PAM-N modulation penalty in pictures

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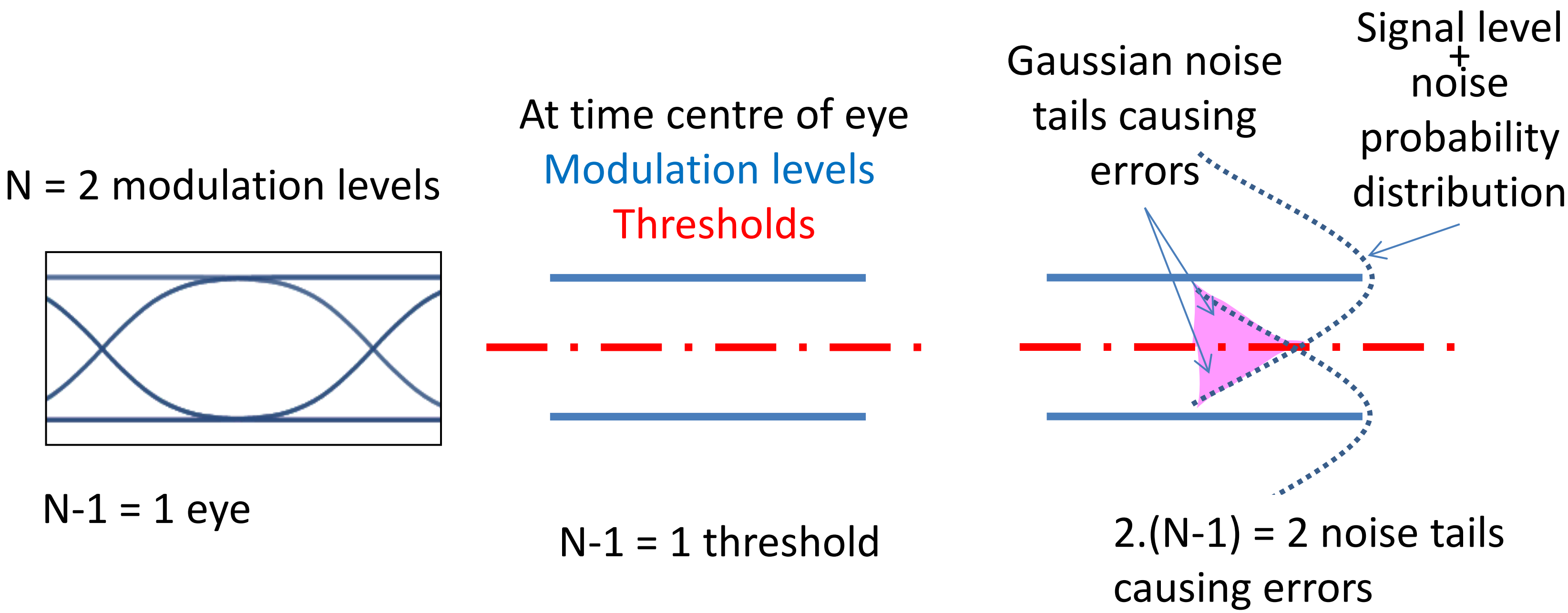
PAM-2 in pictures



- Probability of occurrence of each level = 1/N = 1/2
- Relative probability of error per symbol = 2.(N-1)/N = 1
- The error probability associated with a *single* noise tail on a particular signal level = P_i.½.erfc((OMA/2)/(σ_n.√2))

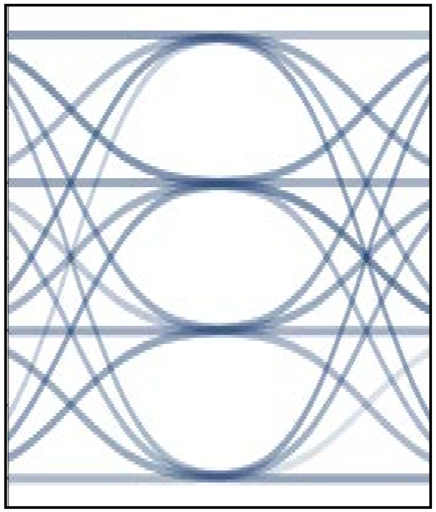
  where P_i is the probability of occurrence of the i^{th} signal level

