally likely from $\{A_1, A_2, A_3, A_4\}$, with probability $\frac{1}{4}$ each. Transmitted pulse is still of highest amplitude, but interfering pulses can have any of the 4 PAM4 amplitudes.



E: Extinction Ratio

$$\begin{aligned}P_1 &= P_1 \\P_2 &= P_1 + \left(\frac{P_4 - P_1}{3}\right) = P_1 + \left(\frac{EP_1 - P_1}{3}\right) = P_1 \left(\frac{E+2}{3}\right) \\P_3 &= P_2 + \left(\frac{P_4 - P_1}{3}\right) = P_1 \left(\frac{2E+1}{3}\right) \\P_4 &= EP_1, \text{ so } A_4^2 = EA_1^2\end{aligned}$$

This leads to

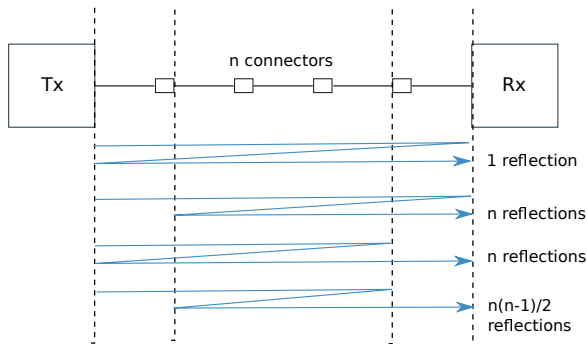
$$\begin{aligned}A_1 &= \sqrt{P_1} = A_4 \frac{1}{\sqrt{E}}, \quad A_2 = \sqrt{P_2} = A_4 \sqrt{\frac{E+2}{3E}} \\A_3 &= \sqrt{P_3} = A_4 \sqrt{\frac{2E+1}{3E}}, \quad A_4 = \sqrt{P_4} = A_4\end{aligned}$$

Now, as in [4], we replace A_4 with

$$D_1 A_4 = \frac{1}{4}(A_1 + A_2 + A_3 + A_4) = A_4 \frac{1}{4} \left(\frac{1}{\sqrt{E}} + \sqrt{\frac{E+2}{3E}} + \sqrt{\frac{2E+1}{3E}} + 1 \right)$$

$$\therefore D_1 = \frac{1}{4} \left(\frac{1}{\sqrt{E}} + \sqrt{\frac{E+2}{3E}} + \sqrt{\frac{2E+1}{3E}} + 1 \right)$$

Appendix C: Derivation of D2 (Attenuation Discount)



- ▶ Signal travels forth, crossing n connectors
- ▶ An interfering term sloshes around – forth, back, and forth – traveling through *additional segments*, relative to the victim.
- ▶ Calculation of S can be replaced with \hat{S} to explicitly model the additional attenuation.

Derivation of D2

Total additional loss of a reflected path scales directly with the number of connectors *between* the interfaces at which the reflections occur. Assume α is the transmission coefficient, and is the same for each segment (loss is evenly distributed). It is the result of a combination of connector insertion loss and fiber attenuation.

$D_2 = \frac{\hat{S}}{5}$ where

$$\begin{aligned}\hat{S} = & \sqrt{R_t R_r} \cdot \sqrt{\alpha^{2n}} + \\ & \sqrt{R_t R_c} \cdot \left(1 + \sqrt{\alpha^2} + \sqrt{\alpha^4} + \dots + \sqrt{\alpha^{2(n-1)}}\right) + \\ & \sqrt{R_r R_c} \cdot \left(1 + \sqrt{\alpha^2} + \sqrt{\alpha^4} + \dots + \sqrt{\alpha^{2(n-1)}}\right) + \\ & \sqrt{R_c R_c} \cdot \left((n-1) + (n-2)\sqrt{\alpha^2} + \dots + \sqrt{\alpha^{2(n-2)}}\right)\end{aligned}$$

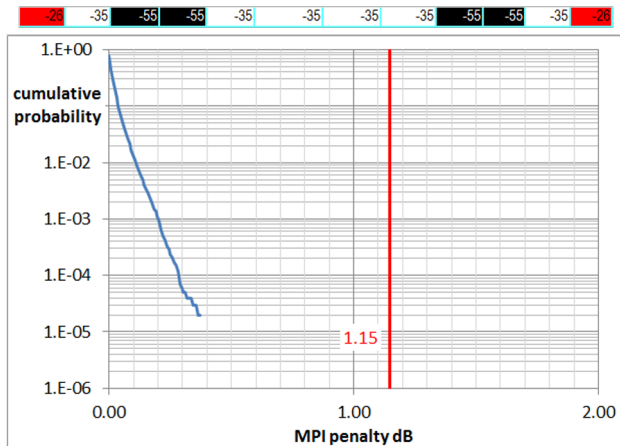
which simplifies to

$$\hat{S} = \sqrt{R_t R_r} \cdot \sqrt{\alpha^{2n}} + \frac{1-\alpha^n}{1-\alpha} \cdot (\sqrt{R_t R_c} + \sqrt{R_c R_r}) + R_c \cdot \left(\frac{n}{1-\alpha} + \frac{\alpha^n - 1}{(1-\alpha)^2}\right)$$

