# RS(544,514) FEC performance with 4:1 interleaving

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IEEE P802.3ck Task Force, July 2018

# Introduction

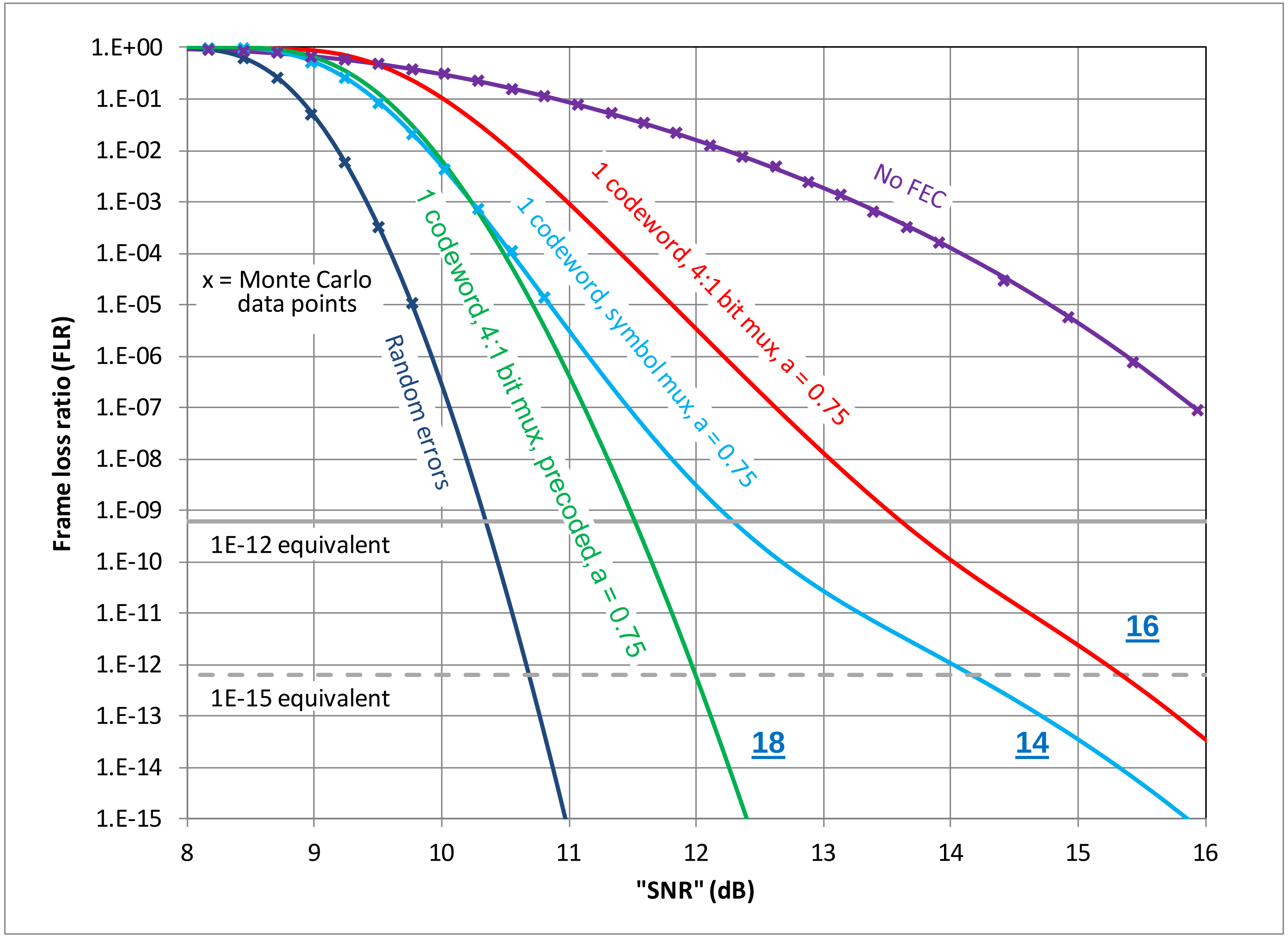
The IEEE P802.3ck Task Force has objectives to define 100 Gb/s lanes for AUI interfaces, backplanes and twin-axial copper cables for 100 Gb/s, 200 Gb/s, and 400 Gb/s Ethernet. If the FEC sublayer in Clause 91 is re-used for 100 Gb/s Ethernet and the Clause 119 PCS is re-used for 200 Gb/s and 400 Gb/s Ethernet, the 100 Gb/s lanes will have to be formed from interleaving four 25 Gb/s lanes.

healey_100GEL_01_0318 contained some analysis of a variety of interleaving schemes and proposed that pre-coding should be used in 100 Gb/s per lane electrical PHYs as a tool to improve error correction performance.

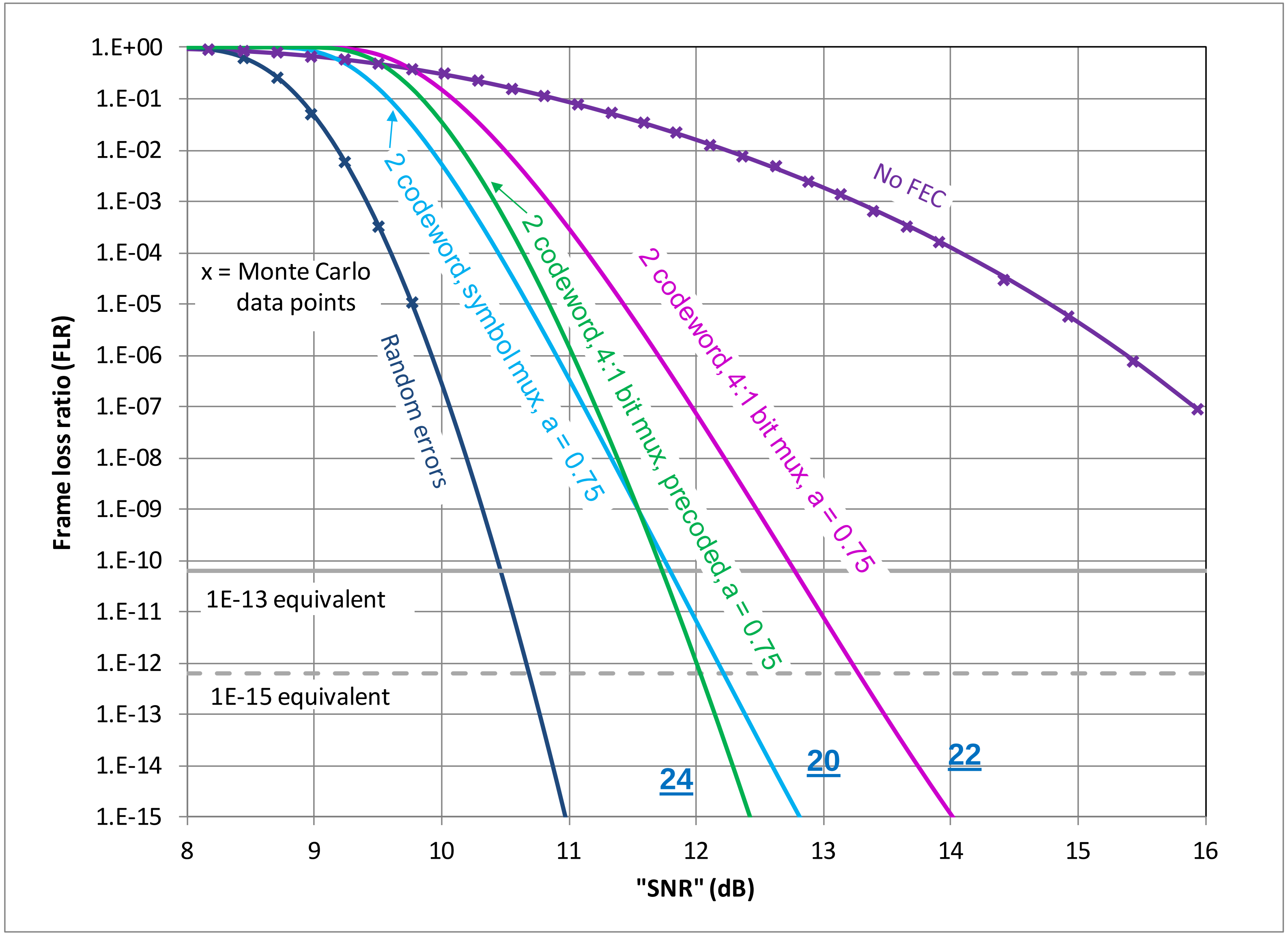
gustlin_3ck_01_0718 contained some concerns about whether the error pattern seen within bursts from practical receivers matches the error pattern where pre-coding turns each burst into two isolated symbol errors and suggested investigating RS symbol multiplexing as an alternative.

