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## Two-Phase Grid Search for Fast COM Calculation

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## Introduction

$>$ COM calculation time is mainly contributed by

- Computation iteration for TX FIR taps, CTLE curves, low-frequency CTLE
- FFE-based model needs extra computation for inversion of large matrix
- For each grid of FOM optimization, inversion of NxN matrix is required for N-tap RX FFE
- Computation cost of NxN matrix is $\mathrm{O}\left(\mathrm{N}^{3}\right)$
$>$ We evaluated a two-phase grid search algorithm to look for much faster COM calculation


## Two-Phase Grid Search for fast FOM Optimization

$>$ Full-Grid Search (Conventional)

- Check all values
- Ex) min / step $/ \max =-0.30 / 0.02 / 0.00$

> Two-Phase Grid Search (New)
- Phase 1: double the step size
phase 1 result

- Phase 2: half the search range with the phase 1 result at the center



## Aggressive vs Conservative Range Reduction in Phase 2

> Two possible schemes

- Aggressive reduction: just a few grids
phase 1 result
- E.g. 5 grids

- Conservative reduction: scaled range
- E.g. half range

$\sim$ full range $(=0.30)=16$ grids
$>$ We propose conservative reduction
- Aggressive reduction may cause a result different from the full-grid search
- In fact, we observe gDC of phase 1 is -16 dB , whereas gDC of full-grid search is -11 dB
- To get the same result as the full-grid search, we need a quite wide range in phase 2


## \# of Grids by Conservative Range Reduction

$>$ For each parameter,

- \# of grids in phase 1 is about half, because of double step
- \# of grids in phase 2 is about half, because of half range
- Hence, total \# of grids for each parameter does not change from the full-grid search
$>$ For multiple (e.g. 4) parameters,
- Total \# of grids in phase 1 is about $\left(\frac{1}{2}\right)^{4}=\frac{1}{16}$ because of double step
- Total \# of grids in phase 2 is about $\left(\frac{1}{2}\right)^{4}=\frac{1}{16}$ because of half range
- Hence, total \# of grids for 4 parameters is about $\left(\frac{1}{2}\right)^{4} \times 2=\frac{1}{8}$ of the full-grid search
$>$ Namely, although the \# of grids for each parameter does not change, total \# of grids for $N$ parameters will reduce by a factor of $\frac{1}{2^{(N-1)}}$


## Requirements for Min \# of Grids and Options

$>$ We applied the algorithm only to parameters with $\geq 6$ grids

- With $\leq 5$ grids (i.e. $\leq 4$ segments), check all full grids in phase 1 and 2
- Ex) min / step / max $=-0.10 / 0.02 / 0.00$ (example of min \# of grids)

- Phase 1 (double the step size) phase 1 result

- Phase 2 with Option A: min 5 grid

- Phase 2 with Option B: min 3 grid



## Experimental Implementation

$>$ In order to evaluate how close results we can get to the full-grid search, we experimentally implemented the algorithm as a wrapping function

- The wrapping function calls the COM tool function for multiple times
- For each phase
- For each package length
- It has unnecessary overhead
- S-parameter files are loaded and analyzed for multiple times
- COM value for phase 1 is unnecessarily calculated
> Once we have consensus, we can work on the full implementation


