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Evaluation of FEC Performance with Symbol and Bit muxing Scenarios

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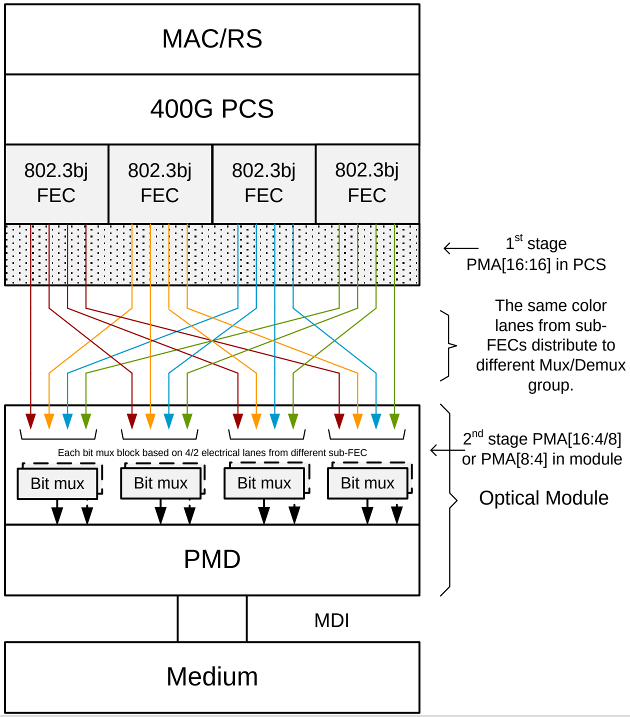
Motivation

- When burst errors happens in electrical and optical lanes, interleaving data from parallel FEC in 400GE ease this problem.
- The method of using FEC Orthogonal Multiplexing(FOM) in 400GE was presented in January meeting 2014 at Indian Well (wang_400_01a_0114.pdf), which also enables protocol-agnostic optical modules and empowers lower cost & broad market potential.



- This contribution includes the quantitative analysis of FOM.
- Four Muxing methods are illustrated: (use 4:1 muxing as example)
  - Orthogonal symbol mux
  - Orthogonal bit mux
  - Non-orthogonal symbol mux
  - Non-orthogonal bit mux
- Similar calculation applies to 2:1 Muxing

Post FEC BER Calculation

- **Random Error Model**

$$SER_{pre} = 1 - (1 - BER_{pre})^{m}$$

$$P_{UE} = \sum_{i=t+1}^{n} \frac{i}{n} * \binom{n}{i} * SER_{pre}^{i} * \left(1 - SER_{pre}\right)^{n-i}$$

$$BER_{post} \approx P_{UE} / m$$

- **Burst Error Model (Gilbert model)**
  - Error propagation is modeled by probability calculation*

$$BER_{post} = \sum_{i=t+1}^{rll_{\max}} \sum_{all\ E} p(rll = i, E) * W(E) * BER_{pre} * (1 - BER_{pre})^{n - rll_{\max} - i}$$

Average weight of burst error

All the combinations of the error pattern

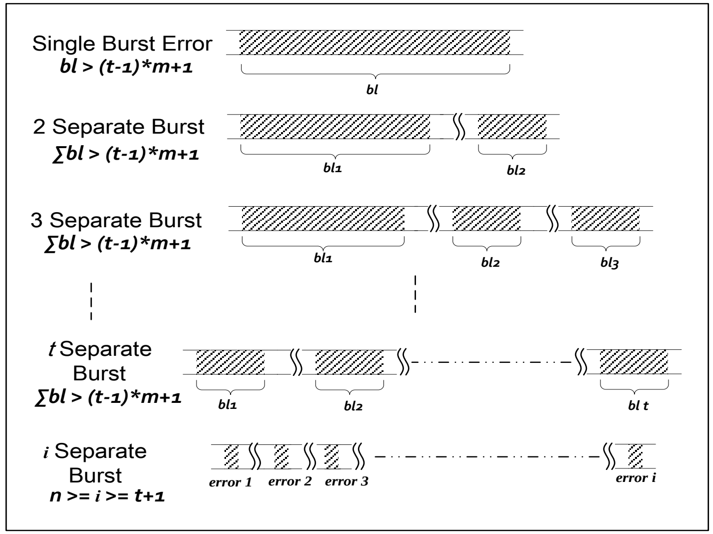
Burst error propagation length

Length of uncorrectable burst errors

* Cathy Ye Liu and Joe Caroselli, "Modeling and Mitigation of Error Propagation of Decision Feedback Equalization in High Speed Backplane Transceivers." Proceedings of DesignCon 2006.

