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Preliminary Studies on DFE Error Propagation, Precoding, and their Impact on KP4 FEC Performance for PAM4 Signaling Systems



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Outline

- 1/(1+D) precoding for PAM4 link systems
 - 1/(1+D) precoding implementations and working mechanism
 - Burst error removal conditions for PAM4 and examples
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 - Error propagation modelling for a 1-tap and for *N*-tap DFE
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 - Silicon measured DFE data
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1/(1+D) precoding for PAM4 links



Pete Anslow, "*FEC performance with PAM4 precoding*", IEEE P802.3bs Task Force, Waikoloa, July 2015

• When there are no symbol errors, $c_k = b_k$ and $c_{k-1} = b_{k-1}$, then,

 $b_k = (a_k - b_{k-1}) + 4 \cdot n$ and $d_k = (c_k + c_{k-1}) + 4 \cdot m$, where n, m = 0, 1

• Thus, the precoding decoder output, d_k , is equal to the precoding encoder input, a_k

1/(1+D) precoding – how errors are affected



• For two continuous symbol errors, $c_k = b_k + \alpha$ ($\alpha \neq 0$), and $c_{k-1} = b_{k-1} + \beta$ ($\beta \neq 0$) (Typically, $\alpha, \beta = \pm 1$, but with *skip-level errors*, they could be $\pm 1, \pm 2, \pm 3$)

 $d_{k} = (c_{k} + c_{k-1}) + 4 \cdot m = ((b_{k} + \alpha) + (b_{k-1} + \beta)) + 4 \cdot m = a_{k} + (\alpha + \beta) + 4 \cdot m$

- Thus, for precoding to work, $\alpha + \beta = 0 \pmod{mod}$ must be satisfied
 - For NRZ, this is always true, since $1+(-1) = 1+1 = (-1)+1 = (-1)+(-1) = 0 \pmod{1}$
 - However, for PAM4 this zero-sum error pattern does not always hold
- Now, if $c_k = b_k$, but $c_{k-1} = b_{k-1} + \alpha \ (\alpha \neq 0)$, i.e., errors is terminated

 $d_{\mathbf{k}} = a_{\mathbf{k}} + \alpha \ (\alpha \neq \mathbf{0}) + \mathbf{4} \cdot \mathbf{m} \neq a_{\mathbf{k}} + \mathbf{4} \cdot \mathbf{m}$

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• A correct symbol following an incorrect one becomes incorrect, regardless of the error pattern

PAM4 burst error removal example



Sudeep Bhoja, et al, "Precoding proposal for PAM4 modulation", 100 Gb/s Backplane and Cable Task Force IEEE 802 3 IEEE 802.3 Chicago September 2011

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Precoder Input : tx(n)

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- Precoder Output : p(n)
- DFE, Slicer Output : d(n)
- Error Event : p(n) d(n)

- 0 -1 1 -1 -1

Decoder Output after 1+D at Rx : r(n)



- In this example it is seen that a burst of 14 PAM4 symbol errors are corrected by precoding, except
 - The very first symbol error and the correct symbol error at the end of the burst block
- There are no holes in the burst errors block in this example

Entry Error

Burst error removal explained

- Slicer output: d(n) = p(n) + e(n)
- Precoding decoder output



3

0 3

$$r(n) = d(n) + d(n-1) + 4 \cdot m = p(n) + e(n) + p(n-1) + e(n-1) + 4 \cdot m$$
$$= tx(n) + e(n) + e(n-1) + 4 \cdot m = tx(n) \quad \text{if } e(n) + e(n-1) = 0 \pmod{4}$$

• In the example

Error Event : p(n) – d(n)



Error

Decoder Output after 1+D at Rx : r(n)
- 2 1 2 2 0 3 2 0 1 3 3 0 0 0 0 3

Error

What if errors assume a different signature

- Still assuming no skip-level errors, but the error pattern is changed slightly
 - Precoder Input : tx(n)

-2 2 2 2 0 3 2 0 1 3 3 0 0 0 0 2 3 0 3

• Precoder Output : p(n)

- DFE, Slicer Output : d(n)
 - $-01111^{3}30223^{1}0^{2}313^{1}1^{3}3^{1}0312$
- Error Event : p(n) d(n)

 $-01 -117^{-1} -111 -11 -17^{-1}7^{-1} -11 -17^{-1}7^{-1} -10000$

Decoder Output after 1+D at Rx : r(n)

 $-212200^23201^333^100^2003^1303$

- The burst errors are only conditionally removed, while the block length is not reduced
- The question is, can errors ever behave like this?