## Evaluation Conditions

$>$ Equalizer configuration

- RX model
- DFE-based model (b1max $=0.85, \mathrm{Nb}=16$ for all conditions, $\mathrm{Nb}=20 / 24 / 28$ for limited conditions)
- FFE-based model (b1max $=0.7, \mathrm{Nb}=1$, pre=3, post=16/20/24/28 for limited conditions)
- TX FIR tap range
- $c(-3) \in[-0.06: 0.02: 0], c(-2) \in[0: 0.02: 0.12], c(-1) \in[-0.34: 0.02: 0], c(1) \in[-0.1: 0.05: 0]$
- $c(0) \geq 0.54$
- RXCTLE
- gDC $\in[-20: 0], g D C 2 \in[-6: 0]$
> Package Model (Tx and Rx)
- $z_{p}=12 \mathrm{~mm}$ or $30 \mathrm{~mm}, C_{d}=110 f F, C_{p}=80 f F, R_{d}=50 \Omega$
> Noise, jitter
- $\eta_{0}=8.20 E-9 V^{2} / G H z, S N R_{T X}=33 \mathrm{~dB}, \sigma_{\mathrm{RJ}}=0.01 \mathrm{UI}, \mathrm{A}_{\mathrm{DD}}=0.02 \mathrm{UI}, \mathrm{R}_{\mathrm{LM}}=0.95$
> Channels
- Publicly available 115 KR/CR channels at web page (see the detail in the back up)
> COM Tool version
- v2.54 (before Rich's speed up) and v2.56mod (after Rich's speed up)
- V2.56mod is expected to have same results as v2.57

> - For v2.56mod, "break" was changed to "continue" according to Rich's intention of speed-up fix
> Computer used for evaluation

- CPU Intel Core i5-8250U @ 1.60GHz 1.80GHz, Memory 8GB, OS Windows 10 Pro


## Two-Phase Search vs Full-Grid Search

> Option A gave 100\% same results as the full-grid search
$>$ Option B gave $95 \%(219 / 230)$ same results as the full-grid search

- COM went up in 3 cases by up to 0.27 dB , and went down in 8 cases by up to 0.51 dB
- This is because EQs are optimized by FOM
- FOM went down in 11 cases by up to 0.16 dB , never went up


These results were completely same for v 2.54 and v 2.56 mod

## Phase 1 (i.e. double step) vs Full-Grid Search

>COM value of Full-Grid Search is not necessarily the best

- In 7 out of 230 cases, phase 1 result was better by up to 0.3 dB
- In most (205 out of 230) cases, phase 1 result was worse by up to 0.9 dB
> Statistically, the COM value is likely improved with a finer step, but this is not always the case



These results were completely same for v 2.54 and v 2.56 mod

## Discussion on Full-Grid Search: Do we really need it?

$>$ Two-phase search gives COM value close to the full-grid search

- We empirically verified that it is same for 230 cases with option A
- Theoretically, two-phase search may be still sub optimal
$>$ However, full-grid search does not necessarily give the best COM as well
- Because EQ parameters are optimized by FOM
$>$ With two-phase search, we may choose a higher COM value from results of phase 1 and phase 2
- This COM value may be higher than COM value by the full-grid search
- This option is not possible for the full-grid search, unless we optimize EQ parameters by COM instead of FOM
> Although two-phase search does not necessarily give the same result as the full-grid search, that is likely the case, and the phase 1 result may give a higher COM value than the full-grid search


## Measured Exec Time with Overhead

$>$ Average speed up is $2.64 x$ (option $A$ ) or $3.29 x$ (option $B$ ) for $v 2.54$

- Or 3.66x (option A) or $4.22 x$ (option B) when combined with Rich's speed up

|  | Measured Execution Time with Overhead (min) |  |  |  |  |  | Speed up (vs FG254) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  | by RM <br> FG256 | by Two phase |  | Combined |  |
|  | FG254 | FG256 | A254 | B254 | A256m | B256m |  | A254 | B254 | A256m | B256m |
| max | 37.47 | 18.22 | 15.70 | 13.63 | 11.74 | 10.76 | 3.59x | 3.69x | 5.96x | 5.34x | 6.41x |
| min | 3.88 | 3.38 | 1.56 | 1.22 | 1.37 | 1.11 | 0.95x | 1.98x | 2.32x | 2.79x | 2.91x |
| average | 16.56 | 7.70 | 6.27 | 5.04 | 4.52 | 3.93 | 2.15x | 2.64x | 3.29x | 3.66x | 4.22x |