Post FEC BER Calculation(Cont'd)

- **Uncorrectable Burst Error Patterns for RSFEC(*n,k,t,m*)**
  - Single burst error (>*t* symbols)
  - Separate burst errors (∑***bl*** > t symbols)
    - *2, 3, … t* separate burst errors.
  - More than *t* separate burst errors



- **Post BER**
  - Assuming
    - Rllmax = 17bit, b=0.5*;
    - G(x) is the probability for having a burst error with length of x symbol.
    - M(x) is the probability for having a burst error longer than x symbol, i.e. $M(x) = \sum_{i=x}^{rll\max/m} G(i)$
  - BER_{Post}

$$BER_{post} = \binom{n}{1} * M(t+1) * W(E) + \binom{n}{2} * \sum_{i=1}^{t} G(i) * M(L-i) * W(E) + \binom{n}{3} * \sum_{i=1}^{t} \sum_{j=1}^{t-i} G(i) * G(j) * M(L-i) * W(E) + \ldots + \sum_{i=t+1}^{n} \binom{n}{i} * G(i) * W(E)$$

Probability of having more than *t+1* separate burst errors

Probability of having three burst error whose total length is longer than *t+1* symbols

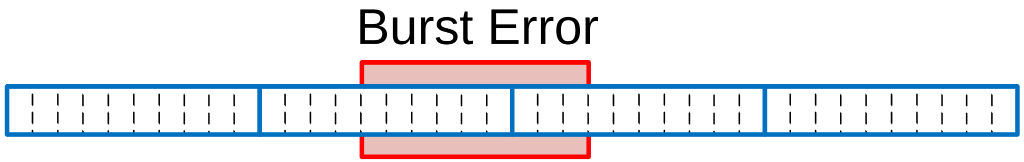
Probability of having two burst error whose total length is longer than *t+1* symbols

Probability of having a single burst error longer than *t+1* symbols

**refer to Cideciyan_02a_1111, liu_01_1105*

Error Symbol Number and Probability

- On Single RS(528,514)

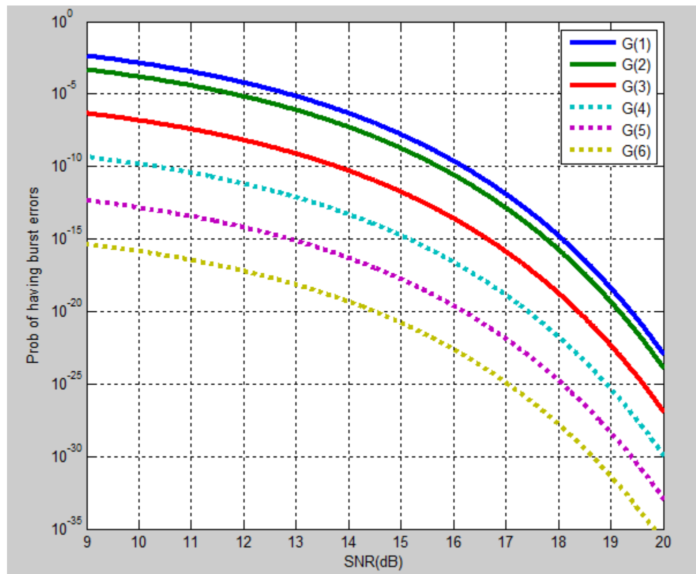


- **For a burst with length of *bl* bits, the number of error symbols and corresponding probability can be calculated as below,**

$$Error\ Symbol\ Number = \begin{cases} ceil(\frac{bl}{m})+1; & of\ \ prob_1 = \frac{|bl\%m-1|}{m} \\ ceil(\frac{bl}{m}); & of\ \ prob_2 = 1-prob_1 \end{cases}$$

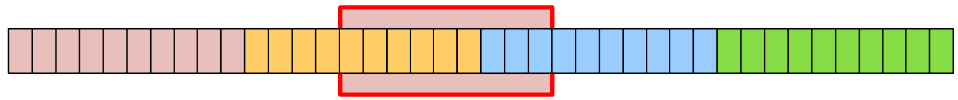
Eq 1-1

- **Offset of burst error in symbols affects the error symbol number.**
  - E.g., a 2 bit burst error may cause 2 error symbols by a probability of 10% , and may become 1 error symbol by 90%.
- **From *Eq1-1*, G(x) curves in figure shows**
  - G(1)>G(2)>G(3)>0, and G(4),…,G(7)=0 @Rllmax=17bit
  - G(1)>G(2)>G(3)>G(4)>G(5)>G(6), and G(7)=0 @Rllmax=50bit