Error propagation model for 1-tap DFE



$$y_k = a_k + h_1 \cdot (a_{k-1} - \hat{a}_{k-1}) \\ + n_k + ISI_{res}$$

- P_{EP} (a.k.a. "a"), the error propagation probability for the next symbol if the current symbol is incorrect, can be derived explicitly. This is plotted to the right
- Assuming no skip-level errors due to noise, for a 1-tap DFE (at h1), the error event should satisfy the *zero-sum error pattern*, since
 - if $(a_{k-1} \hat{a}_{k-1}) = +1$, $(a_k \hat{a}_k) = -1$ or 0 (terminated)
 - if $(a_{k-1} \hat{a}_{k-1}) = -1$, $(a_k \hat{a}_k) = +1$ or 0 (terminated)
- However, for a multiple-tap DFE architecture, the zerosum error pattern cannot be guaranteed



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Average length of burst errors

• The average burst symbol error length (*K*) can be derived as

$$K = (1 - P_{EP}) \cdot \sum_{n=1}^{\infty} n \cdot P_{EP}^{n-1} = 1/(1 - P_{EP})$$

- Thus, for h1/h0=1, P_{EP} = 0.75, K = 1/(1-0.75) = 4
- This equation does not apply with multi-tap DFE
 - *P*_{EP} is no longer a constant; it becomes a strong function of error signature in the previous *N_b* (DFE tap number) symbols and the data pattern in that period as well
 - Burst error length definition will be discussed later; the average burst error length does not reflect the true picture of error distribution



Error propagation for an N-tap DFE receiver

• For an *N*-tap DFE, the signal at the slicer can be expressed as

$$y_k = a_k + \sum_{m=1}^N h_m \cdot (a_{k-m} - \hat{a}_{k-m}) + n_k + ISI_{res}$$

• An example on how symbol errors propagate for a 3-tap DFE receiver is shown. The notation follows

 $\widehat{a}_{k-3} \ \widehat{a}_{k-2} \ \widehat{a}_{k-1}$

- If *p_n* are known, then we can compute burst error length
- It is obvious that not every symbol has to be incorrect for the errors to continually propagate



A 2-tap DFE error propagation example

- To get a closed form solution for *N*-tap DFE is unlikely
 - Some assumptions applying to 1-tap DFE are not true any more in general
- For a 2-tap DFE, the assumptions still more or less hold
 - It is seen that, for a given h1, whether h2 is positive or negative, although the same magnitude, the impact on burst error length is completely different
 - This can be interpreted physically. But with more taps it gets harder and harder
 - Later we will see that the average error burst length (the "*a*" concept) is not a good indicator on how bad the error propagation is. It is not a good indicator on error propagation impact on system performance with FEC, either



A 3-tap DFE error propagation example

- For h1=0.7, SER ratios are computed
 - The min{SER ratio} is ~2; this is much smaller than the value for a single tap at 0.7, which is >3
 - The max{SER ratio} is >5.7; this is much larger than the value for a single tap at 0.7, or even equal to 1.0
- Thus, using the average burst error length to reversely estimate error propagation probability, "*a*", will not lead to correct conclusions







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Burst error length definition discussions

- From the above the analysis we conclude that precoding can remove all the errors, when the zero-sum error pattern is met, except
 - 1. The very first error that remains uncorrected
 - 2. The very first correct symbol following a wrong symbol
- However, a burst error length is not necessarily a continuous block of wrong symbols. More appropriately, a burst error length is defined as a contiguous sequence of symbols such that the first and last symbols are in error and there exists no contiguous subsequence of N_b correctly received symbols within the error burst. Let's call it BEL
 - N_b is typically set to the number of DFE taps
 - When $N_b = 1$, there are no holes in the burst



- Thus, a burst error can contain multiple continuous errors, interleaved with correct symbols
 - The contribution of precoding is very difficult to assess without simulations

Link simulations for error propagation and precoding

- Simulation is in the time domain, performed in a symbol-by-symbol manner
 - The simulation is done in Matlab
- A total of 10 cycles of PRBS31Q, Gray-coded and mapped to PAM4, are simulated
 - Over 10 billion PAM4 symbols
- Random noise and minor residual ISI are adjusted
 - Such that the base SER (PAM4 symbol error ratio) without DFE error propagation is around a desired pre-set target
- For each setup there are 3 sets of simulations, done in parallel across the 3 setups:
 - 1. Base link simulation without DFE error propagation
 - 2. DFE is enabled, but precoding is off
 - 3. DFE is enabled, and precoding is on
- The simulated errors are further post-processed to evaluate KP4 FEC performance

