Estimating MPI Penalty

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January 6, 2016

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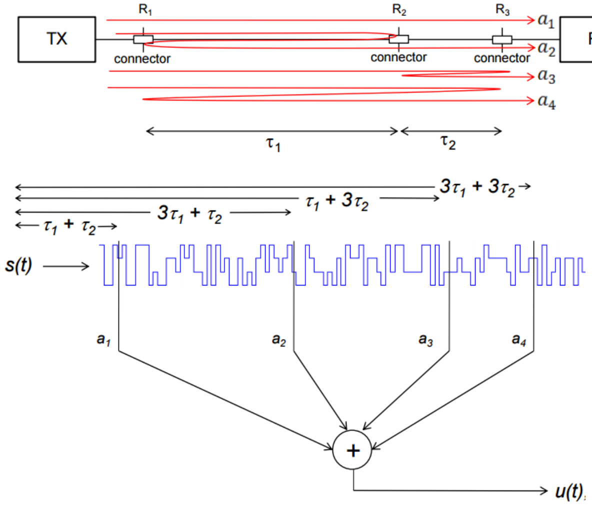
Introduction

- Optical link power penalty associated with MPI (Multi-Path Interference) is difficult to measure experimentally. The worst-case outcome, an outage, has a very low probability of occurring. However, when it occurs, it can severely impair link performance for a relatively long period of time.
- Comprehensive, closed-form analytical solution is also difficult.
- Given our schedule constraints, we may have to rely on a combination of approximation and simulation to estimate a "sufficiently" conservative value of MPI penalty.
- This presentation focuses on approximation – starts with an upper bound and then dials it down judiciously.

Plan of This Presentation

1. Describe deterministic upper bound.
2. Introduce a discount factor.
3. Estimate and recommend a value of discount factor.
4. Review some results.

Upper Bound: Model



The received signal $u(t)$ is the sum of these delayed replicas of transmitted signals.

Received power is $|u(t)|^2$.

Model

- For PAM-m, amplitudes A_i, $i = 1..m$, are transmitted.
- Received signal field $u(t) = B_0 e^{j\omega t} + \sum_{k=1}^{N} \sqrt{R^2} B_k e^{j(\omega t + \tilde{\theta}_k)}$, where
  - B_0 is the victim amplitude; B_k are the interfering amplitudes
  - $\tilde{\theta}_k$ is a random variable in $[0, 2\pi)$. It accounts for various path lengths of interference etalons, as well as spectral width / phase noise. For a more granular treatment of $\tilde{\theta}$ that separately accounts for phase noise and path length, see reference [1].
  - N is the number of interfering terms. $N = p(p-1)/2$, where p is the number of reflectance points in a link: n number of connectors + 2 PMD reflectance points.
  - PMD reflectance is assumed equal to connector reflectance R.
- We make two worst-case assumptions:
  - $B_j = A_m$ for all $j \in [0, N]$. Victim is at highest PAM amplitude, and all interfering terms are of highest PAM amplitude.
  - $\tilde{\theta}_k = \tilde{\theta}$, i.e., it is common to all interferers

Model

- Therefore, $u(t) = A_m e^{j\omega t}(1 + NRe^{j\tilde{\theta}})$ where $NRe^{j\tilde{\theta}}$ is the interference term.
- $I(t) = |u(t)|^2 \approx {A_m}^2(1 + 2NRcos\tilde{\theta})$ where $2NRcos\tilde{\theta}$ is the noise intensity term.
- Since $cos\tilde{\theta}$ is bounded within [-1,1], peak-to-peak noise intensity $\leq 4NR{A_m}^2$.
- MPI Penalty, dB $= 10\log_{10}(\frac{OMA_{inner}}{OMA_{inner} - 4NR{A_m}^2})$
- Substitute $OMA_{inner} = \frac{A_m^2 - A_1^2}{m-1}$, extinction ratio $E = \frac{A_m^2}{A_1^2}$
- MPI Penalty, dB $= 10\log_{10}(\frac{1}{1-x})$, $x = (m-1)4NR(\frac{E}{E-1})$
- This is an upper bound. *The reward of this conservative choice is elimination of outage risk.*