Error Symbol Number and Probability

- Orthogonal Symbol Mux 4:1

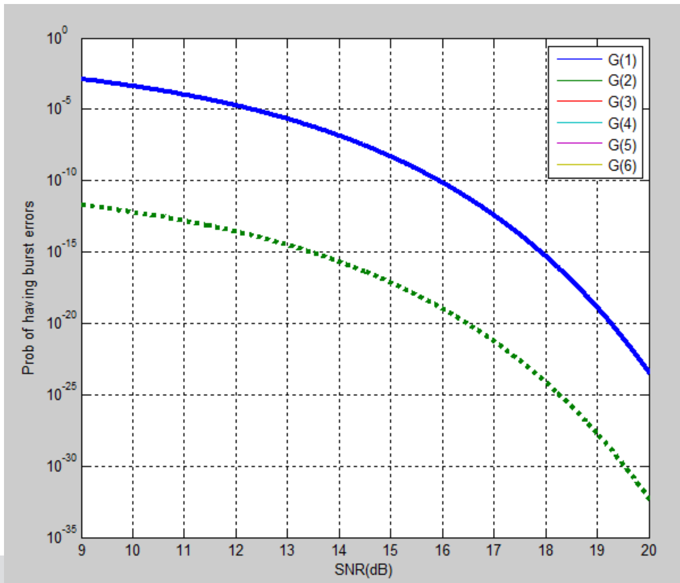


- **A burst with length of *bl* bits becomes x/x+1 symbols according to *Eq1-1*;**
- **The number of error symbols on each FEC lane can be calculated as**

$$Error\ Symbol\ \ Number = \begin{cases} ceil(\frac{x+1}{4}); & of\ \ prob_a = ((x\%4)/4) * prob_2 + (((x+1)\%4)/4) * prob_1; \\ floor(\frac{x}{4}); & of\ \ prob_b = 1 - prob_a; \end{cases}$$

Eq 2-1

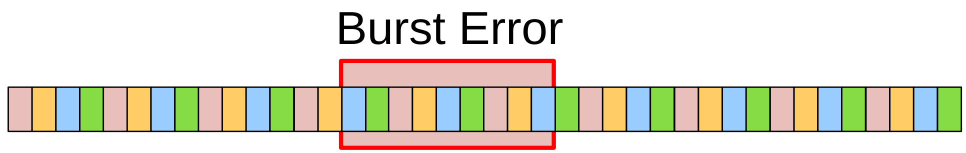
- For example, a 2bit burst become
  - Before 4:1 orthogonal symbol mux
    - 2 error symbol by 10% and 1 error symbols by 90%.
  - After
    - 1 error by 27.5% and 0 errors by 72.5%.



- **G(x) curves in figure shows**
  - G(1)> 0 and G(2),G(3),..,G(7)=0 @Rllmax=17bit;
  - G(1)> G(2)>0 and G(3),G(4)..,G(7)=0 @Rllmax=50bit;
  - Much less probability of having long burst in codeword.

Error Symbol Number and Probability

- Orthogonal Bit Mux 4:1

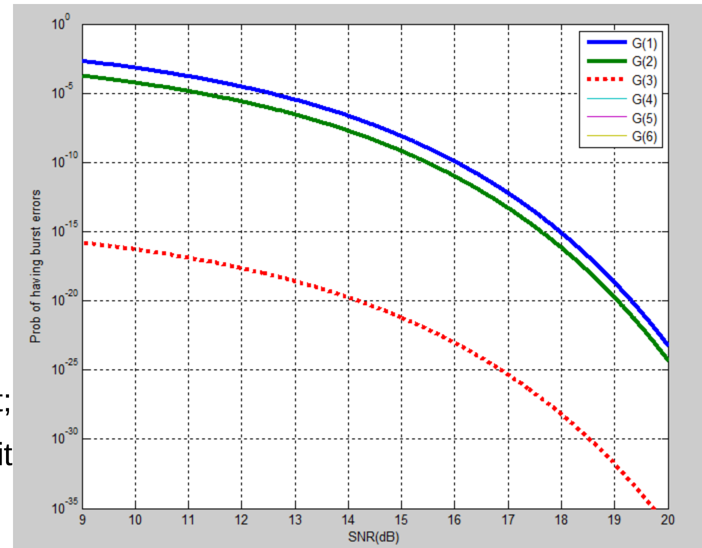


- **A burst with length of *bl* bits divides to shorter burst errors with length of floor(*bl*/4) or ceil(*bl*/4),**

