Structure and Needed Properties of Reasonable Polarization Mode Dispersion Emulators for Coherent Optical Fiber Transmission

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Overview

- Physically reasonable modeling of polarization mode dispersion (PMD)
- Number of retarders and differential group delay (DGD) sections
- Type of time-variable retarders
- Speed of time-variable retarders
- Type and DGD range of DGD sections
- Example PMD emulator (PMDE) for coherent 80 km ZR link
- PMDE for long-range, high-end coherent transmission
- Availability of components and subsystems
- Conclusions
- Takeaway for coherent 80 km ZR link

This presentation is based on https://arxiv.org/abs/1903.05248.
In its reference list, all references cited in this presentation can be found.

Physically reasonable modeling of polarization mode dispersion (PMD)

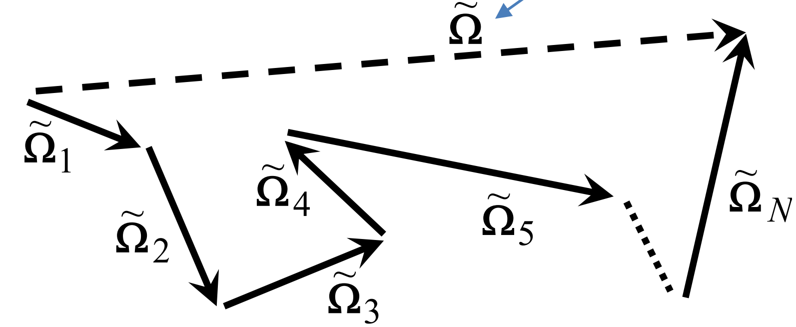
- Poole & Wagner, 1988: Existence of polarization mode dispersion (PMD) and differential group delay (DGD), calculated from condition for <u>first-order independence of output state-of-polarization (SOP), principal states-of-polarization (PSP) given as field vectors.</u>
- Noe et al., 1999: Derivation of PMD from <u>small-signal intensity modulation transfer function. PSPs given as Stokes vectors. PMD or DGD profile results from concatenation of individual PMD or DGD vectors.</u>

$\left|\tilde{\mathbf{\Omega}}\right| = \tau$

DGD

Total PMD vector, input-referred = 1st-order PMD

Individual PMD vectors, input-referred local

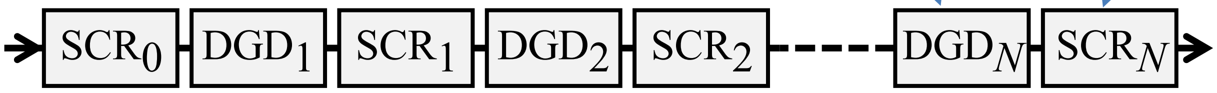


$$\tilde{\mathbf{\Omega}} = \sum_{i=1}^{N} \tilde{\mathbf{\Omega}}_i \qquad \tilde{\mathbf{\Omega}}_i = \left(\prod_{j<i} \mathbf{G}_j^T \right) \mathbf{\Omega}_i$$

Product of inverse rotation matrices of all retarders, including DGD sections, which precede the current DGD section. Sections twist as a function of ω!

N DGD sections

$N+1$ time-variable retarders/scramblers



Physically reasonable modeling of PMD

- Foschini & Poole, 1991: Taylor expansion of PMD vector to define higher-order PMD

$$\tilde{\boldsymbol{\Omega}}(\omega)=\tilde{\boldsymbol{\Omega}}(\omega_0)+(\omega-\omega_0)\tilde{\boldsymbol{\Omega}}'(\omega_0)+\frac{(\omega-\omega_0)^2}{2!}\tilde{\boldsymbol{\Omega}}''(\omega_0)+\ldots$$

PMD vector at reference frequency = 1st-order PMD

2nd-order PMD, SOPMD

<u>Example:</u> 2 DGD sections with total length τ_{max}

$\tilde{\boldsymbol{\Omega}}(\omega)$ gyrates about $\tilde{\boldsymbol{\Omega}}_1$ as a function of ω.

$$\tilde{\boldsymbol{\Omega}}(\omega)=\begin{bmatrix}1\\0\\0\end{bmatrix}\tau_1+\begin{bmatrix}\cos\delta\\ \sin\delta\cos((\omega-\omega_0)\tau_1)\\ \sin\delta\sin((\omega-\omega_0)\tau_1)\end{bmatrix}(\tau_{max}-\tau_1)$$

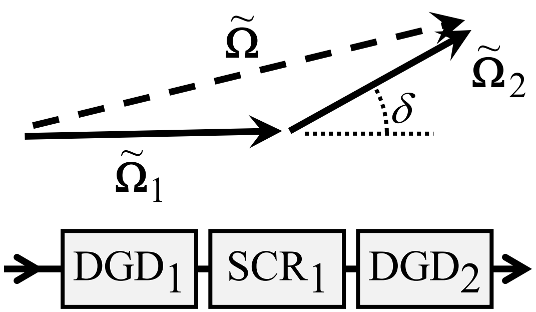
$$\left|\tilde{\boldsymbol{\Omega}}(\omega)\right|=\sqrt{\tau_1^2+(\tau_{max}-\tau_1)^2-2\tau_1(\tau_{max}-\tau_1)\cos(\pi-\delta)}$$

$$\delta=0\quad\Rightarrow\quad\boxed{\left|\tilde{\boldsymbol{\Omega}}(\omega)\right|_{max}=\tau_{max}}$$