This presentation analyses the performance of a variety of 4:1 interleaving schemes using a development of the principles explained for the NRZ case in Annex 1 of anslow_3bs_02_1114.

# 100G all curves



# 400G all curves



# Results for RS(544,514) 100G all gain used for PAM4 part 1

From the curves shown on the previous pages, if all of the coding gain were to be used for the PAM4 link, the BERs required to give FLRs equivalent to that of a BER of 1E-12 (for 100G) or 1E-13 (for 400G) and 1E-15 are:

| | At slicer output | | At FEC input | |
|---|---|---|---|---|
| 100G | FLR = 6.2E-10 | FLR = 6.2E-13 | FLR = 6.2E-10 | FLR = 6.2E-13 |
| No FEC | 1E-12 | 1E-15 | | |
| 1 codeword, 4:1 bit mux, a = 0.75 | 2.55E-6* | 7.55E-9* | | |
| 1 codeword, symbol mux, a = 0.75 | 5.89E-5* | 4.93E-7* | | |
| 1 codeword, 4:1 bit mux, pre-coded, a = 0.75 | 2.47E-4* | 1.03E-4* | 1.23E-4 | 5.14E-5 |
| Random errors | 3.76E-4 | 2.34E-4 | | |
| 400G | FLR = 6.2E-11 | FLR = 6.2E-13 | FLR = 6.2E-11 | FLR = 6.2E-13 |
| No FEC | 1E-13 | 1E-15 | | |
| 2 codeword, 4:1 bit mux, a = 0.75 | 2.03E-5* | 6.03E-6* | | |
| 2 codeword, symbol mux, a = 0.75 | 1.69E-4* | 7.57E-5* | | |
| 2 codeword, 4:1 bit mux, pre-coded, a = 0.75 | 1.70E-4* | 9.70E-5* | 8.51E-5 | 4.85E-5 |
| Random errors | 3.20E-4 | 2.34E-4 | | |

Note – these values are the BER **including** the additional errors due to the bursts. To account for burst errors, the values marked with “*” have been multiplied by 4 when a = 0.75.

# Results for RS(544,514) 100G all gain used for PAM4 part 2

From the curves shown on the previous pages, if all of the coding gain were to be used for the PAM4 link, the $SER_{in}$ and SNR required to give FLRs equivalent to that of a BER of 1E-12 (for 100G) or 1E-13 (for 400G) and 1E-15 are:

| 100G | For FLR = 6.2E-10 | | | For FLR = 6.2E-13 | | |
|---|---|---|---|---|---|---|
| | $SER_{in}$ | SNR (dB) | SNR + 6.99 | $SER_{in}$ | SNR (dB) | SNR + 6.99 |
| 1 codeword, 4:1 bit mux, a = 0.75 | 1.28E-6 | 13.60 | 20.59 | 3.77E-9 | 15.34 | 22.33 |
| 1 codeword, symbol mux, a = 0.75 | 2.95E-5 | 12.28 | 19.27 | 2.47E-7 | 14.16 | 21.15 |
| 1 codeword, 4:1 bit mux, pre-coded, a = 0.75 | 1.23E-4 | 11.52 | 18.51 | 5.14E-5 | 12.00 | 18.99 |

| 400G | For FLR = 6.2E-11 | | | For FLR = 6.2E-13 | | |
|---|---|---|---|---|---|---|
| | $SER_{in}$ | SNR (dB) | SNR + 6.99 | $SER_{in}$ | SNR (dB) | SNR + 6.99 |
| 2 codeword, 4:1 bit mux, a = 0.75 | 1.01E-5 | 12.77 | 19.76 | 3.02E-6 | 13.27 | 20.26 |
| 2 codeword, symbol mux, a = 0.75 | 8.44E-5 | 11.74 | 18.73 | 3.79E-5 | 12.16 | 19.15 |
| 2 codeword, 4:1 bit mux, pre-coded, a = 0.75 | 8.51E-5 | 11.73 | 18.72 | 4.85E-5 | 12.03 | 19.02 |

Where:

$SER_{in}$ is the symbol error ratio due to noise only (does not include bursts)
SNR (dB) is the “SNR” in equation (1) on page 12
SNR + 6.99 is the SNR as defined on page 5 of healey_100GEL_01_0318

# Conclusion

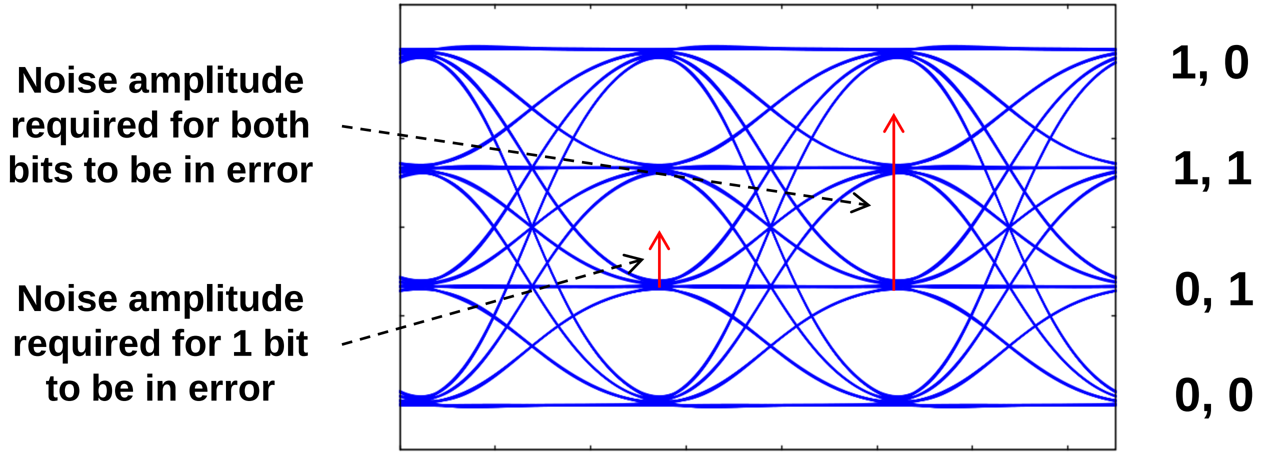
For 400G using a Clause 119 PCS, the performance at 1E-13 equivalent BER with worst case RS symbol multiplexing is almost the same as for pre-coding (with an error pattern where pre-coding turns each burst into two isolated symbol errors). At 1E-15 equivalent BER worst case symbol multiplexing is slightly worse than pre-coding.

For 100G using a Clause 91 FEC sublayer, the performance at 1E-13 equivalent BER with RS symbol multiplexing is better than that with 4:1 bit multiplexing, but still significantly worse than that for pre-coding (with an error pattern where pre-coding turns each burst into two isolated symbol errors). At 1E-15 equivalent BER symbol multiplexing is much worse than pre-coding.

# Annex

# Gray coding

Assume the use of Gray coding (see IEEE Std 802.3-2018 120.5.7) as illustrated below:




If noise causes any of the 4 levels to be mistaken for an adjacent level, this causes one of the two bits to be in error.

If there is just enough Gaussian noise to cause a BER of 3.8E-4* due to single level errors, then the probability of that noise causing both bits to be in error is 2E-24.

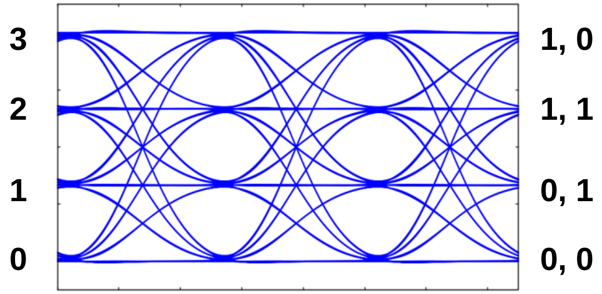
This analysis therefore assumes that only one of the two bits is in error.

* FLR = 6.2E-10 (equivalent to BER = 1E-12 with random errors) after RS(544,514) FEC

# Burst error model 1

The NRZ burst analysis in anslow_3bs_02_1114 page 12 assumed that if a **bit** is in error, the worst case probability that the next **bit** is also in error is 0.5.  If we assume, for Gray coded PAM4, that an error in a particular symbol only causes the decision on the next symbol to move up or down one level, then the possibilities are:

| Correct level | Received level | | Error pattern | |
|---|---|---|---|---|
| | One up | One down | One up | One down |
| 3 | 3 | 2 | ✓, ✓ | ✓, ✗ |
| 2 | 3 | 1 | ✓, ✗ | ✗, ✓ |
| 1 | 2 | 0 | ✗, ✓ | ✓, ✗ |
| 0 | 1 | 0 | ✓, ✗ | ✓, ✓ |




Since two of the eight possibilities result in both bits being correct, these states terminate the burst.  Therefore for Gray coded PAM4, if a **symbol** is in error, the worst case probability that the next **symbol** is also in error is 0.75.