Case 1: DFE-less, AWGN dominated channel

- AWGN dominated channel, with minor contributions from residual ISI
 - AWGN is adjusted to achieve SER ~ 1e-6, 1e-5, 1e-4, 1e-3
- The maximum KP4 symbol error count is plotted for different SER
 - SER ~ 1e-3 seems to be a good rule of thumb for KP4 FEC correction capability due to AWGN
- SER statistics for 1e-3 is computed, as an example
 - The computed *Variance/mean* = 0.9982 ~ 1.0, a good approximation to Poisson distribution



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Case 1: DFE-less, AWGN dominated channel (Con't)

- Precoding impact on KP4 FEC performance is studied for SER = 1e-3
 - Precoding roughly doubled the SER, as expected from theory

(SER with precoding = 1.9957e-3)/ (SER without precoding = 9.9899e-4) \approx 2



- Precoding for FEC performance
 - Precoding weakened FEC performance
 - Without precoding the maximum KP4 FEC symbol error count is 14
 - With precoding the maximum KP4 FEC symbol error count is 21
 - There are a lot more beyond 14
- Thus, for AWGN dominated channels, 1/(1+D) precoding should be avoided

Case 2: DFE-less, residual ISI dominated channel

- Two residual ISI distributions are simulated
 - AWGN impact now is secondary to residual ISI impact

Residual ISI	ISI-1	ISI-2
Precoding = 0	9.9968E-6	7.9991e-6
Precoding = 1	1.8214e-5	1.3961e-5





Case 3: DFE-less, single residual ISI dominated channel

- Consider only a signal cursor residual ISI dominating the channel errors
 - Precoding in does not help SER or maximum FEC symbol errors, link in Cases 1 and 2
 - PAM4 symbol error distributions without precoding is less a problem than with precoding



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Case 4a: 1-tap DFE at the first post-cursor

- First, the limit case, h1/h0 = 1, is studied for base SER = 1.0050e-4
 - The SER after precoding is reduced by roughly half from that without precoding
 - Theoretically, the SER with precoding is always twice of the base SER

Conditions	SER
Precoding = 0	4.0227E-4
Precoding = 1	2.0047E-4

- It is observed that
 - Without error propagation the error distribution (blue) essentially follows binominal distribution (red)
 - With error propagation the error distribution is very different (black)
 - Precoding effectively reduced the error occurrence (with the given setting) for this setup (green)



Case 4a: 1-tap DFE at the first post-cursor (Con't)

- Different h1/h0 values are simulated
 - It is seen that after precoding the SER is essentially the same, as stated on the previous slide

h1/h0	0.30	0.50	0.60	0.75	1.00
Precoding = 0	1.0556e-4	1.6125e-4	2.4126e-4	3.7143e-4	4.0227E-4
Precoding = 1	2.0051e-4	2.0050e-4	2.0049e-4	2.0046e-4	2.0047E-4

- The impact on KP4 FEC performance: red curve moved from above to below the blue curve
 - Precoding works well for h1/h0 > ~0.6 (but the exact value is a function of many conditions)
 - Precoding makes the overall FEC performance worse for h1/h0 < -0.6



Average burst error length discussions



• For a 1-tap DFE at h1, the average burst error length and the "a" concept work perfectly

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Case 4b: 1-tap DFE at the first post-cursor (Con't)

- Set h1=-0.5 for base SER=1e-5
 - The SER ratio after and before precoding is 1.6319 (for h1=+0.5, this ratio is 1.2434). Thus, h1 tap magnitude and polarity both matter
- PAM4 symbol error statistics
 - Precoding makes the number of errors in a window larger; precoding increases the probability of errors



Conditions	SER
Precoding = 0	1.6142e-5
Precoding = 1	2.6342e-5

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Maximum burst symbol error length



Case 4c: DFE with only one non-zero tap

- For $h_k/h0 = 1$ (k=1, 2, 3), and no other non-zero taps
 - When precoding is not applied, the overall SER is about 4x of the base SER (1.0050e-4) for all 3 examples, as expected
 - When precoding is enabled, it is seen that
 - For k=1, the new SER halved the base SER; precoding helps
 - For k=2,3, the new SER doubled the SER without precoding which can be proven theoretically; precoding hurts
- DFE tap locations
 - The farther away the tap, the more negative impact on error propagation
 - The error signature is not obviously reflected from the post-DFE SER
 - The concept of "*a*" only applies for a 1-tap DFE at the first post-cursor

