Optical Link Model for PAM-4 Multimode Channels with equalizers

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Motivation for a Optical Link Model

- Provide a methodology to estimate reaches and penalties for:
  - Multimode fiber links using VCSEL based transceivers using PAM-4
    - 50GBASE-SR, 100GBASE-SR2, 200GBASE-SR4, 400GBASE-SR8
    - 400GBASE-SR4.2, BiDi, extended reaches transceivers and others
    - Operation at 850 nm or longer wavelengths
    - Restricted to OM3, OM4 and OM5
- Development of an open portable tool in Excel or other software applications
    - Tool to support and facilitate the development of PAM-4 PMDs

Objectives

- Initiate the development of the new MMF link model
  - Agreement to provide a link model during the last IEEE Plenary meeting
    - Continuation of work presented in (Castro_NGMMF_01_0318.pdf)
- To develop a model for worst-case links
  - The equations and algorithms should provide penalties worse than the penalties that could be obtained by a full numerical simulation of a link with identical parameters
  - The optical parameters i.e. laser, receiver, & fiber parameters, will be selected to represented on average worst-case compliant transceivers and fibers
    - The optical parameters used here only serve as an illustration of the model functionalities
- The model should be accurate but simple enough to be implemented in a spreadsheet with Visual Basic (VBA) with/without dynamic link libraries (DLL)
  - Open: source code can be inspected
  - Portable and accepted by experts in the field (optical-link modeling)
- This presentation focuses on describing the algorithms and equations for:
  - Intersymbol interference (ISI) and jitter penalties
  - Relative Intensity Noise (RIN)
  - Equalizer and Noise Enhancement
  - Modal Noise, eye skew, and other penalties will be shown in posterior presentations

Outline

- General Remarks
  - Discuss 3 approaches for link models: worst-case-eye opening, statistical eye amplitude (proposed), and numerical simulation
- Description of proposed link model
  - Description of key components of the model
    - Algorithm, basic equation and nomenclature for Gaussian channel.
- Analytical solutions for the 5-tap Equalizer
  - FFE taps and Noise Enhancement Factors (NEF) for RIN and AWGN derived.
- ISI & Jitter eye reductions
  - Eye amplitude and eye opening due to bandwidth limitations and jitter. The bandwidth limitations caused by laser, fiber, and detector
- RIN
  - Effect of multi-level signaling on noise
- MPN
  - Effect of multi-level signaling on noise
- MN
- Baseline Wander
- Penalties & Margins

Outline (continuation)

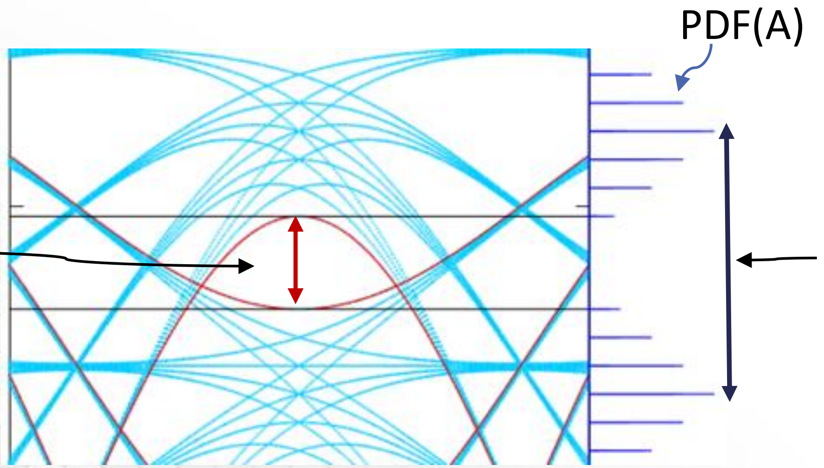
- Comparison of proposed methods
  - Comparison among worst-case-eye opening, statistical eye amplitude and numerical simulation
- Summary & Conclusions
- References
- Annexes
  - Annex I Link Model Algorithm
    - Putting all together.
      - General Flow
    - Input/output Parameters per block
  - Annex II
    - Examples to evaluate accuracy of Statistical Eye Amplitude method for estimation of PDF(A)
  - Annex III
    - Table: Summary of Equations
  - Annex IV
    - Derivation of SER, BER based on Penalties and Margin

General Remarks

General Remarks

- Development of a reasonable worst-case link model
  - Previous link models compute penalties based on worst-case eye opening (figure below)
    - Good estimator for non-equalized PAM-2, but too pessimistic for equalized PAM-4
  - We propose a different method for equalized PAM-4 links
    - Use parameters from the probability density function of the received signal {PDF(A)} based on ISI and jitter and the statistical eye amplitude method
  - In this presentation we describe and compare the 3 approaches:
    - Worst-case eye opening
    - Statistical eye amplitude
    - Numerical simulation
  - We propose to use the statistical worst-case eye opening for PAM-4 equalized optical link

Current approach "Worst-Case eye opening"

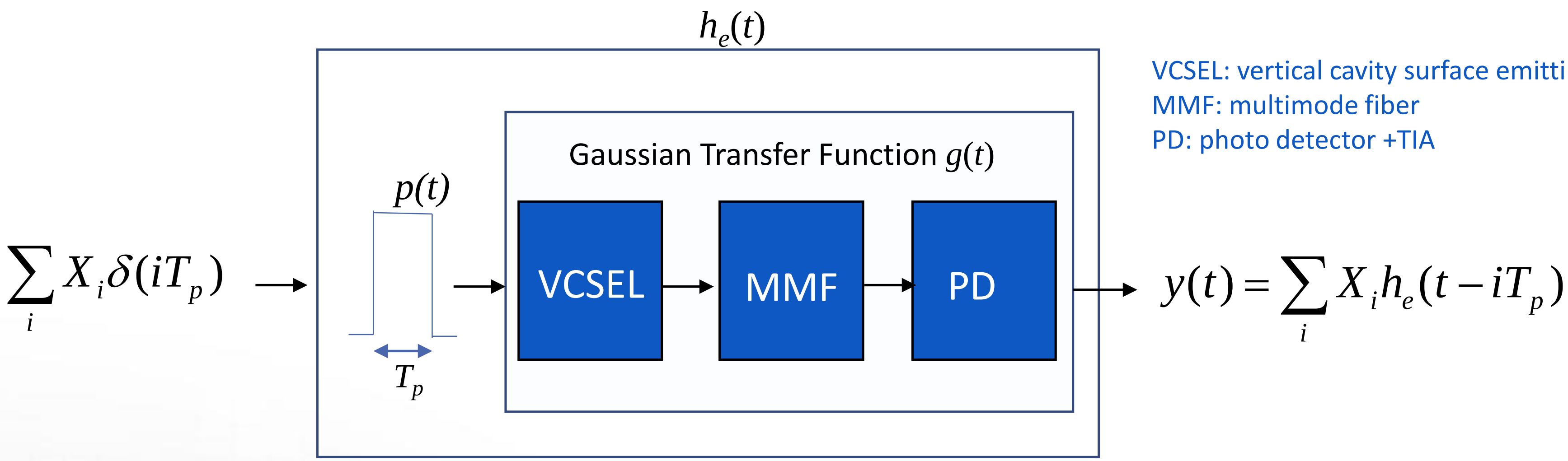


Proposed "**statistical eye amplitude**" method: the information from the eye amplitude PDF is included in the evaluation to provide a reasonable worst-case link

Link Model Description

Link Model Description

- The optical link model assumes a Gaussian transfer function.



VCSEL: vertical cavity surface emitting laser
MMF: multimode fiber
PD: photo detector +TIA

- *X_i represents the symbol sequence, T_p, the symbol duration (after DCD correction), $p(t)$, the pulse function, $g(t)$ the gaussian pulse response, $h_e(t)$ is the system impulse response, given by,*

$$h_e(t) = p(t) \otimes g(t) = 0.5[\mathrm{erf}(k\frac{[2t + T_p]}{T_C}) + \mathrm{erf}(k\frac{[-2t + T_p]}{T_C})] \qquad \text{(A.1)}$$

- *where $\mathrm{erf}(t)$ is the error function, k = 0.9062 is a scaling constant.*

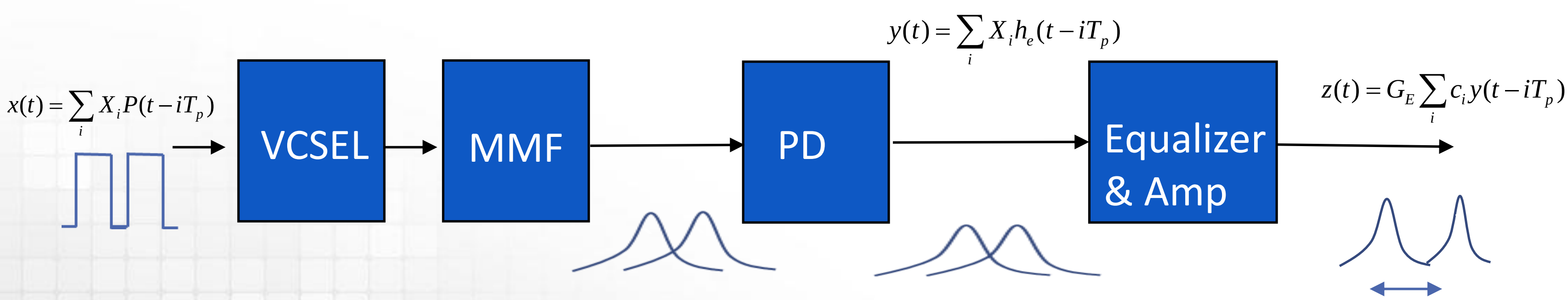
Link Model Description

- T_C is the 10-90% overall system rise time, which comprises the laser (T_S), fiber, and the photo-receiver (T_R), given by

$$T_C = \sqrt{T_e^{\,2} + T_R^{\,2}} \qquad \text{(A.2)}$$

where, $T_e = \sqrt{T_S^{\,2} + (480\ 10^3)^2 (BW_M^{\ -2} + BW_{Ch}^{\ -2}\ )}$ is the 10-90% rise time component attributed to laser and fiber

- Equalizer type and assumed location



- Impulse response including equalizer with 5 taps (symmetric) described in next section

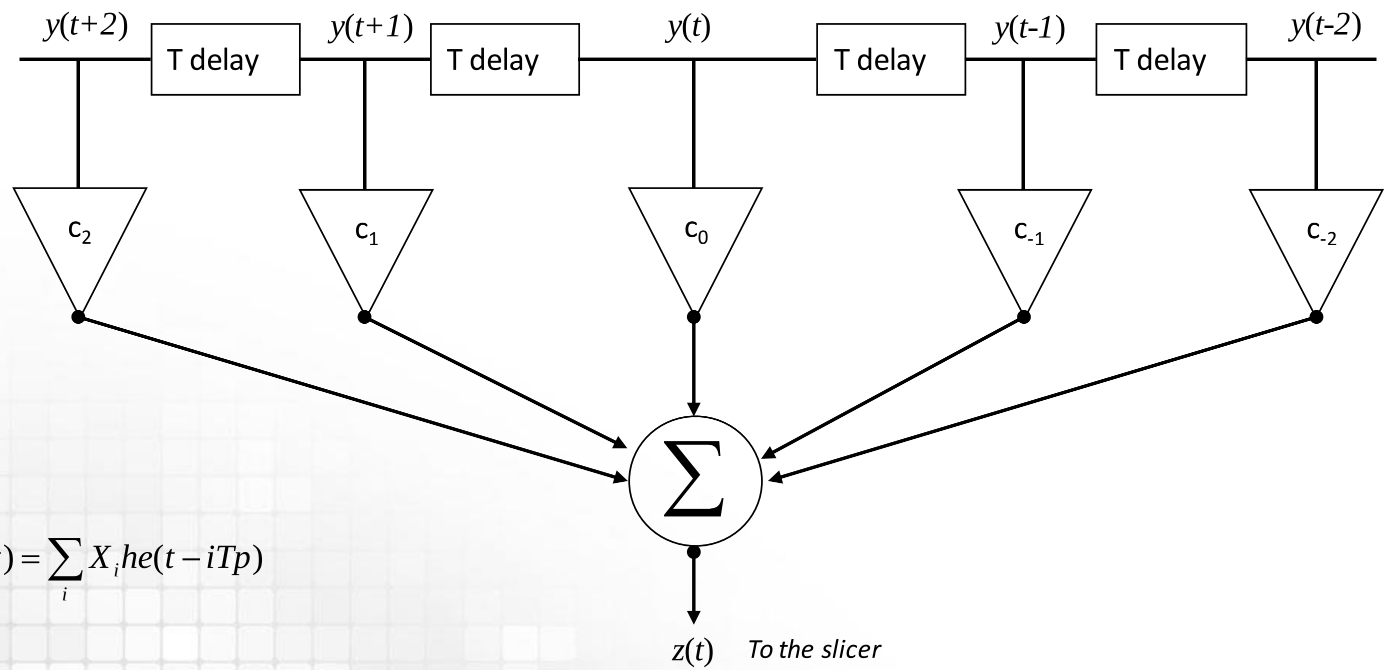
$$h_f(t) = \sum_{i=-2}^{2} c_i h_e(t - iT_p) \qquad \text{(A.3)}$$

Analytical Solution for a 5-Tap Equalizer (FFE)

T-spaced equalizer with 5 Taps

- The impulse response of the equalized channel is given by (A.3), where
- The equalizer structure is given below, and the analytical derivation of the tap weights is shown in the next slides

T-spaced equalizer with 5 Taps



$$y(t) = \sum_i X_i he(t - iTp)$$

T-spaced equalizer with 5 Taps

- The tap weights are computed using mean minimum square error (MMSE)method:

$$\begin{vmatrix} c_{-2} \\ c_{-1} \\ c_0 \\ c_1 \\ c_2 \end{vmatrix} = [H^T H + (\frac{n_{total}}{s})^2 I]^{-1} \begin{vmatrix} h_{-2} \\ h_{-1} \\ h_0 \\ h_1 \\ h_2 \end{vmatrix} \quad \text{(B.1)}$$

Where, I is the identity or unit matrix, n_{total} is the total noise of the channel, and H is the matrix given by

$$\begin{aligned} h_0 &= 1 \\ h_1 &= h_{-1} = h_e(T_p)/h_e(0) \\ h_2 &= h_{-2} = h_e(2T_p)/h_e(0) \\ &\cdots \\ h_n &= h_{-n} = h_e(nT_p)/h_e(0) \end{aligned} \quad \text{(B.2)}$$

$$H^T = \begin{bmatrix} h_{-2} & h_{-1} & h_0 & h_1 & h_2 & 0 & 0 & 0 & 0 \\ 0 & h_{-2} & h_{-1} & h_0 & h_1 & h_2 & 0 & 0 & 0 \\ 0 & 0 & h_{-2} & h_{-1} & h_0 & h_1 & h_2 & 0 & 0 \\ 0 & 0 & 0 & h_{-2} & h_{-1} & h_0 & h_1 & h_2 & 0 \\ 0 & 0 & 0 & 0 & h_{-2} & h_{-1} & h_0 & h_1 & h_2 \end{bmatrix} \quad \text{(B.3)}$$