# Burst error model 2

The second aspect of this table is that of the six possibilities giving bits in error, two have errors in the first bit while four have errors in the second bit.

| Correct level | Received level | | Error pattern | |
|---|---|---|---|---|
| | One up | One down | One up | One down |
| 3 | 3 | 2 | ✓, ✓ | ✓, ✗ |
| 2 | 3 | 1 | ✓, ✗ | ✗, ✓ |
| 1 | 2 | 0 | ✗, ✓ | ✓, ✗ |
| 0 | 1 | 0 | ✓, ✗ | ✓, ✓ |

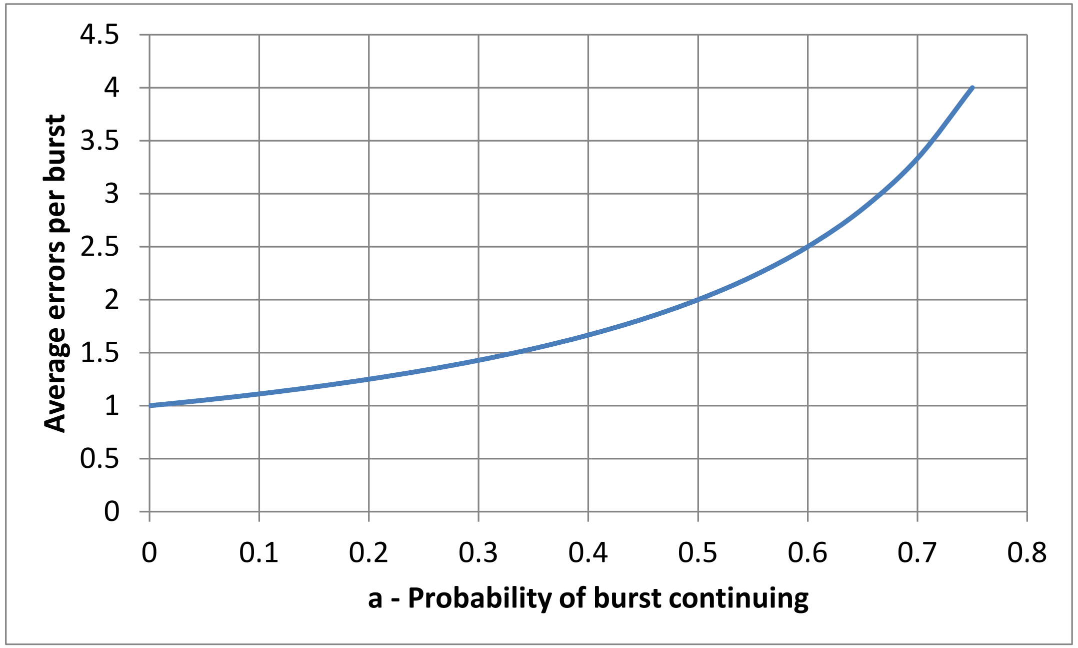
The analysis in the remainder of this contribution therefore assumes that if a given symbol is in error, the probability of a bit error in the first bit is 1/3 and in the second bit is 2/3.

# Burst error model 3

The "SNR" shown on the X axis of the results slides is related to the noise induced input SER via the following equation:

$$SER_{in} = \frac{3}{4} erfc\left(\sqrt{\frac{SNR}{2}}\right) \qquad (1)$$

Which does not include the additional errors due to the bursts. The average number of errors in a burst is related to the probability of the burst continuing "a" as shown below:




For a = 0.75, the $BER_{in}$ including bursts is 4 x the $BER_{in}$ due to noise.

# Single burst bound

As pointed out in anslow_01_0815_logic, for a non-interleaved scheme, a single burst that lasts for ~74 PAM4 symbols has a high probability of causing errors in 16 FEC symbols (which is uncorrectable). With a = 0.75, the probability of a burst this long is 0.75^74 = 5.7E-10. When this is combined with the probability that the codeword has at least one error in it, a simple lower bound for the FLR can be calculated.

If a is the probability of the burst continuing, a more accurate calculation for the probability that a single burst is uncorrectable is:

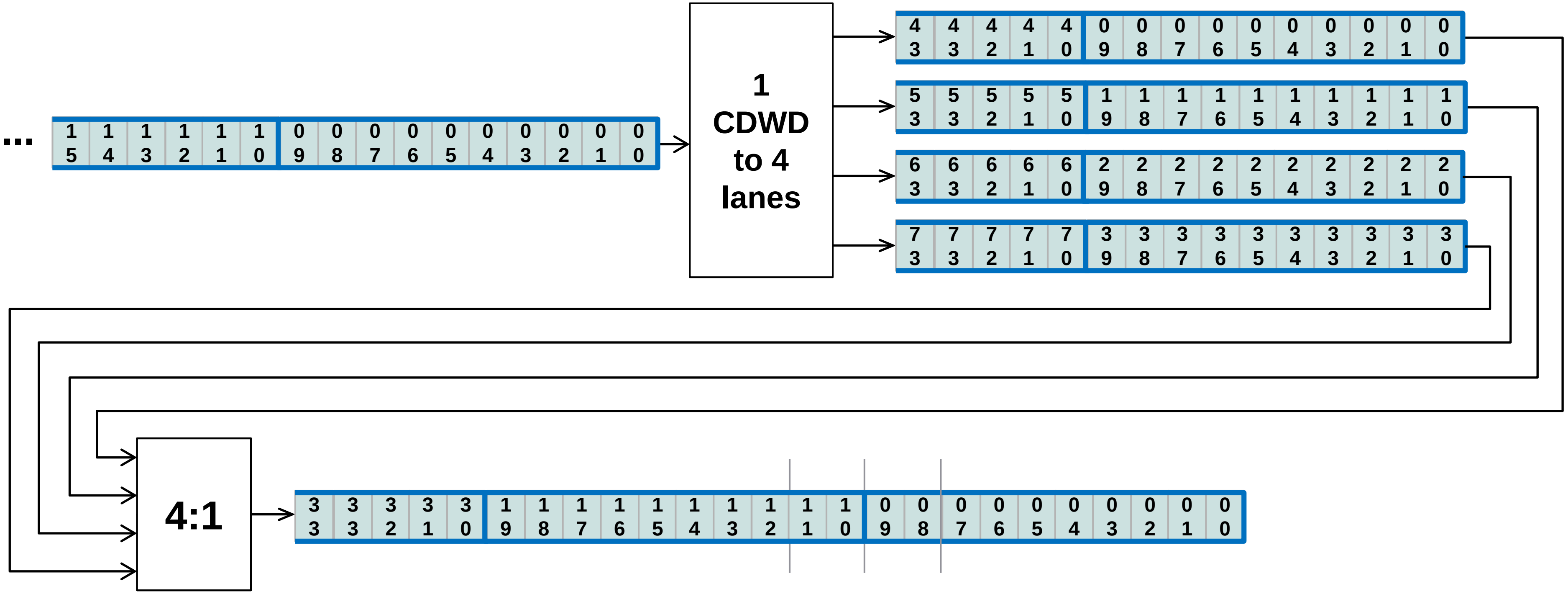
$$P_{uncorr} = 1/5*a^{71}*(1-a) + 2/5*a^{72}*(1-a) + 3/5*a^{73}*(1-a) + 4/5*a^{74}*(1-a) + a^{75}*(1-a) + a^{76}*(1-a) + a^{77}*(1-a) + \ldots$$

For a = 0.75, this evaluates to 8.2E-10.

This bound is plotted as a dotted line on page 15.

# Clause 91 100G with symbol mux PMA

Round robin distribution of FEC symbols to the FEC lanes. Symbol multiplex in the PMA.




PMA must find FEC symbol boundaries.