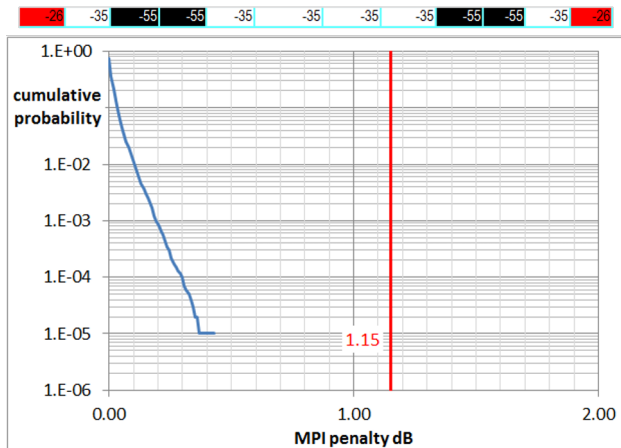
Other, simpler approximations of D_2 are possible.

Appendix D: Simulation Plots for Various Cases

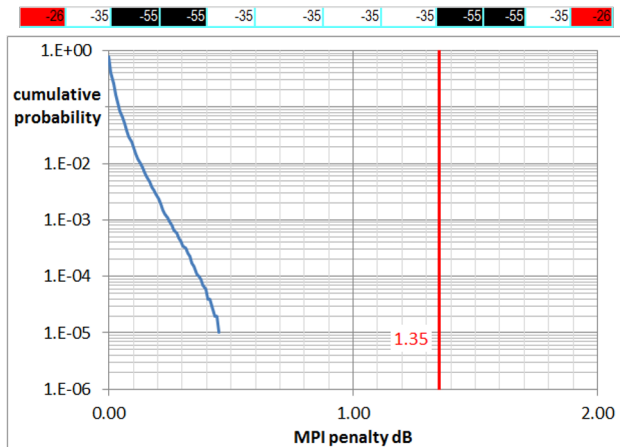
4 dB mid-span



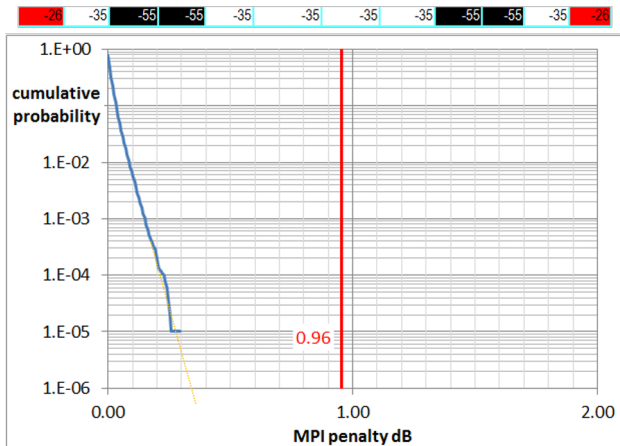
4 dB distributed



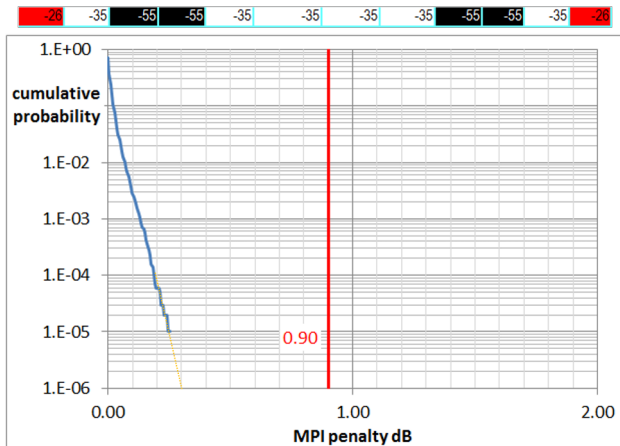
4 dB just after Tx



6.3 dB mid-span



6.3 dB distributed



6.3 dB just after Tx