T-spaced equalizer with 5 Taps

- The tap weights are given by *:*

$$c_0 = 1 \tag{B.4}$$

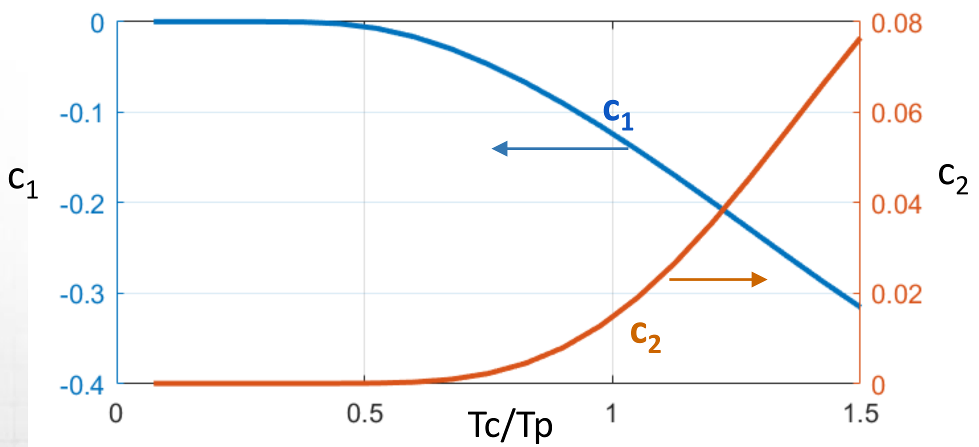
$$c_1 = c_{-1} = \frac{h_1(S^2 + 4Sh_1^2 + Sh_2^2 - 4Sh_2 + 2h_1^4 + 6h_1^2h_2^2 - 8h_1^2h_2 + 2h_1^2 - 2h_2^4 - 2h_2^3 + 3h_2^2 - 1)}{S^2 - 6Sh_1^2h_2 + Sh_1^2 + Sh_2^2 + 2Sh_2 + 2S - 6h_1^4h_2 + 2h_1^4 + 9h_1^2h_2^2 - 2h_1^2h_2 - 3h_1^2 - 2h_2^4 - 2h_2^3 + h_2^2 + 2h_2 + 1} \tag{B.5}$$

$$c_2 = c_{-2} = \frac{S^2h_2 + Sh_1^2h_2 - 3Sh_1^2 + 4Sh_2^3 + 2Sh_2^2 + 2h_1^4h_2 - 3h_1^4 - 6h_1^2h_2^3 - 2h_1^2h_2^2 + 5h_1^2h_2 + h_1^2 + 4h_2^5 + 4h_2^4 - 2h_2^2 - h_2}{S^2 - 6Sh_1^2h_2 + Sh_1^2 + Sh_2^2 + 2Sh_2 + 2S - 6h_1^4h_2 + 2h_1^4 + 9h_1^2h_2^2 - 2h_1^2h_2 - 3h_1^2 - 2h_2^4 - 2h_2^3 + h_2^2 + 2h_2 + 1} \tag{B.6}$$

where,

$$S = \left(\frac{n_{total}}{h_e(0)s'}\right)^2 = \left(\frac{1}{Qh_e(0)P^{alloc/10}}\right)^2 \tag{B.7}$$

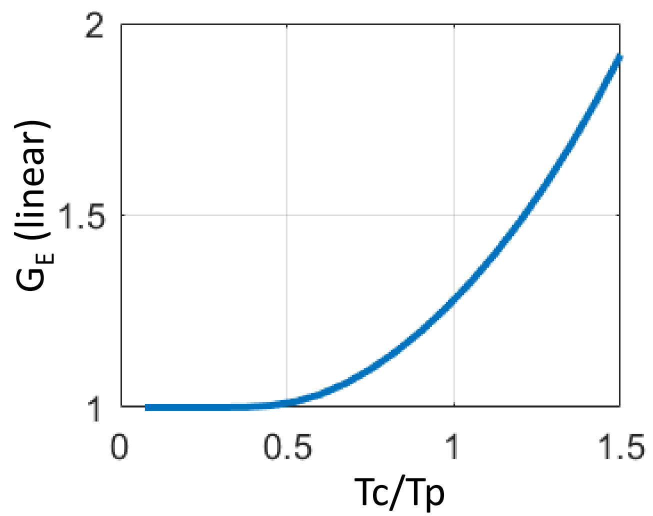
Where Q is the targeted Q factor



Equalizer Gain and Noise Enhanced Factor for White Noise

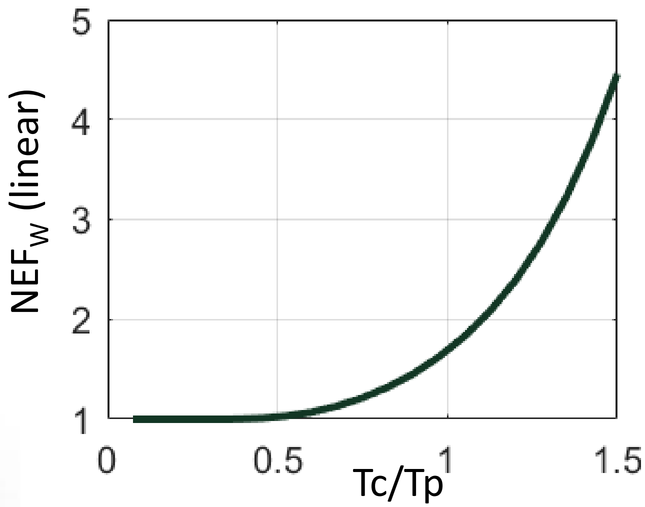
- The equalizer gain is given by***:***
  - *For this example, it is assumed Q=3.6972, P_alloc=$10^{0.5}$ (or 5 dB)*

$$G_E = \frac{1}{\sum_{i=-2}^{2} c_i} \qquad \text{(B.8)}$$



- The Noise Enhance factor for white noise (AWGN) is given by
  - *Assumed noise after the slice is AWGN*

$$NEF_W = G_E^{\,2} \sum_{i=-2}^{2} c_i^{\,2} \qquad \text{(B.9)}$$



**See more on this topic, including equations, in Fibre Channel MSQS2 (references)*

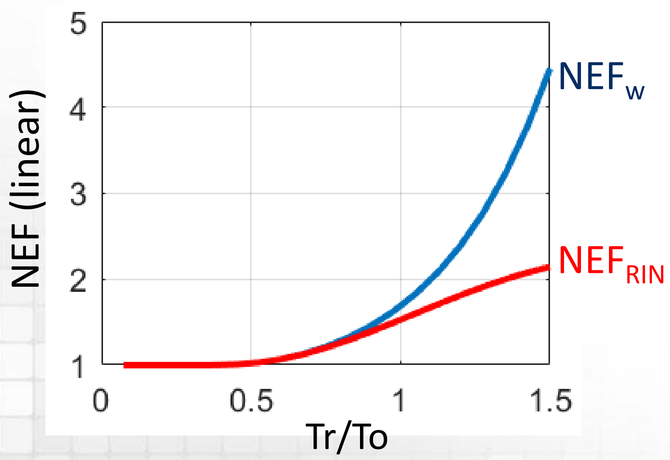
Noise Enhanced Factor for Relative Intensity Noise

- The equalizer gain is given by:
  - *Similar assumptions for Q and power budget allocation than previous slide*

$$NEF_{RIN} = G_E{}^2 \{ \sum_{i=-2}^{2} c_i^2 \int_{-\infty}^{\infty} e^{-4\pi^2 f^2 (\frac{Tc}{2.563})^2} df + \sum_{i=-2}^{2} \sum_{k=-2, k\neq i}^{2} c_i c_k \int_{-\infty}^{\infty} e^{-j(i-k)2\pi f T_P} e^{-4\pi^2 f^2 (\frac{Tc}{2.563})^2} df \} \quad \text{(B.10)}$$

- Which can be solved analytically, producing,

$$NEF_{RIN} = NEF_W + 4(c_1 c_2 + c_1) e^{-(\frac{1.2816 T_p}{T_c})^2} + (4c_2 + 2c_1^2) e^{-(2\frac{1.2816 T_p}{T_c})^2} + 4c_2 c_1 e^{-(3\frac{1.2816 T_p}{T_c})^2} + 2c_2^2 e^{-(4\frac{1.2816 T_p}{T_c})^2} \} \quad \text{(B.11)}$$

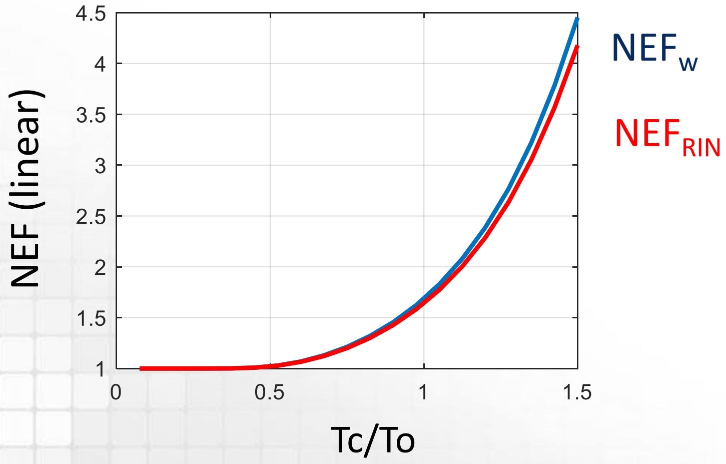


Noise Enhanced Factor for Relative Intensity Noise

- Previous results are in agreement with MSQS2 for the 3 taps case
- Results indicates that RIN NEF are lower than the white noise NEF
    - This occurs since we assume that the RIN is filtered by the composite effect of fiber, laser, and receiver. See transfer function (square) term in C.19

$$e^{-4\pi^2 f^2\left(\frac{Tc}{2.563}\right)^2}$$

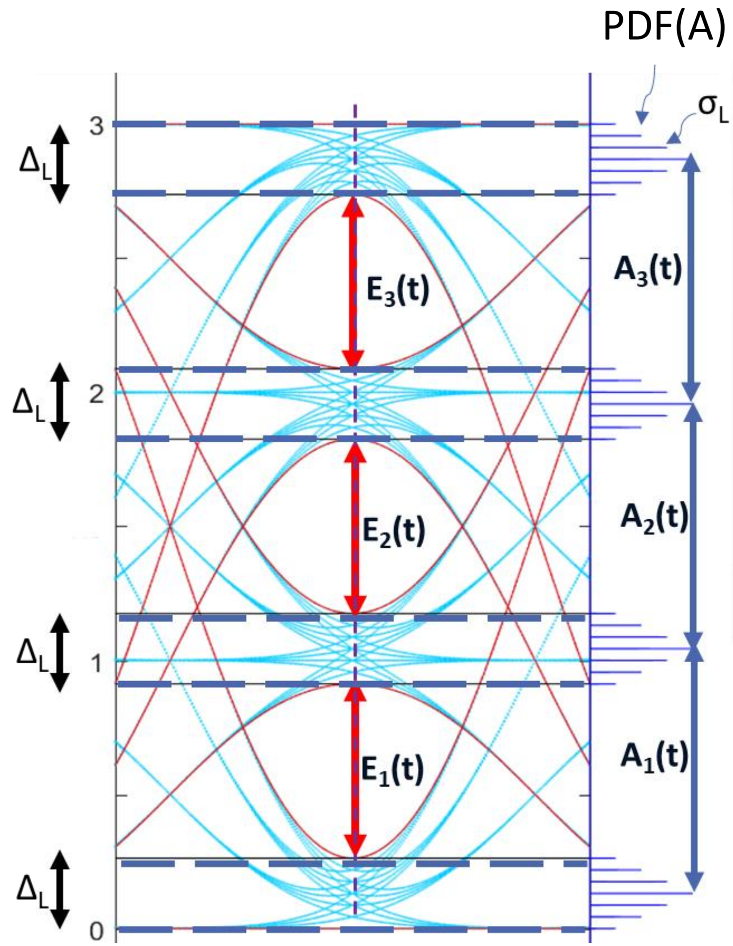
    - In our model the effect of fiber and laser on the reduction of the pulse are already included by the term ρ_{Te} (B.1) .
    - Therefore, the transfer function should include only the receiver. By interchanging the term T_C *with* T_P we obtain a worst case NEF for the RIN, which is now very similar to the NEF for white noise



ISI & Jitter

Eye Parameters used to estimate ISI and Jitter

- Ideally, an accurate way to estimate the link performance is to simulate the complete link using a very large test pattern or to use known channel characteristics to estimate statistics of the received signals.
    - Probability density functions (PDF) of the eye at the positions defined by sampling and jitter without noise could be obtained. See for example PDF(A) in the figure, where A_i represents the eye amplitude for the 3 eye and i is the index (i.e. i=3 represents the top eye).
    - The noise can be convolved to the obtained PDF, and the performance of the link can be estimated
    - However, an Excel spreadsheet could require excessive time to run this model or make necessary to use DLLs.
- Previous link models have used the <u>worst-case eye opening method</u> to provided a good estimate of the worst-case links with modest complexity and computational requirements.
    - The worst-case eye opening method, uses, $E_i(t)$ (shown in the Figure), to estimate ISI and Jitter penalties, where $t=JT_p$ accounts for the jitter and J=effective jitter.



Eye Parameters used to estimate ISI and Jitter

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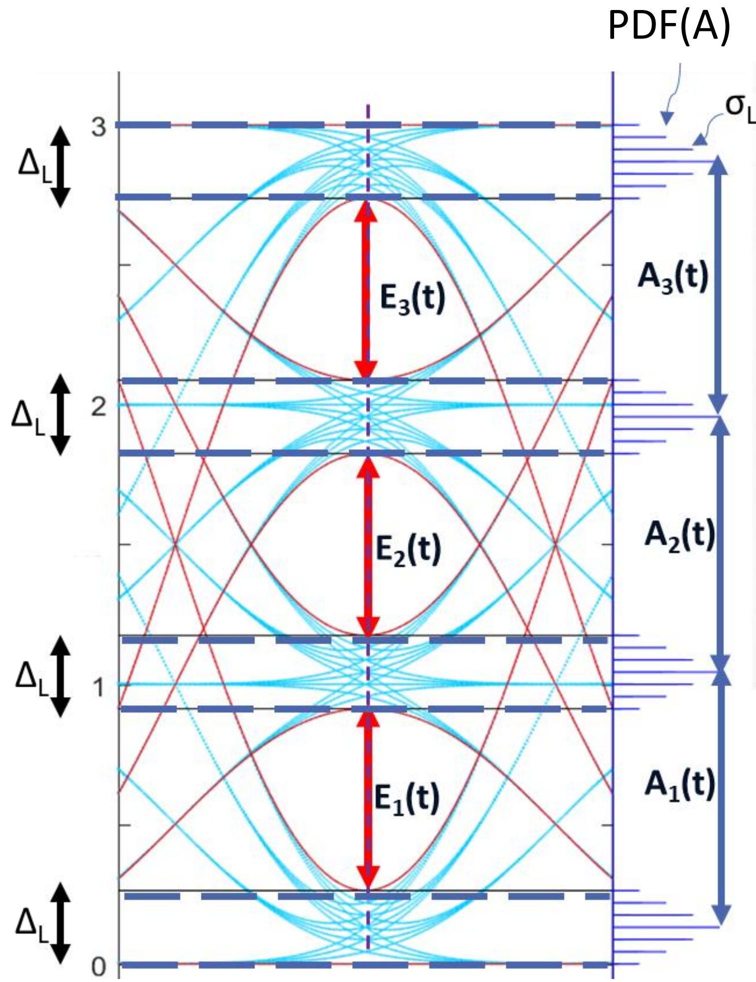
- However, for equalized PAM-4, the worst-case eye opening method significantly penalizes the link by placing all the noise in the worst case traces of the eye (red traces). This produces over pessimistic results for reaches and BER.
- For equalized PAM-4, it will be shown that the <u>statistical eye amplitude</u> method, is a better estimator than the <u>worst-case eye opening</u> method.
    - The statistical eye amplitude method tends to match the numerical simulation with less computational requirements using previous link model theory and a new set of analytical expressions.
    - The statistical eye amplitude method is <u>not</u> any better than the eye closure method for non-equalized PAM-2. More on this in subsequent slides.
- The statistical eye amplitude method uses information of the eye diagram traces such as level thickness Δ_L , mean and standard deviation σ_L to estimate ISI and Jitter, and signal dependent noise penalties
    - The eye amplitude of the Gaussian channel $A_i(t)$, and variations can be obtained with relative simple equations.
    - It becomes clear that since the eye amplitude is larger than the eye opening (Ai>Ei) this method is less pessimistic.
    - However, the opening of the eye is compensated by the increase of signal dependent noise due to σ_L.