Non-zero DFE tap	h1/h0 = 1	h2/h0 = 1	h3/h0 = 1
Precoding = 0	4.0227E-4	4.0942E-4	4.0351e-4
Precoding = 1	2.0047E-4	8.0124E-4	8.0235e-4



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Case 5: 5-tap DFE, with h1=0

- Two 5-tap DFE's are constructed as shown
 - Tap h1 is set to 0, emulating some designs
 - The resulting SERs are always higher when precoding is turned on
- For both configurations the link shall work better when precoding is not used with or without FEC

Base = 1.0324e-6	DFE-1	DFE-2	
Precoding = 0	1.8212e-6	1.8268e-6	
Precoding = 1	3.5600e-6	3.5465e-6	



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Case 6: 8-tap DFE

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- Precoding only made the link worse •
 - The error propagation in terms of the average SER increase is • around 2x without precoding; and around 2.5x with precoding
 - With only the first DFE the average SER increase is around • 3.3. Now, with some extra DFE tap with given setting, the average error count increase is smaller



X: 11

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Y: 1

Base	DFE enabled		KP4 Max Sy	mbol Errors
Noise and ISI	precoding = 0	precoding = 1	precoding = 0	precoding = 1
1.0324e-6	2.0233e-6	2.4987e-6	3	4
1.0090e-5	1.9809e-5	2.5024e-5	5	5
1.0050E-4	1.9952e-4	2.5380e-4	9	11



Case 7: 10-tap DFE

- The 10-tap DFE is constructed as shown
- Two base SER's are simulated, 1e-6 and 1e-4
 - With precoding the SER is always worse
- Statistically, precoding hurts KP4 FEC performance for the example of SER = 1e-4
 - The impact on KP4 symbol error count between precoding on and off is the same from the limited simulations

	DFE enabled				
Noise and ISI	precoding = 0	precoding = 1			
1.0324e-6	1.8831e-6	2.1355e-6			
1.0050E-4	1.9247e-4	2.1980e-4			
	KP4 Max Sy	mbol Errors			
Noise and ISI	KP4 Max Sy precoding = 0	mbol Errors precoding = 1			
Noise and ISI 1.0324e-6	KP4 Max Sy precoding = 0 4	mbol Errors precoding = 1 4			





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Case 8: Alternating DFE tap configurations

- To see how bad error propagation can be, for a base SER at 1e-5, four DFE lengths are tried
 - 5-tap = [0.7, -0.1, 0.1, -0.1, 0.1]
 - 7-tap = [0.7, -0.1, 0.1, -0.1, 0.1, -0.1, 0.1]
 - 9-tap = [0.7, -0.1, 0.1, -0.1, 0.1, -0.1, 0.1, -0.1, 0.1]
 - 11-tap = [0.7, -0.1, 0.1, -0.1, 0.1, -0.1, 0.1, -0.1, 0.1, -0.1, 0.1]
- It is observed that, for *this set of* alternating DFE tap coefficients, precoding always improves performance in SER, BEL, and KP4 FEC max symbol error count
 - However, the performance is so bad that the FEC completely fails even for the 7-tap case
 - Does precoding always help in the alternating DFE tap configurations?

DFE tap	SER		BEL		Max FEC SE	
configurations	PreC=0	PreC=1	PreC=0	PreC=1	PreC=0	PreC=1
5-tap	4.2673e-5	2.2012e-5	39	20	8	7
7-tap	6.2625e-5	3.2712e-5	104	58	21	17
9-tap	1.5739e-4	1.0047e-4	607	333	122	98
11-tap	8.8070e-4	7.2101e-4	3071	2378	544	508

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Case 8: Alternating DFE tap configurations (Con't)

- Reducing h1 to 0.4 from 0.7, and repeat the simulation for two cases
 - 5-tap = [0.4, -0.1, 0.1, -0.1, 0.1]
 - 11-tap = [0.4, -0.1, 0.1, -0.1, 0.1, -0.1, 0.1, -0.1, 0.1, -0.1, 0.1]
- It is observed that if h1 values is reduced or the rest DFE taps become larger relative in magnitude, precoding effect diminishes
 - Precoding could degrade overall link performance; its effect should be analyzed case by case