Accounting for PMD Reflectances Separately

- It is helpful to separate out reflectance values of transmitter, receiver, and connectors, because it enables us to explore various scenarios.
- For n connectors between Tx and Rx, We can count various reflections separately and add them up [4].
  - One reflection between Tx and Rx
  - n reflections between Tx and n connectors
  - n reflections between Rx and n connectors
  - $n(n-1)/2$ reflections among n connectors
- MPI Penalty, dB $= 10\log_{10}(\frac{1}{1-x})$, $x = (m-1)4S(\frac{E}{E-1})$, where $S = \sqrt{R_t R_r} + n\sqrt{R_t R_c} + n\sqrt{R_r R_c} + \frac{n(n-1)}{2}R_c$ R_c, R_t, R_r are discrete reflectances of connectors, transmitter and receiver, respectively. Table 1 lists a few examples.

MPI Penalty, Upper Bound

Table 1: MPI Penalty, Upper Bound, for 2, 4 and 6 connectors. PAM4, Ext. Ratio 6 dB. All values in dB. No discount factor applied ($D = 1$).

Cases	Tx	Rx	Conn	Pmpi(2)	Pmpi(4)	Pmpi(6)
DR4 D1.0	20	26	35	0.81	1.44	2.31
Case A	26	26	26	1.20	4.01	-
Case B	20	20	26	3.20	-	-
Case C	26	26	35	0.47	0.89	1.47
Case D	35	35	35	0.13	0.34	0.66
Case E	26	26	55	0.20	0.23	0.26
Case F	26	26	45	0.26	0.36	0.47
Case G	20	26	55	0.40	0.45	0.49
Case H	20	26	45	0.49	0.64	0.80

Discount Factor

- We now introduce an arbitrary discount factor D, to compensate for the highly conservative nature of this upper bound – but without raising the outage risk.
- MPI Penalty, dB $= 10\log_{10}(\frac{1}{1-x}), x = D(m-1)4S(\frac{E}{E-1})$ where $0 < D \leq 1$

- How should we determine the appropriate value of D?
  - Precedents: Look in past IEEE link models
  - Estimation: Derive a simple approximation
  - Simulation: Perform Monte Carlo analysis
  - Measurement: Preferred but hard to get it right
  - A combination of the above, using good judgment. This presentation includes the first two.

Discount Factor: Precedents

- In the past, IEEE link models have used a similar discount factor called Reflection Noise factor [3].
- From Notes: *"Reflection noise factor of 0.6 introduced to avoid undue pessimism. The value needs further consideration."*

Table 2: Reflection Noise Factors Used in past IEEE Link Models*

File	Tab	Cell	Value
10GEPBud3_1_16a.xls	LX4_SMF	L10	0.6
	1310S	L10	0.6
	1550S40km	L10	0.6
EFM0_0_2.7.xls	1000LX10SMF	L11	0.2
	1000BX10.1490	L11	0.6
	1000PX10.1310	L11	0.2

*Binary NRZ, 2 PMD reflectances only (no connectors)

Estimation of Discount Factor

- Let's consider two discounts, using simple approximations.
- Amplitude Discount
  - At 25 GBaud, a PAM symbol occupies only 8 meters of fiber. If we assume that interfering terms are from fairly independent symbols, where each symbol has PAM amplitude from $\{0,1,2,3\}$, we can scale down the magnitude of interference.
  - Risk Scenario: A long burst of PAM 3 symbols.
- Attenuation Discount
  - We can view a link as made of multiple segments, where each segment represents a combination of connector insertion loss and fiber attenuation. Interfering terms get more attenuated than signal, as they get bounced around the link.
  - Risk Scenario: A short link with low connector insertion losses.