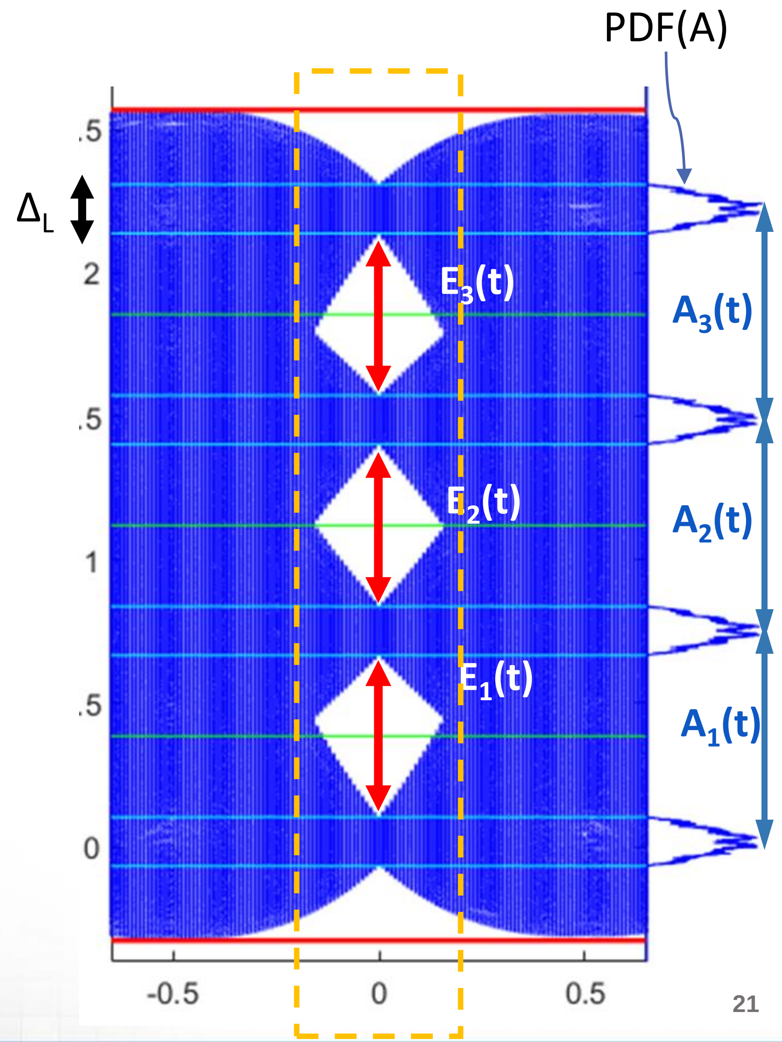


ISI and Jitter Penalties for Equalized Link using worst-case-eye opening method

- The vertical Eye opening is computed using:

$$\Delta_L = (\mathrm{M}-1)\sum_{k=1}^{7}\left|\mathrm{h}_f(0.5\mathrm{JT}_p+\mathrm{kT}_p)\right| + \left|\mathrm{h}_f(-0.5\mathrm{JT}_p+\mathrm{kT}_p)\right| \quad \text{(C.1)}$$

$$\sigma_l(JT_p) = \frac{\sqrt{5}}{2}\sqrt{\sum_{i=1}^{3} h_f(0.5J\,T_p+iT_p)^2 + h_f(-0.5J\,T_p+iT_p)^2} \quad \text{(C.2)}$$



- Eye opening is computed using:

$$E(JTp) = \mathrm{h}_f(0.5\mathrm{JT}_p) - \Delta_L \quad \text{(C.3)}$$

- The ISI and jitter penalties are computed using:

$$P_{ISI}^{\ i} = -10\log_{10}[E(JT_p)] \quad \text{(C.4)}$$

$$P_{Jitter}^{\ i} = -10\log_{10}[E(JT_p)] - P_{ISI}^{\ i} \quad \text{(C.5)}$$

ISI and Jitter Penalties for Equalized Link using the statistical-eye amplitude method

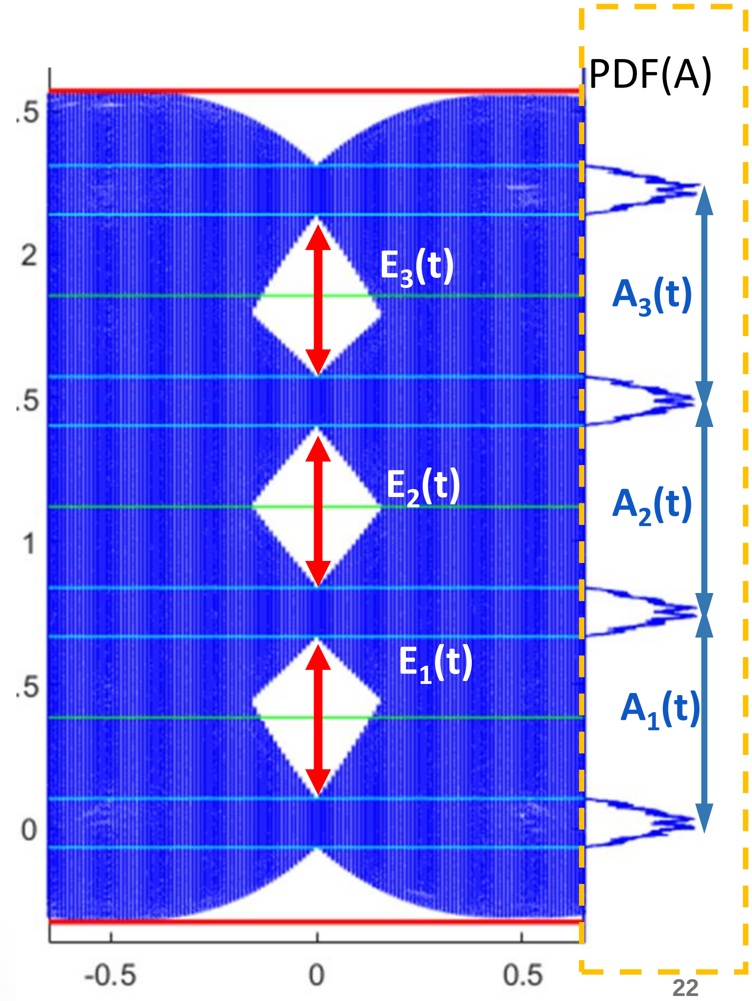
- Eye amplitude is computed using:

$$A(JTp) = h_f(0.5JT_p) \qquad \text{(C.6)}$$

- The ISI and jitter penalties are computed using:

$$P_{ISI}^{\,i} = -10\log_{10}[A(JT_p)] \qquad \text{(C.7)}$$

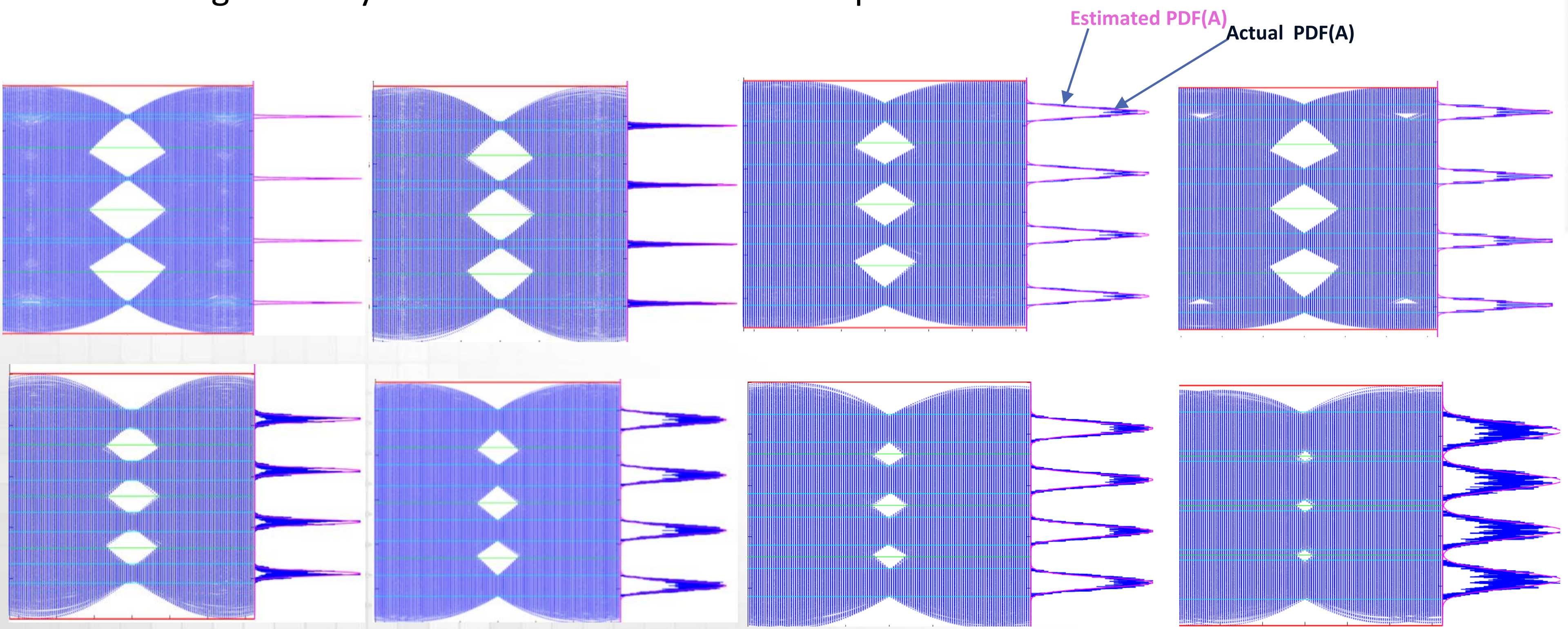
$$P_{Jitter}^{\,i} = -10\log_{10}[A(JT_p)] - P_{ISI}^{\,i} \qquad \text{(C.8)}$$



For non equalized channels previous equations can be simplified as shown in Journal of Lightwave Tech. Vol 34 (16), April 2016

Verification of the Statistical Eye Amplitude Method

- The numerical PDF(A) with the Gaussian approximation was compared for a broad range of ISI and jitter conditions (Details and traces in log scale in Annex II).
  - The estimation of PDF(A) (magenta traces) was very close to the envelope of the numerical simulated PDF(A).
  - Results (details of conditions in Annex II) show that the statistical amplitude method is a significantly better estimator for PAM-4 equalized links.



RIN_{OMA}

RIN_{OMA}

- IEEE 802.3ae (clause 52.9.6) defines RIN_{OMA} as the ratio of the electrically noise power normalized to a 1 Hz bandwidth and the electrical power of a square wave modulation
- The RIN_{OMA} (outer) can be measured using a scope and a square wave pattern (i.e., 8 zeros and 8 threes as indicated in 802.3bs Table 139-9). The equations to relate the measured RIN parameters to our PAM-4 equalized optical link will be shown in this section.
  - The normalized variance due to RIN can be obtained using

$$\sigma_{RIN-OMA}{}^{2} = \frac{n_0{}^2 + n_3{}^2}{2(3\Delta/2)^2 \rho_{Te}{}^2} = \frac{\mathrm{K}_{RIN} 10^{-RIN/10} 10^6}{\sqrt{0.477/BW_{RX}{}^2)}} \qquad \text{(D.1)}$$

  - Where 3Δ is the optical modulation amplitude without any dispersion penalties, K_{RIN}=0.7, BW_{Rx} is the bandwidth of the receiver, and $\rho_{Te} = A_i(JT_p)$ is the reduction in the optical signal attributed only to fiber and laser (T_e).
  - Note that previous link models use the composed bandwidth (laser, fiber and receiver)
  - We consider it better to separate the effect of the optical pulse dispersion from the electrical
    - Before the PD the variations on optical signal power due to dispersion reduces the noise (proportionality assumed). The reduction is proportional to the term ρ_{Te} which only accounts for the laser and fiber.
    - The PD bandwidth BW_{RX} also limits the power of the noise (higher spectral components attenuated).

RIN_{OMA}

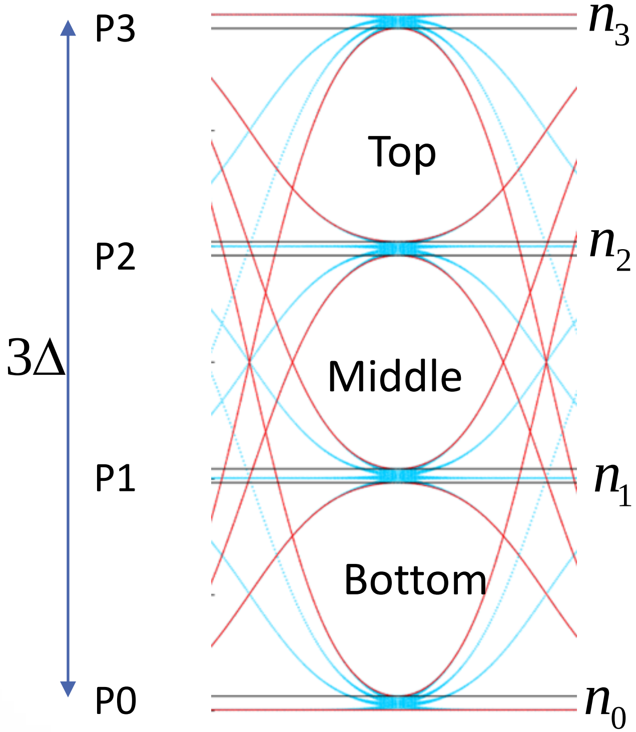
- Assuming the laser operates in a regime where RIN variance scales with the square of the optical power, the terms P_0, P_1, P_2, P_3, which represent the optical power of the levels can be obtained as a function of the VCSEL extinction ratio (ER).

$$\Delta = \frac{P_3 - P_0}{3} = P_0 \frac{ER-1}{3} \qquad \text{(D.2)}$$

$$P_1 = P_0 + \Delta = P_0 \frac{ER+2}{3}, \qquad \text{(D.3)}$$

$$P_2 = P_0 + 2\Delta = P_0 \frac{2ER+1}{3}, \qquad \text{(D.4)}$$

$$P_3 = ER\ P_0 \qquad \text{(D.5)}$$



RIN$_{OMA}$

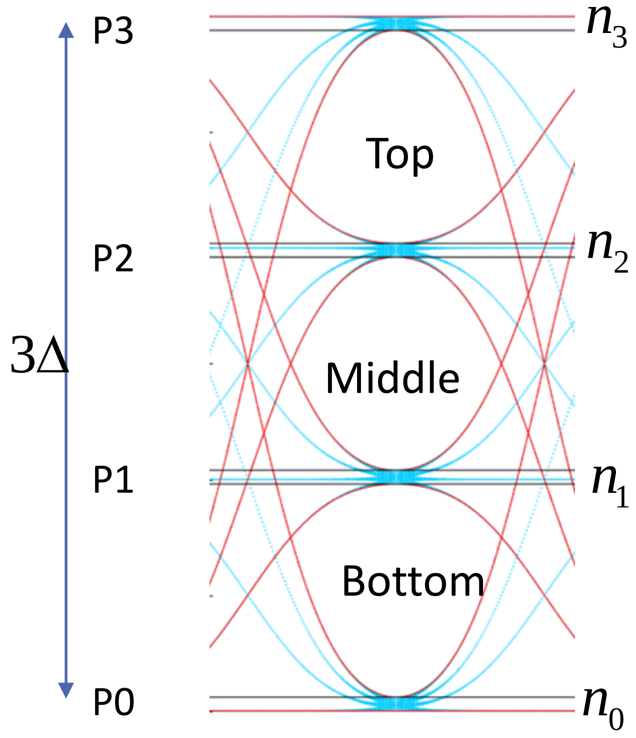
- The RIN variances (no-normalized) for each signal level are derived.

