

RS(544,514) FEC performance with 4:1 interleaving (updated)

Pete Anslow, Ciena
Including data kindly provided by Rich Mellitz

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Introduction

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The IEEE P802.3ck Task Force has objectives to define 100 Gb/s lanes for AUI interfaces, backplanes and twin-axial copper cables for 100 Gb/s, 200 Gb/s, and 400 Gb/s Ethernet. If the FEC sublayer in Clause 91 is re-used for 100 Gb/s Ethernet and the Clause 119 PCS is re-used for 200 Gb/s and 400 Gb/s Ethernet, the 100 Gb/s lanes will have to be formed from interleaving four 25 Gb/s lanes.

[healey_100GEL_01_0318](#) contained some analysis of a variety of interleaving schemes and proposed that pre-coding should be used in 100 Gb/s per lane electrical PHYs as a tool to improve error correction performance.

[gustlin_3ck_01_0718](#) contained some concerns about whether the error pattern seen within bursts from practical receivers matches the error pattern where pre-coding turns each burst into two isolated symbol errors and suggested investigating RS symbol multiplexing as an alternative.

This presentation analyses the performance of a variety of 4:1 interleaving schemes using a development of the principles explained for the NRZ case in Annex 1 of [anslow_3bs_02_1114](#) and prompted by discussions with Geoff Zhang from Xilinx now includes some multi-tap DFE results.

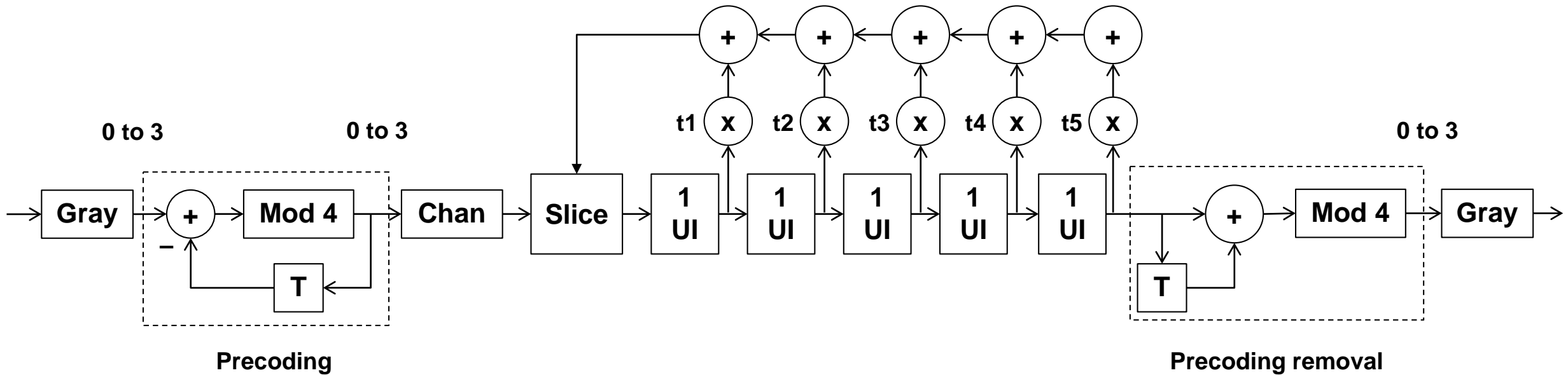
Slides that are changed from those in [anslow_3ck_adhoc_01_072518](#) have a “*” in the top right corner.

Precoding

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The results with precoding in [anslow_3ck_adhoc_01_072518](#) were for a 1 tap DFE where it is expected that precoding will have the effect of converting an error burst of any length into just two PAM4 symbol errors.

To try to help answer the question of what the performance will be for a realistic multi-tap DFE with precoding, the Monte Carlo model used for previous presentations was extended to include a 5-tap DFE as below.



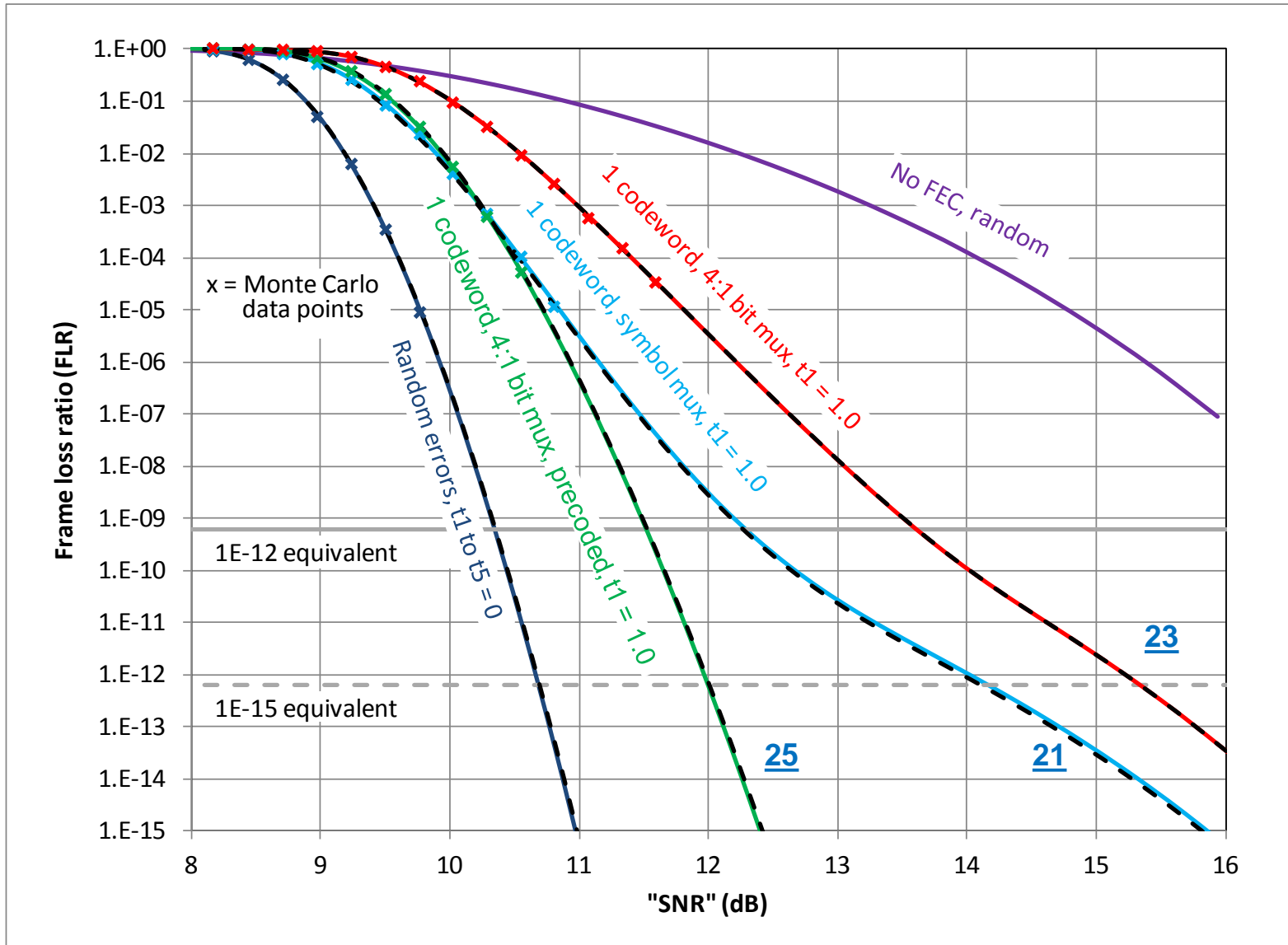
Model validation

The new Monte Carlo model was then used to re-create the previously simulated (1-tap DFE) results.

The error statistics (probability of n FEC symbol errors per burst) were also extracted and used to create dashed curves at lower FLR than could be simulated directly by the Monte Carlo program.

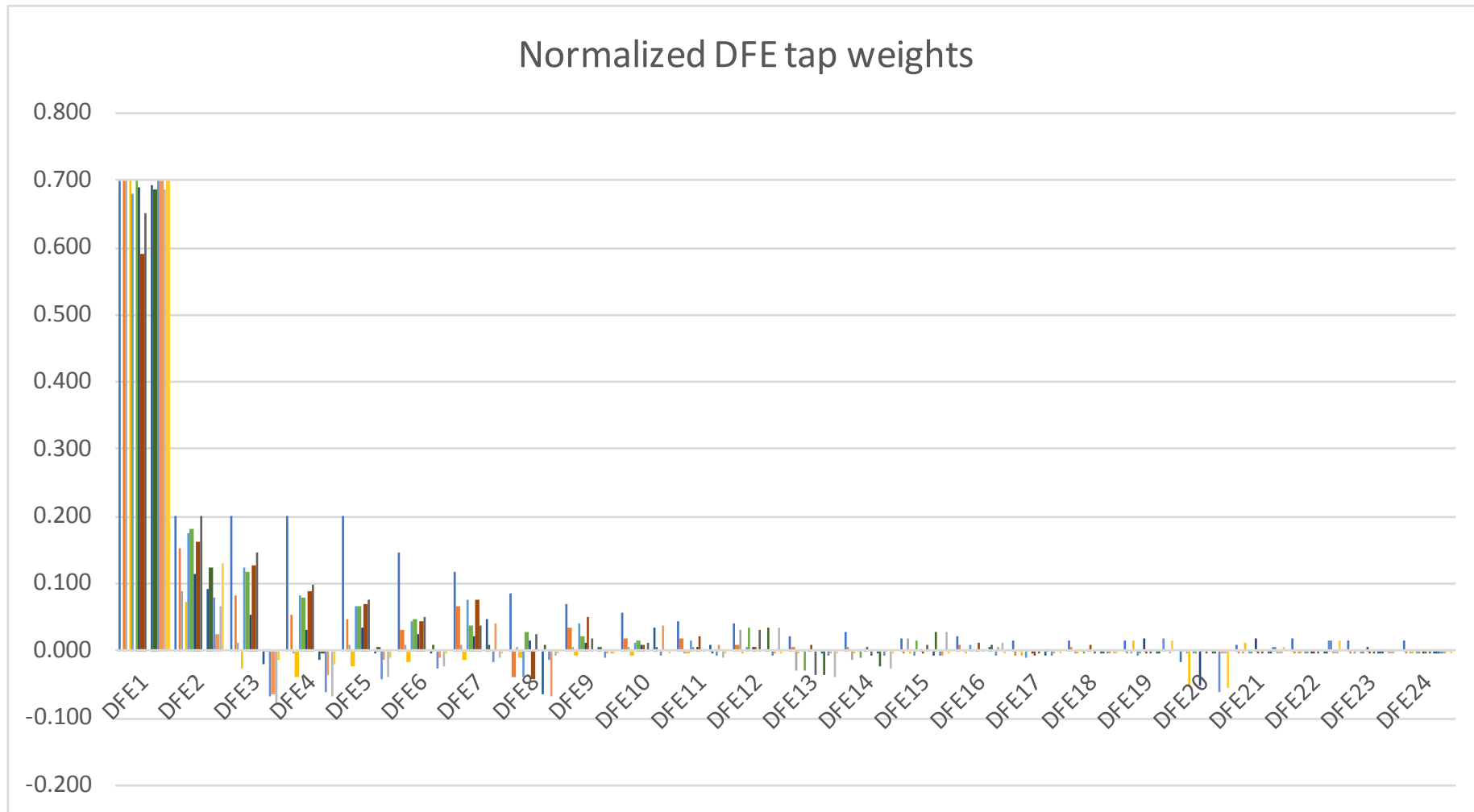
The Monte Carlo values (x markers) and dashed curves generated as above are plotted on the next slide together with the solid curves generated using probability analysis as per the principles explained for the NRZ case in Annex 1 of [anslow_3bs_02_1114](#).