## Breakdown of Execution Time (v2.54)

$>$ Measured execution time with overhead

- Full grid 994s
- Option A 376s (2.64x)
- Option B 302s (3.29x)

| $\begin{gathered} \text { Load } \\ \text { S4P files } \end{gathered}$ | FOM 12mm |  |  |  | $\mathrm{COM}$$12 \mathrm{~mm}$ |  | FOM 30mm |  |  |  | $\begin{gathered} \mathrm{COM} \\ 30 \mathrm{~mm} \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 40.0s | 465.2s |  |  |  | 6.6 s |  | 475.1s |  |  |  | 7.0s |
| $\begin{gathered} \text { Load } \\ \text { S4P files } \end{gathered}$ | $\begin{aligned} & \text { P1 FOM } \\ & 12 \mathrm{~mm} \end{aligned}$ | $\begin{gathered} \hline \mathrm{P} 1 \mathrm{COM} \\ 12 \mathrm{~mm} \end{gathered}$ | $\begin{gathered} \text { Load } \\ \text { S4P files } \end{gathered}$ | $\begin{gathered} \text { P2 FOM } \\ 12 \mathrm{~mm} \end{gathered}$ | $\begin{gathered} \text { P2 COM } \\ 12 \mathrm{~mm} \end{gathered}$ | $\begin{gathered} \text { Load } \\ \text { S4P files } \end{gathered}$ | $\begin{aligned} & \text { P1 FOM } \\ & 30 \mathrm{~mm} \end{aligned}$ | Pi COM 30 mm | $\begin{gathered} \text { Load } \\ \text { S4P files } \end{gathered}$ | $\begin{gathered} \text { P2 FOM } \\ 30 \mathrm{~mm} \end{gathered}$ | $\begin{aligned} & \text { P2 COM } \\ & 30 \mathrm{~mm} \end{aligned}$ |
| 40.0s | 39.8 s | 6.6 s | 40.0s | 57.5s | 6.6 s | 40.0s | 39.5s | 7.0s | 40.0s | 52.0s | 7.0s |
| $\begin{gathered} \text { Load } \\ \text { S4P files } \end{gathered}$ | $\begin{aligned} & \text { P1 FOM } \\ & 12 \mathrm{~mm} \end{aligned}$ | $\begin{gathered} \hline \mathrm{P} 1 \mathrm{COM} \\ 12 \mathrm{~mm} \end{gathered}$ | $\begin{gathered} \text { Load } \\ \text { S4P files } \end{gathered}$ | P2 FOM 12mm | $\begin{array}{\|c} \hline \text { P2 COM } \\ 12 \mathrm{~mm} \end{array}$ | $\begin{gathered} \text { Load } \\ \text { S4P files } \end{gathered}$ | $\begin{gathered} \text { P1 FOM } \\ 30 \mathrm{~mm} \\ \hline \end{gathered}$ | $\begin{gathered} \hline \mathrm{P}_{1} \mathrm{COM} \\ 30 \mathrm{~mm} \end{gathered}$ | $\begin{gathered} \text { Load } \\ \text { S4P files } \end{gathered}$ | $\begin{gathered} \text { P2 FOM } \\ 30 \mathrm{~mm} \end{gathered}$ | $\begin{gathered} \hline \text { P2 COM } \\ 30 \mathrm{~mm} \end{gathered}$ |
| 40.0s | 39.8s | 6.6 s | 40.0s | 18.6s | 6.6 s | 40.0s | 39.6s | 7.0s | 40.0s | 16.9s | 7.0s |

- Estimated execution time without overhead
- Full grid

| Load S4P files | FOM 12mm | $\begin{aligned} & \mathrm{COM} \\ & 12 \mathrm{~mm} \end{aligned}$ | FOM 30mm | $\begin{gathered} \mathrm{COM} \\ 30 \mathrm{~mm} \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: |
| 40.0s | 198.1s | 6.5 s | 212.4 s | 6.8 s |

- Option A 242s (4.10x)

| Load | P1 FOM | P2 FOM | COM | P1 FOM | P2 FOM | COM |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| S4P files | 12 mm | 12 mm | 12 mm | 30 mm | 30 mm | 30 mm |
| 40.0 s | 39.8 s | 57.5 s | 6.6 s | 39.5 s | 52.0 s | 7.0 s |