$$Burst\ Length\,(bits) = \begin{cases} ceil(\frac{bl}{4}); & of\ \ prob_1 = (bl\%4)/4; \\ floor(\frac{bl}{4}); & of\ \ prob_2 = 1 - prob_1; \end{cases}$$

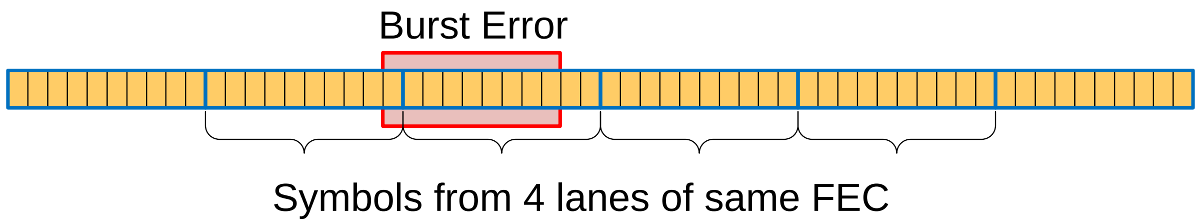
Eq 3-1

- **Then the number of error symbols caused by shorter burst on each FEC lane can be calculated by *Eq1-1*.**
- **G(x) curves in figure shows ,**
  - G(1)>G(2)>0, and G(3), G(4),G(5),G(6),G(7)=0;@Rllmax=17bit;
  - G(1)>G(2)>G(3)> 0, and G(4),G(5),G(6),G(7)=0;@Rllmax=50bit