100G previous result validation (1-tap DFE)



DFE tap weights

Rich Mellitz kindly provided the normalized DFE tap weights as calculated by COM for some channels submitted to IEEE 802.3. For details of the values see [26](#)



Multi-tap DFE

In the P802.3cd draft, Table 136-18 for 50GBASE-CR, 100GBASE-CR2, and 200GBASE-CR4 and Table 137-6 for 50GBASE-KR, 100GBASE-KR2, and 200GBASE-KR4 have a normalized DFE coefficient magnitude limit of 0.7 for tap 1 and 0.2 for all other taps.

This, together with the tap information shown on the previous slide (which has the same limits) led to the selection of:

$$t_1 = 0.7, t_2 = 0, t_3 = 0.2, t_4 = 0, t_5 = 0.2$$

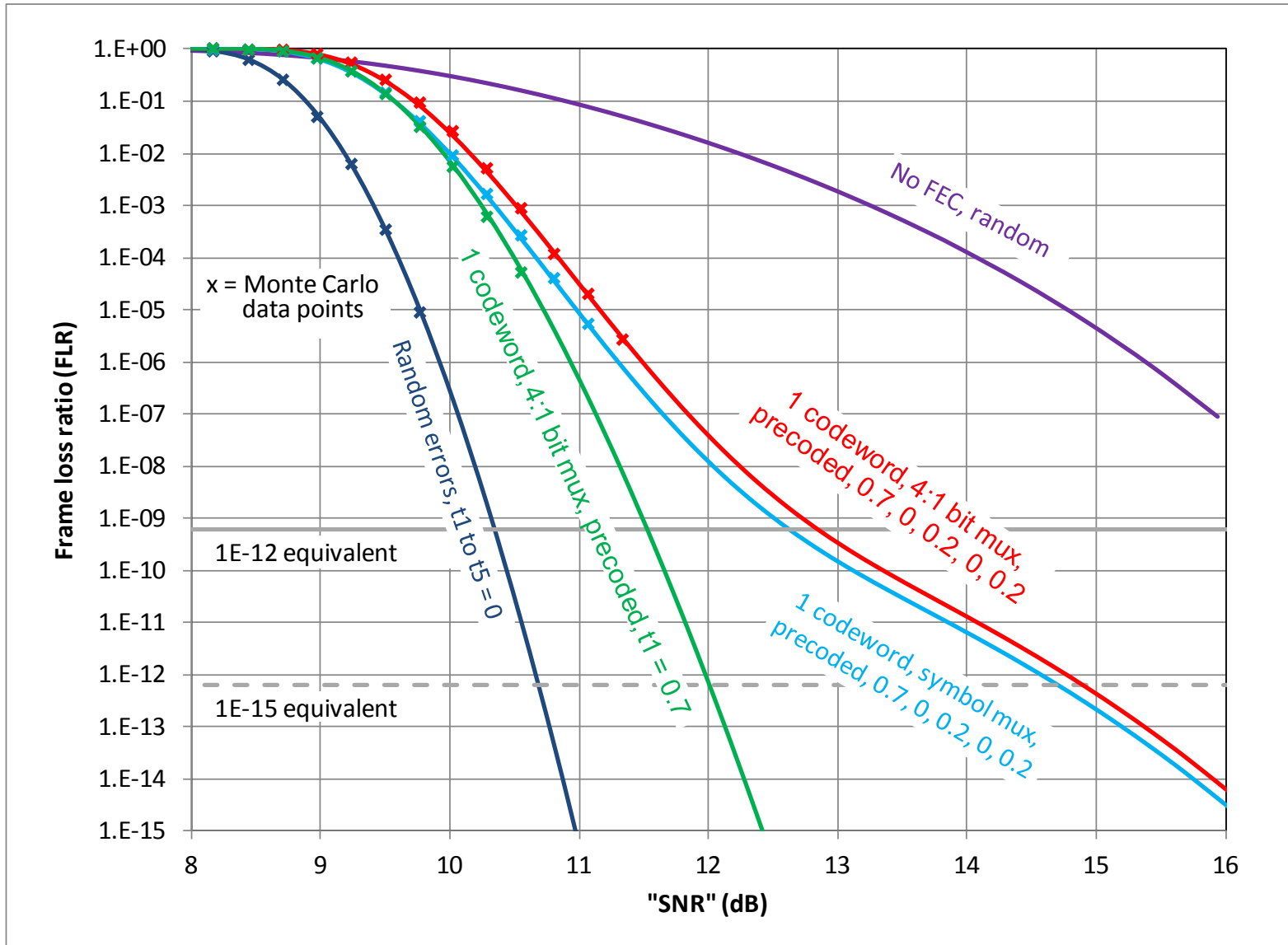
$$t_1 = 0.7, t_2 = -0.1, t_3 = 0.1, t_4 = -0.1, t_5 = 0.1$$

as “realistic” but possibly near worst case candidate sets of tap weights.

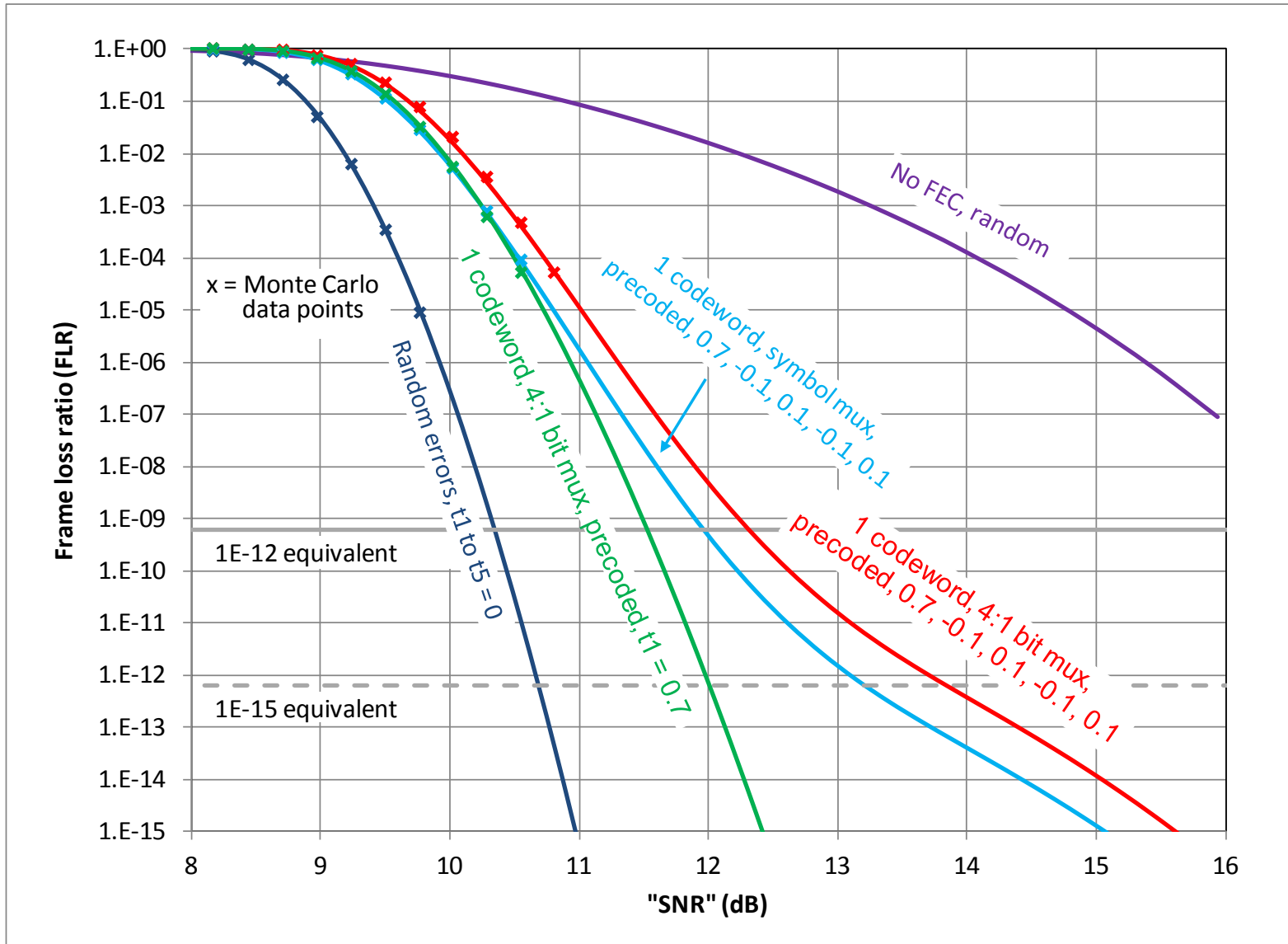
The Monte Carlo values (x markers) and solid curves generated from the error statistics for these sets of tap weights with precoding turned on for both bit and FEC symbol interleaving are plotted on the next two slides together with the 1-tap DFE with precoding result shown earlier.

As a check, slide [10](#) shows the result for $t_1 = 0.7, t_2 = -0.1, t_3 = 0.1, t_4 = -0.1, t_5 = 0.1$ with precoding turned off.

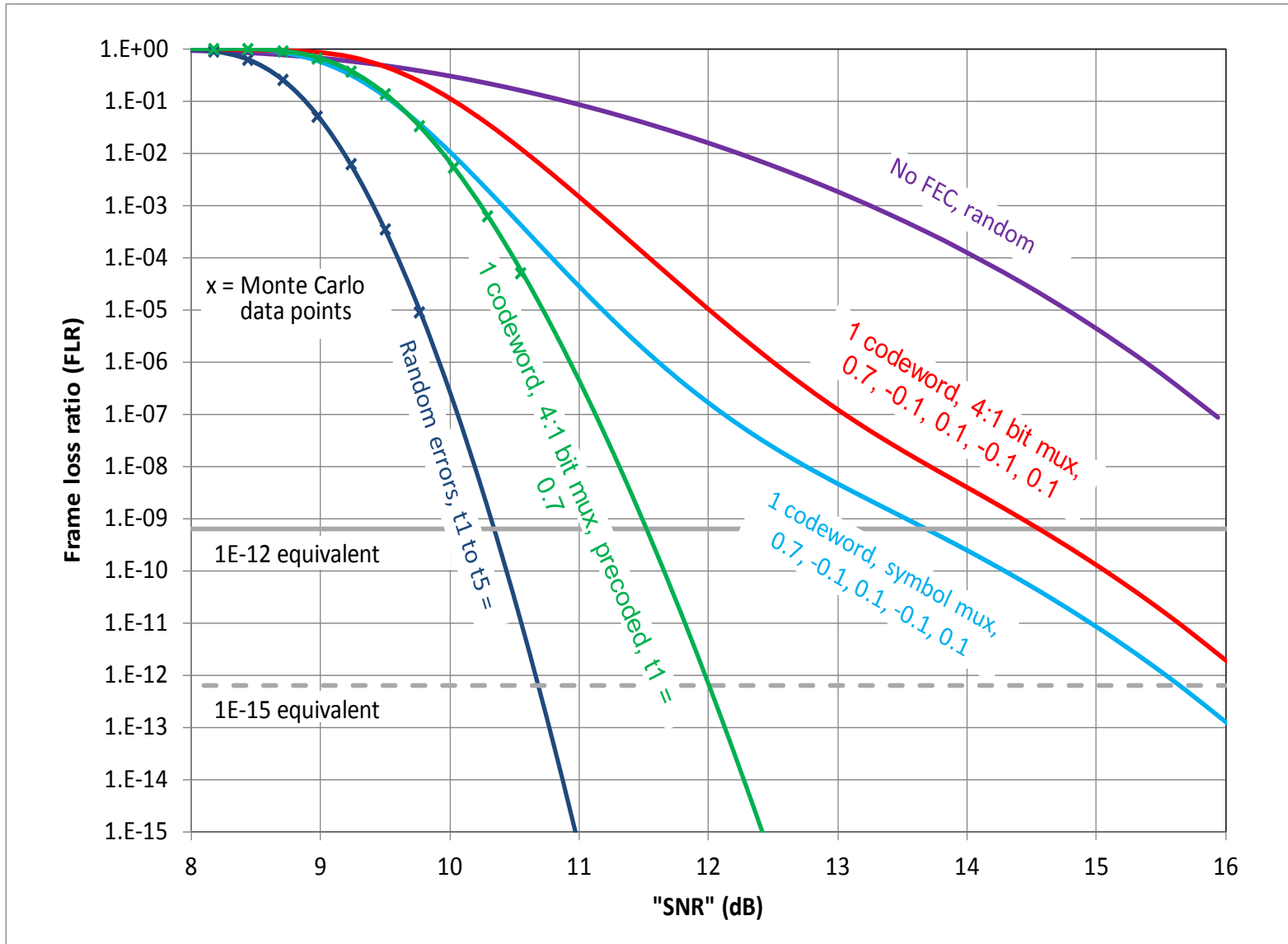
100G 5-tap DFE results (0.7, 0, 0.2, 0, 0.2) with precoding