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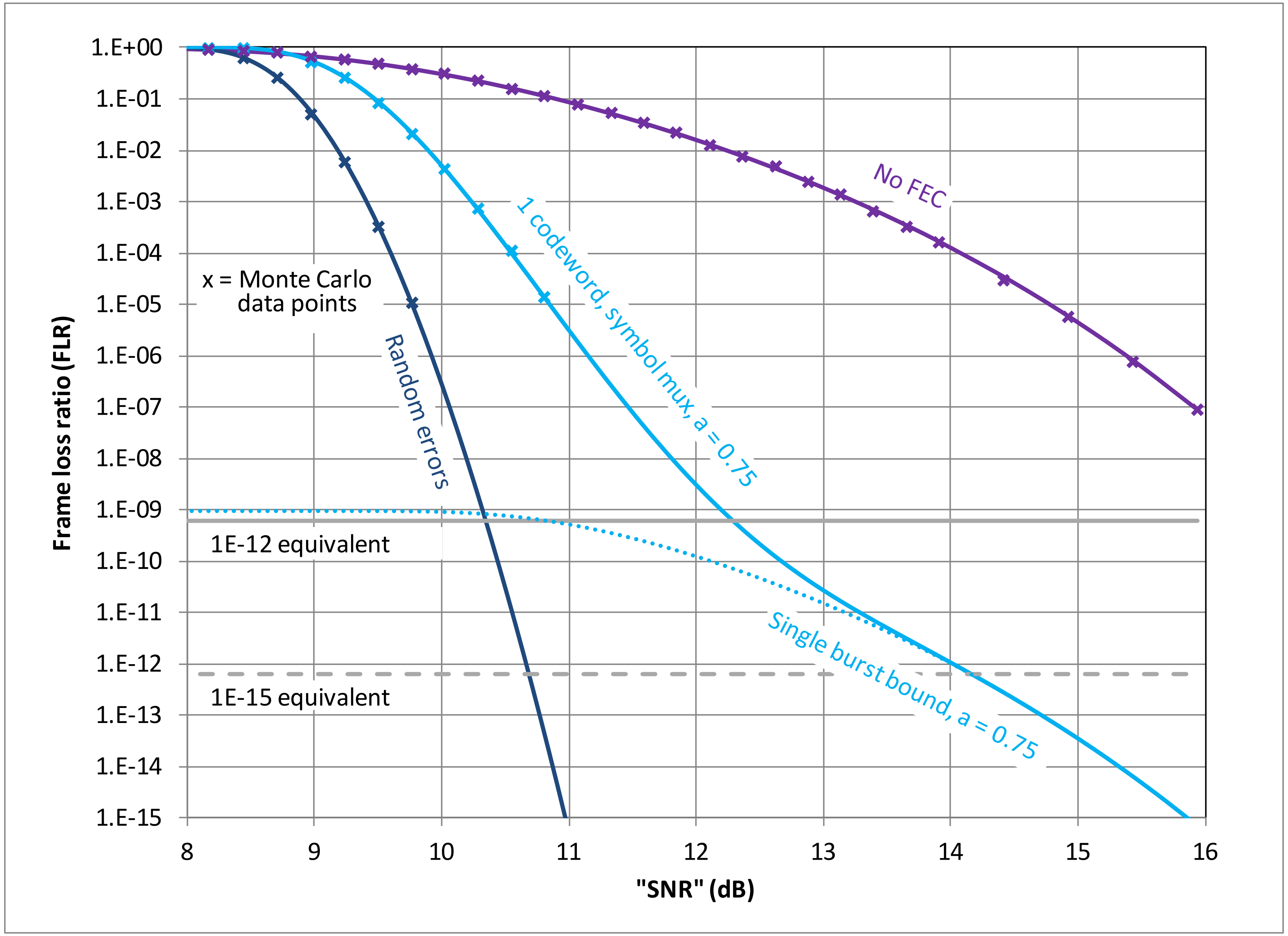
# Clause 91 100G with symbol mux PMA

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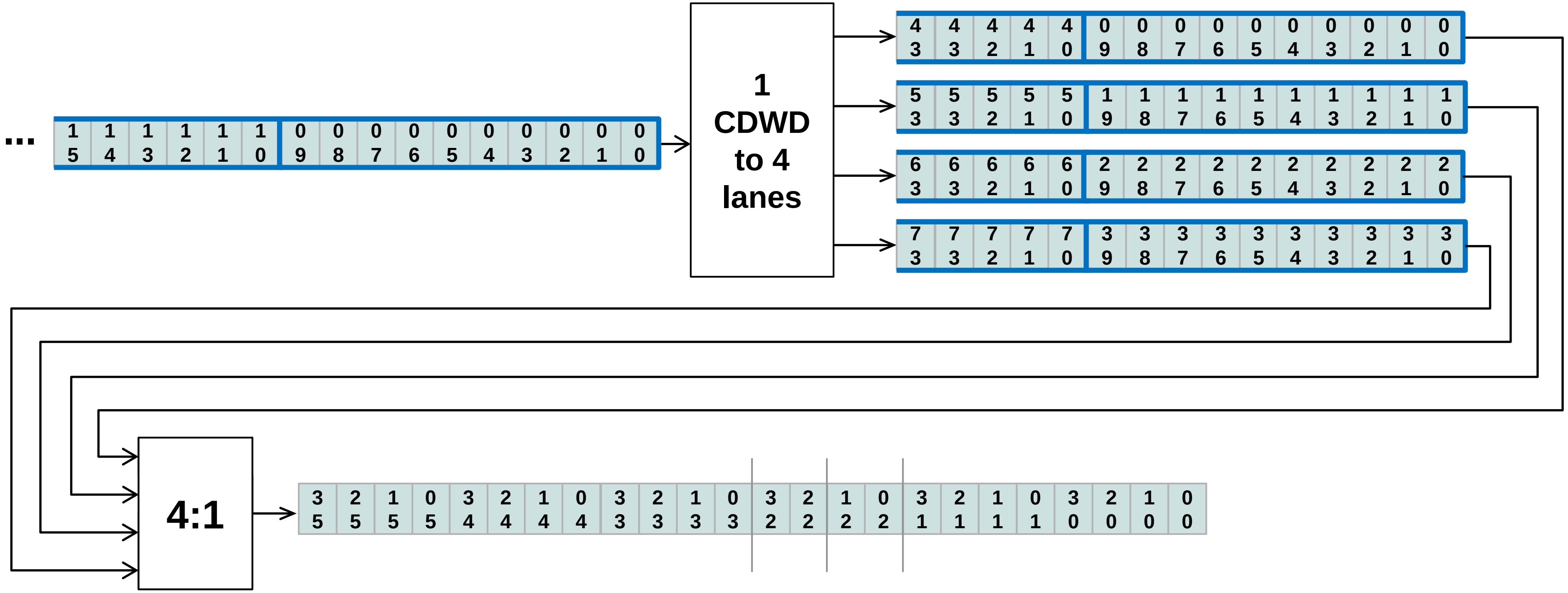
# Clause 91 100G with bit mux PMA

Round robin distribution of FEC symbols to the FEC lanes. Bit multiplex in the PMA.

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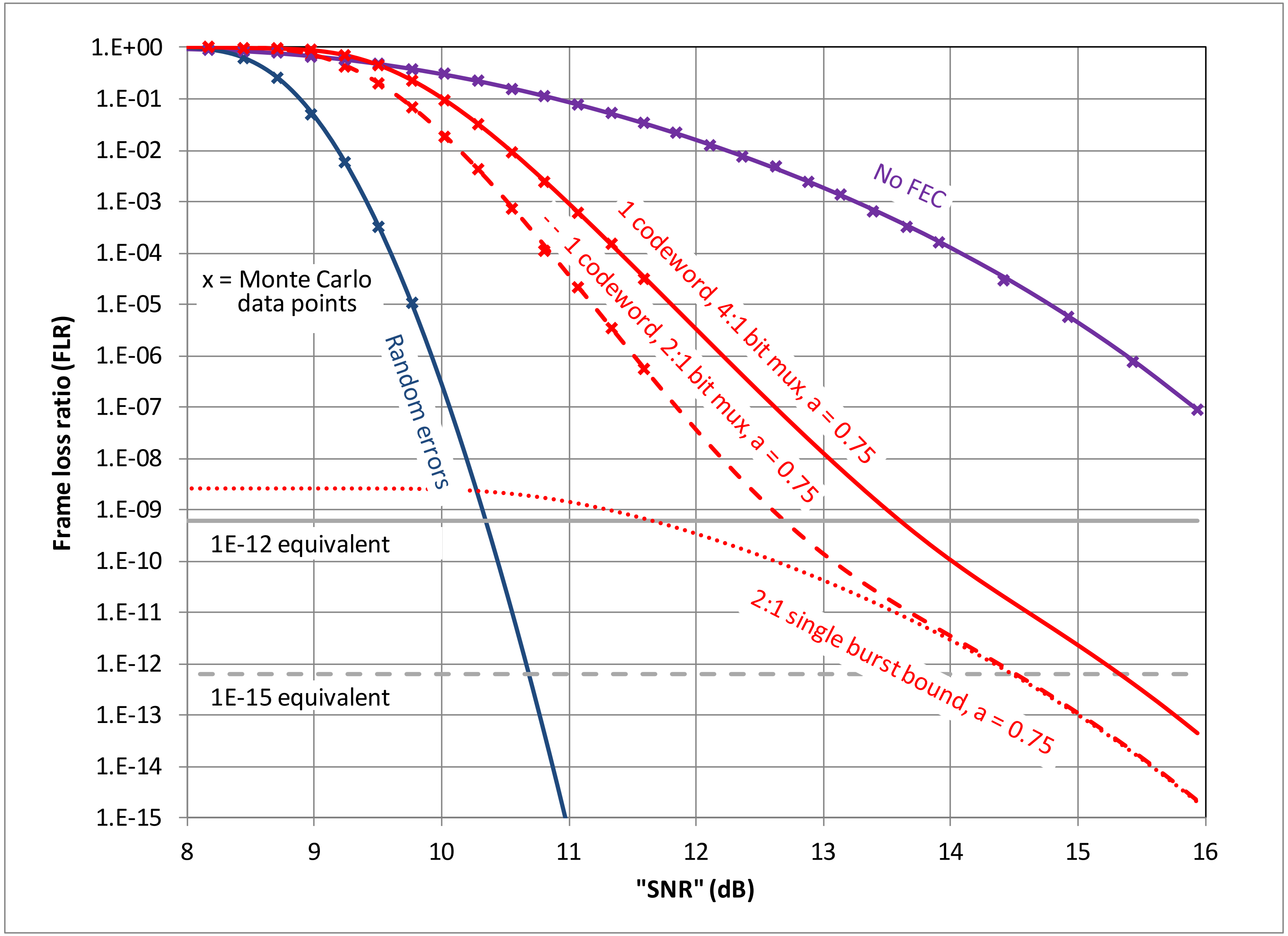
# Clause 91 100G with bit mux PMA