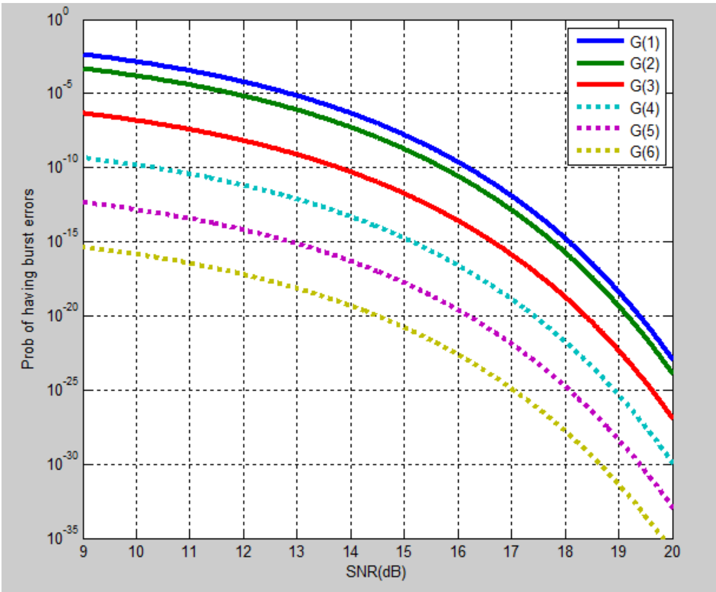


Error Symbol Number and Probability

- Non-Orthogonal Symbol Mux 4:1

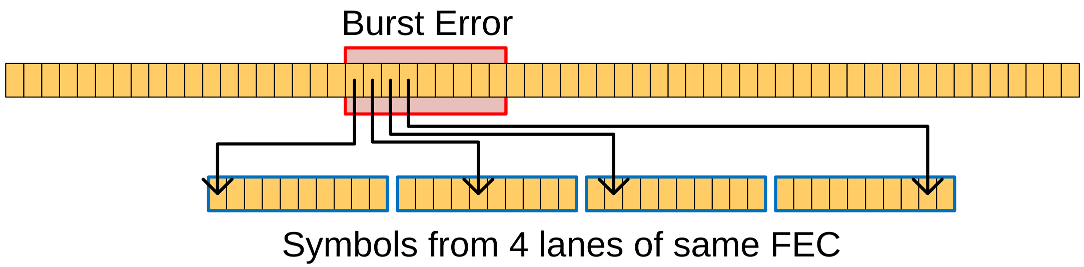


- **In worst case, symbols from 4 lanes belong to the same RS FEC codeword;**
- **The worst performance should be same as using one RS(528,514).**
- **A burst with length of *bl* bits becomes x/x+1 error symbols by *Eq1-1*;**
- **Add up error symbols from 4 lanes in one codeword.**
- **G(x) curves in figure shows,**
  - G(1)>G(2)>G(3)>0, and G(4),…,G(7)=0 @Rllmax=17bit;
  - G(1)>G(2)>G(3)>G(4)>G(5)>G(6)>0, and G(7)=0 @Rllmax=50bit;