- Option B 168s (5.90x)

| Load | P1 FOM | P2 FOM | COM | P1 FOM | P2 FOM | COM |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| S4P files | 12 mm | Pmm <br> 12 mm | 30 mm | 12mm | 30 mm |  |
| 40.0 s | 39.8 s | 18.6 s | 6.6 s | 39.6 s | 16.9 s | 7.0 s |

## Breakdown of Execution Time (v2.56mod)

$>$ Measured execution time with overhead

- Full grid 462s (2.15x)
- Option A 271s (3.66x)
- Option B 236s (4.22x)

| $\begin{gathered} \text { Load } \\ \text { S4P files } \end{gathered}$ | FOM 12mm |  |  |  | $\begin{gathered} \mathrm{COM} \\ 12 \mathrm{~mm} \end{gathered}$ |  | FOM 30mm |  |  |  | $\begin{gathered} \mathrm{COM} \\ 30 \mathrm{~mm} \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 38.5s | 198.1s |  |  |  | 6.5s |  | 212.4 s |  |  |  | 6.8 s |
| $\begin{gathered} \text { Load } \\ \text { S4P files } \end{gathered}$ | $\begin{gathered} \text { P1 FOM } \\ 12 \mathrm{~mm} \end{gathered}$ | $\begin{gathered} \hline \mathrm{P} 1 \mathrm{COM}^{12 \mathrm{~mm}} \end{gathered}$ | $\begin{gathered} \text { Load } \\ \text { S4P files } \end{gathered}$ | $\begin{gathered} \text { P2 FOM } \\ 12 \mathrm{~mm} \end{gathered}$ | $\begin{gathered} \hline \begin{array}{c} \text { P2 COM } \\ 12 \mathrm{~mm} \end{array} \\ \hline \end{gathered}$ | $\begin{gathered} \text { Load } \\ \text { S4P files } \end{gathered}$ | $\begin{gathered} \text { P1 FOM } \\ 30 \mathrm{~mm} \end{gathered}$ | $\begin{gathered} \hline \mathrm{P} 1 \mathrm{COM}_{30 \mathrm{~mm}} \end{gathered}$ | $\begin{gathered} \text { Load } \\ \text { S4P files } \end{gathered}$ | $\begin{gathered} \text { P2 FOM } \\ \text { 30mm } \end{gathered}$ | $\begin{gathered} \hline \text { P2 COM } \\ 30 \mathrm{~mm} \end{gathered}$ |
| 38.5 s | 17.5s | 6.4 s | 38.5s | 29.1 s | 6.5 s | 38.5 s | 17.4s | 6.8 s | 38.5 s | 26.7s | 6.8 s |
| $\begin{gathered} \text { Load } \\ \text { S4P files } \end{gathered}$ | $\begin{gathered} \text { P1 FOM } \\ 12 \mathrm{~mm} \end{gathered}$ | $\begin{gathered} \hline \mathrm{P} 1 \mathrm{COM} \\ 12 \mathrm{~mm} \end{gathered}$ | $\begin{gathered} \text { Load } \\ \text { S4P files } \end{gathered}$ | $\begin{gathered} \text { P2 FOM } \\ 12 \mathrm{~mm} \end{gathered}$ | $\begin{gathered} \hline \begin{array}{c} \text { P2 COM } \\ 12 \mathrm{~mm} \end{array} \\ \hline \end{gathered}$ | $\begin{gathered} \text { Load } \\ \text { S4P files } \end{gathered}$ | $\begin{gathered} \text { P1 FOM } \\ \text { 30mm } \end{gathered}$ | $\begin{gathered} \hline \mathrm{P}_{1} \mathrm{COM} \\ 30 \mathrm{~mm} \end{gathered}$ | $\begin{gathered} \text { Load } \\ \text { S4P files } \end{gathered}$ | $\begin{gathered} \text { P2 FOM } \\ 30 \mathrm{~mm} \end{gathered}$ | $\begin{gathered} \hline \text { P2 COM } \\ 30 \mathrm{~mm} \end{gathered}$ |
| 38.5s | 17.6s | 6.4 s | 38.5 s | 10.4s | 6.5 s | 38.5 s | 17.4s | 6.8 s | 38.5s | 9.5 s | 6.8 s |