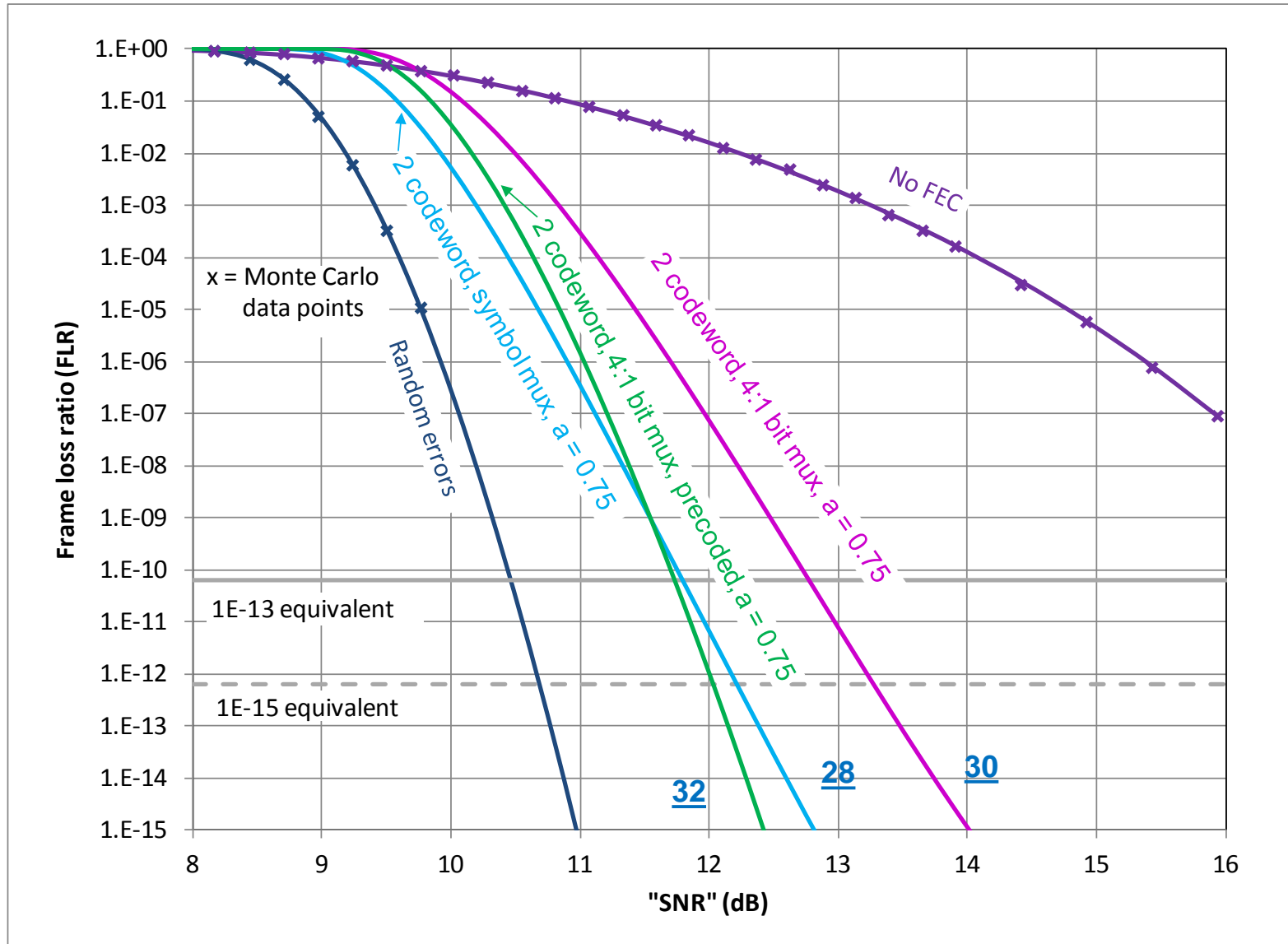
100G 5-tap DFE results (0.7, -0.1, 0.1, -0.1, 0.1) with precoding



100G 5-tap DFE results (0.7, -0.1, 0.1, -0.1, 0.1) no precoding



400G all curves



Results for RS(544,514) 100G all gain used for PAM4 part 1

From the curves shown on the previous pages, if all of the coding gain were to be used for the PAM4 link, the BERs required to give FLRs equivalent to that of a BER of 1E-12 (for 100G) or 1E-13 (for 400G) and 1E-15 are:

	At slicer output		At FEC input	
100G	FLR = 6.2E-10	FLR = 6.2E-13	FLR = 6.2E-10	FLR = 6.2E-13
No FEC	1E-12	1E-15		
1 codeword, 4:1 bit mux, t1 = 1	2.55E-6*	7.55E-9*		
1 codeword, symbol mux, t1 = 1	5.89E-5*	4.93E-7*		
1 codeword, 4:1 bit mux, pre-coded, t1 = 0.7	2.47E-4*	1.03E-4*	1.23E-4	5.14E-5
1 codeword, symbol mux, pre-coded, t1 = 1, t3 & t5 = 0.2				
Random errors	3.76E-4	2.34E-4		
400G	FLR = 6.2E-11	FLR = 6.2E-13	FLR = 6.2E-11	FLR = 6.2E-13
No FEC	1E-13	1E-15		
2 codeword, 4:1 bit mux, a = 0.75	2.03E-5*	6.03E-6*		
2 codeword, symbol mux, a = 0.75	1.69E-4*	7.57E-5*		
2 codeword, 4:1 bit mux, pre-coded, a = 0.75	1.70E-4*	9.70E-5*	8.51E-5	4.85E-5
Random errors	3.20E-4	2.34E-4		

Note – these values are the BER **including** the additional errors due to the bursts. To account for burst errors, the values marked with “*” have been multiplied by 4 when a = 0.75.

Results for RS(544,514) 100G all gain used for PAM4 part 2

From the curves shown on the previous pages, if all of the coding gain were to be used for the PAM4 link, the SER_{in} and SNR required to give FLRs equivalent to that of a BER of $1E-12$ (for 100G) or $1E-13$ (for 400G) and $1E-15$ are:

100G	For FLR = $6.2E-10$			For FLR = $6.2E-13$		
	SER_{in}	SNR (dB)	SNR + 6.99	SER_{in}	SNR (dB)	SNR + 6.99
1 codeword, 4:1 bit mux, $t1 = 1$	1.28E-6	13.60	20.59	3.77E-9	15.34	22.33
1 codeword, symbol mux, $t1 = 1$	2.95E-5	12.28	19.27	2.47E-7	14.16	21.15
1 codeword, 4:1 bit mux, pre-coded, $a = 0.75$	1.23E-4	11.52	18.51	5.14E-5	12.00	18.99
1 codeword, symbol mux, pre-coded, $t1 = 1, t3 \& t5 = 0.2$						

400G	For FLR = $6.2E-11$			For FLR = $6.2E-13$		
	SER_{in}	SNR (dB)	SNR + 6.99	SER_{in}	SNR (dB)	SNR + 6.99
2 codeword, 4:1 bit mux, $a = 0.75$	1.01E-5	12.77	19.76	3.02E-6	13.27	20.26
2 codeword, symbol mux, $a = 0.75$	8.44E-5	11.74	18.73	3.79E-5	12.16	19.15
2 codeword, 4:1 bit mux, pre-coded, $a = 0.75$	8.51E-5	11.73	18.72	4.85E-5	12.03	19.02

Where:

SER_{in} is the symbol error ratio due to noise only (does not include bursts)

SNR (dB) is the "SNR" in equation (1) on page [19](#)

SNR + 6.99 is the SNR as defined on page 5 of [healey_100GEL_01_0318](#)

Conclusion

For 400G using a Clause 119 PCS, the performance at $1E-13$ equivalent BER with worst case RS symbol multiplexing is almost the same as for precoding with a 1-tap DFE. At $1E-15$ equivalent BER worst case symbol multiplexing is slightly worse than pre-coding with a 1-tap DFE.

400G analysis for a multi-tap DFE is in progress.

For 100G using a Clause 91 FEC sublayer, the performance at $1E-12$ equivalent BER with RS symbol multiplexing is better than that with 4:1 bit multiplexing, but still significantly worse than that for precoding with a 1-tap DFE. At $1E-15$ equivalent BER symbol multiplexing is much worse than precoding with a 1-tap DFE.

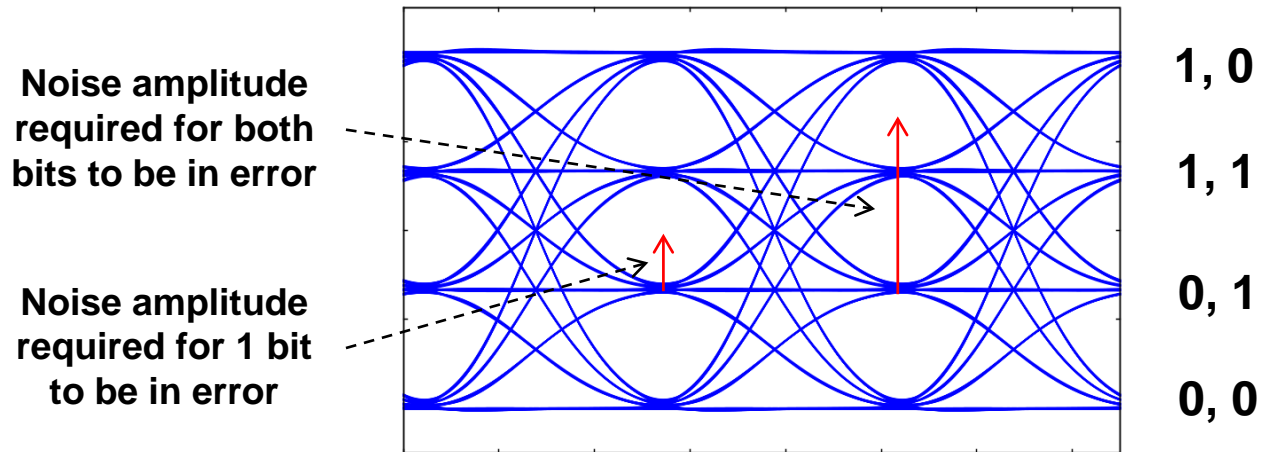
100G with a multi-tap DFE ($t_1 = 0.7$, $t_2 = 0$, $t_3 = 0.2$, $t_4 = 0$, $t_5 = 0.2$) and precoding shows a penalty of ~ 1 dB at $1E-12$ BER equivalent and ~ 2.5 dB at $1E-15$ BER equivalent compared to precoding with a 1-tap DFE.