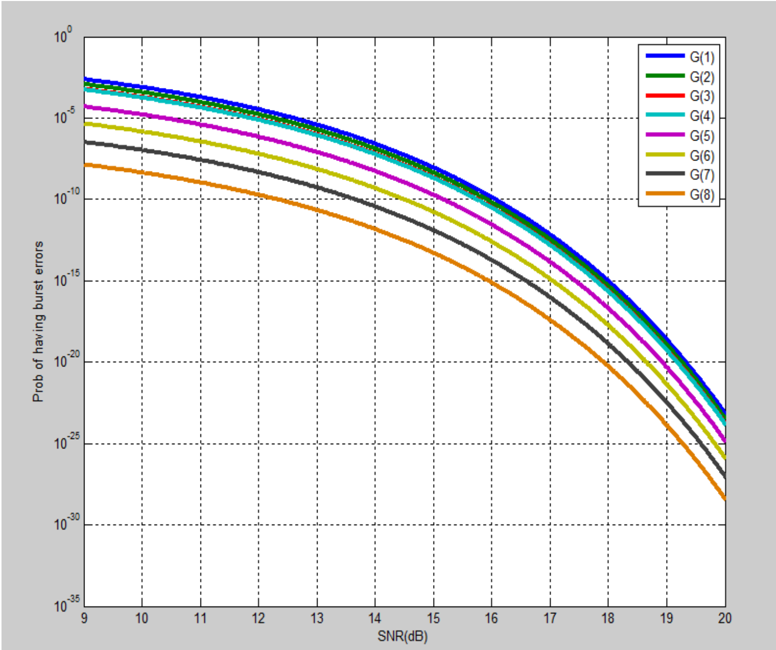


Error Symbol Number and Probability

- Non-Orthogonal Bit Mux 4:1

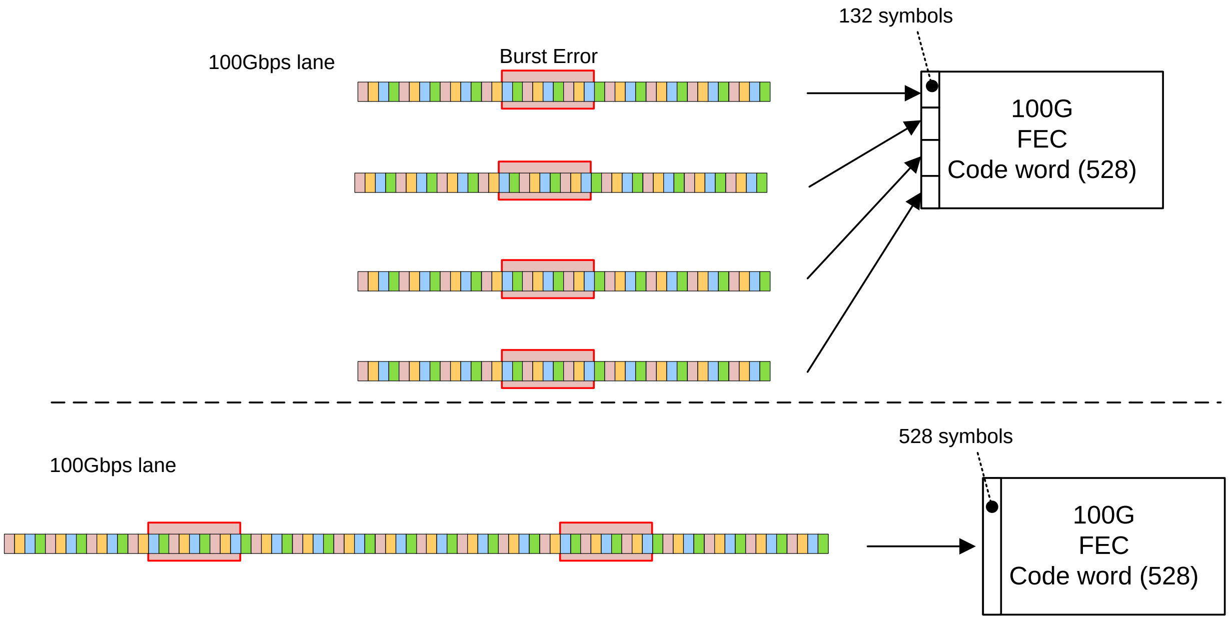


- **A burst with length of *bl* bits becomes shorter bursts on each lane by *Eq3-1*;**
- **Use *Eq1-1* to get number of error symbols on each lane;**
- **Add up errors on all 4 lanes in one codeword.**
- **G(x) curves in figure shows,**
  - G(1)>G(2)>…>G(7)>G(8)>0 @Rllmax=17bit;
  - G(1)>G(2)>…>G(7)>G(8)>0 @Rllmax=50bit;
  - Larger probability than in other muxing methods.