$>$ Estimated execution time without overhead

- Full grid 462s (2.15x)
$\begin{array}{ccccc}\text { Load } \\
\text { S4P files }\end{array} \quad$ FOM 12mm \(\left.\begin{array}{c}COM <br>

12 \mathrm{~mm}\end{array}\right]\) FOM 30mm | COM |
| :---: |
| 38.5 s |

- Option A 143s (6.97x)

| Load | P1 FOM | P2 FOM | COM | P1 FOM | P2 FOM | COM |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| S4P files | 12 mm | 12 mm | 12 mm | 30 mm | 30 mm | 30 mm |
| 38.5 s | 17.5 s | 29.1 s | 6.5 s | 17.4 s | 26.7 s | 6.8 s |

- Option B 107s (9.32x)

| Load | P1 FOM | P2 FOM | COM | P1 FOM | P2 FOM | COM |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| S4P files | 12 mm | 12 mm | 12 mm | 30 mm | 30 mm | 30 mm |
| 38.5 s | 17.6 s | 10.4 s | 6.5 s | 17.4 s | 9.5 s | 6.8 s |

## Estimated Exec Time without Overhead

$>$ Average speed up will be 4.10x (option A) or $5.90 x$ (option B) for v2.54

- Or 6.97x (option A) or $9.32 x$ (option B) when combined with Rich's speed up

|  | Estimated Execution Time without Overhead (min) |  |  |  |  |  | Speed up (vs FG254) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  | by RM FG256 | by Two phase |  | Combined |  |
|  | FG254 | FG256 | A254 | B254 | A256m | B256m |  | A254 | B254 | A256m | B256m |
| max | 37.47 | 18.22 | 9.80 | 7.29 | 5.65 | 4.69 | 3.59x | 5.21x | 7.49x | 9.11x | 12.82x |
| min | 3.88 | 3.38 | 1.00 | 0.66 | 0.85 | 0.58 | 0.95x | 3.01x | 4.57x | 4.04x | 6.70x |
| average | 16.56 | 7.70 | 4.04 | 2.81 | 2.38 | 1.78 | 2.15x | 4.10x | 5.90x | 6.97x | 9.32x |



## Execution Time by RX Model

> DFE-based model is much faster than FFE-based

- FFE-based execution time increases with \# of taps
- DFE-based model is $4.4 x$ faster than FFE-based with 24 post taps


## $>$ Both DFE- and FFE-based models will achieve speed up


\#n: n-tap DFE or 3-pre/n-post FFE
This is average exec time for $\mathrm{CH} 3, \mathrm{CH} 76, \mathrm{CH} 89$ with one case of package trace lengths using COM tool v2.56mod


This is estimated exec time without overhead.

## Summary

$>$ Two-phase search will speed up COM tool by 4.10x ~ 5.90x

- Option A
- $100 \%$ same results as conventional full-grid search
- 4.10x speed up by two-phase search
- $6.97 x$ speed up combined with RM's speed up
- Option B
- $95 \%$ same results as conventional full-grid search
- COM may go up or down, because EQs are optimized by FOM
- $5.90 x$ speed up by two-phase search
- $9.32 x$ speed up combined with RM's speed up
> Small variation of COM has been existing due to FOM-based optimization
- Two-phase search does not introduce extra variation for all channels simulated
> DFE-based model is observed to be much faster than FFE-based model
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## Back up