Annex

Gray coding

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Assume the use of Gray coding (see IEEE Std 802.3-2018 120.5.7) as illustrated below:



If noise causes any of the 4 levels to be mistaken for an adjacent level, this causes one of the two bits to be in error.

If there is just enough Gaussian noise to cause a BER of $3.8E-4^*$ due to single level errors, then the probability of that noise causing both bits to be in error is $2.8E-23$.

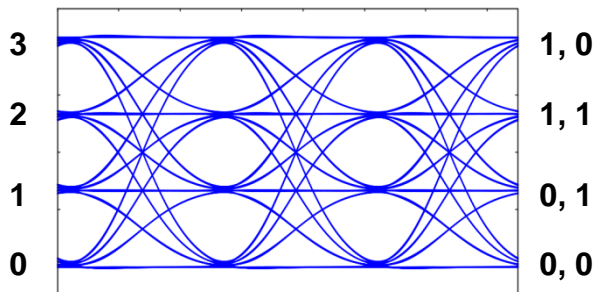
This analysis therefore assumes that only one of the two bits is in error.

* FLR = $6.2E-10$ (equivalent to BER = $1E-12$ with random errors) after RS(544,514) FEC

Burst error model 1

The NRZ burst analysis in [anslow_3bs_02_1114](#) page 12 assumed that if a **bit** is in error, the worst case probability that the next **bit** is also in error is 0.5. If we assume, for Gray coded PAM4, that an error in a particular symbol only causes the decision on the next symbol to move up or down one level, then the possibilities are:

Correct level	Received level		Error pattern	
	One up	One down	One up	One down
3	3	2	✓, ✓	✓, ✗
2	3	1	✓, ✗	✗, ✓
1	2	0	✗, ✓	✓, ✗
0	1	0	✓, ✗	✓, ✓



Since two of the eight possibilities result in both bits being correct, these states terminate the burst. Therefore for Gray coded PAM4, if a **symbol** is in error, the worst case probability that the next **symbol** is also in error is 0.75.

Burst error model 2

The second aspect of this table is that of the six possibilities giving bits in error, two have errors in the first bit while four have errors in the second bit.

Correct level	Received level		Error pattern	
	One up	One down	One up	One down
3	3	2	✓, ✓	✓, ✗
2	3	1	✓, ✗	✗, ✓
1	2	0	✗, ✓	✓, ✗
0	1	0	✓, ✗	✓, ✓

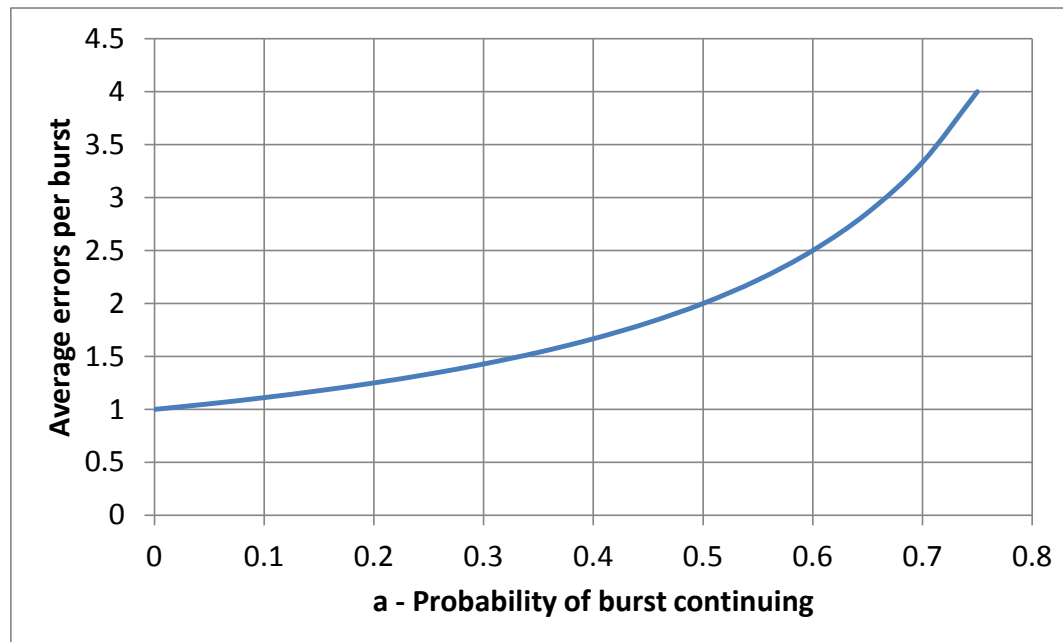
The analysis in the remainder of this contribution therefore assumes that if a given symbol is in error, the probability of a bit error in the first bit is $1/3$ and in the second bit is $2/3$.

Burst error model 3

The “SNR” shown on the X axis of the results slides is related to the noise induced input SER via the following equation:

$$SER_{in} = \frac{3}{4} \operatorname{erfc} \left(\sqrt{\frac{SNR}{2}} \right) \quad (1)$$

Which does not include the additional errors due to the bursts. The average number of errors in a burst is related to the probability of the burst continuing “a” as shown below:



For $a = 0.75$, the BER_{in} including bursts is 4 x the BER_{in} due to noise.

Single burst bound

As pointed out in [anslow_01_0815_logic](#), for a non-interleaved scheme, a single burst that lasts for ~74 PAM4 symbols has a high probability of causing errors in 16 FEC symbols (which is uncorrectable). With $a = 0.75$, the probability of a burst this long is $0.75^{74} = 5.7E-10$. When this is combined with the probability that the codeword has at least one error in it, a simple lower bound for the FLR can be calculated.

If a is the probability of the burst continuing, a more accurate calculation for the probability that a single burst is uncorrectable is:

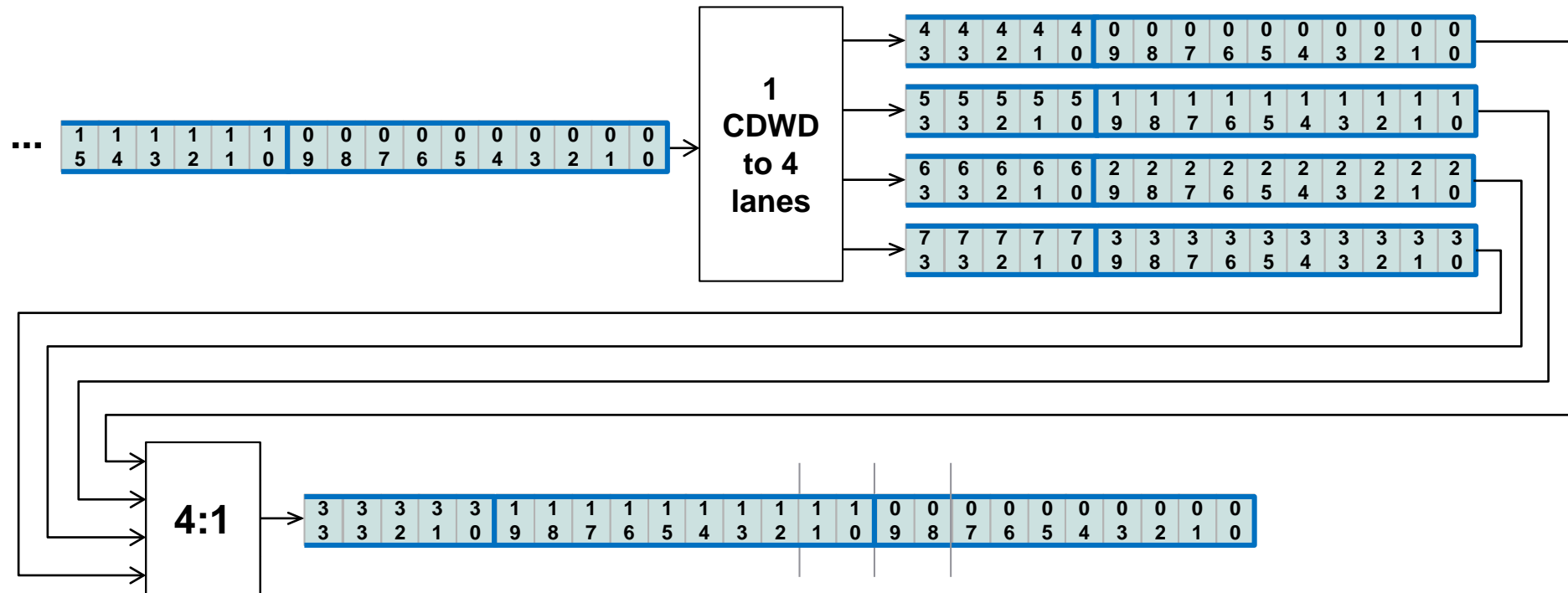
$$P_{\text{uncorr}} = 1/5 * a^{71} * (1-a) + 2/5 * a^{72} * (1-a) + 3/5 * a^{73} * (1-a) + 4/5 * a^{74} * (1-a) + a^{75} * (1-a) + a^{76} * (1-a) + a^{77} * (1-a) + \dots$$

For $a = 0.75$, this evaluates to $8.2E-10$.

This bound is plotted as a dotted line on page [22](#).

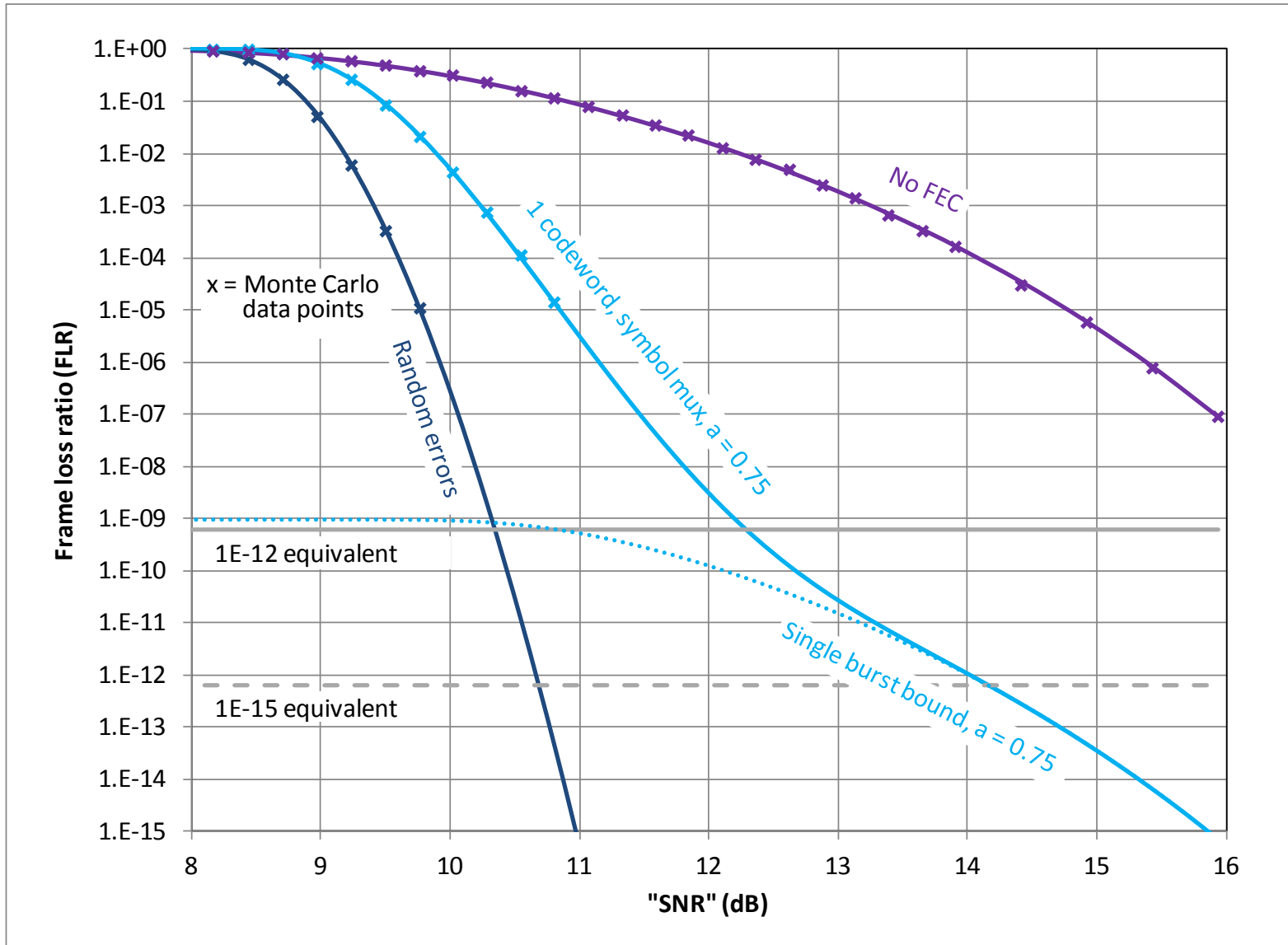
Clause 91 100G with symbol mux PMA

Round robin distribution of FEC symbols to the FEC lanes. Symbol multiplex in the PMA.