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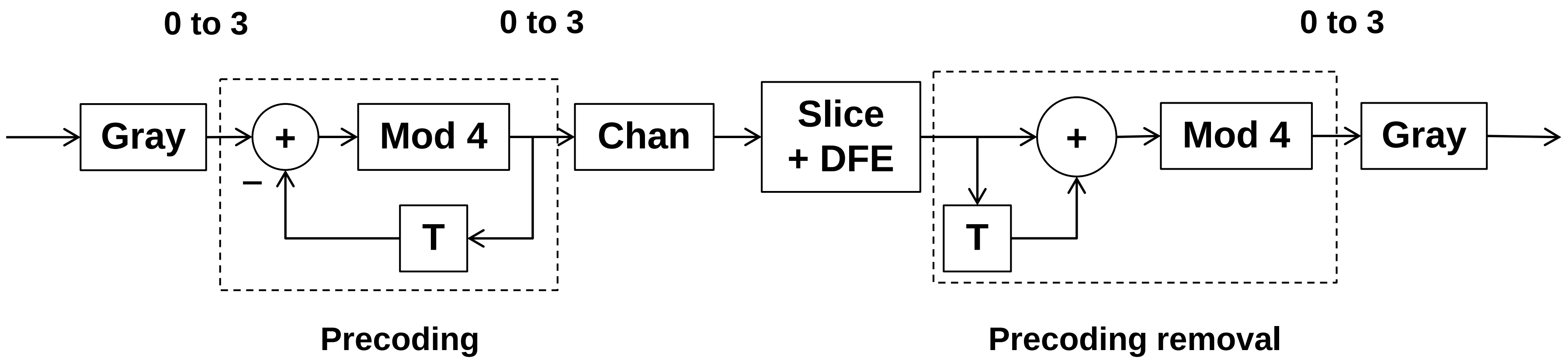
# Pre-coding

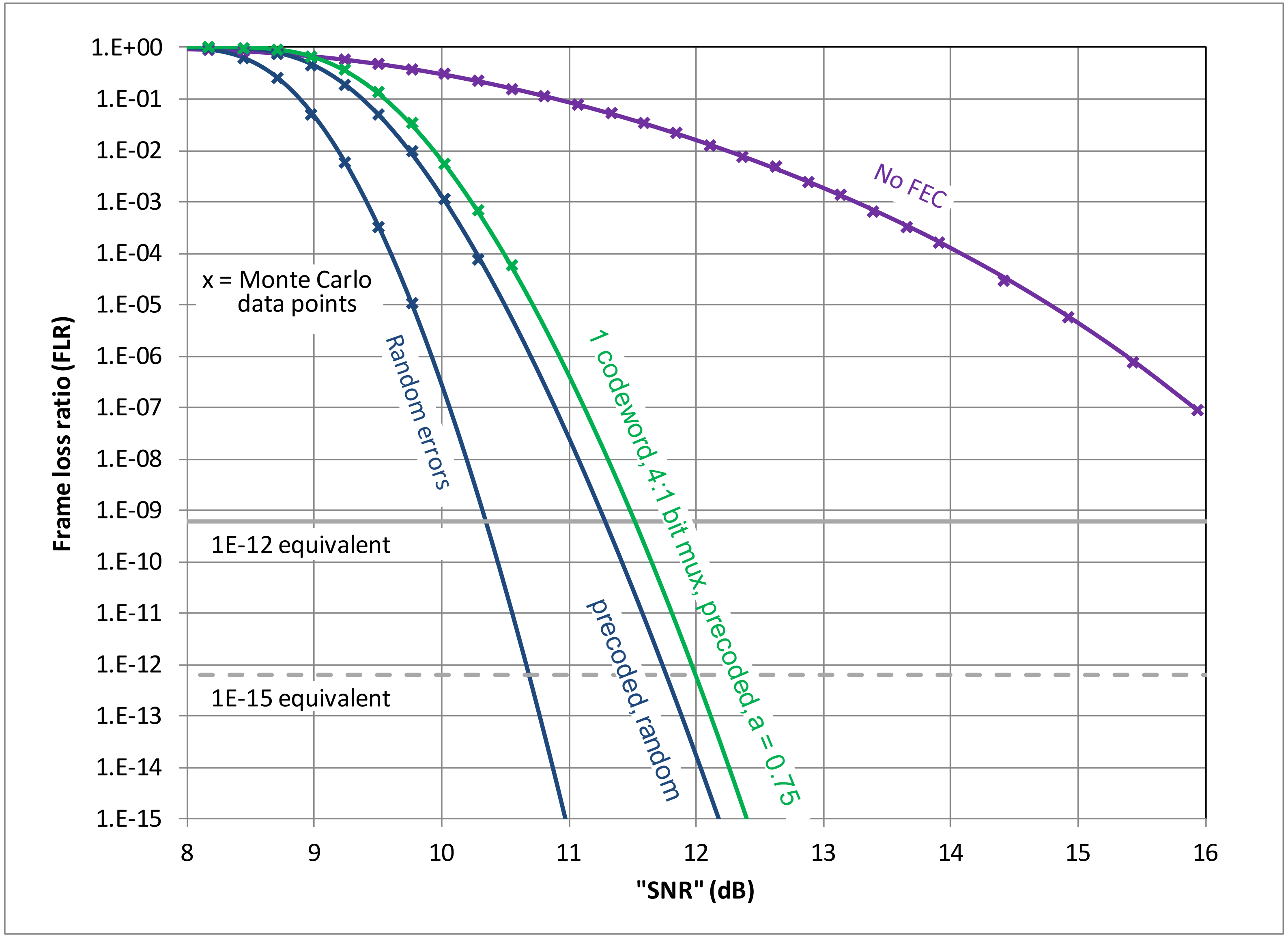
Pre-coding as defined in 802.3cd 120.5.7.2 was assumed.  This is performed as illustrated below.




See page 5 of parthasarathy_01_0911 for a worked example.

A “feature” of this pre-coding process is that a single random errored PAM4 symbol at the slicer output turns into two errored PAM4 symbols after the pre-coding is removed.