## Channels Used for Simulation

＞Simulation was done for the following publicly available 115 KR／CR channels

| CH \＃ | Group | Description | Reference Document |
| :---: | :---: | :---: | :---: |
| $1-2$ | RM1 | Two Very Good 28dB Loss Ideal Transmission Lines | mellitz＿3ck＿adhoc＿02＿072518．pdf |
| $3-8$ | RM2 | 24／28／32dB Cabled Backplane Channels including Via | mellitz＿3ck＿adhoc＿02＿081518．pdf |
| $9-10$ | RM3 | Synthesized CR Channels（2．0m and 2．5m 28AWG Cable） | mellitz＿100GEL＿adhoc＿01＿021218．pdf |
| $11-13$ | RM4 | Best Case 3＂，13＂，18＂Tachyon Backplane | mellitz＿100GEL＿adhoc＿01＿010318．pdf |
| $14-15$ | NT1 | Orthogonal or Cabled Backplane Channels | tracy＿100GEL＿03＿0118．pdf |
| 16 | AZ1 | Orthogonal Backplane Channel | zambell＿100GEL＿01a＿0318．pdf |
| $17-19$ | HH1 | Initial Host 30dB Backplane Channel Models | heck＿100GEL＿01＿0118．pdf |
| $20-35$ | HH2 | 16／20／24／28dB Cabled Backplane Channels | heck＿3ck＿01＿1118．pdf |
| $36-54$ | UK1 | Measured Traditional Backplane Channels | kareti＿3ck＿01a＿1118．pdf |
| $55-73$ | UK2 | Measured Cabled Backplane Channels |  |
| $74-88$ | UK3 | Measured Orthogonal Backplane Channels | zambell＿3ck＿01＿1118．pdf |
| $89-115 ~$ | AZ2 | Measured Orthogonal Backplane with Varied Impedances |  |

All channel data are taken from IEEE 100GEL Study Group and P802．3ck Task Force－Tools and Channels pages．
i．e．http：／／www．ieee802．org／3／100GEL／public／tools／index．html and http：／／www．ieee802．org／3／ck／public／tools／index．html

## COM parameters (DFE16)

| Table 93A-1 parameters |  |  |  |
| :---: | :---: | :---: | :---: |
| Parameter | Setting | Units | Information |
| f b | 53.125 | GBd |  |
| $f$ min | 0.05 | GHz |  |
| Delta_f | 0.01 | GHz |  |
| C_d | [1.1e-4 1.1e-4] | nF | [TX RX] |
| z_p select | [12] |  | [test cases to run] |
| z_p ${ }^{\text {(TX) }}$ | [12 30; 1.8 1.8; 0 0; 000$]$ | mm | [test cases] |
| z_p (NEXT) | [12 30; 1.81.8;0 0;0 0] | mm | [test cases] |
| z_p (FEXT) | [12 30; 1.81.8;0 0;0 0] | mm | [test cases] |
| z_p (RX) | [12 30; 1.81.8;0 0; 00 ] | mm | [test cases] |
| C_p | [0.8e-40.8e-4] | nF | [TX RX] |
| C_v | [00] | nF | [TX RX] |
| R_0 | 50 | Ohm |  |
| R_d | [ 50 50] | Ohm | [TX RX] |
| A_V | 0.41 | V |  |
| A_fe | 0.41 | V |  |
| A_ne | 0.6 | V |  |
| L | 4 |  |  |
| M | 32 |  |  |
| filter and Eq |  |  |  |
| f_r | 0.75 | *fb |  |
| $\mathrm{c}(0)$ | 0.54 |  | min |
| $\mathrm{c}(-1)$ | [-0.34:0.02:0] |  | [min:step:max] |
| $\mathrm{c}(-2)$ | [0:0.02:0.12] |  | [min:step:max] |
| c(-3) | [-0.06:0.02:0] |  | [min:step:max] |
| c(-4) | [0] |  | [min:step:max] |
| c (1) | [-0.1:0.05:0] |  | [min:step:max] |
| N_b | 16 | UI |  |
| b_max(1) | 0.85 |  |  |
| b_max(2..N_b) | 0.2 |  |  |
| g_DC | [-20:1:0] | dB | [min:step:max] |
| f_z | 21.25 | GHz |  |
| f_p1 | 21.25 | GHz |  |
| f_p2 | 53.125 | GHz |  |
| g_DC_HP | [-6:1:0] |  | [min:step:max] |
| f_HP_PZ | 0.6640625 | GHz |  |
| ffe_pre_tap_len | 0 | UI |  |
| ffe_post_tap_len | 0 | UI |  |
| Include PCB | 0 | logical |  |