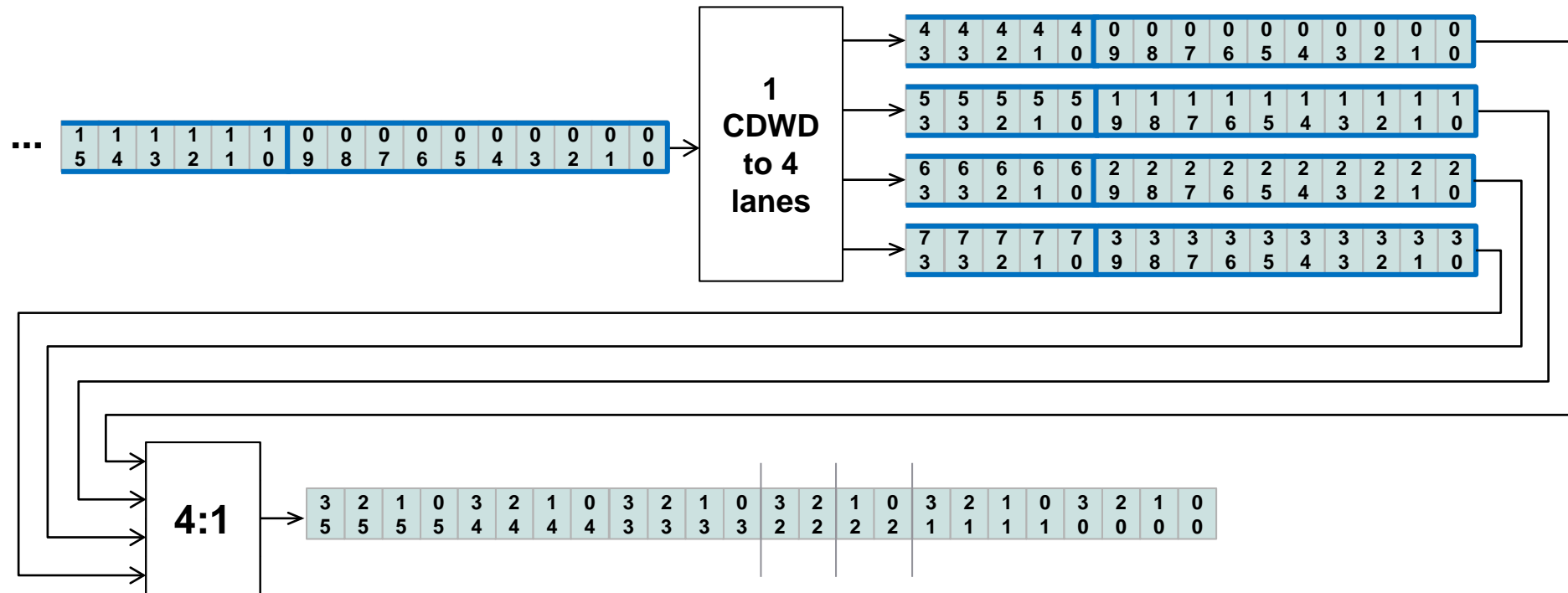
PMA must find FEC symbol boundaries.

Clause 91 100G with symbol mux PMA

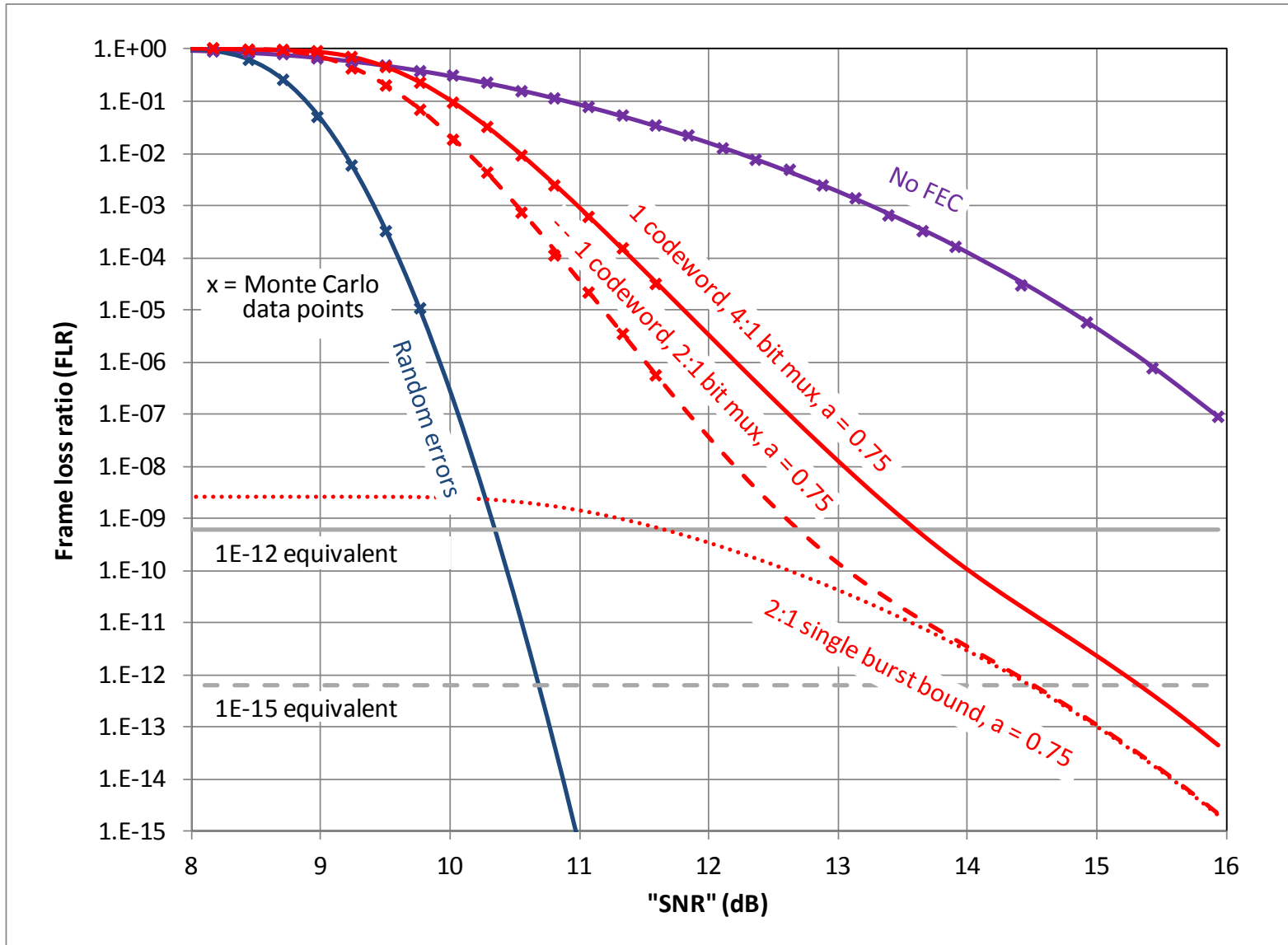


Clause 91 100G with bit mux PMA

Round robin distribution of FEC symbols to the FEC lanes. Bit multiplex in the PMA.



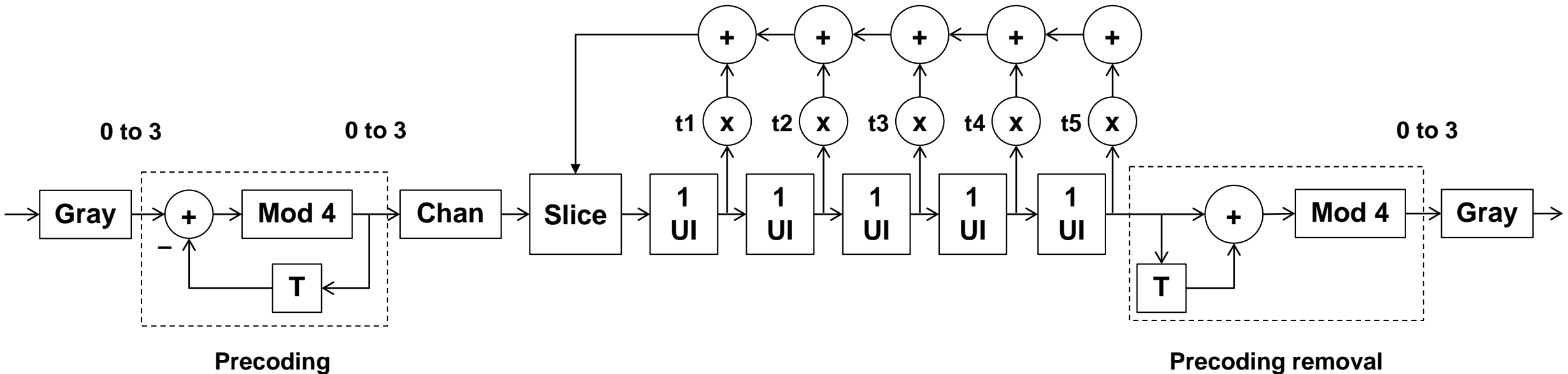
Clause 91 100G with bit mux PMA



Precoding

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Precoding as defined in 802.3cd 120.5.7.2 was assumed. This is performed as illustrated below.



See page 5 of [parthasarathy_01_0911](#) for a worked example.

A “feature” of this precoding process is that a single random errored PAM4 symbol at the slicer output turns into two errored PAM4 symbols after the precoding is removed.