$$n_0^2 = \rho_{Te}^2 \frac{2(3\Delta/2)^2}{1+ER^2} \sigma_{RIN-OMA}^2 \quad \text{(D.6)}$$

$$n_1^2 = \rho_{Te}^2 2(3\Delta/2)^2 \frac{(ER+2)^2}{9(1+ER^2)} \sigma_{RIN-OMA}^2 \quad \text{(D.7)}$$

$$n_2^2 = \rho_{Te}^2 2(3\Delta/2)^2 \frac{(2ER+1)^2}{9(1+ER^2)} \sigma_{RIN-OMA}^2 \quad \text{(D.8)}$$

$$n_3^2 = \rho_{Te}^2 \frac{2(3\Delta/2)^2}{1+ER^2} ER^2 \sigma_{RIN-OMA}^2 \quad \text{(D.9)}$$



RIN$_{OMA}$

- Computation of average RIN relative to each eye amplitude of PAM-4
  - *For the Top eye (assumed worst-case)*

$$\sigma_T^{\ 2} = \frac{n_2^{\ 2} + n_3^{\ 2}}{2(\Delta/2)^2 \rho_{Te}} = \frac{1}{2(\Delta/2)^2}\{\frac{2(3\Delta/2)^2}{1+ER^2}[ER^2 + \frac{(2ER+1)^2}{9}]\sigma_{RIN-OMA}^{\ 2}\} \quad \text{(D.10)}$$

$$\sigma_T^{\ 2} = \frac{9ER^2 + (2ER+1)^2 \sigma_{RIN-OMA}^{\ 2}}{1+ER^2} = \rho_{eye-RIN}^{\ 2}\sigma_{RIN-OMA}^{\ 2} \quad \text{(D.11)}$$

  - *Similarly for the middle and bottom eye*

$$\sigma_M^{\ 2} = \frac{(ER+2)^2 + (2ER+1)^2}{1+ER^2}\sigma_{RIN-OMA}^{\ 2} \quad \text{(D.12)}$$

$$\sigma_B^{\ 2} = \frac{(ER+2)^2 + 9}{1+ER^2}\sigma_{RIN-OMA}^{\ 2} \quad \text{(D.13)}$$

  - *Assuming* $ER = \frac{P_3}{P_0} = 2$ *(worst-case)* $\rho_{eye-RIN}^{\ 2} = 12.2$

$$\boxed{\sigma_T^{\ 2} = 12.2\sigma_{RIN-OMA}^{\ 2}} \quad \text{(D.14)}$$

This is the variance used as worst case

$$\sigma_M^{\ 2} = 8.2\sigma_{RIN-OMA}^{\ 2} \quad \text{(D.15)}$$

$$\sigma_B^{\ 2} = 5\sigma_{RIN-OMA}^{\ 2} \quad \text{(D.16)}$$

RIN_{OMA}

- Based on the stated assumptions, the top eye RIN variance and other signal dependent noises increase significantly, **12.2** times.
  - If we assume ER=2 and RIN variance scale proportional to the square of the optical power square.
    - The noise increases 9 times for a very large ER
- It is open to discussion if the value is too pessimistic and should be reduced in the model
  - The value for the middle eye (~8.2) could be considered
- The effect of the equalizer is included by multiplying the NEF to the noise terms previously derived
  - For example for the assumed worst-case eye (top eye), the equation (D.14) is modified to

$$\sigma_T{}^2 = NEF\,12.2\sigma_{RIN-OMA}{}^2 \qquad \text{(D.17)}$$

RIN_{OMA} Penalty for Worst-case Eye-closure Method

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- For non-stressed channels (short fiber) the Q, modulation amplitude, and noise are related using:

$$Q = \frac{\Delta_{Min}}{2n_W} \qquad \text{(D.18)}$$

where, Δ_{Min} is the minimum optical modulation amplitude required by the receiver using a non stressed pattern to produce a BER (Q).

- After propagation through the link the minimum value Q factor should be maintained

$$(\frac{\Delta}{2})ISI = Q(\sqrt{n_{AWGN}{}^2 + n_{RIN}{}^2} = Q(\sqrt{(\frac{\Delta_{\min}}{2Q})^2 + \sigma_T{}^2(\frac{\Delta}{2}\rho_{Te})^2} \qquad \text{(D.19)}$$

where, Δ is the optical modulation amplitude required to maintain the BER (Q), and ISI is the ISI penalties in linear scale. For the worst-case eye opening method ISI = Ei(JTp).

- Solving $(\Delta/2)^2 ISI^2[1-(\frac{Q\,\sigma_{RIN}}{ISI}\rho_{Te})^2]^2 = (\Delta_{min}/(2))^2$ and using $P_{RIN} = -10\log 10(\frac{\Delta_{\min}}{ISI\,\Delta})$ we obtain:

$$P_{RIN} = -10\log 10(\sqrt{1-(\frac{Q\,NEF\,\rho_{eye-RIN}\,\rho_{Te}\,\sigma_{RIN}}{ISI})^2}) \qquad \text{(D.20)}$$

where $\rho_{eye-RIN}{}^2 = 12.2$

RIN_{OMA} Penalty for Statistical Eye Amplitude Method

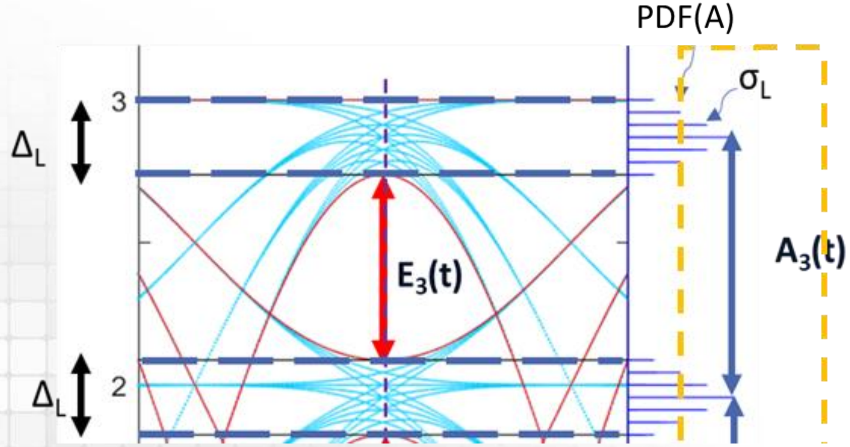
- The eye amplitude $A_3(t)$ is used to represent the worst eye (top eye).
- We use $A_3(t)$ instead of $E_3(t)$, which is different than previous link models
- A good approximation for PDF(Y), that facilitates the implementation in an Excel spreadsheet is given by,

$$\overline{PDF}(Y) = e^{-[A_3(JT_P)-th]^2/2\sigma_Y^2} \;/\; \sqrt{2\pi\sigma_Y^{\,2}} \qquad \text{(D.21)}$$

where,

$$\sigma_Y^{\,2} = \sqrt{n_{AWGN}^{\;\;2} + \sigma_T^{\,2}(\frac{\Delta}{2}\rho_{Te})^2 + f_\Delta(\frac{\Delta}{2})^2\sigma_L^{\,2}} \qquad \text{(D.22)}$$

where f_Δ is a factor to minimize cross central penalties. Ideally should be 1. But if so that increase cross-central penalty (Pc). Using 1/3 it reduces it. Personally, I would use 1 but I want to understand the value of having a low Pc in this new model approach.



RIN_{OMA} Penalty for Statistical Eye Amplitude Method

- For non-stressed channels Q, the modulation amplitude and noise are related using:

$$Q = \frac{\Delta_{Min}}{2n_W} \qquad \text{(D.23)}$$

Where Δ_{Min} is the minimum optical modulation amplitude required by the receiver, using a non stressed pattern to produce a BER (Q). The noise is the receiver noise, not penalties due to fiber (very short optical link)

The variance of the histogram is also assumed to be low due to the short link (low ISI).

- After propagation through the link the minimum value Q factor should be maintained

$$(\frac{\Delta}{2})ISI = Q(\sqrt{(\frac{\Delta_{\min}}{2Q})^2 + \sigma_T{}^2(\frac{\Delta}{2}\rho_{Te})^2 + f_\Delta\,(\frac{\Delta}{2})^2\sigma_L{}^2} \qquad \text{(D.24)}$$

where Δ is the optical modulation amplitude required to maintain the BER (Q), σ_L is the eye level variation standard deviation, and ISI is the ISI penalties in linear scale. For the statistical eye amplitude method ISI = $A_i(JT_p)$. Note that σ_L is dependent on ISI.

- Solving (D.24) and using $P_{RIN} = -10\log 10(\frac{\Delta_{\min}}{ISI\,\Delta})$ we obtain:

$$P_{RIN} = -10\log 10(\sqrt{1-(\frac{Q\,\sigma_{RIN}}{ISI}\rho_{eye-RIN}\rho_{Te})^2 + f_\Delta(\frac{\sigma_L}{ISI})^2}) \qquad \text{(D.25)}$$

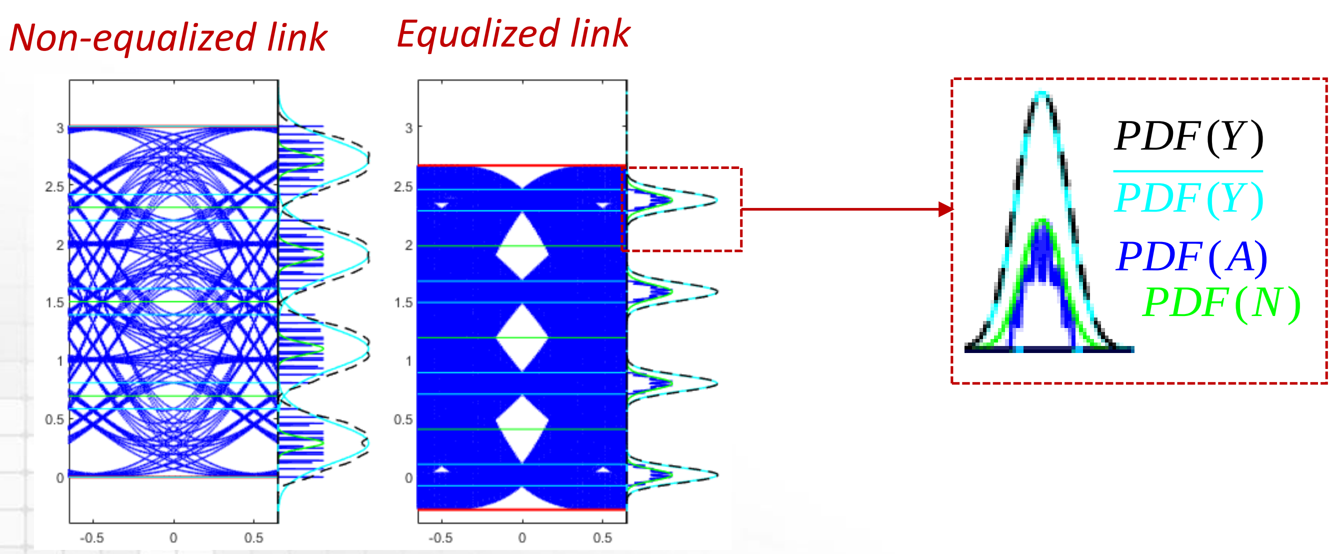
RIN_{OMA} Statistical eye amplitude vs Numerical Simulation

- A numerical link simulation should produce PDF(Y) given by

$$PDF(Y) = PDF(A) \otimes PDF(N) \qquad \text{(D.26)}$$

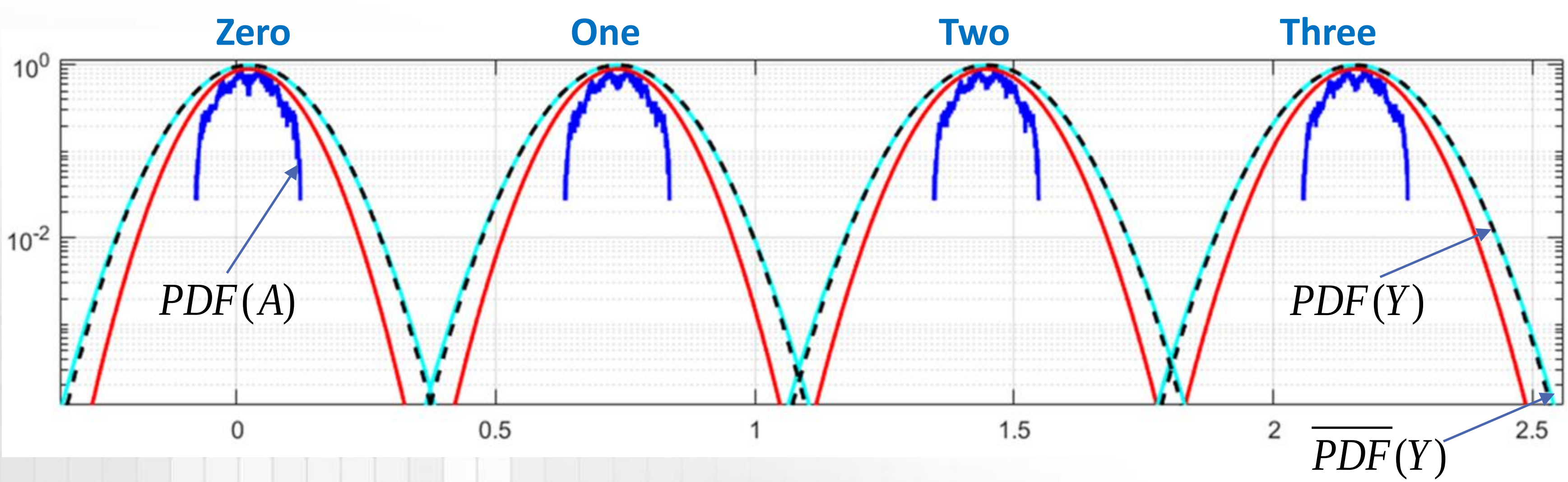
where $\otimes$ is the convolution operator PDF(N) is a Gaussian representation of the noise with standard deviation computed before

- The Figure below shows an example for a 100 m PAM-4 equalized and non-equalized link.
  - The PDF with cyan traces represent the results from the statistical eye amplitude method. The black traces represent results from the numerical simulation. The blue traces represent the exact PDF(A) for non-equalized and equalized links.



RIN_{OMA} Statistical eye amplitude vs Numerical Simulation

- Another example is shown in the Figure below
- Comparison of the $\overline{PDF}(Y)$ (cyan trace) and $PDF(Y)$ (dashed black trace) for a PAM-4 signal propagating over 100 m is illustrated. The total noise (Gaussian) is represented by the red-traces and the PDF of the amplitude, $PDF(A)$, (no noise only ISI and jitter) is shown by the blue traces.
- For PAM-4 equalized links there is very good agreement between the estimated $\overline{PDF}(Y)$ with numerically simulated $PDF(Y)$.



Mode Partition Noise

- We use the same equations from the Fibre Channel PI-6 link model for this noise.

$$\sigma_{MPN}^{\ 2} = NEF\left[\frac{k_{MPN}}{\sqrt{2}}\left(1 - e^{-(\pi BDL\Delta\lambda_{RMS})^2}\right)\right]^2 \qquad \text{(F.26)}$$

- Some changes:
  - Noise scaling factor to take into account the eye reduction due to PAM-4. The factor $\rho_{eye} = 3$ to be multiplied to the standard variation of MPN
  - Used K_{MPN} = 0.1 instead of 0.3
    - There is work presented by Panduit and other groups such as Georgia Tech that might support this reduction.