## $\because: 0^{\circ}$

## Algorithm Option A (min 5 grids)

```
> Phase 1
step = org_step; max = org_max; min = org_min;
nseg = round( (org_max - org_min) / org_step );
if (nseg > 4)
    step = org_step * 2; % double the step size
end
> Phase 2
    step = org_step; max = org_max; min = org_min;
nseg = round( (org_max - org_min) / org_step );
if (nseg > 4)
        qnseg = ceil( nseg / 4 ); % round up to the same or upper integer
        min = max(org_min, phase1_result - org_step * qnseg);
        max = min(org_max, phase1_result + org_step * qnseg);
    end
```


## Algorithm Option B (min 3 grids)

```
> Phase 1
step = org_step; max = org_max; min = org_min;
nseg = round( (org_max - org_min) / org_step );
if (nseg > 4)
    step = org_step * 2; % double the step size
end
> Phase 2
    step = org_step; max = org_max; min = org_min;
nseg = round( (org_max - org_min) / org_step );
if (nseg > 4)
        qnseg = floor( nseg / 4 ); % round down to the same or lower integer
        min = max(org_min, phase1_result - org_step * qnseg);
        max = min(org_max, phase1_result + org_step * qnseg);
    end
```


## Difference between Full-grid Search and Option B

| $\begin{gathered} \text { PKG } \\ \text { zp } \end{gathered}$ | CH \# | Total IL | Fitted IL | ICN mV | TX FIR |  | gDC |  | gDC2 |  | FOM |  |  | COM |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | Full Grid | Option B | FG | OB | FG | OB | Full Grid | Opt. B | OB - FG | Full Grid | Opt. B | OB - FG |
| 12 | 6 | 29.19 | 22.98 | 0.88 | [-0.02 0.08-0.24 0.66 0] | [0 0.04-0.2 0.76 0] | -10 | -10 | -3 | -3 | 16.7815 | 16.7263 | -0.0552 | 6.3031 | 6.1431 | -0.1600 |
|  | 28 | 32.02 | 25.11 | 1.60 | [-0.02 0.08-0.26 0.64 0] | [0 0.04-0.22 0.74 0] | -11 | -12 | -2 | -2 | 15.1575 | 14.9940 | -0.1635 | 4.7614 | 4.4805 | -0.2809 |
|  | 39 | 29.82 | 20.95 | 1.77 | [-0.02 0.08-0.26 0.64 0] | [0 0.04-0.22 0.74 0] | -7 | -8 | -2 | -2 | 12.9105 | 12.8301 | -0.0804 | 2.9139 | 2.5452 | -0.3687 |
|  | 44 | 34.64 | 25.54 | 1.77 | [0 0.04-0.22 0.74 0] | [-0.02 0.08-0.26 0.64 0] | -11 | -10 | -3 | -3 | 11.9066 | 11.7440 | -0.1626 | 2.1359 | 2.4066 | 0.2707 |
|  | 54 | 45.31 | 35.09 | 1.77 | [0 0.04-0.24 0.72 0] | [0 0.02-0.22 0.76 0] | -17 | -17 | -4 | -4 | 7.8329 | 7.7267 | -0.1062 | -0.82785 | -1.3414 | -0.51355 |
| 30 | 1 | 35.14 | 28.01 | 0.00 | [-0.02 0.08-0.24 0.66 0] | [0 0.04-0.2 0.71-0.05] | -19 | -19 | -2 | -2 | 15.1463 | 15.1367 | -0.0096 | 3.9172 | 3.9172 | 0.0000 |
|  | 13 | 37.73 | 30.34 | 2.65 | [0 $0.04-0.280 .68$ 0] | [0 0.02-0.26 0.72 0] | -16 | -18 | -3 | -3 | 8.2306 | 8.2003 | -0.0303 | -1.4008 | -1.4817 | -0.0809 |
|  | 26 | 23.83 | 17.82 | 2.26 | $[-0.020 .08-0.240 .61-0.05]$ | [0 0.04-0.22 0.74 0] | -8 | -10 | -2 | -2 | 14.842 | 14.7685 | -0.0735 | 4.1102 | 3.8900 | -0.2202 |
|  | 33 | 36.35 | 29.42 | 1.55 | [-0.02 0.08-0.26 0.64 0] | [-0.02 0.08-0.26 0.64 0] | -17 | -17 | -4 | -3 | 11.8882 | 11.8771 | -0.0111 | 1.1499 | 1.1202 | -0.0297 |
|  | 48 | 36.97 | 27.52 | 1.77 | [0 $0.04-0.240 .720]$ | [0 0.06-0.26 0.68 0] | -15 | -15 | -4 | -3 | 11.0199 | 10.9512 | -0.0687 | 1.4218 | 1.5975 | 0.1757 |
|  | 76 | 30.98 | 24.34 | 1.12 | [-0.02 0.08-0.26 0.64 0] | [0 0.04-0.22 0.74 0] | -13 | -14 | -4 | -4 | 12.7397 | 12.5900 | -0.1497 | 2.1359 | 1.8518 | -0.2841 |