# Clause 91 100G with bit mux PMA and pre-coding

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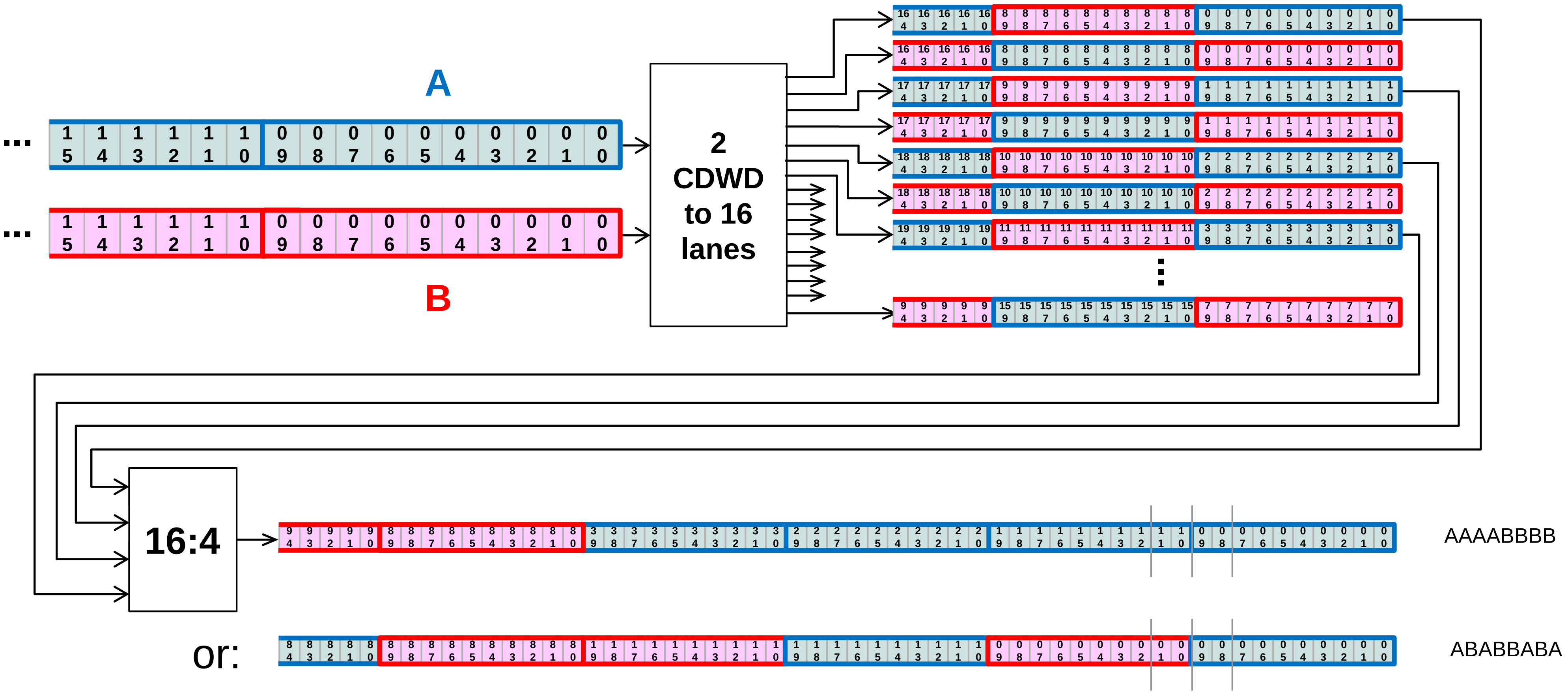
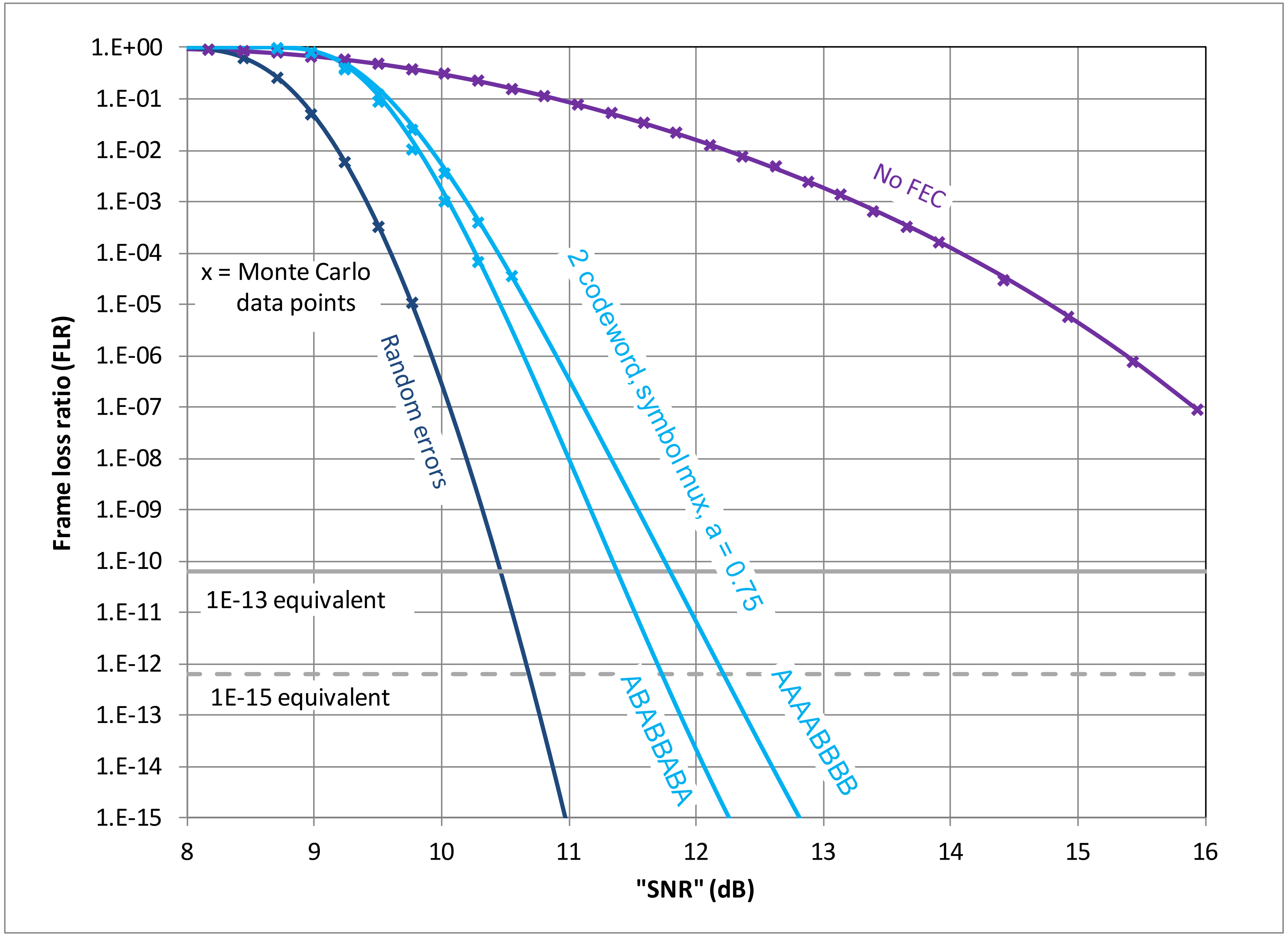
# Clause 119 400G with symbol mux PMA

Symbol interleave from 2 FEC codewords. Symbol multiplex in the PMA.




If one codeword is uncorrectable, the other is marked bad also.

# Clause 119 400G with symbol mux PMA

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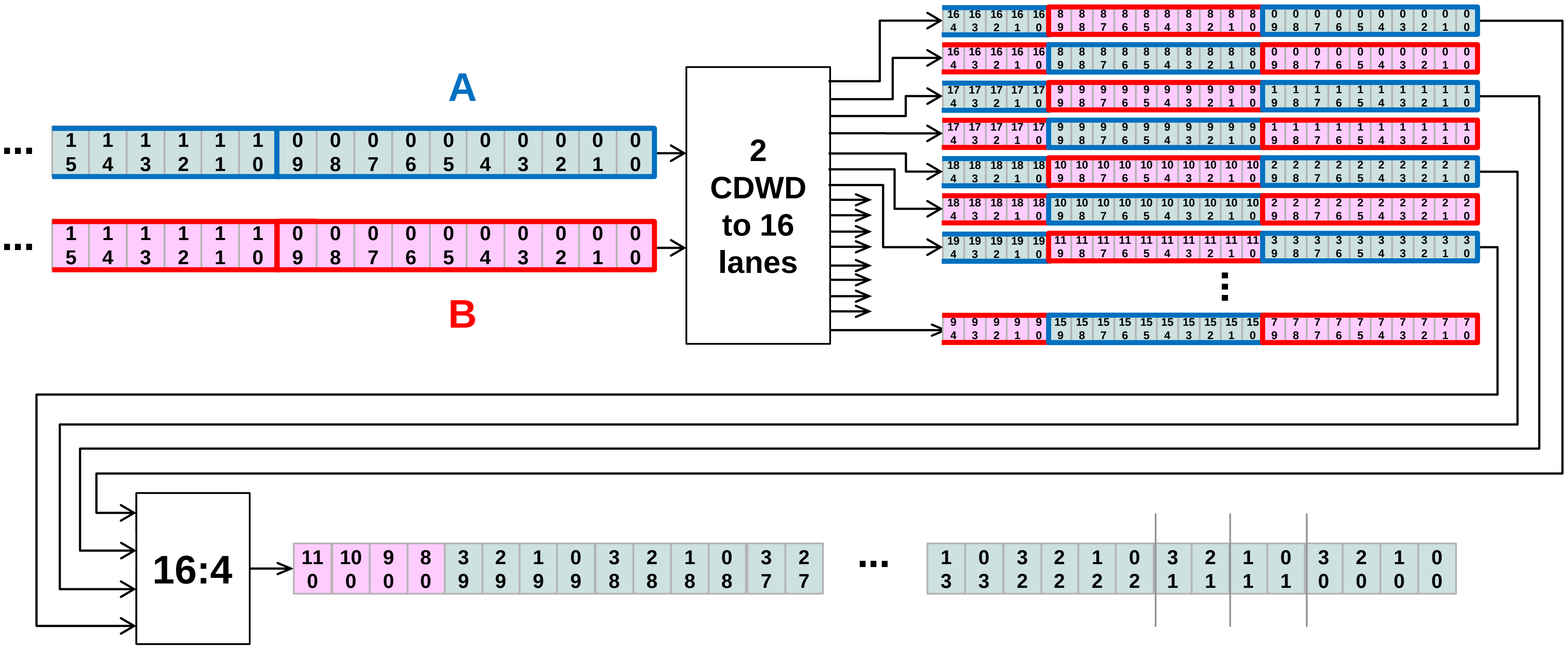
# Clause 119 400G with bit mux PMA