- Not additional changes are proposed for now

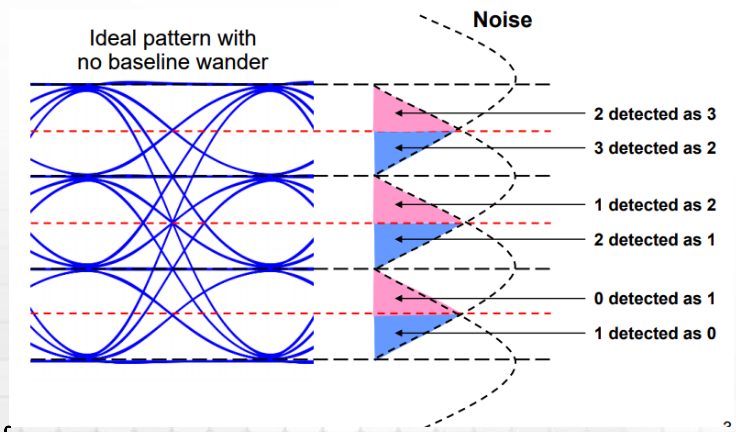
Modal Noise

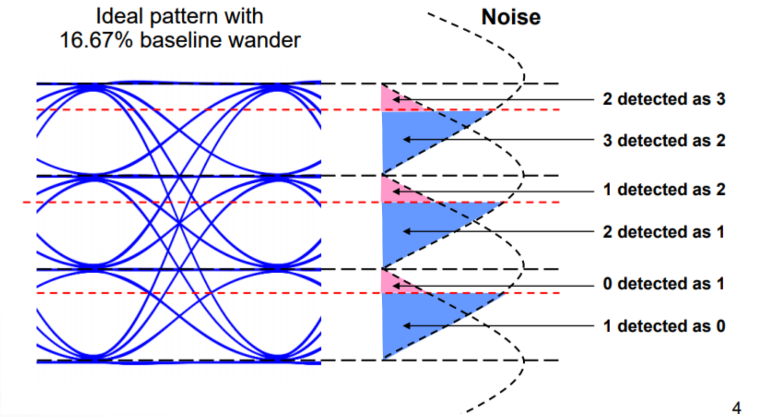
- Modal noise is caused by mode selective loss and power fluctuation in the fiber modes. Essentially, there are two regimes:
  - Coherent MN caused by the interference of signals traveling through different paths (modes), with different losses.
    - Requires mode selecting losses and mode delays below the VCSEL coherent time the VCSEL.
  - Non-coherent MN caused by laser mode power fluctuation (mode partition) and selective losses.
    - The laser modes couple differently to the fiber modes. Laser power fluctuation among its modes, translates fiber mode power variations. When all the modes reach the receiver there are no overall power variations. However, when there are mode selective losses MN is produced.
- Both MN regimes can coexist to some degree depending on the fiber length. MN scales with optical power. The eye reduction increase this noise relative to the eye sigma by $\rho_{eye} = 3$
- We are studying what should the valued of this noise be. Values used in examples shown in this presentation are not final.

Baseline Wander

- Baseline wander causes an offset between the eyes and the decision points of each eye increasing the errors as shown in the diagrams below (from first reference).
- Baseline wander magnitude depends on LF cutoff frequency, symbol rate, coding, and pulse shape among other parameters.
- Assuming a pulsed shaped PAM-M signal, it can be computed as:

$$\sigma_W{}^2 = \frac{T_p}{2\tau}\frac{1}{(M-1)^2 M}\sum_{i=0}^{M-1} i^2 = \frac{2\pi T_p B_{HighPass-cutoff}}{2}\frac{(M-1)(2M-1)M}{6(M-1)^2 M} = \frac{(2M-1)}{6(M-1)}\pi T_p B_{HighPass-cutoff} \qquad \text{(G.1)}$$





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Baseline Wander

- For NRZ baseline wander was defined as the instantaneous offset (in %) in the signal generated by AC coupling at the Baud rate / 10,000.
- If this cut-off frequency is still used for PAM-4 the baseline wander variance is

$$\sigma_W{}^2 \approx \frac{7}{180000}\pi = 1.22E-4 \qquad \text{(G.2)}$$

- Baseline wander is a signal dependent noise so it should be increased due to the eye reduction. The noise relative to half eye is given by

$$n_w{}^2 = (\frac{\Delta}{2}(M-1)\sigma_W)^2 \qquad \text{(G.3)}$$

- Assumptions
  - Baseline wander is represented as a truncated Gaussian function

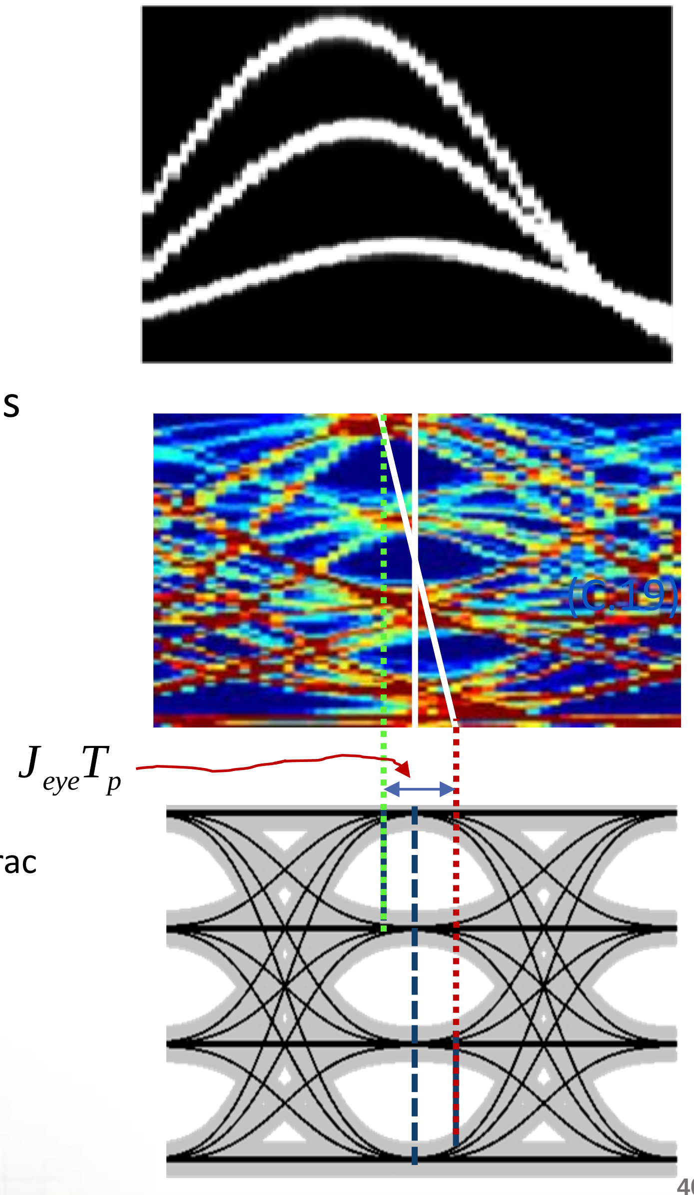
Eye Skew

Eye Skew

- Eye diagram skew is produced during transmission of PAM-4 signal using direct modulated VCSELs
  - Depends on the bias current and transmitted level
    - See T11/16-152v0 and T11/15-263v0
    - Levels 3 rise time is faster than level one (Fig. A)
  - Produce non-optimum sampling time
- Eye skew can be modeled using the laser rate equations
- However, those models could be challenging to implement in Excel without increasing the response delays
- In the link model presented, we treat eye skew as a component of data dependent jitter.

$$J = J_0 + J_{eye} \qquad \text{(H.1)}$$

where, J_o is the effective jitter without eye skew based on delta Dirac model



Total noise variances

- For both methods (worst-case eye diagram & statistical eye amplitude) the sum of variances are represented by

$$n_x{}^2 = n_{RIN}{}^2 + n_{MPN}{}^2 + n_{MN}{}^2 + n_W{}^2 \qquad \text{(I.1)}$$

- Based on previous slides (G.1) can be converted in

$$n_x{}^2 = NEF(\frac{\Delta}{2})^2[(\rho_{Te}\ \rho_{eye-RIN}\sigma_{MPN})^2 + (\rho_{eye}\ ISI\,\sigma_{MPN}\ )^2 + (\rho_{eye}\ ISI\,\sigma_{MN}\ )^2 + \rho_{eye}{}^2 1.22E-4] \qquad \text{(I.2)}$$

- The normalized variance used to compute the penalties is given by,

$$\sigma_x{}^2 = \frac{n_x{}^2}{(\Delta/2)^2} = NEF\,\rho_{eye}{}^2[(\rho_{Te}\ \frac{\rho_{eye-RIN}}{\rho_{eye}}\sigma_{MPN})^2 + (\ ISI\,\sigma_{MPN}\ )^2 + (ISI\,\sigma_{MN}\ )^2 + 1.22E-4] \qquad \text{(I.3)}$$

Total Penalties

- The computation of the penalties using the total noise variances is a critical step to determine the margin of the link.
- Most of the other Penalties for individual effects: MN, MPN, RIN are not impacting the computation of margin.
- For the worst-case eye closure method the computation of the penalty for the targeted Q factor, Q_T, is given by,

$$P_x = -10\log 10\left(\sqrt{1-\left(\frac{Q_T\ \sigma_X}{ISI}\right)^2}\right) \qquad \text{(J.1)}$$

- For the statistical eye amplitude method the penalty is given by,

$$P_x = -10\log 10\left(\sqrt{1-\frac{Q_T{}^2(\sigma^2{}_X+\sigma_L{}^2)}{ISI^2}}\right) \qquad \text{(J.2)}$$

  - Note that for both the terms ISI are different since one use the eye closure and the other one the amplitude as shown in previous slide

- The total penalties are given by

$$P_t = P_{att} + P_X - P_w \qquad \text{(J.3)}$$

  - Where P_{att} is the attenuation of the link and P_W the penalty for baseline wander

Total Penalties and Margin

- The optical link budget is given by,

$$\theta = OMA_o^{Tx} - OMA_o^{Rx} - 10\log 10(3) \qquad \text{(J.4)}$$

  - Where OMA terms are the transmitter power and receiver sensitivity.

- The margin is computed using

$$\Omega = \theta - L_{con} - P_T + G_E - \sqrt{NEF_W} \qquad \text{(J.5)}$$

  - The reach is estimated for Ω=0 length that produce zero margins.

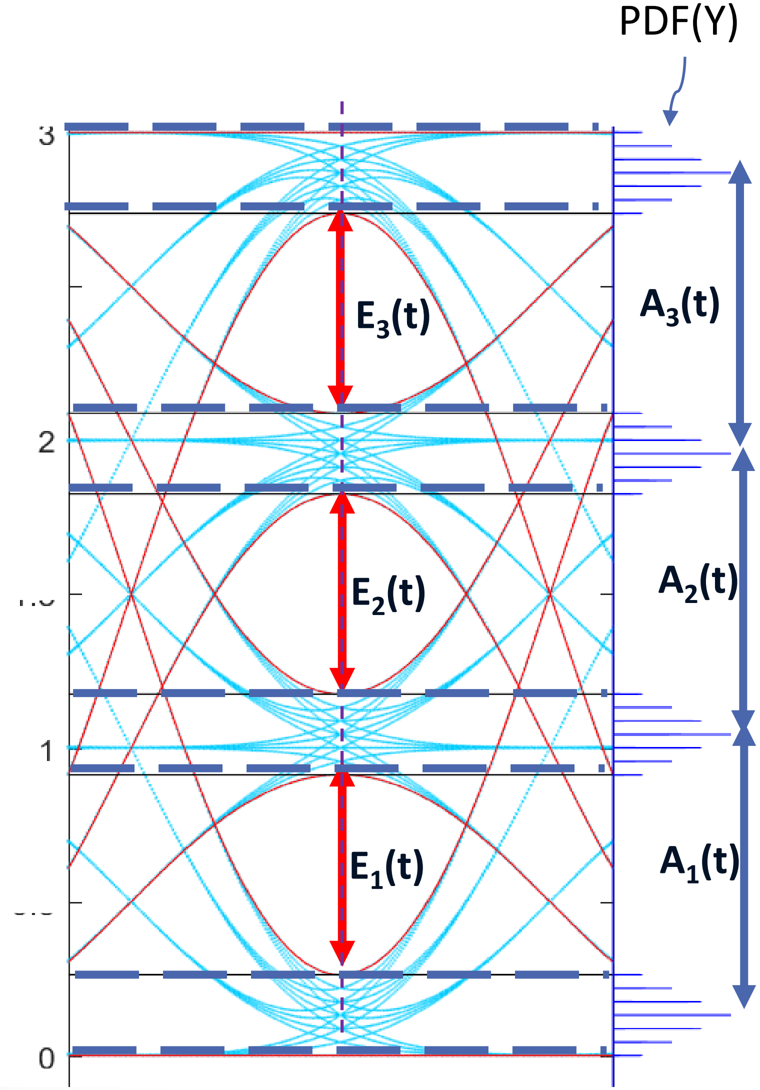
Verification of worst-case eye opening method and statistical eye amplitude with Numerical Simulations

PANDUIT

Numerical Verification of equations for ISI and Jitter

- *The eye diagram was obtained from numerical simulation*
  - *SSPRQ sequence or longer simulated*
  - *PDF of traces at the decision point computed*
    - *PDF(y) for all the traces*
    - *Eye closure and amplitude computed*

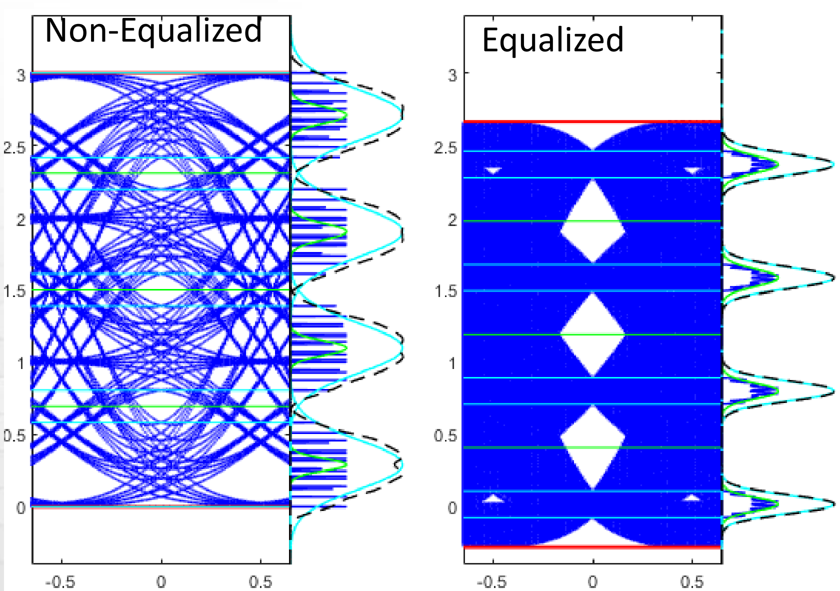
- *Comparison of both methods, worst-case eye closure and statistical amplitude were compared with the numerical simulation as shown in next slide*