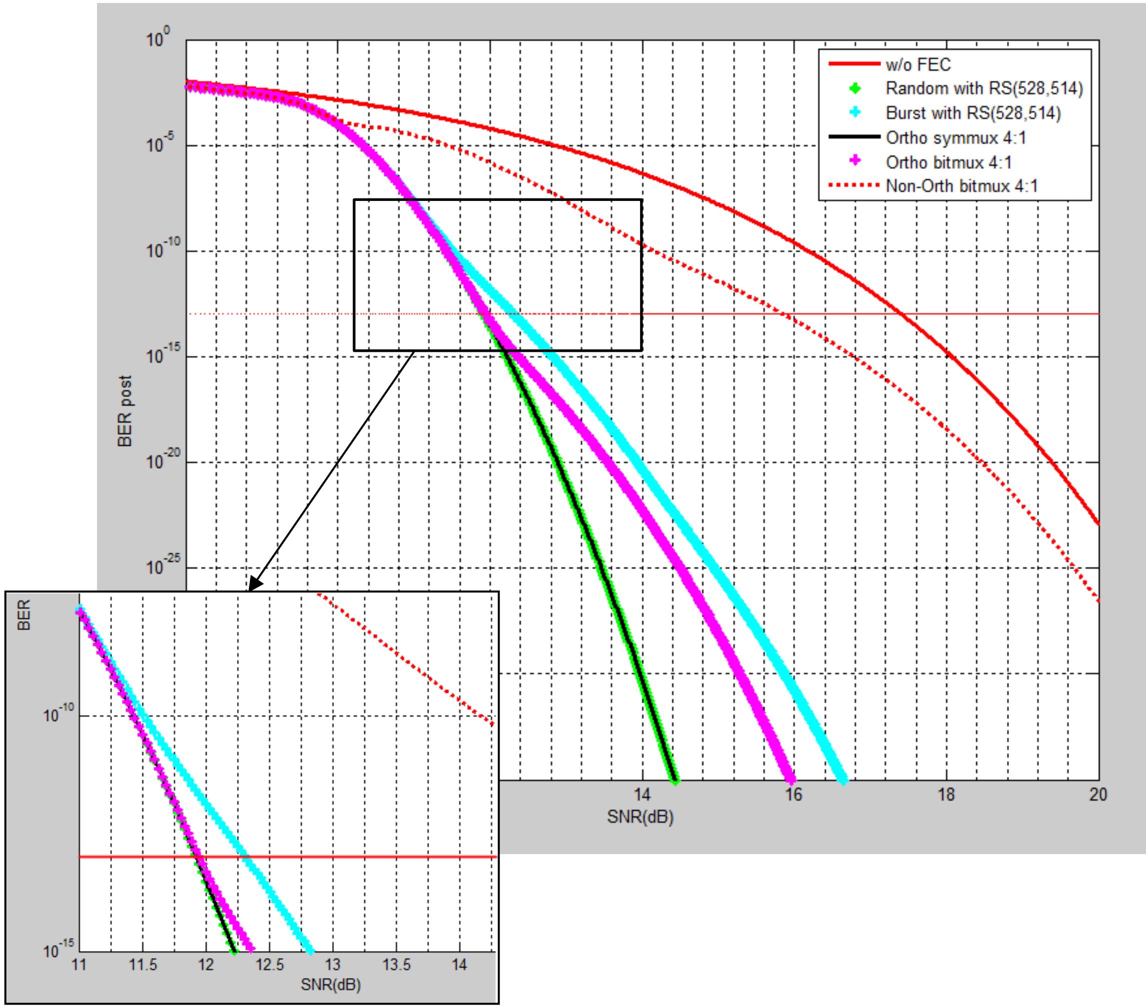
Consideration on Errors From Multi-lane

- **While every lane has same error probability, errors from a single lane or from multiple lanes can be considered identically.**
  - Use $C_n^i = \binom{n}{i}$ to present the chances of having *i* errors in a codeword.



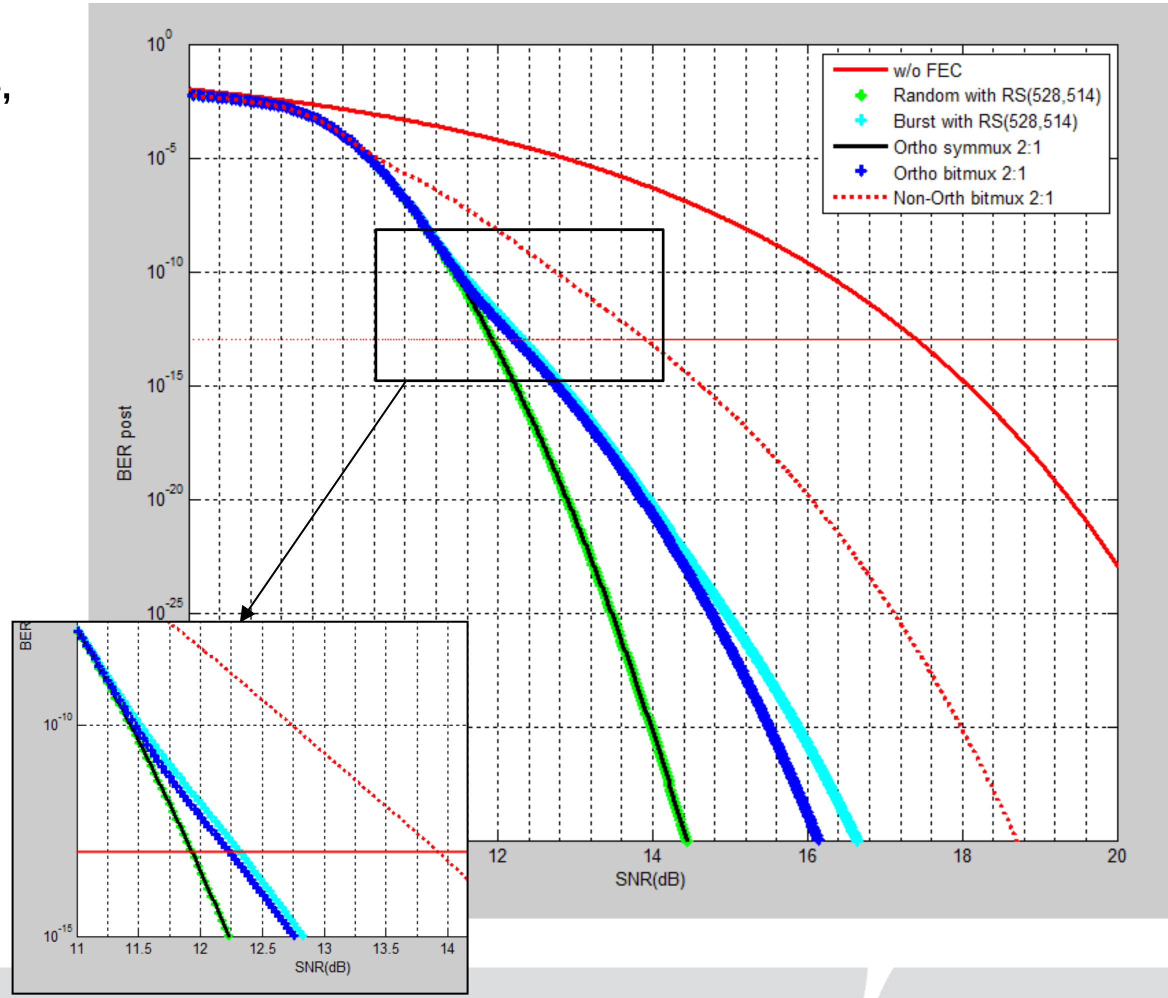
4:1 Mux Post FEC BER vs. SNR_{NRZ}

- **Orthogonal symbol mux has excellent performance, with the same coding gain as single RS(528,514) random error curve.**
- **Orthogonal bitmux also has better coding gain than 802.3bj CR4/KR4 burst error curve.**
- **Non-orthogonal bitmux is not acceptable.**
- **By orthogonal bitmux, to obtain BER_{post} = 1e-13; BER_{in} should be 3.4e-5;**