## Better results of Phase 1 than Full-Grid Search

| $\begin{array}{\|l} \text { PKG } \\ \text { zp } \end{array}$ | CH \# | Total IL | Fitted IL | ICN mV | TX FIR |  | gDC |  | gDC2 |  | FOM |  |  | COM |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | Full Grid | Phase 1 | FG | P1 | FG | P1 | Full Grid | Phase 1 | P1-FG | Full Grid | Phase 1 | P1-FG |
| 12 | 10 | 33.61 | 27.84 | 1.91 | [0 0.02-0.2 0.780] | [0 0.04-0.22 0.740] | -13 | -14 | -3 | -4 | 11.4581 | 11.3270 | -0.1311 | 1.7556 | 1.8196 | 0.0640 |
|  | 17 | 38.31 | 29.74 | 2.05 | [0 0.06-0.26 0.68 0] | [-0.02 0.08-0.26 0.64 0] | -14 | -14 | -2 | -2 | 10.8454 | 10.6599 | -0.1855 | 1.5144 | 1.5975 | 0.0831 |
|  | 18 | 37.57 | 29.62 | 2.03 | [0 0.06-0.26 0.68 0] | [-0.02 0.08-0.26 0.64 0] | -14 | -14 | -2 | -2 | 10.8957 | 10.6698 | -0.2259 | 1.6184 | 1.6815 | 0.0631 |
|  | 44 | 34.64 | 25.54 | 1.77 | [00.04-0.22 0.74 0] | [-0.02 0.08-0.26 0.64 0] | -11 | -10 | -3 | -2 | 11.9066 | 11.6283 | -0.2783 | 2.1359 | 2.1804 | 0.0445 |
|  | 46 | 36.12 | 27.09 | 1.77 | [0 0.06-0.30.64 0] | [-0.02 0.08-0.3 0.6 0] | -9 | -10 | -3 | -2 | 10.1452 | 9.9526 | -0.1926 | 0.50056 | 0.54669 | 0.04613 |
|  | 88 | 40.48 | 33.04 | 0.69 | [0 0.06-0.26 0.68 0] | [-0.02 0.08-0.26 0.64 0] | -15 | -16 | -4 | -4 | 11.3821 | 11.2469 | -0.1352 | 0.91515 | 0.92481 | 0.00966 |
| 30 | 48 | 42.16 | 27.52 | 1.77 | [00.04-0.24 0.72 0] | [-0.02 0.08-0.26 0.64 0] | -15 | -14 | -4 | -4 | 11.0199 | 10.8494 | -0.1705 | 1.4218 | 1.7237 | 0.3019 |