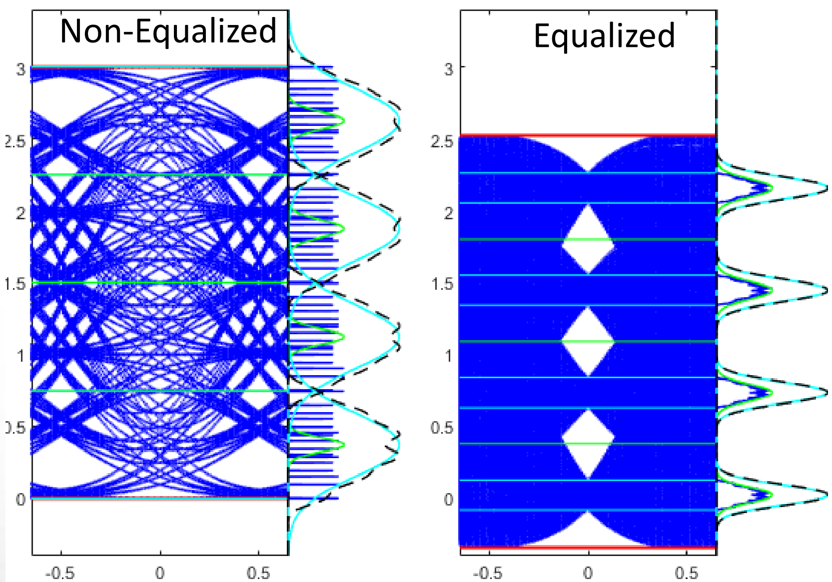
Numerical Verification of equations for ISI and Jitter

Tx Power OMA	Rx Sensitivity OMA	Lambda_c	Lambda_z	Q
-0.1	*-11.07*	*840*	*1316*	*3.6972*
Base_Rate	**Spect. Width**	**Ext. Ratio**	**DCD_DJ**	**Effect_DJ...**
26.5625	*0.55*	*2*	*1.73*	*0.12*
RIN_OMA	**Ts_20_80**	**k_RIN**	**k_MPN**	**TX_r**
-133	*20*	*0.7*	*0.1*	*12*
RX_r	**Baseline Wander_SD**	**R_NF**	**BWrec**	**BWtest**
12	*0.012*	*0*	*19000*	*21675*

12.5 m OM4

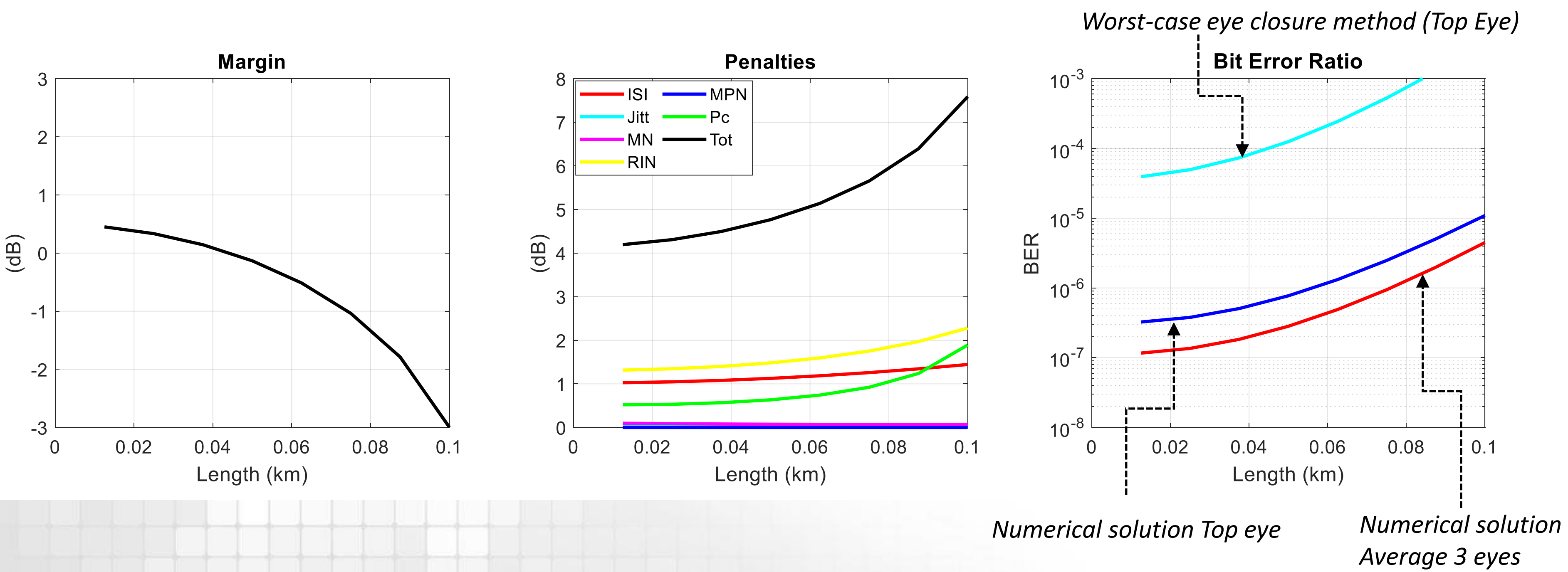


100 m OM4



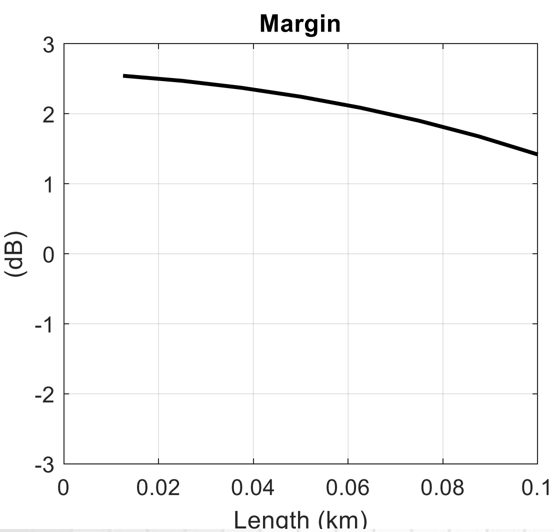
Worst-Case eye closure method vs numerical simulation

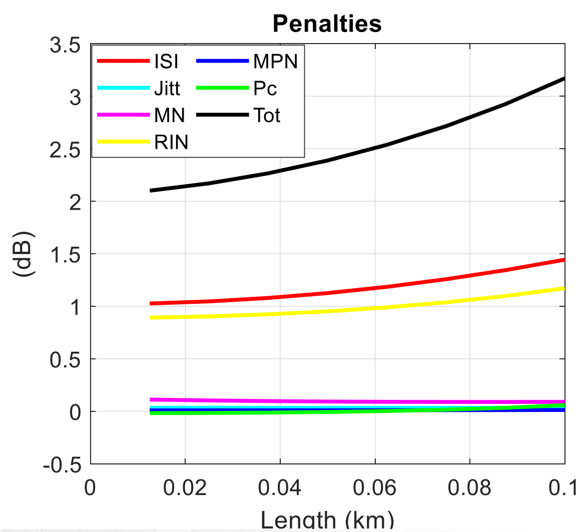
- The input parameters used in this method predict negative margins for less than 50 m
- This result can be explained by looking at the BER plot
  - The red and blue traces are results from numerical simulation. The blue trace represents the top eye and the red trace the average of the 3 eyes
  - The cyan trace represents the BER of the top eye estimated using the worst-case eye closure method
  - The extra penalties of this method compared with the numerical simulated top eye is 1.9 dB
  - The extra margin compared to the average simulated BER is 2.1 dB

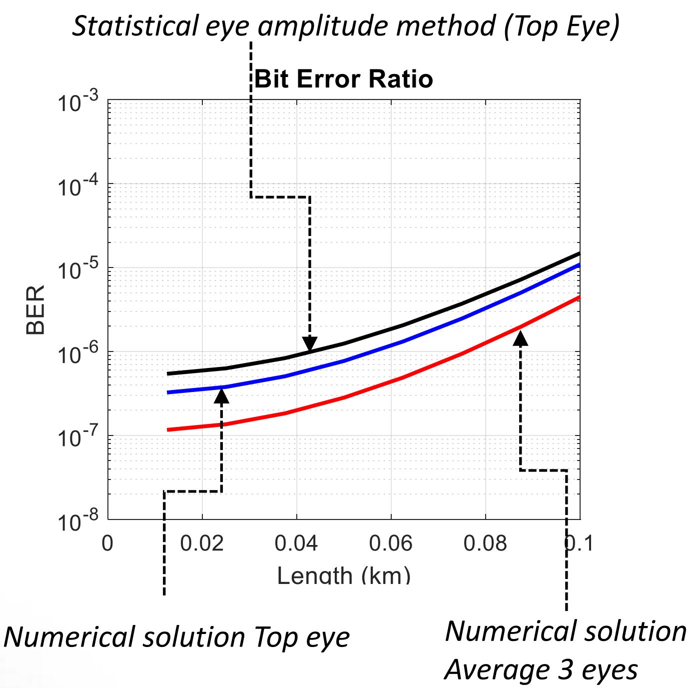


Statistical eye-amplitude method vs simulation

- For the input parameter used, this method predicts negative margins for less than 50 m.
- This result can be explained by looking at the BER Figure
    - The red and blue traces are results from numerical simulation. The blue trace represents the top eye and the red trace the average of the 3 eyes.
    - The black trace represents the BER of the top eye estimated using the statistical eye closure method.
    - The extra penalties of this method compared with the numerical simulated top eye is 0.1 dB
    - The extra margin compared to the average simulated BER is 0.27 dB





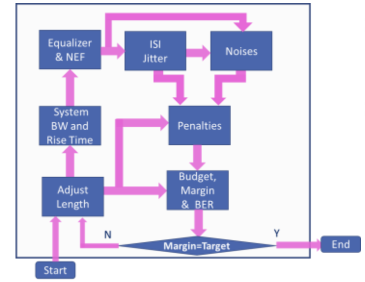


Summary & Conclusions

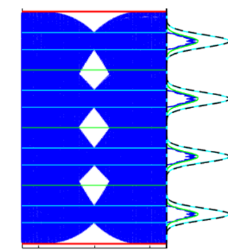
- Described Panduit's proposed link model for PAM-4 signals over multimode fiber
- Proposed a new method to evaluate penalties
  - Previous link models used worst-case eye closure, and alternative method statistical-eye amplitude method demonstrated more accurate for Equalized PAM-4 link
    - For non-equalized PAM-2 the worst-case eye closure method is better
- Developed and mathematically derived model to facilitate description
  - The equalized PAM-4 link model requires a larger sets of equations due to the multiple signal levels and the equalization
  - Most of the equations/algorithms disclosed in this presentation.
  - Equalizer with 5 (or 3 taps) solved analytically to simplify implementation.
  - Noise enhanced factor for AWGN and signal dependent noised derived and compared
  - Labeled all equations to facilitate discussion
  - The model components that disclosed in this presentation focused on ISI, JITTER and RIN Penalties
    - More work to be presented for MN and MPN
    - More examples to be shared in future presentations

Summary & Conclusions

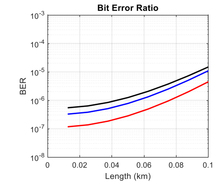
- Verified that analytical equations for ISI and Jitter match numerical solution
  - Comparison of worst-case eye closure method vs numerical simulation shows additional penalties of around 2 dB
  - Comparison of statistical eye-amplitude method vs numerical simulation shows additional penalties of around 0.2 dB



- The link model using the proposes statistical eye-amplitude method still represents worst-case conditions
  - All signal dependent noise variances are increased 9 times to account for the eye reduction
  - The RIN variance is increased 12.2 times
  - It can be tuned to provide required margins, i.e. 0.5 dB or 1 dB to account for penalties not considered in the presented link model



- The presented link model can be implemented in a Excel /VBA spreadsheet.
  - A version of the more accurate model can also be implemented but it will require to use DLLs.
  - Panduit volunteer to implement the model after its revision. See annex I for general description of the link model modules data flow.



Request to the participants and reviewers

- Not all the information was disclosed in this presentation
- This is a proposal, please do not start using equations of the model until the technical contributors/reviewers agree.
- The participants that will like to provide <u>technical</u> comment are encouraged to send us their comments by email or we can schedule meeting to discuss (jmca.Panduit.com)
    - We tried to number all the equations to facilitate discussion.
    - If different equations are proposed, please, it will be appreciated if a technical justification with references if possible are provided

References

Fiber Optics Technology
Panduit Corporation
E-mail: jmca@panduit.com

Prior work for equalized channels and PAM-4

- Previous work for 1Gbps and 10Gbps using NRZ link models
  - Del Hanson, David Cunningham, Piers Dawe and David Dolfi (for 10G)
- Prior works for equalized channels :
  - D. Cunningham proposed a 3-tap equalizer for PI-6 (12-044v1, 12-123v0)
    - However, required several sheets (one per link length) and valid only for NRZ
  - PAM-4 power budget penalties require more sophisticated equations than NRZ
  - Equalization taps need to be efficiently computed for each length in one sheet
- In Fibre Channel, PAM-4 has been modeled using additional software packages
  - For Python languages 16-013 v0, 16-012v0
  - For Matlab 15-263v0
  - An Excel VBA was proposed in T11-2016-065v0
    - Fully implemented PI-6P (32GFC NRZ)
- Investigation of 60 Gb/s 4-PAM Using an 850 nm VCSEL and Multimode Fiber
  - Journal of Lightwave Tech. Vol 34 (16) , April 2016