2:1 Mux Post FEC BER vs. SNR_{NRZ}

- **Orthogonal symbol mux has excellent performance, with the same code gain as single RS(528,514) random error curve.**
- **Orthogonal bitmux also has similar code gain as 802.3bj CR4/KR4 burst error curve.**
- **Non-orthogonal bitmux is not acceptable.**
- **By orthogonal bitmux, to obtain BER_{post} = 1e-13; BER_{in} should be 1.9e-5;**



Summary

- **This slides present a method to evaluate how different symbol and bit mux affects the FEC code gain against burst error in link.**
- **Orthogonal symbol and bit mux has better performance than 802.3 CR4/KR4.**
- **By FOM bitmux 2:1, requirement of BER_{in} = 1.9e-5 to satisfy BER_{post} = 1e-13 is proposed.**
- **Further analysis on RS(544,514) will be undertaken.**

Thank you

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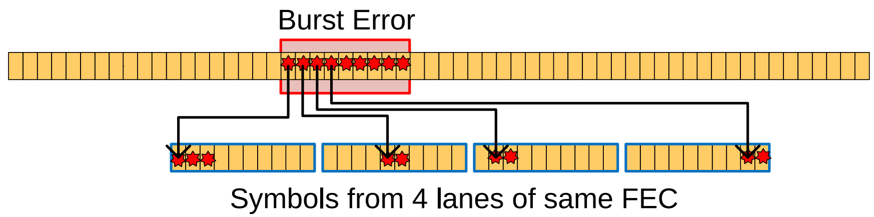
Why orthogonal bit mux can obtain better performance as symbol mux based 802.3bj CR4/KR4?

- **When a burst error happens on bitmux links, it will divide into shorter burst on each FEC. Thus on each FEC data stream, the probability of having this short burst is much smaller than using no mux at all.**
  - For instance, a 16 bit long burst will become 4 bit burst on each FEC lane in 4:1 mux scenario; the probability of having 16bit burst is BER*b^(16-1), while the chance of having 4bit burst is BER*b^(4-1); So on each FEC data lane, The probability of having same size burst error has been reduced by 1000 times. (~b^12 = 0.00024).
- **On page 7, with ortho - bitmux 4:1 , only G(1),G(2) > 0@assuming Rllmax =17bit; which means that maximally 1 or 2 symbols will be broken when a up-to-17bit burst error happens.**
- **By Comparison, the CR4/KR4 performance on page 5 with Rllmax=17bit, shows the value of G(1),G(2), G(3) > 0, which means when a up-to-17bit burst occurs, 1 or 2 or 3 symbols may be contaminated. Also note that G(x) value on this page is greater than ortho bitmux 4:1. That's why bitmux 4:1 is better than CR4/KR4.**

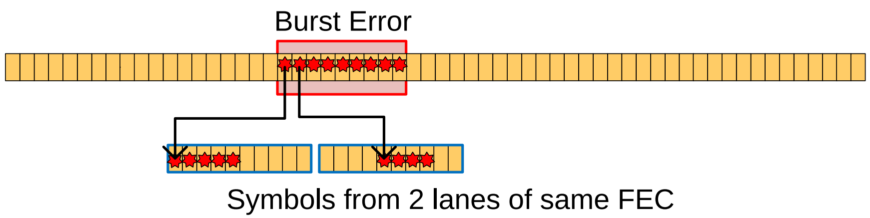
Why non-orthogonal bit mux 4:1 is worse than non-orthogonal bit mux 2:1?

- **By common sense, mux 4:1 would have better performance than mux 2:1, the result of orthogonal bitmux conforms with this conclusion.**
- **However with non-orthogonal bitmux, 2:1 mux outperform the 4:1 mux.**
- **Why ?**



Max: 8 symbols error

Min: 4 symbols error



Max: 4 symbols error

Min: 2 symbols error

- **Only orthogonal interleaving will reduce the error probability on each FEC;**