Amplitude Discount

- Amplitude Discount Factor
  $D_1 = \frac{1}{4}(\frac{1}{\sqrt{E}} + \sqrt{\frac{E+2}{3E}} + \sqrt{\frac{2E+1}{3E}} + 1)$
- See Appendix B for derivation of D_1
- MPI Penalty, dB $= 10\log_{10}(\frac{1}{1-x}), x = D_1(m-1)4S(\frac{E}{E-1})$

Table 3: Amplitude Discount Factor D_1 for PAM4

E (dB)	D_1
4	0.82
6	0.77
8	0.73
100	0.60

Attenuation Discount

- Attenuation Discount Factor $D_2 = \frac{\hat{S}}{S}$
- See Appendix C for derivation of D_2
- MPI Penalty, dB $= 10\log_{10}(\frac{1}{1-x}), x = D_2(m-1)4S(\frac{E}{E-1})$

Table 4: Attenuation Discount Factor D_2 for various scenarios. Assumptions: Connector reflectance 35 dB, Tx reflectance 26 dB, Rx reflectance 26 dB.

Scenario	SegAttn (dB)	α	n	D_2
DR4	0.30	0.933	2	0.93
	0.65	0.861	4	0.77
FR8	0.25	0.944	4	0.90
	0.57	0.877	6	0.73
LR8	0.50	0.891	4	0.81
	0.88	0.817	6	0.63

Choosing the Value of Discount Factor

- Since D_1 and D_2 are results of unrelated effects, we can take D as a product of D_1 and D_2: $D = D_1 D_2$
- From Tables 3 and 4, a range of $0.50 \leq D \leq 0.70$ seems like a good starting point of discussion.
- Different considerations for different link types
  - DR4: Lower D_1 (external modulation), but higher D_2 (lower attenuation, fewer connectors). $0.60 \leq D \leq 0.72$
  - FR8: Higher D_1 (leave room for direct modulation), moderate D_2 (mid-range attenuation and connector count). $0.60 \leq D \leq 0.74$
  - LR8: Lower D_1 (external modulation) as well as lower D_2 (higher attenuation, more connectors). $0.49 \leq D \leq 0.62$
- In the following pages, results for $D = 0.5$, $D = 0.6$ and $D = 0.7$ are presented.

Results for D = 0.5

Table 5: MPI Penalty, Upper Bound, for 2, 4 and 6 connectors, modified by Discount Factor $D = 0.5$. PAM4, Ext. Ratio 6 dB. All values in dB.

Cases	Tx	Rx	Conn	Pmpi(2)	Pmpi(4)	Pmpi(6)
DR4 D1.0	20	26	35	0.39	0.66	1.00
Case A	26	26	26	0.56	1.56	3.59
Case B	20	20	26	1.31	3.20	8.62
Case C	26	26	35	0.23	0.42	0.67
Case D	35	35	35	0.07	0.17	0.32
Case E	26	26	55	0.10	0.11	0.13
Case F	26	26	45	0.13	0.18	0.23
Case G	20	26	55	0.20	0.22	0.24
Case H	20	26	45	0.24	0.31	0.38

Results for D = 0.6

Table 6: MPI Penalty, Upper Bound, for 2, 4 and 6 connectors, modified by Discount Factor $D = 0.6$. PAM4, Ext. Ratio 6 dB. All values in dB.

Cases	Tx	Rx	Conn	Pmpi(2)	Pmpi(4)	Pmpi(6)
DR4 D1.0	20	26	35	0.47	0.80	1.23
Case A	26	26	26	0.68	1.95	4.88
Case B	20	20	26	1.63	4.27	-
Case C	26	26	35	0.28	0.51	0.82
Case D	35	35	35	0.08	0.20	0.39
Case E	26	26	55	0.12	0.14	0.15
Case F	26	26	45	0.16	0.21	0.27
Case G	20	26	55	0.24	0.26	0.29
Case H	20	26	45	0.29	0.37	0.46

Results for D = 0.7

Table 7: MPI Penalty, Upper Bound, for 2, 4 and 6 connectors, modified by Discount Factor $D = 0.7$. PAM4, Ext. Ratio 6 dB. All values in dB.