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ANNEX I

Link Model Algorithm Input /Output parameters

Link Model Algorithm
Input/output Parameters per block

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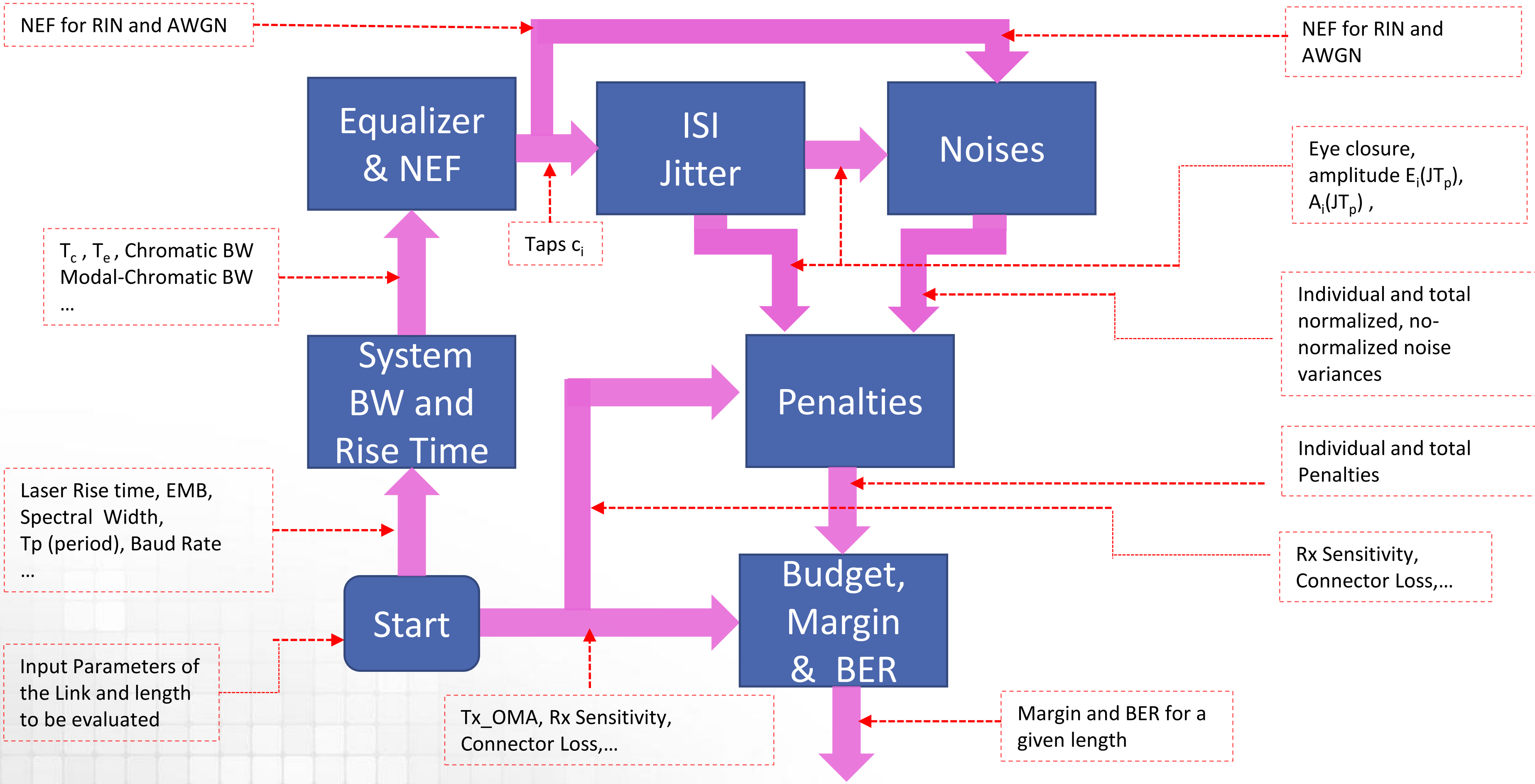
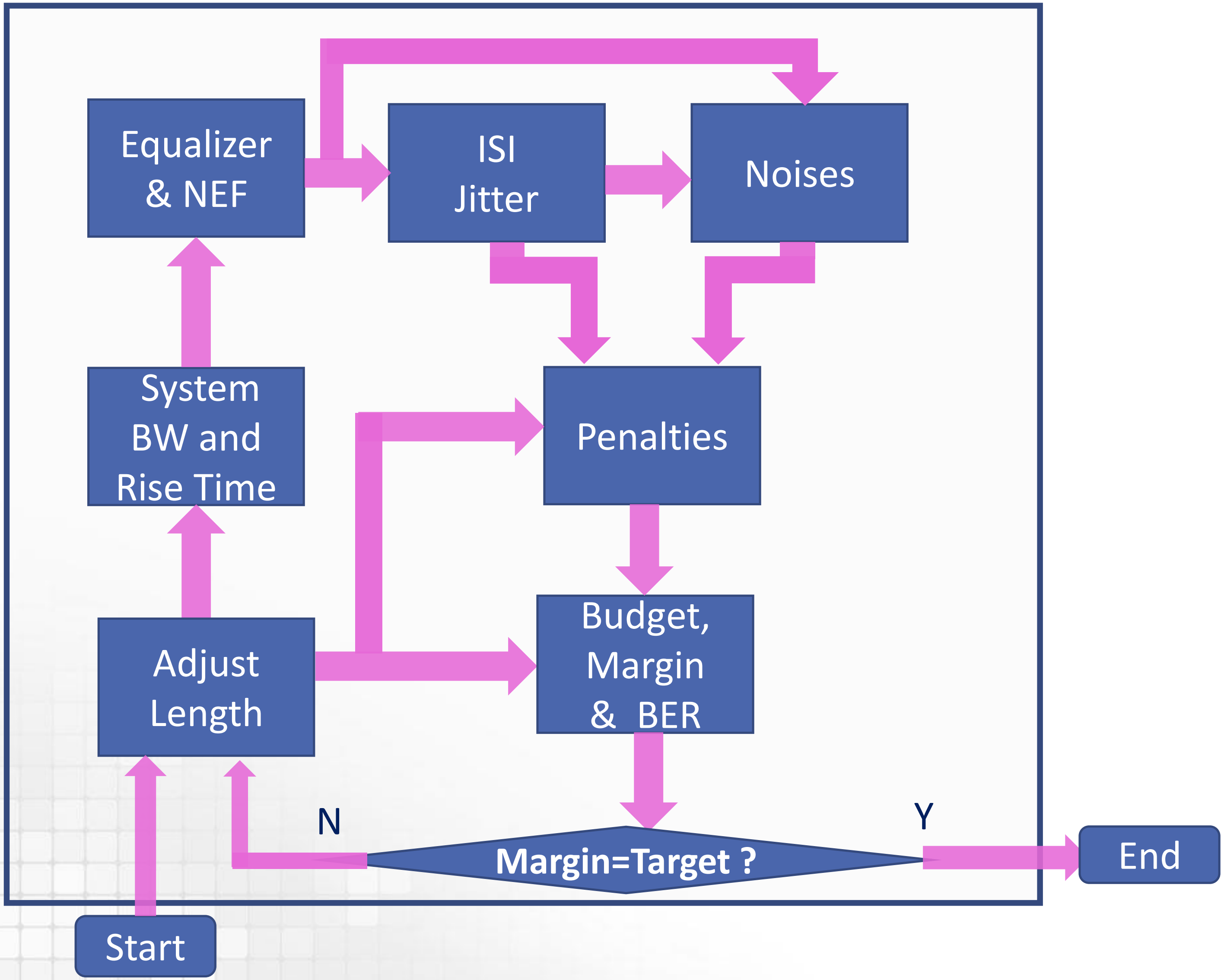


Data flow for one Length

Link Model Algorithm



Adjust link length until margin equals zero (or a targeted value)

ANNEX II

Examples to evaluate accuracy of Statistical Eye Amplitude method for estimation of PDF(A)

Example I

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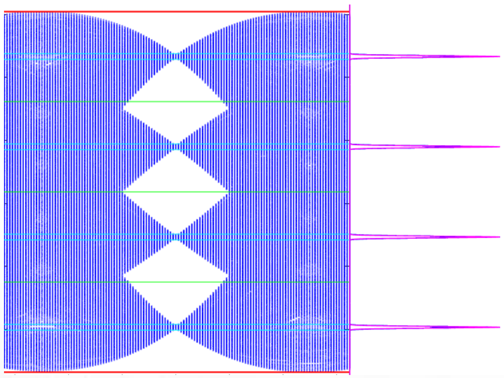
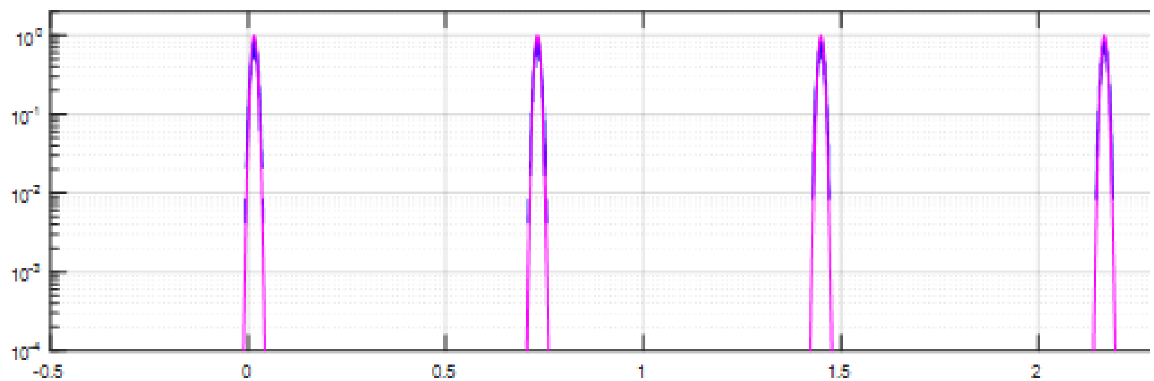
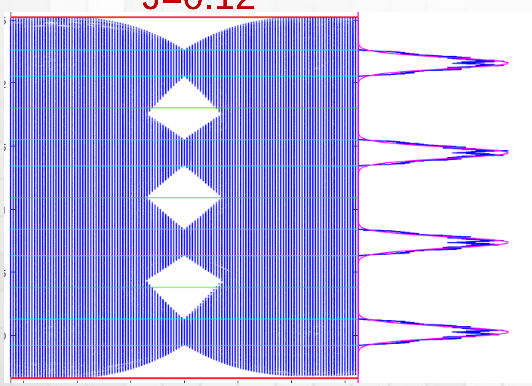
- Blue traces PDF(A) from numerical simulation.
- Magenta traces represent the Gaussian estimation
- Parameters (partial list)
  - Ts=20 ps, Spectral With=0.55 nm, EMB=4700 MHz-km, Length =100m

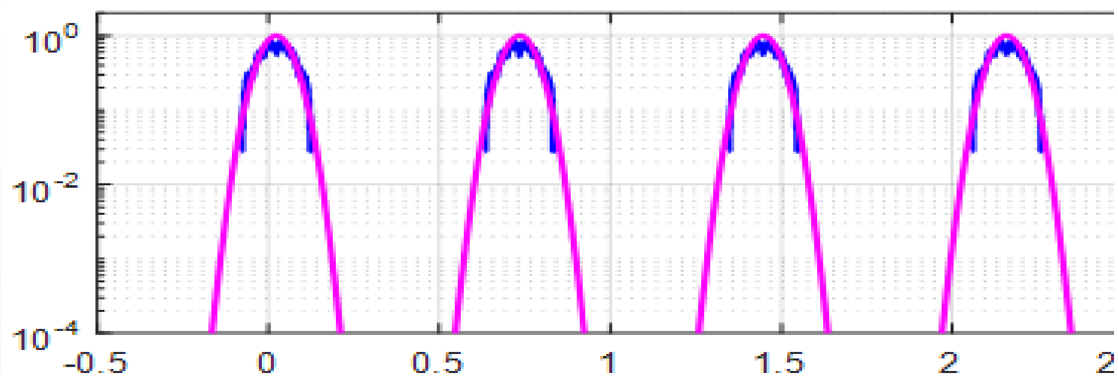
J=0





J=0.12

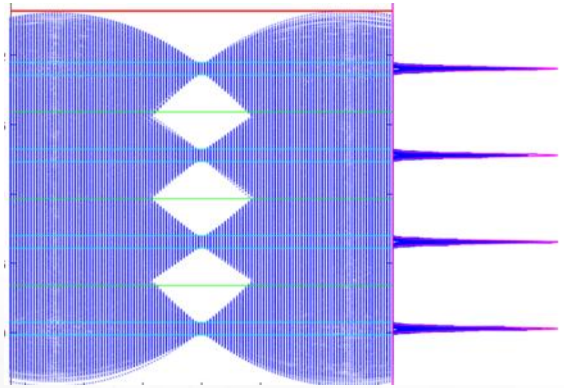


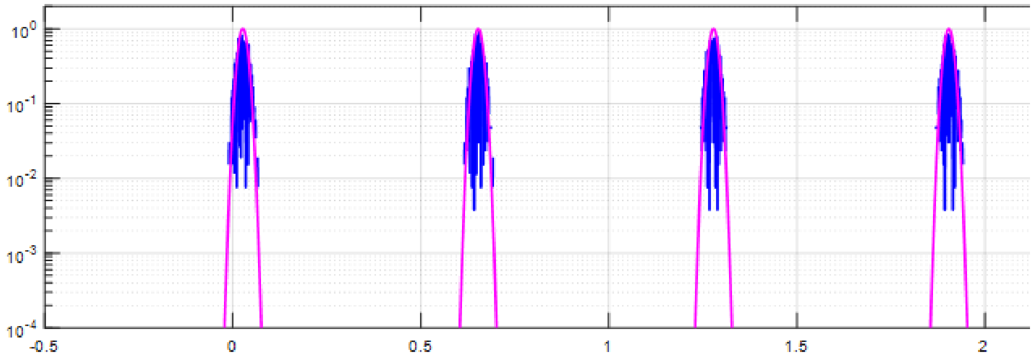


Example II

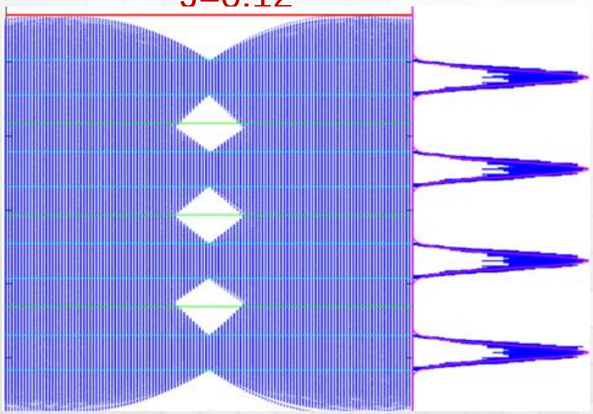
- Blue traces PDF(A) from numerical simulation.
- Magenta traces represent the Gaussian estimation
- Parameters (partial list)
  - Ts=20 ps, Spectral With=0.55 nm, EMB=2000 MHz-km, Length =100m

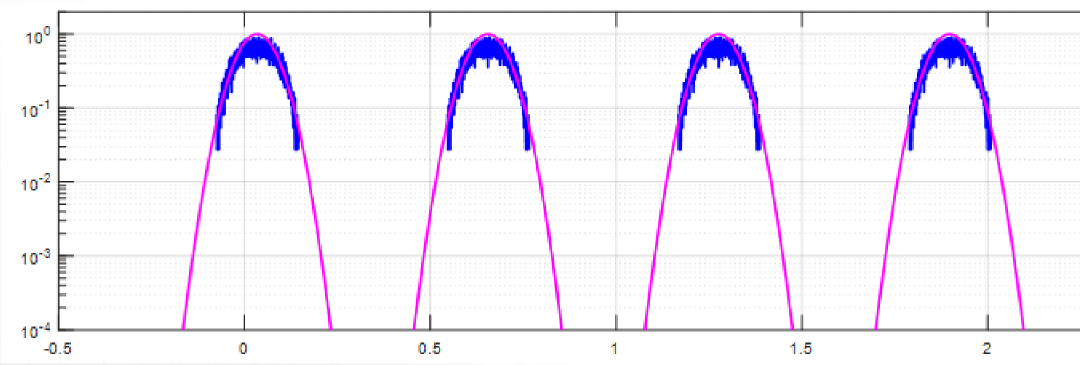
J=0





J=0.12

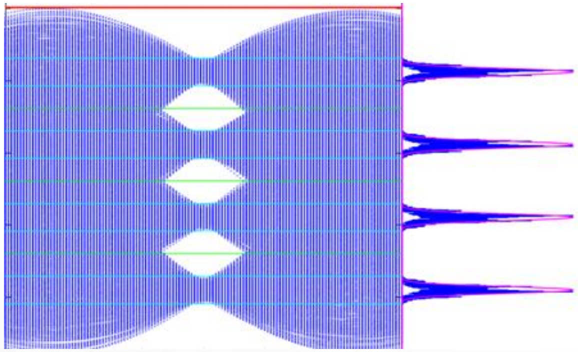


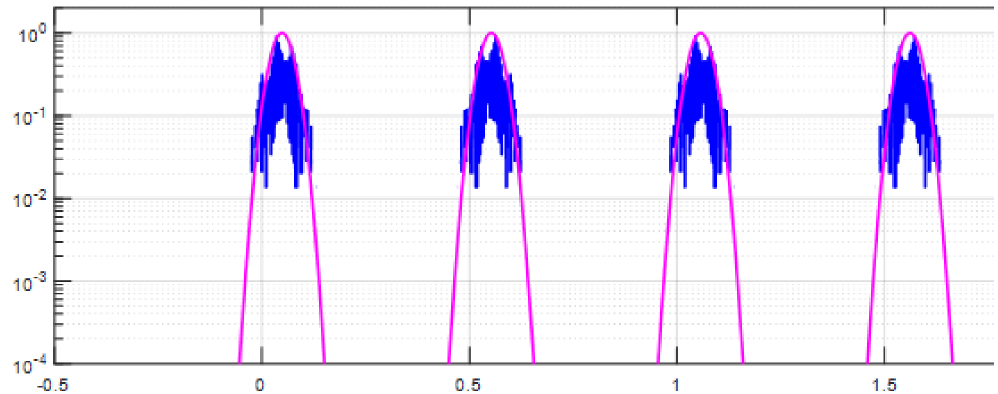


Example III extreme case (Margin <<0)