Symbol interleave from 2 FEC codewords. Bit multiplex in the PMA.




If one codeword is uncorrectable, the other is marked bad also.

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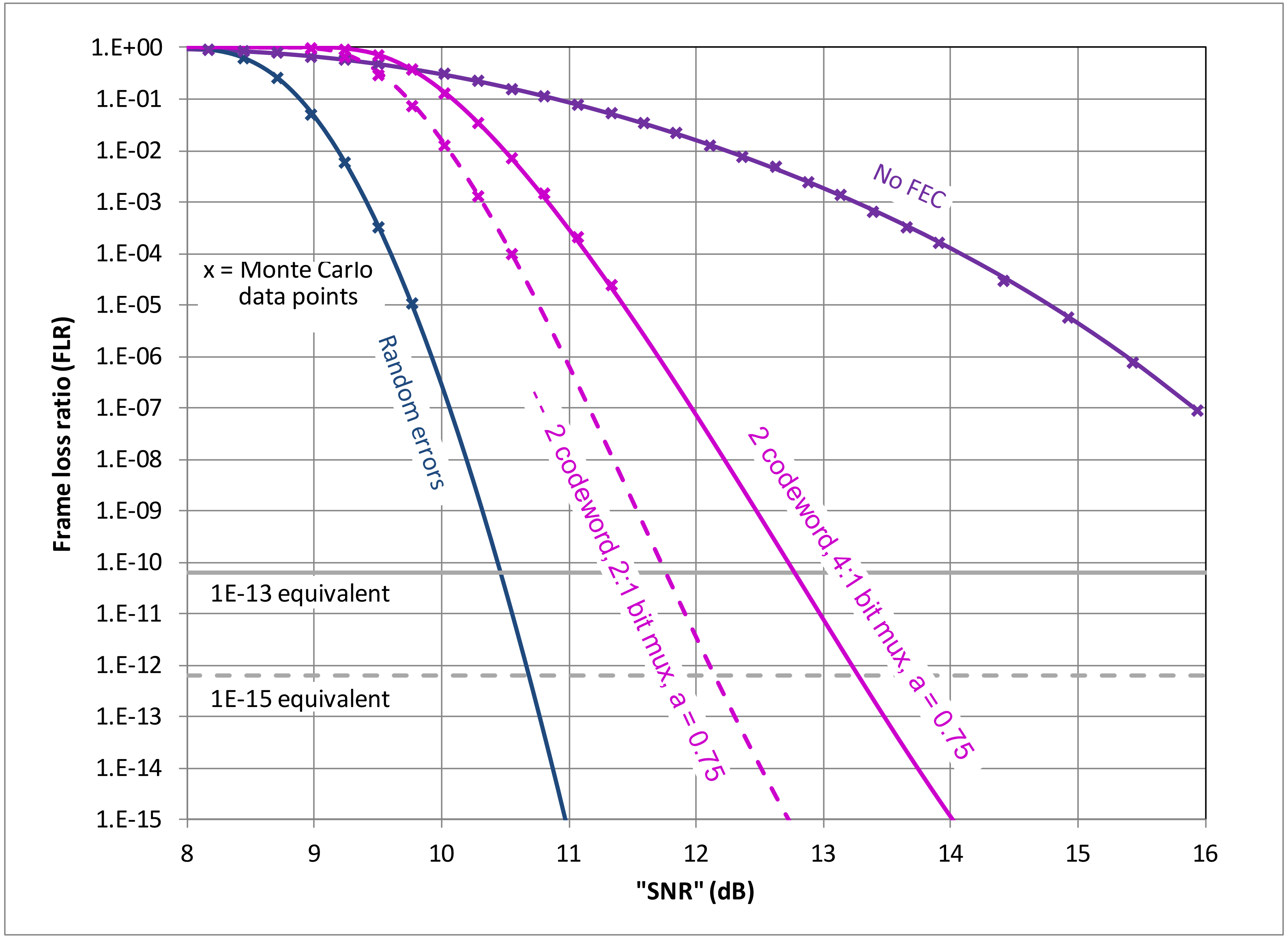
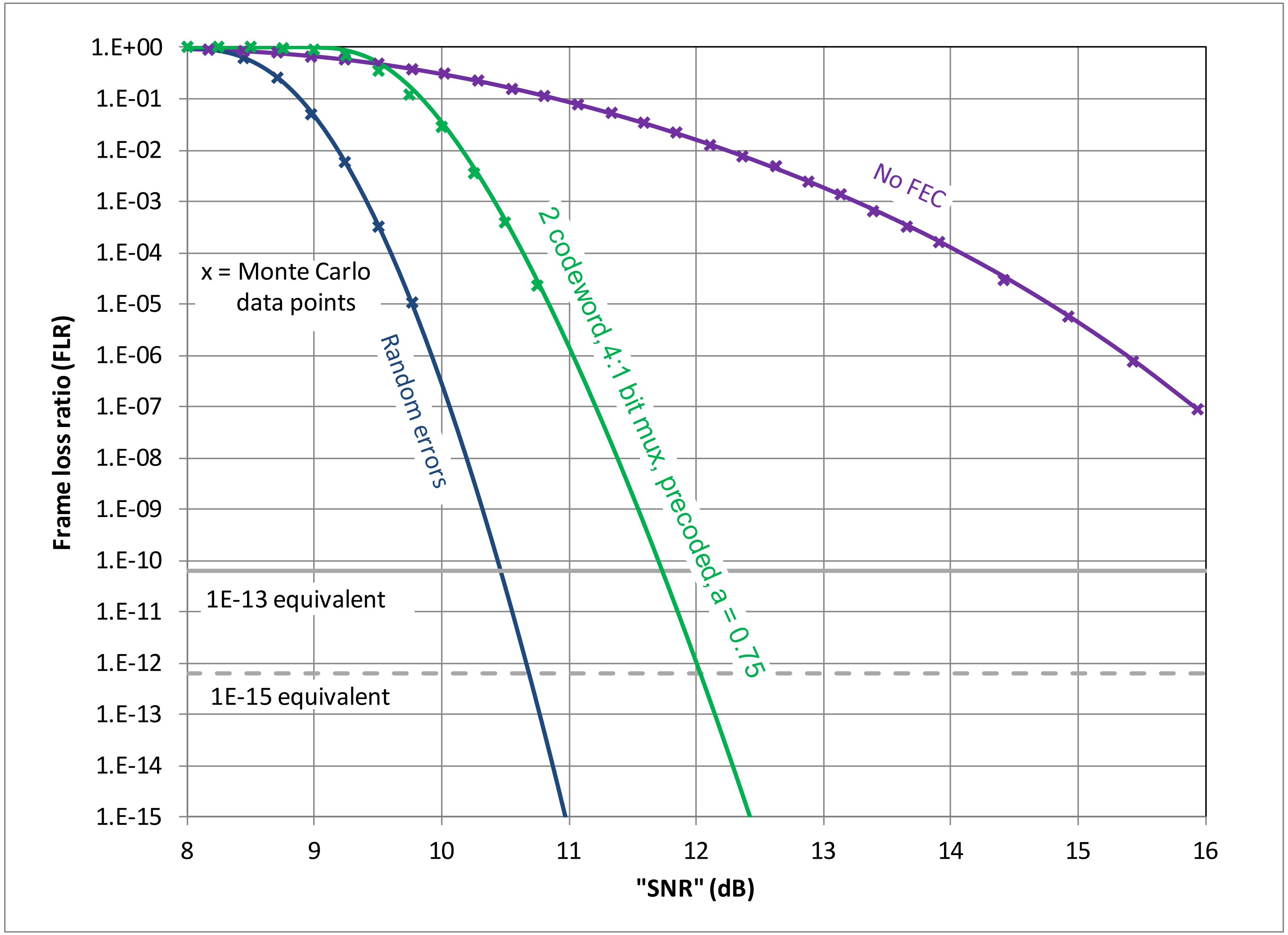
# Clause 119 400G with bit mux PMA