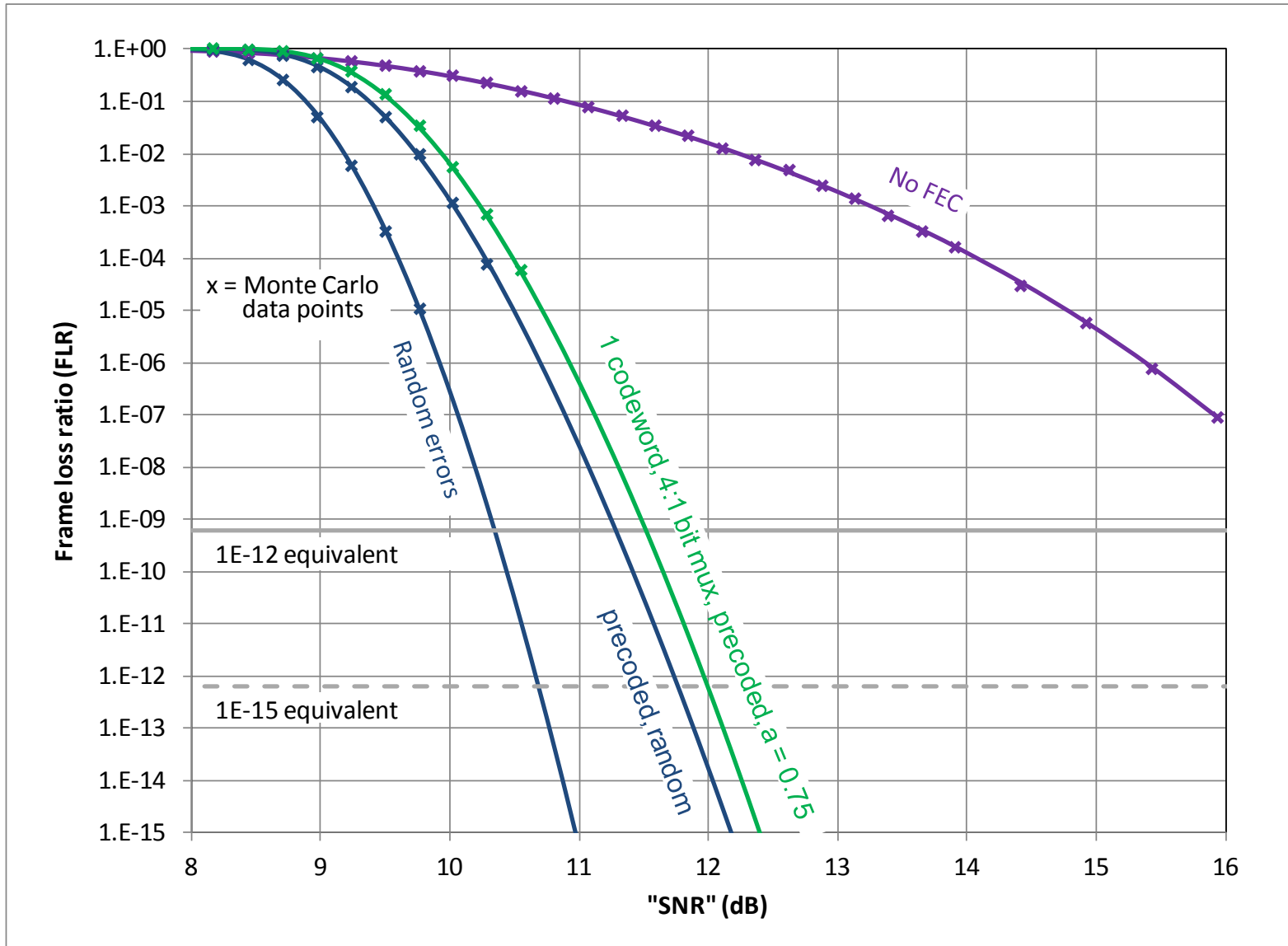
DFE tap weights



Rich Mellitz kindly provided the normalized DFE tap weights as calculated by COM for some channels submitted to IEEE 802.3.

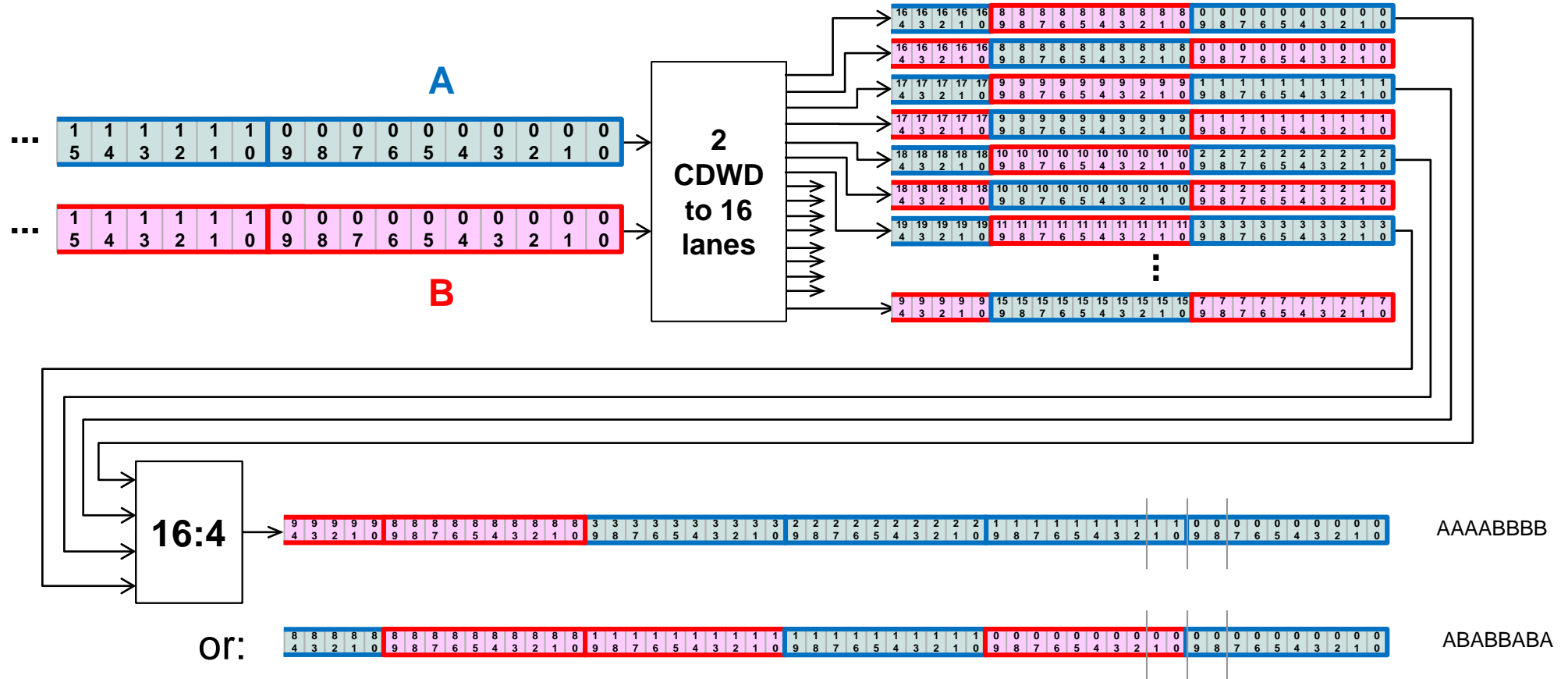
channel	pkg len	ll (dB)	DFE1	DFE2	DFE3	DFE4	DFE5	DFE6	DFE7	DFE8	DFE9	DFE10	DFE11	DFE12	DFE13	DFE14	DFE15	DFE16	DFE17	DFE18	DFE19	DFE20	DFE21	DFE22	DFE23	DFE24
tracy_100GEL_05_0118\B56_Thru_CblBP	30 mm	23.641	0.700	0.200	0.200	0.200	0.200	0.147	0.116	0.086	0.071	0.056	0.044	0.042	0.023	0.027	0.019	0.022	0.014	0.017	0.017	-0.018	0.010	0.020	0.016	0.017
mellitz_3ck_02_081518_CBP\CaBP_BGAVia_Opt2_32dB	12 mm	30.146	0.700	0.154	0.083	0.055	0.048	0.033	0.066	-0.041	0.035	0.017	0.017	0.007	0.005	0.005	-0.004	0.009	-0.007	0.004	-0.005	0.000	-0.002	-0.002	-0.001	-0.003
mellitz_3ck_02_081518_CBP\CaBP_BGAVia_Opt2_32dB	20 mm	30.146	0.700	0.089	0.014	-0.001	0.009	0.009	0.008	0.007	0.007	0.006	-0.002	0.031	-0.029	-0.014	0.018	0.003	0.003	-0.001	-0.001	-0.001	-0.001	-0.002	-0.001	-0.001
mellitz_3ck_02_081518_CBP\CaBP_BGAVia_Opt2_32dB	30 mm	30.146	0.700	0.072	-0.027	-0.039	-0.023	-0.017	-0.012	-0.009	-0.006	-0.006	-0.005	-0.005	-0.005	-0.005	-0.005	-0.004	-0.007	-0.001	0.014	-0.051	0.013	-0.002	0.003	-0.003
mellitz_3ck_02_081518_CBP\CaBP_BGAVia_Opt2_28dB	12 mm	26.347	0.680	0.176	0.123	0.083	0.067	0.043	0.074	-0.040	0.041	0.013	0.016	0.005	0.004	0.003	-0.006	0.008	-0.010	0.003	-0.007	-0.001	-0.004	-0.004	-0.003	-0.005
mellitz_3ck_02_081518_CBP\CaBP_BGAVia_Opt2_28dB	20 mm	26.347	0.699	0.181	0.117	0.080	0.066	0.048	0.037	0.027	0.021	0.015	0.005	0.035	-0.028	-0.009	0.014	0.003	0.002	-0.002	-0.002	-0.002	-0.002	-0.003	-0.003	-0.002
mellitz_3ck_02_081518_CBP\CaBP_BGAVia_Opt2_28dB	30 mm	26.347	0.691	0.116	0.053	0.032	0.033	0.026	0.021	0.016	0.013	0.010	0.007	0.005	0.003	0.002	0.001	0.001	-0.002	0.002	0.018	-0.050	0.018	-0.002	0.005	-0.002
mellitz_3ck_02_081518_CBP\CaBP_BGAVia_Opt2_24dB	12 mm	22.569	0.590	0.161	0.127	0.087	0.069	0.043	0.077	-0.043	0.051	0.010	0.020	0.007	0.007	0.006	-0.003	0.012	-0.008	0.007	-0.005	0.002	-0.001	-0.001	0.000	-0.002
mellitz_3ck_02_081518_CBP\CaBP_BGAVia_Opt2_24dB	20 mm	22.569	0.650	0.200	0.145	0.097	0.075	0.051	0.037	0.025	0.018	0.011	0.001	0.033	-0.035	-0.007	0.008	0.000	-0.001	-0.004	-0.004	-0.004	-0.004	-0.004	-0.005	-0.004
mellit...072518/Z0d_100_206in_0p13dBpi_twinax26_smooth	12 mm	27.980	0.692	0.091	-0.020	-0.013	-0.001	-0.003	0.047	-0.066	0.005	0.034	0.009	0.000	-0.002	0.000	-0.008	0.005	-0.006	-0.005	-0.002	-0.003	-0.003	-0.003	-0.002	-0.004
mellit...072518/Z0d_100_206in_0p13dBpi_twinax26_smooth	20 mm	27.980	0.688	0.125	0.002	-0.002	0.005	0.008	0.009	0.008	0.006	0.005	-0.003	0.033	-0.037	-0.025	0.027	0.010	0.001	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002
mellit...072518/Z0d_100_206in_0p13dBpi_twinax26_smooth	30 mm	27.980	0.700	0.079	-0.067	-0.062	-0.041	-0.026	-0.017	-0.012	-0.010	-0.008	-0.007	-0.007	-0.007	-0.006	-0.006	-0.006	-0.007	-0.003	0.018	-0.060	0.005	0.016	0.003	-0.003
mellitz...72518_channels--Z0d_100_14p25in_2dBpi_meg6_rf	12 mm	27.980	0.700	0.026	-0.065	-0.037	-0.015	-0.011	0.041	-0.067	-0.003	0.038	0.008	0.000	-0.001	0.001	-0.006	0.006	-0.003	-0.003	0.000	-0.001	-0.001	-0.001	0.000	-0.002
mellitz...72518_channels--Z0d_100_14p25in_2dBpi_meg6_rf	20 mm	27.980	0.685	0.067	-0.082	-0.068	-0.040	-0.022	-0.011	-0.006	-0.004	0.002	-0.012	0.035	-0.040	-0.025	0.029	0.013	0.002	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	0.000
mellitz...72518_channels--Z0d_100_14p25in_2dBpi_meg6_rf	30 mm	27.980	0.700	0.129	-0.015	-0.021	-0.011	-0.005	-0.002	-0.001	-0.001	-0.001	-0.002	-0.002	-0.003	-0.003	-0.004	-0.002	-0.005	0.000	0.016	-0.055	0.006	0.016	0.004	-0.001