  and σ_n is the RMS of the Gaussian noise

  – Note: the symbol error ratio equals the bit error ratio for PAM-2

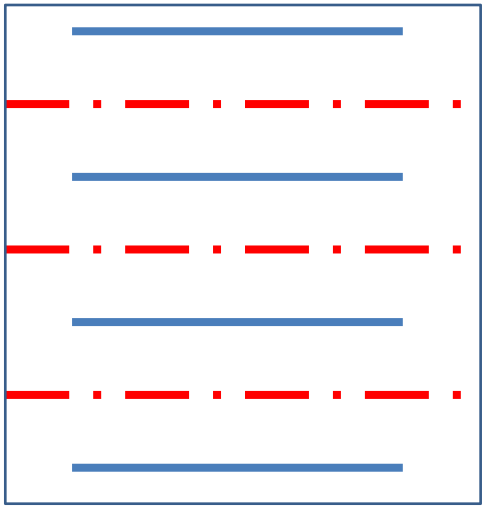
PAM-4 in pictures

N = 4 modulation levels



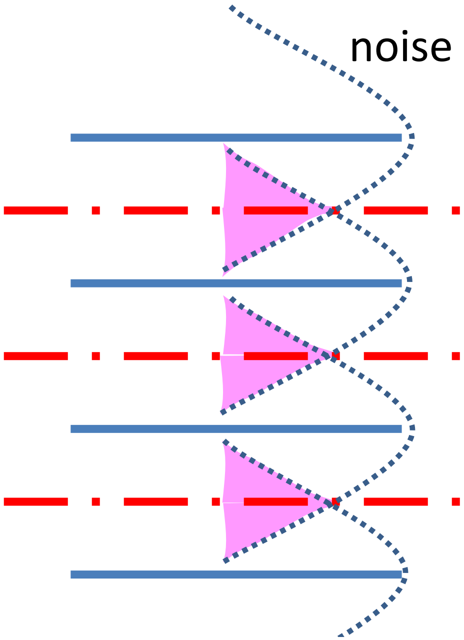
N-1 = 3 eyes

At time centre of eye
Modulation levels
Thresholds



N-1 = 3 thresholds

noise



2.(N-1) = 6 noise tails causing errors

- Probability of occurrence of each level = 1/N = 1/4
- Relative probability of errors per symbol = 2.(N-1)/N = 3/2
  - *The symbol error ratio increases !*
  - For Gray coded PAM-4, 1 symbol error produces 1 bit error; but each symbol translates to $\log_2(N)$ = 2 bits, so the ratio of SER (symbol error ratio) to BER is: SER/BER = $\log_2(N)$ = 2
- Relative probability of errors per bit = $2.(N-1)/(N.\log_2(N))$ = 3/4
  - *The bit error ratio decreases !*

BER to power penalty

For a target BER of 2.4×10^{-4}:

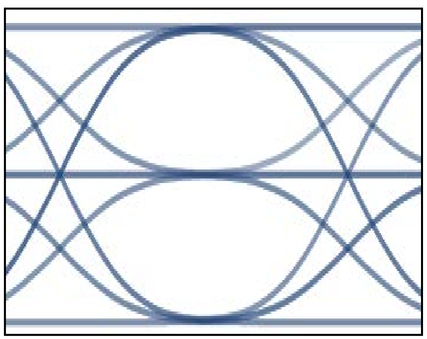
- For ideal NRZ, the Q required is 3.492
- For ideal PAM-4, the Q required is 3.414
  - a negative Q penalty of ~0.098 dB
  - total modulation penalty for PAM-4 is 4.678 dB

    (including impact of OMA scaling, higher symbol error rate, lower bit error rate, for a given outer eye OMA and fixed receiver noise, at BER of 2.4×10^{-4})

Back up

PAM-3 cartoon

N = 3 modulation levels

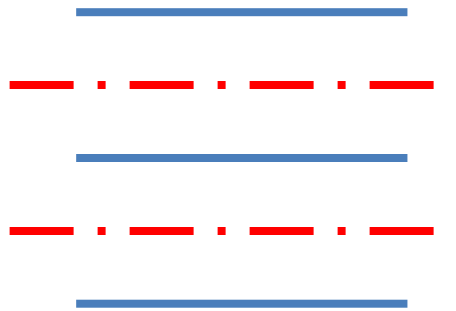


N-1 = 2 eyes

At time centre of eye

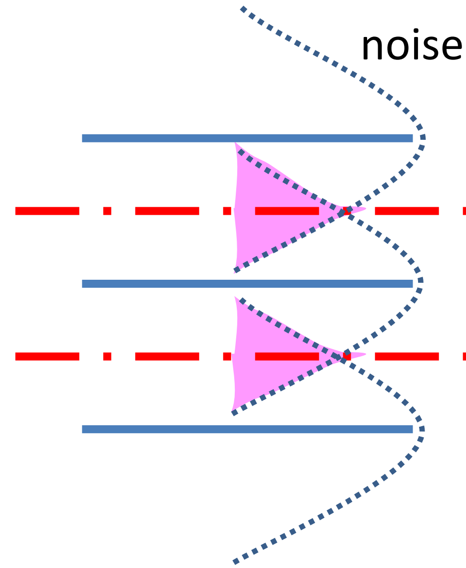
Modulation levels

Thresholds



N-1 = 2 thresholds

noise



2.(N-1) = 2 noise tails causing errors

- Probability of occurrence of each level = 1/N = 1/3
- Relative probability of error per symbol = 2.(N-1)/N = 4/3
- Relative probability of errors per bit = 2.(N-1)/(N.$\log_2$(N)) = 0.841

General PAM-N modulation penalty in words

Ideal Transmitter

- Negligible noise, negligible ISI, equally spaced modulation levels

Ideal Receiver

- Gaussian noise, perfect timing and slicing; Negligible added ISI/jitter
- Errors due to a noise tail crossing more than one threshold are ignored

PAM-N

- N levels with equal probability of occurrence; N-1 eyes; N-1 thresholds; $\log_2 N$ bits/symbol
- 2.(N-1) noise tails causing symbol errors.
- 2.(N-1)/N times the number of **symbol** errors generated for PAM-N (compared to PAM-2) for same inner-eye OMA to noise ratio
- 2.(N-1)/(N.$\log_2 N$) times the number of **bit** errors generated for PAM-N (compared to PAM-2) for same inner-eye OMA to noise ratio

- For PAM-N, this allows a decrease in Q (cf. NRZ) to meet a given target BER
  - a negative power penalty, which increases in magnitude as target BER increases (slope of BER curve); **~ 0.098 dB for PAM4 at a BER of 2.4×10^{-4} .**
  - reduces the **PAM-4 modulation penalty to 4.678 dB** (compared to the OMA scaling penalty of 4.771 dB).

Simulation showing BER vs OMA for ideal Tx