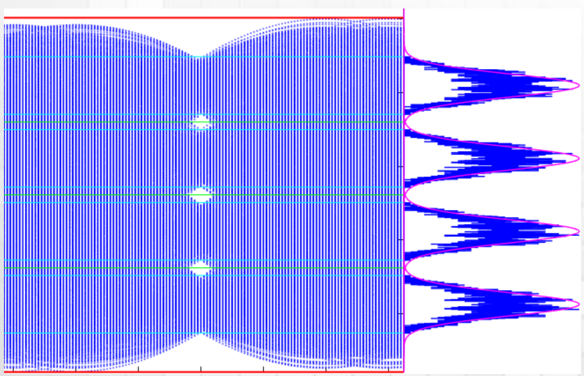
- Blue traces PDF(A) from numerical simulation.
- Magenta traces represent the Gaussian estimation
- Parameters (partial list)
  - Ts=25 ps, Spectral With=0.65 nm, EMB=1800 MHz-km, Length =100m

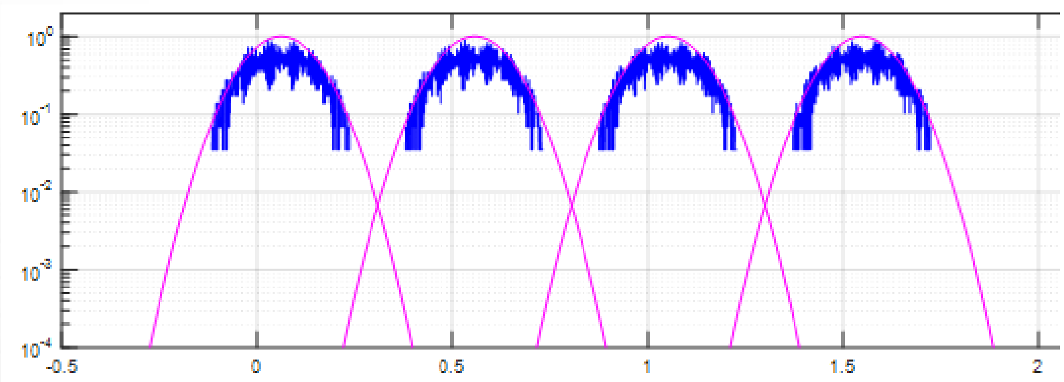
J=0





J=0.2





Annex III

Table: Summary of Equations

Model Penalties and Parameters

#	Parameter	Symbol	No Equalizer case	With equalizer	Notes
1	PAM levels	M	4		
2	Effective Symbol Period	T_P	[1/BASE_RATE-DCD]		DCD is duty cycle distortion
3	System Rise Time 10%-90%	T_C	$\sqrt{T_e^2 + T_R^2)}$		T_S is the laser rise time, T_R the Rx risetime
4	Equivalent fiber rise time10%-90%	T_e	$\sqrt{T_S^2 + (480\ 10^3)^2 (BW_M^{-2} + BW_{Ch}^{-2}\ )}$		BW_M is the EMB, BW_{Ch} is the chromatic bandwidth
5	Impulse Response	$h_e(t)$ $h_f(t)$	$h_e(t) = \frac{1}{2}[\mathrm{erf}(k\frac{2t+T_p}{T_c}) + \mathrm{erf}(k\frac{-2t+T_p}{T_c})$	$h_f(t) = \sum_{i=-2}^{2} c_i h_e(t)$	Ci are the equalizer tap weights, k=0.9062 (see derivation section)
6	OMA_{outer}	3Δ	OMA without dispersion		
7	Eye skew data dep. jitter	J_{eye}	0.04		
8	Effective Jitter	J	$\frac{DJ + CDR\_alloc - DCD}{T_p} + J_{eye}$		
9	Eye level thickness	Δ_L	$(M\text{-}1)\sum_{k=1}^{7}\left\|h_e(\frac{JT_p}{2}+kT_p)\right\|+\left\|h_e(-\frac{JT_p}{2}+kT_p)\right\|$	$(M\text{-}1)\sum_{k=1}^{7}\left\|h_f(\frac{JT_p}{2}+kT_p)\right\|+\left\|h_f(-\frac{JT_p}{2}+kT_p)\right\|$	
10	Eye level variance	σ_L^2	$\frac{\sqrt{5}}{2}\sqrt{\sum_{i=1}^{3} h_e(\frac{JT_p}{2}+iT_p)^2 + h_e(-\frac{JT_p}{2}+iT_p)^2}$	$\frac{\sqrt{5}}{2}\sqrt{\sum_{i=1}^{3} h_f(\frac{JT_p}{2}+iT_p)^2 + h_f(-\frac{JT_p}{2}+iT_p)^2}$	Proxy for variance of the vertical eye PDF
11	Eye Opening	$E_i(JT_P)$	$Mh_e(0.5J)$ -(M-1)	$h_f(0.5JT_p)$ -Δ_L	i is the eye index, i.e, i=3 is the top eye
12	Eye Amplitude	$A_i(T_P)$	$h_e(0.5JT_P)$	$h_f(0.5JT_P)$	
13	Linear isi-jitter factor	ISI	ISI=$E_i(JT_p)$ for the worst-caser eye closure method	ISI=$A_i(JT_p)$ for the statistical eye amplitude method	
14	VCSEL-MMF ISI before equalizer	ρ_{Te}	$0.5[\mathrm{erf}(k\frac{(J+1)T_p}{T_e}) + \mathrm{erf}(k\frac{(-J+1)T_p}{T_e})]$		Reduction in amplitude due to laser and fiber
15	Noise std adjustment due to eye reduction	ρ_{eye}	$(M-1)$		For signal dependent noise different than RIN
16	Noise std adjustment due to eye reduction	$\rho_{eye-RIN}$	$\sqrt{\frac{13ER^2 + 4ER + 1}{1+ER^2}}$		$\sqrt{12.2}$ for the Top eye using ER=2, and equal to ρ_{eye} $ER \to \infty$
17	Extinction Ratio (min)	ER	2		

Model Penalties and Parameters

#	Parameter	Symbol	No Equalizer case	With equalizer	Notes
18	RIN variance (normalized)	$\sigma_{RIN}{}^2$	$\frac{K_{RIN}10^{-RIN_{OMA}/10}10^6}{\sqrt{0.477/BW_{Rx}{}^2)}}$	$NEF\frac{K_{RIN}10^{-RIN_{OMA}/10}10^6}{\sqrt{0.477/BW_{Rx}{}^2)}}$	
19	MPN variance (normalized)	$\sigma_{MPN}{}^2$	$[\frac{k_{MPN}}{\sqrt{2}}(1-e^{-(\pi BDL\Delta\lambda_{RMS})^2})]^2$	$NEF[\frac{k_{MPN}}{\sqrt{2}}(1-e^{-(\pi BDL\Delta\lambda_{RMS})^2})]^2$	$k_{MPN}=0.1$
20	MN variance (normalized)	$\sigma_{MN}{}^2$	$[F_{MN}(L_{con})e^{-K_{MN}t/t_{coh}}]^2$	$NEF[F_{MN}(L_{conn})e^{-K_{MN}t/t_{coh}}]^2$	FMN function to relates MN to connector loss (Lcon). Future work to be presented
21	Baseline wander variance (normalized)	$\sigma_W{}^2$	$\frac{7}{18}\pi T_p B_{HighPass-cutoff}$	$NEF\frac{7}{18}\pi T_p B_{HighPass-cutoff}$	For non-equalize case we use 1.2e-4
22	Noise RIN variance	$n_{RIN}{}^2$	$(\frac{\Delta}{2}\rho_{Te}\ \rho_{eye-RIN}\sigma_{MPN})^2$	$NEF(\frac{\Delta}{2}\rho_{Te}\ \rho_{eye-RIN}\sigma_{MPN})^2$	
23	Noise_MPN variance	$n_{MPN}{}^2$	$(\frac{\Delta}{2}\rho_{eye}\ ISI\ \sigma_{MPN})^2$	$NEF(\frac{\Delta}{2}\rho_{eye}\ ISI\ \sigma_{MPN})^2$	
24	Noise_MN variance	$n_{MN}{}^2$	$(\frac{\Delta}{2}\rho_{eye}\ ISI\sigma_{MN})^2$	$NEF(\frac{\Delta}{2}\rho_{eye}\ ISI\sigma_{MN})^2$	To be reviewed when new MN model is proposed
25	Baseline Wander variance used here	$n_W{}^2$	$(\frac{\Delta}{2}\rho_{eye})^2 1.22E-4$	$NEF(\frac{\Delta}{2}\rho_{eye})^2 1.22E-4$	Does not depend on OMA
26	Total noise sum of variances	$n_X{}^2$	$n_{RIN}{}^2+n_{MPN}{}^2+n_{MN}{}^2+n_W{}^2$		
27	Normalized total Noise	$\sigma_X{}^2$	$NEF\,\rho_{eye}{}^2[(\rho_{Te}\ \frac{\rho_{eye-RIN}}{\rho_{eye}}\sigma_{MPN})^2+(ISI\ \sigma_{MPN})^2+(ISI\ \sigma_{MN})^2+1.22E-4]$		
28	Equalizer Gain	G_E	1	$(1+2c_1+2c_2)^{-1}$	
29	Noise Enhancement Factor for white noise	NEF_W	1	$G_E{}^2\sum_{i=-2}^{2}c_i^2$	
30	Noise Enhancement Factor for signal dependent noise	NEF	1	$G_E{}^2\{\sum_{i=-2}^{2}c_i^2+\sum_{i=-2}^{2}\sum_{k=-2,k\neq i}^{2}c_i c_k e^{-(\frac{1.2816(i-k)T_F}{T_c})^2}\}$	

Model Penalties and Parameters

PANDUIT

#	Parameter	Symbol	Worst-Case Eye-Opening Method	Statistical Eye Amplitude Method
31	ISI Penalty	P_{ISI}	$-10\log 10[E_3(0)]$	$-10\log 10[A_3(0)]$
32	Jitter Penalty	P_J	$-10\log 10([E_3(0.5JT_P)]-P_{ISI}$	$-10\log 10([A_3(0.5JT_P)]-P_{ISI}$
33	ISI-Jitter linear	ISI	$E_3(0.5JT_P)$	$A_3(0.5JT_P)$
34	Cross central penalties factor	f_Δ	0	1 or 1/3 to make minimize Pc. To be discussed. Does not affect the final reach or BER!
35	RIN Penalty	P_{RIN}	$-10\log 10(\sqrt{1-(\frac{Q\,\rho_{Te}\rho_{eye-RIN}\sigma_{RIN}}{ISI^2})^2}$	$-10\log 10(\sqrt{1-\frac{Q^2\rho_{Te}^2(\rho_{eye-RIN}^2\sigma^2_{RIN}+\sigma_L^2 f_\Delta)}{ISI^2}}$
36	MPN penalty	P_{MPN}	$-10\log 10(\sqrt{1-(Q\,\rho_{eye}\sigma_{MPN})^2}$	$-10\log 10(\sqrt{1-Q^2(\rho_{eye}^2\sigma^2_{MPN}+\sigma_L^2 f_\Delta)}$
37	MN penalty	P_{MN}	$-10\log 10(\sqrt{1-(Q\,\rho_{eye}\sigma_{MN})^2}$	$-10\log 10(\sqrt{1-Q^2(\rho_{eye}^2\sigma^2_{MN}+\sigma_L^2 f_\Delta)}$
38	Total noise Variances	n_X^2	$n_{RIN}^2+n_{MPN}^2+n_{MN}^2+n_W^2$	
39	Total noise Variances normalized	σ_X^2	$NEF\,\rho_{eye}^2[(\rho_{Te}\frac{\rho_{eye-RIN}}{\rho_{eye}}\sigma_{MPN})^2+(\ ISI\,\sigma_{MPN}\ )^2+(ISI\,\sigma_{MN}\ )^2+1.22E-4]$	
40	Penalties Mix	P_X	$-10\log 10(\sqrt{1-(\frac{Q\,\sigma_X}{ISI})^2})$	$-10\log 10(\sqrt{1-\frac{Q^2(\sigma^2_X+\sigma_L^2)}{ISI^2}}$
41	Cross-Central Penalties	P_C	$P_X-(P_{ISI}+P_{DC}+P_{MPN}+P_{RIN}+P_{MN}+P_W)$	
42	Power Budget per eye	θ	$OMA_o^{Tx}-OMA_o^{Rx}-10\log 10(3)$	
43	Penalties Total	P_T	$P_{att}+P_X-P_w$	
44	Margin	Ω	$\theta-L_{con}-P_T+G_E-\sqrt{NEF_W}$	

Annex IV

Symbol Error Rate
Bit Error Rate

Symbol Error Ratio (statistical eye-amplitude method)

- The actual Q factor for each length is estimated using the following method.
- The maximum Q_{max} is obtained assuming zero AWGN noise at the detector (or infinite Tx OMA) . This parameter estimates the BER floor of the system

$$Q_{\max} = \frac{ISI}{\sqrt{(\sigma^2{}_X + \sigma_L{}^2)}} \qquad \text{(K.1)}$$

- The Q_{max} determines the BER floor of the link.
- From (H.2) the penalties to achieve a targeted Q, Q_T, were obtained. Here we use similar derivation used for (D.24) to determine the achievable Q factor, Q_a , when there is positive margin.

$$(\frac{\Delta}{2})ISI = Q_a(\sqrt{(\frac{\Delta_{\min}}{2Q_T})^2 + (n_X{}^2 + (\frac{\Delta}{2})^2 \sigma_L{}^2)} \qquad \text{(K.2)}$$

$$(\frac{\Delta}{2})ISI = (\frac{\Delta_{\min}}{2Q_T} Q_a) / (\sqrt{1 + (\frac{Q_a}{ISI})^2 (\sigma_X{}^2 + \sigma_L{}^2)} = (\frac{\Delta_{\min}}{2Q_T} Q_a) / (\sqrt{1 + (\frac{Q_a}{Q_{\max}})^2} \qquad \text{(K.3)}$$

Symbol Error Ratio (statistical eye-amplitude method)

- The penalty to produce Q_a is given by (now in dB)

$$P_a = 10\log 10\left(\frac{Q_a / Q_T}{\sqrt{1-(Q_a / Q_{\max})^2}}\right) \qquad \text{(K.4)}$$

- Using (J.3) and(J.5) and assuming that with Margin=0 , $Q_a = Q_T$ we obtain

$$\Omega = \theta - L_{con} - P_T + G_E - \sqrt{NEF_W} = \theta - L_{con} - (P_{att} + P_X - P_w) + G_E - \sqrt{NEF_W} = 0 \qquad \text{(K.5)}$$

$$P_{X0} = \theta - L_{con} - P_{att} + P_w + G_E - \sqrt{NEF_W} \qquad \text{(K.6)}$$

- Using, $P_X = P_{X0} + \Omega$ we obtain,

$$Q_a = \frac{1}{\sqrt{Q_T^{-2} 10^{-2(\Omega + P_{X0})/10} + Q_{\max}^{-2}}} \qquad \text{(K.7)}$$

Symbol Error Ratio (statistical eye-amplitude method)

- Based on (k.7) is the achievable Q at each length is bounded between f , 0=< Q_a <=Q_{max}
- When the margin is zero If Ω=0 Q_a =Q_T
- From previous equations the SER and BER can be computed. For a targeted BER the Q factor can be obtained as shown in king_01_1215_smf.pdf