Clause 91 100G with bit mux PMA and pre-coding



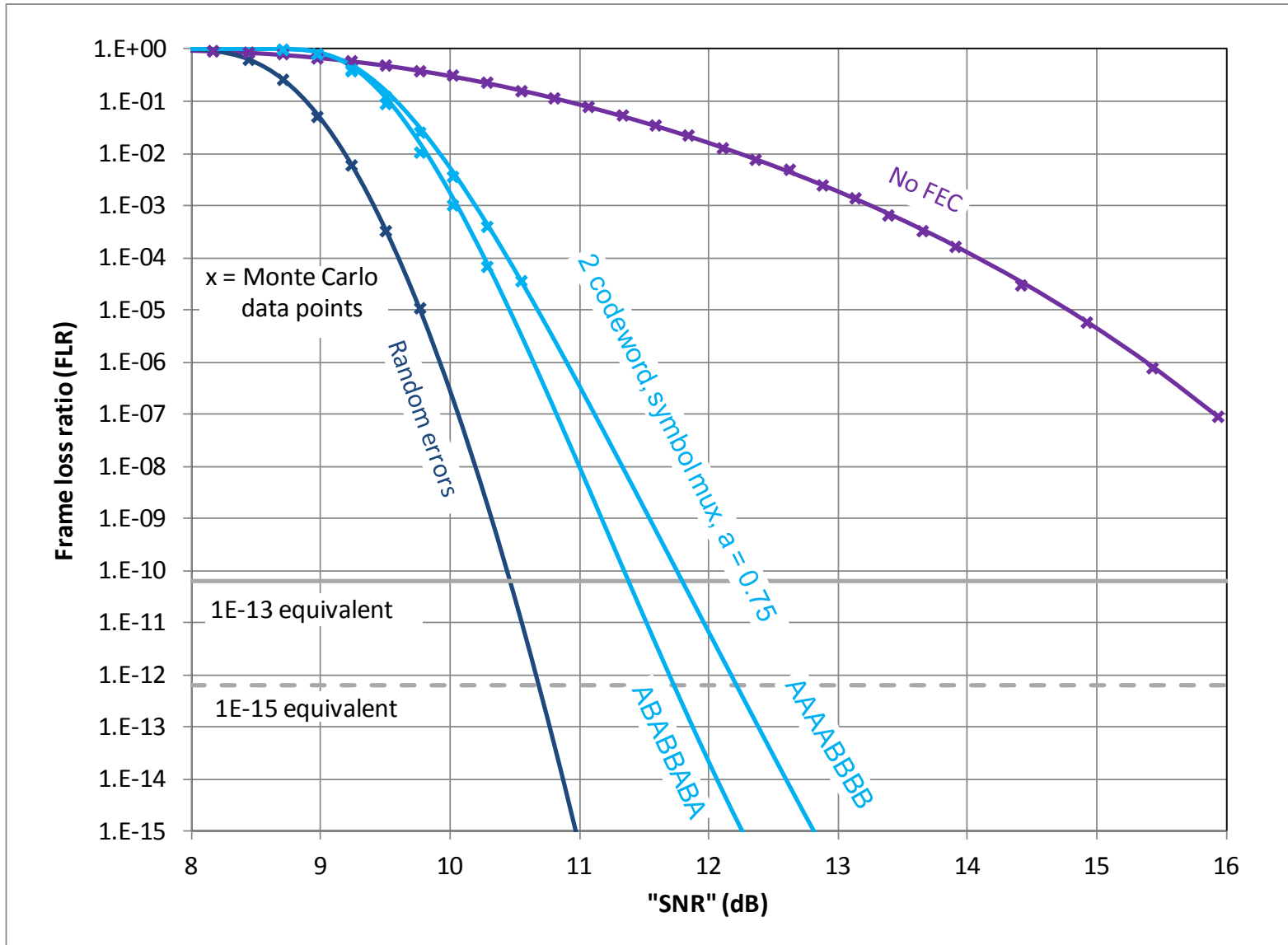
Clause 119 400G with symbol mux PMA

Symbol interleave from 2 FEC codewords. Symbol multiplex in the PMA.



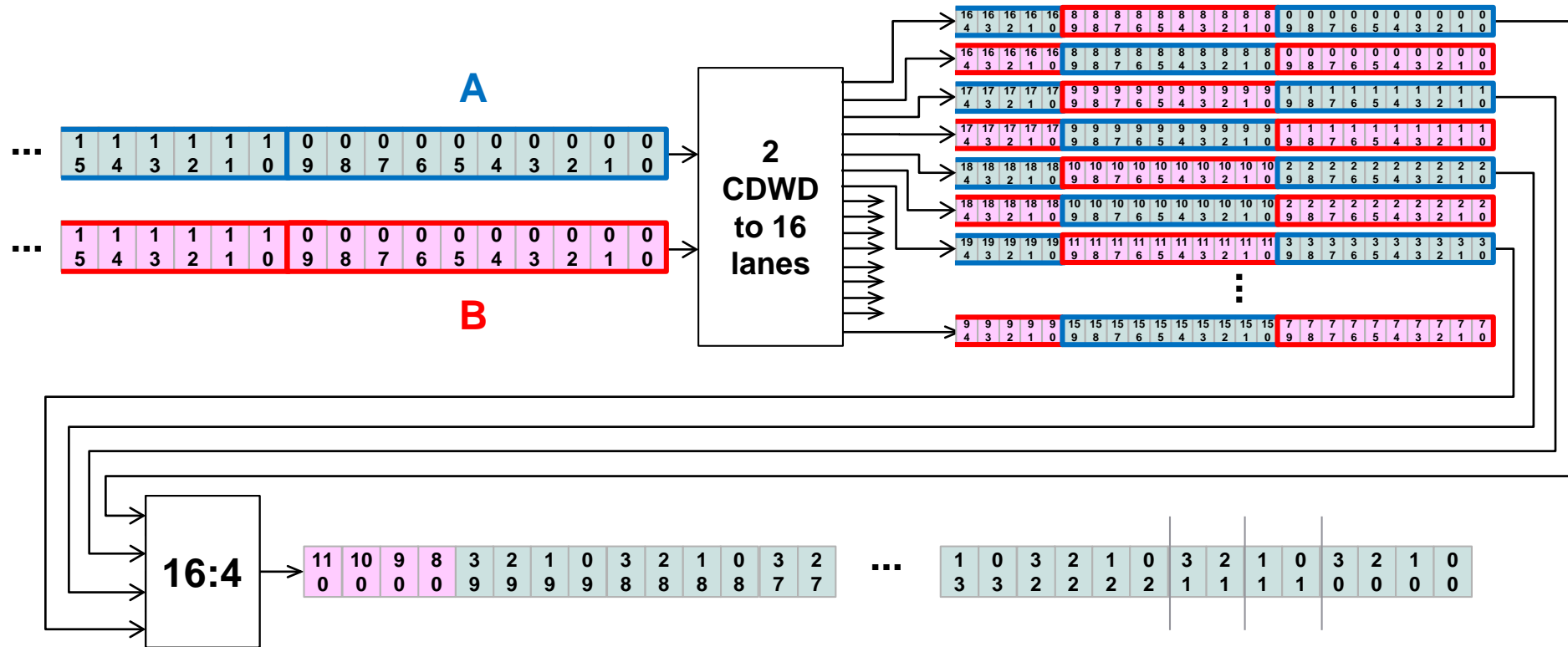
If one codeword is uncorrectable, the other is marked bad also.

Clause 119 400G with symbol mux PMA



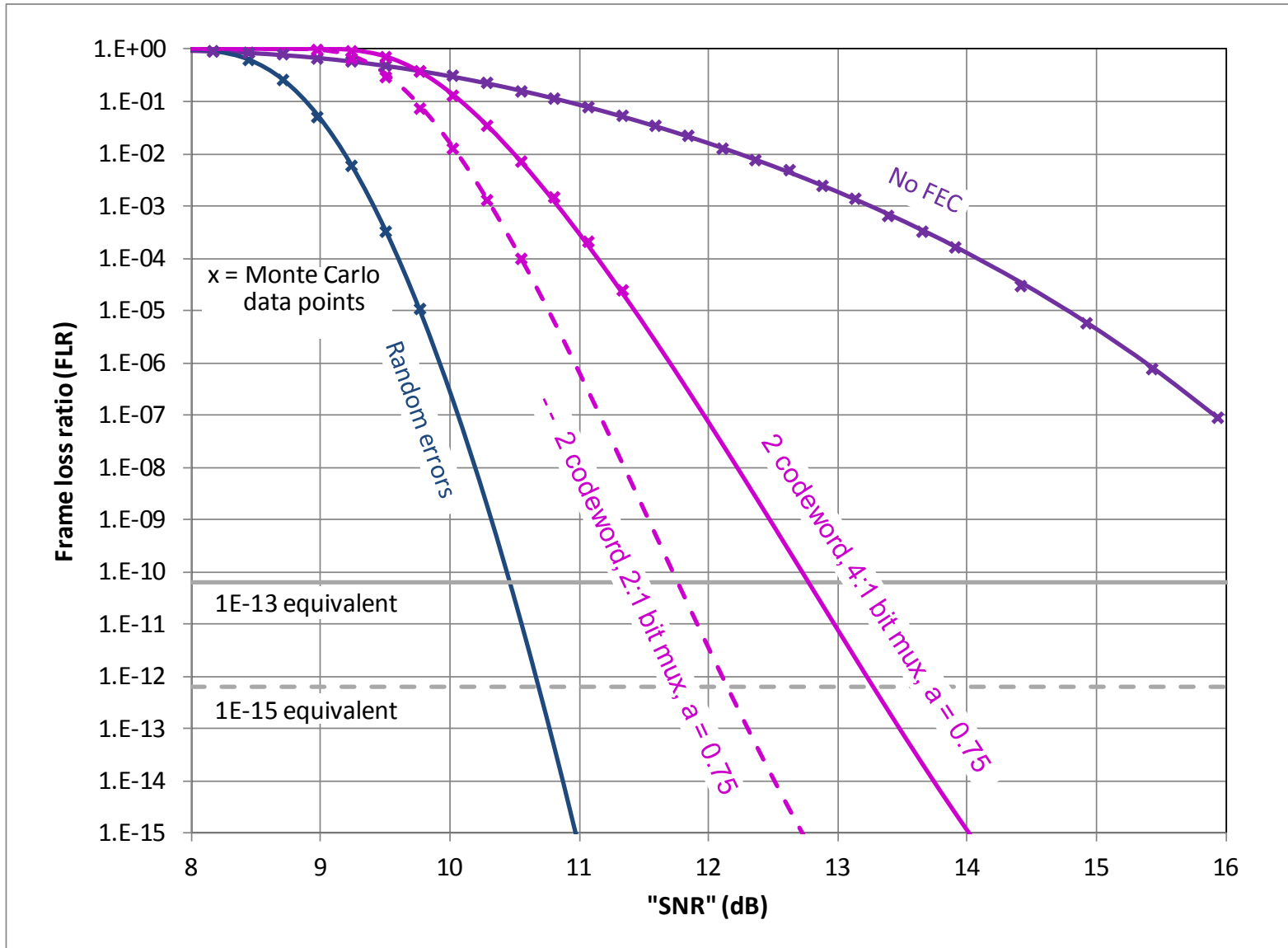
Clause 119 400G with bit mux PMA

Symbol interleave from 2 FEC codewords. Bit multiplex in the PMA.