Cases	Tx	Rx	Conn	Pmpi(2)	Pmpi(4)	Pmpi(6)
DR4 D1.0	20	26	35	0.55	0.95	1.48
Case A	26	26	26	0.80	2.38	6.73
Case B	20	20	26	1.97	5.68	-
Case C	26	26	35	0.32	0.60	0.97
Case D	35	35	35	0.09	0.24	0.45
Case E	26	26	55	0.14	0.16	0.18
Case F	26	26	45	0.18	0.25	0.32
Case G	20	26	55	0.28	0.31	0.34
Case H	20	26	45	0.34	0.44	0.55

Are We Still Being Sufficiently Conservative?

- There's no easy answer. We have to make a subjective judgment call. Here are some points to consider.
- Protection from outage: We are still covered. Here are the pessimistic assumptions we are continuing to make:
  - Laser is ideally monochromatic and coherent, and every single interfering term is temporally aligned and antipodal to the victim signal. All interference has aligned polarization.
- Direction of D_2: D_2 moves in a helpful direction. It is low when link budget is tight.
- Protection from a long string of PAM 3 symbols: Amplitude Discount D_1 is based on transmitted pulse set at the highest PAM level, but not the interfering terms. We are a bit exposed here.
- One option: Take higher end of D_1 but lower end of D_2.

Conclusion

- MPI penalty upper bound guarantees that there will be no outage.
- The price of upper bound is higher values of MPI penalty in link budget.
- Discount factor values in the range of 0.5 to 0.7 appear to be worth considering.
- Measurements and simulations can help us further refine the value.

References

1. “Effects of Phase-to-Intensity Noise Conversion by Multiple Reflections on Gigabit per Second DFB Laser Transmission Systems”, by Gimlett & Cheung, JOLT Vol. 7, No. 6, June 1989.
2. “Measurements and Simulations of Multipath Interference for 1.7 Gbit/s Lightwave System Utilizing Single and Multi-frequency Lasers”, by D. Duff, et al., Proc. OFC, 1989.
3. “The 10G Ethernet Link Model”, by Piers Dawe. http://www.ieee802.org/3/efm/public/sep01/dawe_1_0901.pdf
4. “Improved MPI Upper Bound Analysis”, by Farhood et al. http://www.ieee802.org/3/bm/public/nov12/farhood_01_1112_optx.pdf

Appendix A: Summary of Equations

$$\text{MPI Penalty, dB} = 10\log_{10}(\frac{1}{1-x}) \tag{1}$$

$$x = D(m-1)4S(\frac{E}{E-1}) \tag{2}$$

$$S = \sqrt{R_t R_r} + n\sqrt{R_t R_c} + n\sqrt{R_r R_c} + \frac{n(n-1)}{2}R_c \tag{3}$$

$$D = D_1 D_2 \tag{4}$$

$$D_1 = \frac{1}{4}(\frac{1}{\sqrt{E}} + \sqrt{\frac{E+2}{3E}} + \sqrt{\frac{2E+1}{3E}} + 1) \tag{5}$$

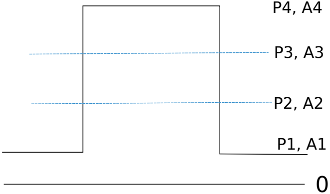
$$D_2 = \frac{\hat{S}}{S} \tag{6}$$

$$\hat{S} = \sqrt{R_t R_r} \cdot \sqrt{\alpha^{2n}} + \frac{1-\alpha^n}{1-\alpha} \cdot \left(\sqrt{R_t R_c} + \sqrt{R_c R_r}\right) + R_c \cdot \left(\frac{n}{1-\alpha} + \frac{\alpha^n - 1}{(1-\alpha)^2}\right) \tag{7}$$

α: transmission coefficient of a link segment, E: extinction ratio, m: number of PAM levels, n: number of connectors, R_c, R_t, R_r: reflectance values of connectors, transmitter and receiver, respectively.

Appendix B: Derivation of D1

- For upper bound, we had assumed $B_j = A_4, \forall j$, for PAM4, in received field $u(t) = B_0 e^{j\omega t} + \sum_{k=1}^{N} \sqrt{R^2} B_k e^{j(\omega t + \tilde{\theta})}$
- Let's change that to $B_0 = A_4$, and B_k, $k \in [1, N]$, equally likely from $\{A_1, A_2, A_3, A_4\}$, with probability $\frac{1}{4}$ each. Transmitted pulse is still of highest amplitude, but interfering pulses can have any of the 4 PAM4 amplitudes.