DFE tap	SER		BEL		Max FEC SE	
configurations	PreC=0	PreC=1	PreC=0	PreC=1	PreC=0	PreC=1
5-tap: h1=0.7	4.2673e-5	2.2012e-5	39	20	8	7
5-tap: h1=0.4	1.3674e-5	2.0415e-5	27	26	7	7
11-tap: h1=0.7	8.8070e-4	7.2101e-4	3071	2378	544	508
11-tap: h1=0.4	5.0022e-5	3.8338e-5	887	492	178	167



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Case 9: DFE from COM settings

- DFE values are from COM based analysis
 - The spreadsheet, "some_DFE_tap_results.xlsx", was simulated and prepared by Richard Mellitz, which I received from Pete Anslow
 - Four of cases are singled out for the simulation in the presentation Link 2: "tracy_100GEL_05_0118\B56_Thru_CblBP", 30mm
 Link 9: "mellitz_3ck_02_081518_CBP\CaBP_BGAVia_Opt2_24dB", 12mm
 Link 10: "mellitz_3ck_02_081518_CBP\CaBP_BGAVia_Opt2_24dB", 20mm
 Link 16: "mellitz...72518_channels--Z0d_100_14p25in_2dBpi_meg6_rtf", 20mm





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Case 9: DFE from COM settings (Con't)

- Observations from the simulation results
 - h1 is dominant, the outcome is somewhat 1-tap-DFE-like
 - Precoding improved in some and degraded in others
 - The impact of precoding is situation dependent

Links	2	9	10	16
Precoding = 0	2.4205e-5	1.8180e-5	2.0154e-5	2.6985e-5
Precoding = 1	2.0317e-5	2.0043e-5	2.0124e-5	2.0231e-5





Case 10: DFE from silicon measurement

- An analog-based RX architecture test-chip designed to meet CEI-56G-MR-PAM4
 - It is seen from the RX architecture highlight, there is a 2-stage CTLE and an AGC, and 10-tap DFE
 - DFE h1 is intended to take care of the loss around the Nyquist frequency
 - The rest 9 DFE taps are mainly for fine tuning what is left over after TX FIR and CLTE
- Lab measurement
 - 13 channels, from backplanes to copper cables
 - Ball-to-ball loss ranges from 15.5 to 34.5dB, while BER ranged from 3.56e-10 to 7.41e-6 (not a direct function of losses), without crosstalk
 - Different amount of crosstalk was applied so that the system achieved BER roughly around 1e-4
 - Next, 4 channels are selected for simulations; the h1/h0 values cover the largest (1.2189) and the smallest (0.4352) and some middle ones



Jay Im, et al, "A 40-to-56Gb/s PAM-4 Receiver with 10-Tap Direct Decision-Feedback Equalization in 16nm FinFET", ISSCC 2017

Precoding impact for Case 10

- DFE tap convergence
 - With TX 3-tap FIR and RX 2-stage CTLE, the resultant DFE tap coefficients all showed h1 domination; 4 cases are selected
- It is seen that in 3 out of 4 cases precoding contributed positively to enhancing the KP4 FEC performance
 - The case in which precoding did not help FEC showed h1<0.5
- However, for a different design also with 10-tap DFE, precoding impact should be studied





Summary and Conclusions

- DFE error propagation for PAM4 signaling is preliminarily studied
 - Beyond 1-tap DFE, error propagation effect is very complicated in general
 - The "*a*" approach for studying FEC performance only works for h1-tap DFE
- For a h1-tap DFE, precoding effect depends on the tap coefficient strength
 - The tap coefficient has to be large enough (roughly >0.6) for the precoding to help
 - With ADC based design, though DFE typically only has 1-tap, its value usually is small
- For designs with a multi-tap DFE, the front-end linear EQ plays a big role
 - TX FIR and RX CTLE should work jointly to help DFE tap coefficient distribution
- The 1/(1+D) precoding is helpful conditionally for PAM4 channel links
 - DFE coefficient signature directly impacts error signature, thus FEC performance
 - Multi-tap DFE coefficient polarity plays an equal role as the DFE tap strengths
 - Precoding effect for a multi-tap DFE architecture should be analyzed case by case
- Nevertheless, precoding is recommended in the standard for PAM4 links
 - Precoding implementation overhead is minimal
 - Use of precoding should be carefully accessed

Adaptable. Intelligent.