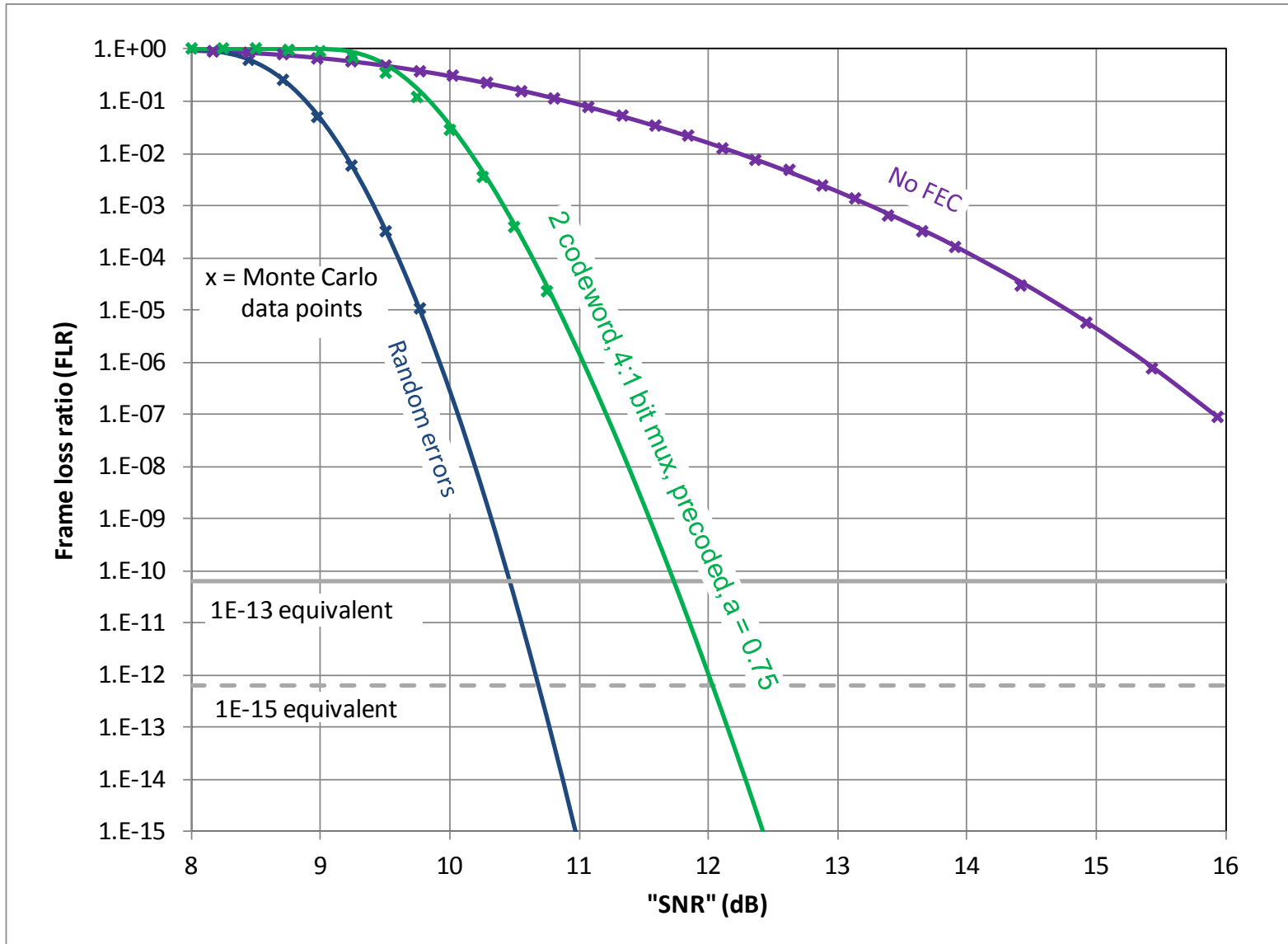


If one codeword is uncorrectable, the other is marked bad also.

Clause 119 400G with bit mux PMA

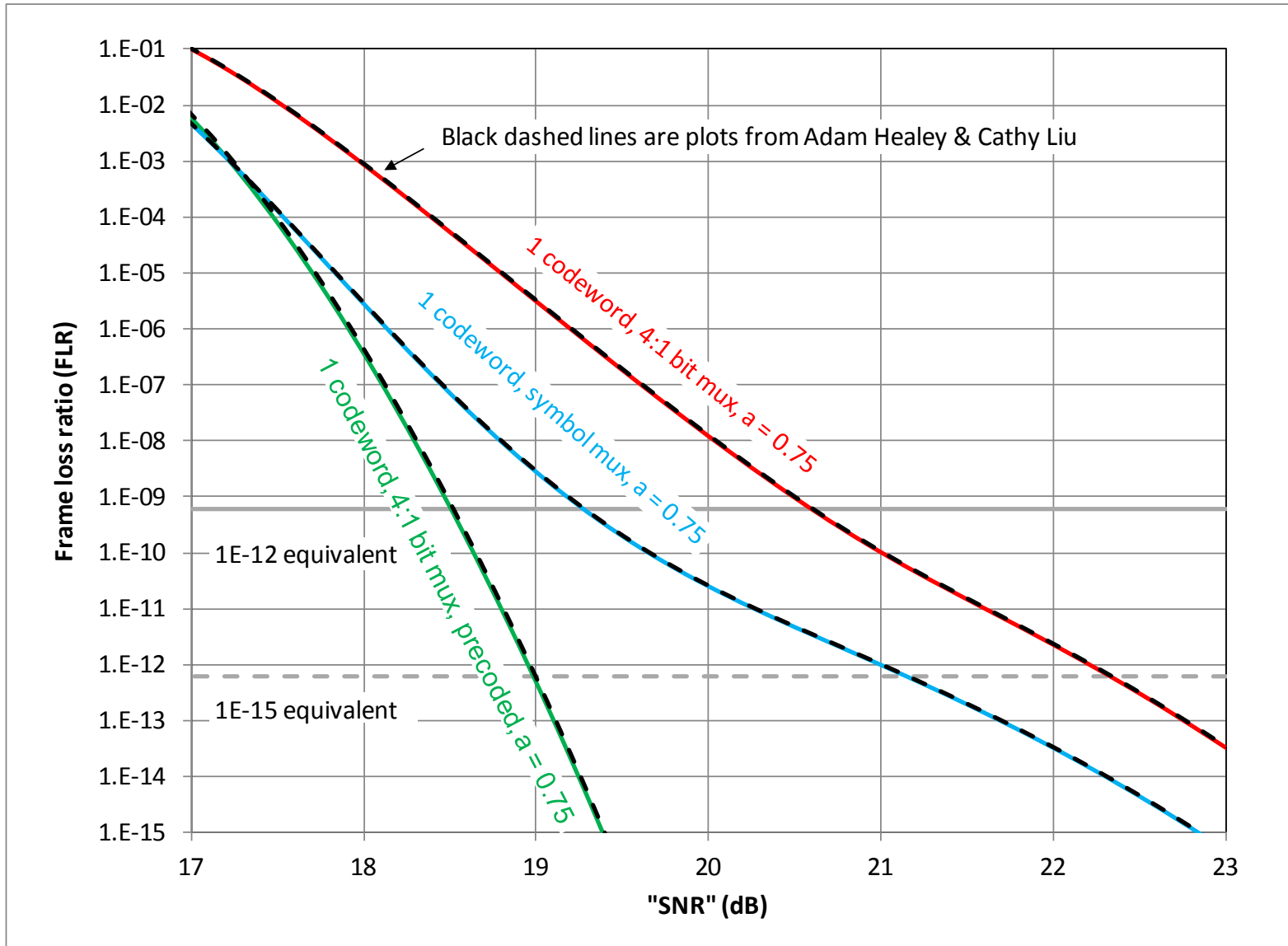


Clause 119 400G with bit mux PMA and pre-coding

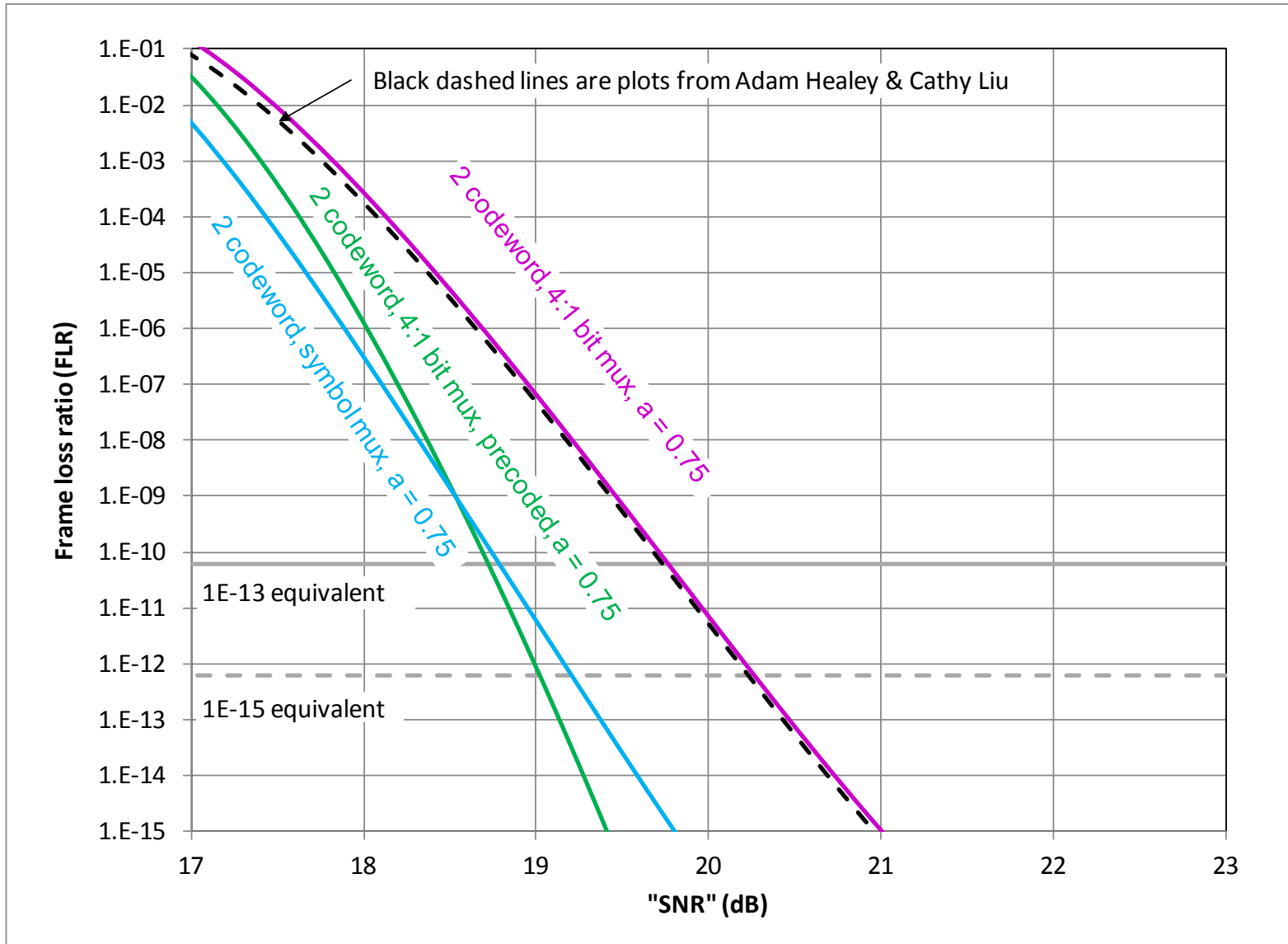


Comparison with
healey_100GEL_01_0318

100G curves re-plotted on same axes as healey_100GEL_01_0318



400G curves re-plotted on same axes as healey_100GEL_01_0318



Thanks!