E: Extinction Ratio

$P_1 = P_1$

$P_2 = P_1 + (\frac{P_4 - P_1}{3}) = P_1 + (\frac{EP_1 - P_1}{3}) = P_1(\frac{E+2}{3})$

$P_3 = P_2 + (\frac{P_4 - P_1}{3}) = P_1(\frac{2E+1}{3})$

$P_4 = EP_1$, so ${A_4}^2 = E{A_1}^2$

This leads to

$$A_1 = \sqrt{P_1} = A_4 \frac{1}{\sqrt{E}},\ A_2 = \sqrt{P_2} = A_4 \sqrt{\frac{E+2}{3E}}$$

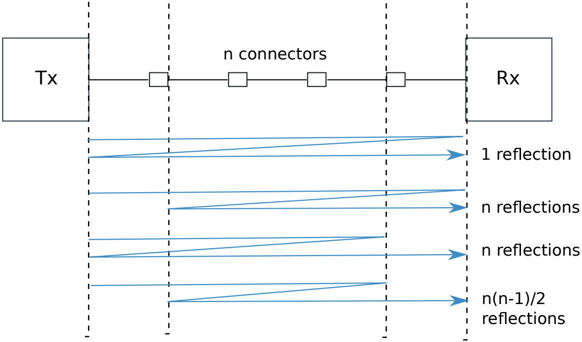
$$A_3 = \sqrt{P_3} = A_4 \sqrt{\frac{2E+1}{3E}},\ A_4 = \sqrt{P_4} = A_4$$

Now, as in [4], we replace A_4 with

$$D_1 A_4 = \frac{1}{4}(A_1 + A_2 + A_3 + A_4) = A_4 \frac{1}{4}\left(\frac{1}{\sqrt{E}} + \sqrt{\frac{E+2}{3E}} + \sqrt{\frac{2E+1}{3E}} + 1\right)$$

$$\therefore D_1 = \frac{1}{4}\left(\frac{1}{\sqrt{E}} + \sqrt{\frac{E+2}{3E}} + \sqrt{\frac{2E+1}{3E}} + 1\right)$$

Appendix C: Derivation of D2



- Signal travels forth, crossing n connectors
- An interfering term sloshes around – forth, back, and forth – traveling through *additional* segments, relative to the victim.
- Calculation of S can be replaced with $\hat{S}$ to explicitly model the additional attenuation.

Derivation of D2

Total additional loss of a reflected path scales directly with the number of connectors *between* the interfaces at which the reflections occur. α is the transmission coefficient of each segment. It is the result of a combination of connector insertion loss and fiber attenuation. $D_2 = \frac{\hat{S}}{S}$ where

$$\begin{aligned}\hat{S} \quad = \quad & \sqrt{R_t R_r} \cdot \sqrt{\alpha^{2n}} + \\ & \sqrt{R_t R_c} \cdot \left(1 + \sqrt{\alpha^2} + \sqrt{\alpha^4} + \cdots + \sqrt{\alpha^{2(n-1)}}\right) + \\ & \sqrt{R_r R_c} \cdot \left(1 + \sqrt{\alpha^2} + \sqrt{\alpha^4} + \cdots + \sqrt{\alpha^{2(n-1)}}\right) + \\ & \sqrt{R_c R_c} \cdot \left((n-1) + (n-2)\sqrt{\alpha^2} + \cdots + \sqrt{\alpha^{2(n-2)}}\right)\end{aligned}$$

which simplifies to

$$\hat{S} = \sqrt{R_t R_r} \cdot \sqrt{\alpha^{2n}} + \frac{1-\alpha^n}{1-\alpha} \cdot \left(\sqrt{R_t R_c} + \sqrt{R_c R_r}\right) + R_c \cdot \left(\frac{n}{1-\alpha} + \frac{\alpha^n - 1}{(1-\alpha)^2}\right)$$

Other, simpler approximations of D_2 are possible.