# Clause 119 400G with bit mux PMA and pre-coding

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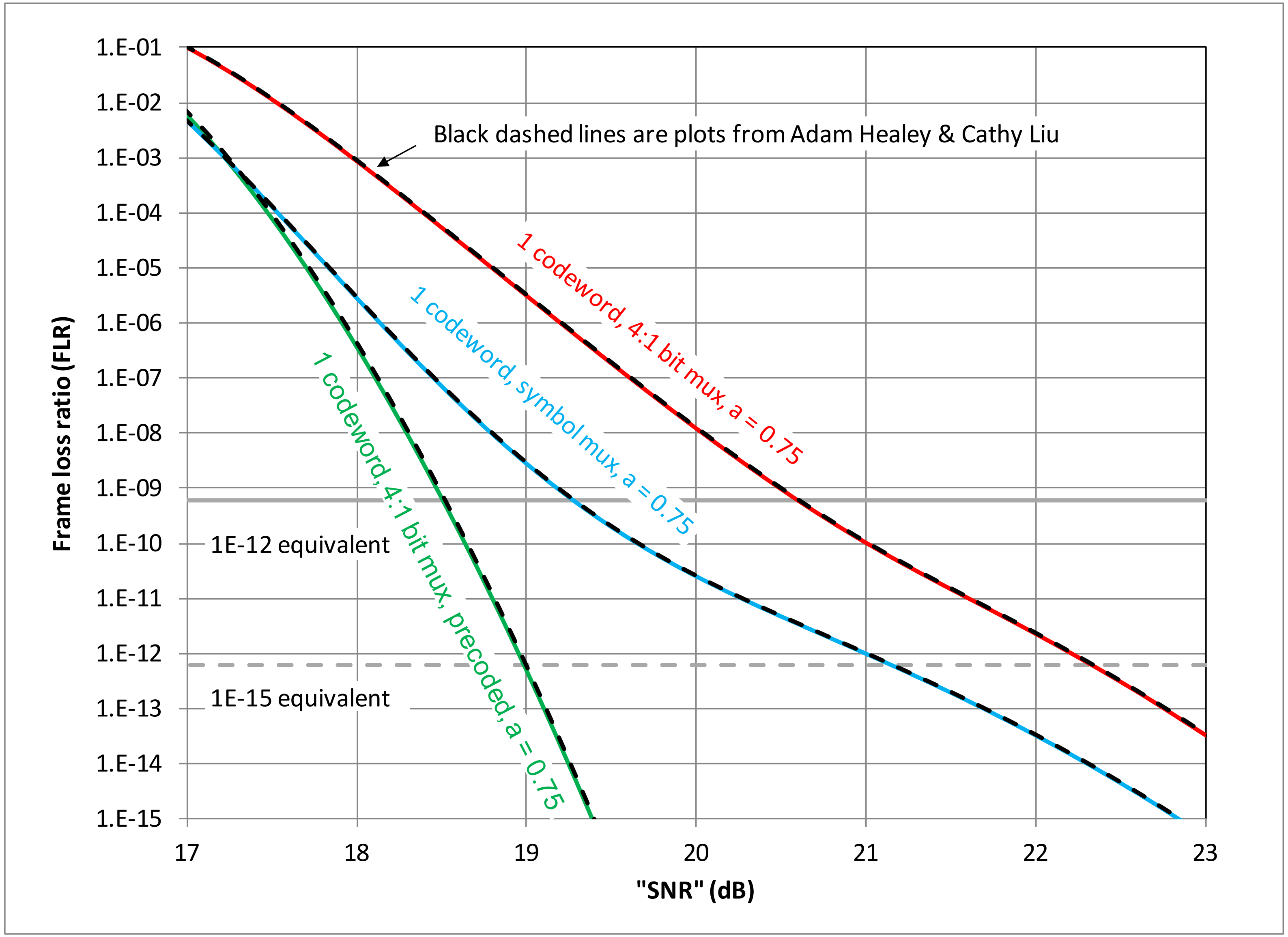
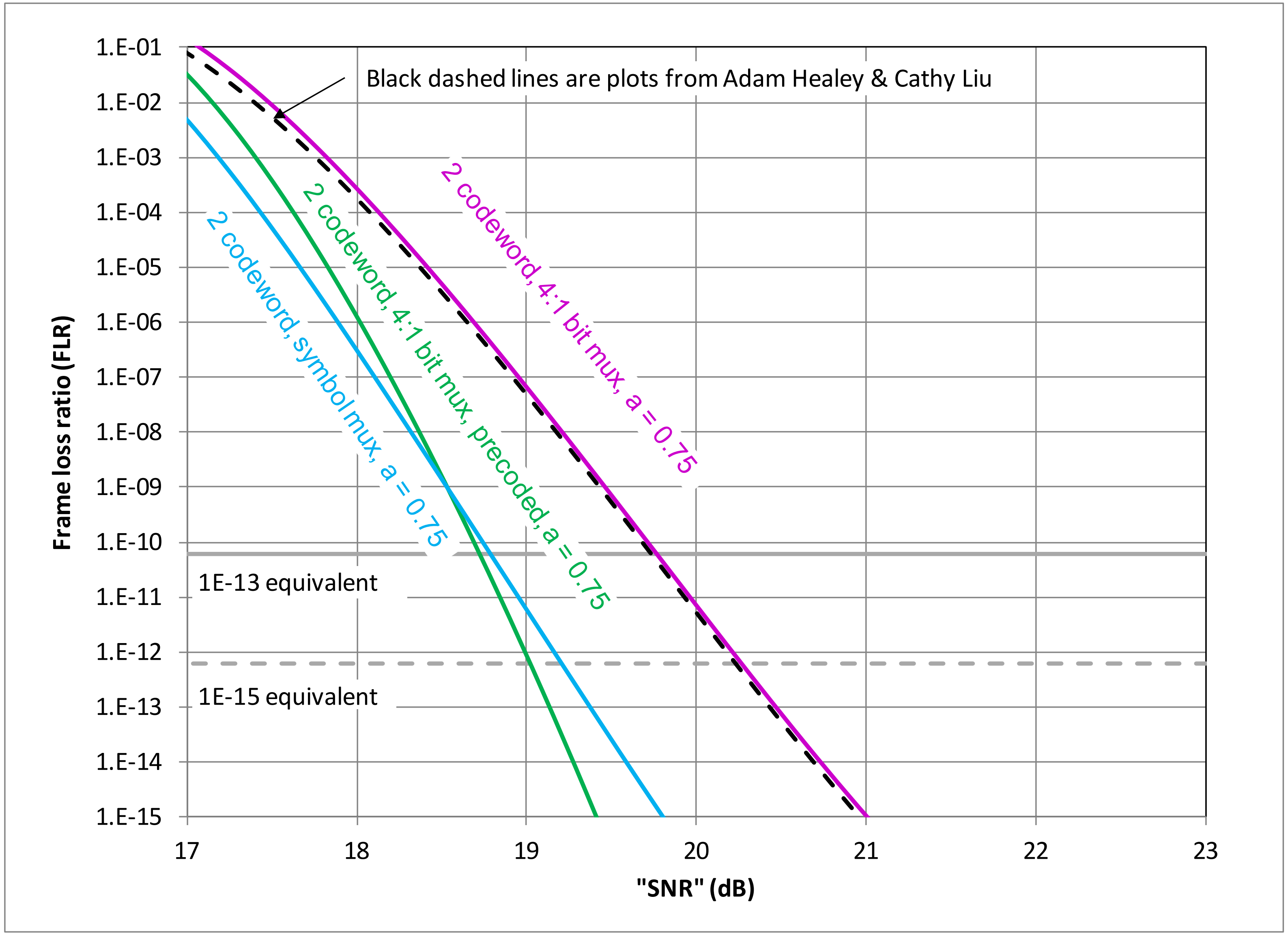
# Comparison with healey_100GEL_01_0318

# 100G curves re-plotted on same axes as healey_100GEL_01_0318



# 400G curves re-plotted on same axes as healey_100GEL_01_0318



Thanks!