2nd-order PMD:

$$\tilde{\boldsymbol{\Omega}}'(\omega)=\begin{bmatrix}0\\ -\sin\delta\sin((\omega-\omega_0)\tau_1)\\ \sin\delta\cos((\omega-\omega_0)\tau_1)\end{bmatrix}\tau_1(\tau_{max}-\tau_1)$$

$$\left|\tilde{\boldsymbol{\Omega}}'(\omega)\right|=\sin\delta\cdot\tau_1(\tau_{max}-\tau_1)$$



Maximum 2nd-order PMD: $\delta=\pi/2$ $\quad\tau_1=\tau_{max}/2=\tau_{max}-\tau_1\quad\Rightarrow$

$$\boxed{\left|\tilde{\boldsymbol{\Omega}}'(\omega)\right|_{max}=\tau_{max}^2/4}\qquad\left|\tilde{\boldsymbol{\Omega}}(\omega)\right|=\tau_{max}/\sqrt{2}$$

$$\boxed{\left|\tilde{\boldsymbol{\Omega}}'(\omega)\right|_{max}=s\cdot\left|\tilde{\boldsymbol{\Omega}}(\omega)\right|_{max}^2}$$

Here $\boxed{s=0.25}$, else $\boxed{\text{Cases found so far: } s\le 1/\pi\approx 0.32}$??

Bent DGD profile with semicircle shape

Physically reasonable modeling of PMD

- Foschini & Poole, 1991: Taylor expansion of PMD vector to define higher-order PMD

$$\tilde{\boldsymbol{\Omega}}(\omega)=\tilde{\boldsymbol{\Omega}}(\omega_0)+(\omega-\omega_0)\tilde{\boldsymbol{\Omega}}'(\omega_0)+\frac{(\omega-\omega_0)^2}{2!}\tilde{\boldsymbol{\Omega}}''(\omega_0)+\ldots$$

PMD vector at reference frequency = 1st-order PMD

2nd-order PMD, SOPMD

However: A truncated Taylor expansion is a very bad approximation of periodic or quasi-periodic phenomena! For large deviations from the reference frequency, infinite PMD is predicted inevitably! But this is totally contradicted by the fact that the expectation value of (1st-order) PMD is the same for all frequencies!

It makes much more sense to model PMD naturally, by concatenated DGD sections!

Suppression of cross polarization by equalizers (= inverted structures) defined by higher-order PMD definition methods

1st-order PMD is the same for any modeling method.

Method order	1	2	3
Gaussian input pulse width [a.u.]	5.6	7.2	9.5
Taylor expansion of PMD vector (TEPV)	10.3 dB	14.8 dB	19.9 dB
Exponential Jones matrix expansion (EMTY)	10.3 dB	12.6 dB	16.1 dB
Fourier expansion of mode coupling (FEMC)	10.3 dB	21.6 dB	35.5 dB

+9.6 dB

+25.2 dB

- dB and pulse width values are averaged over 75 PMD … .
- … all methods, using the value obtained … e p… improvement of high method orders …

This method is very similar to a sequence of cascaded DGD sections.

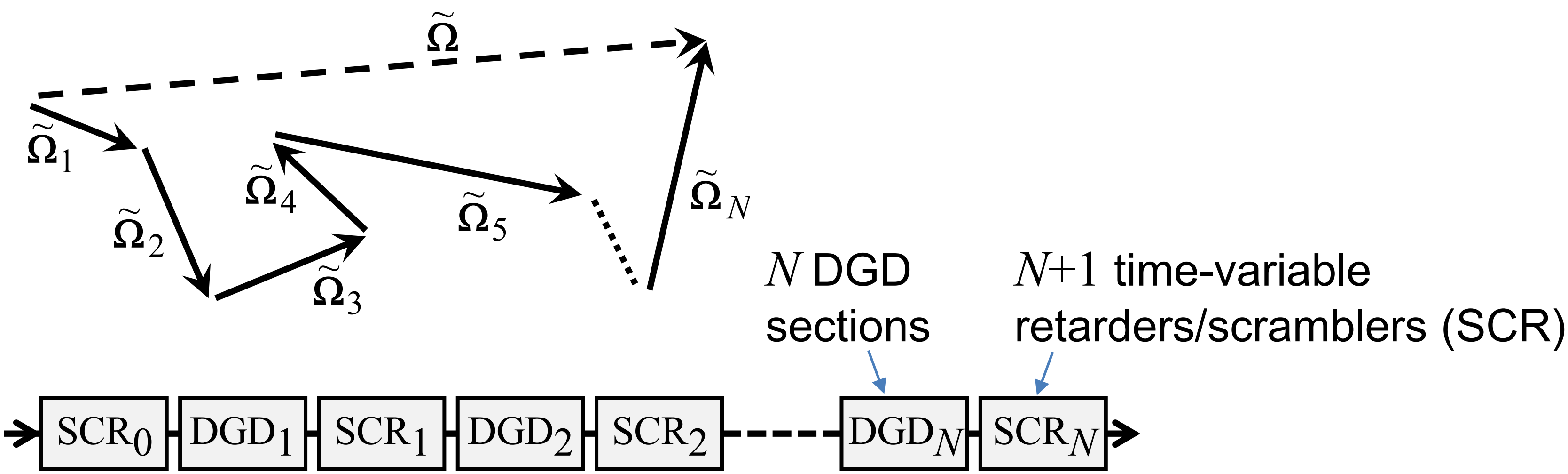
Added parameters or orders give here a much better approximation of natural PMD!

- Extinction improvement of higher-order FEMC over 1st-order PMD seems to be ≥2 times larger (in dB) than that of TEPV or EMTY !
- Reason: FEMC (and SDGD) are closely related to natural PMD, unlike higher-order TEPV and EMTY.
- Drawback: Finding FEMC coefficients is a numerical optimization process ⇒ more research is needed.

Physically reasonable modeling of PMD

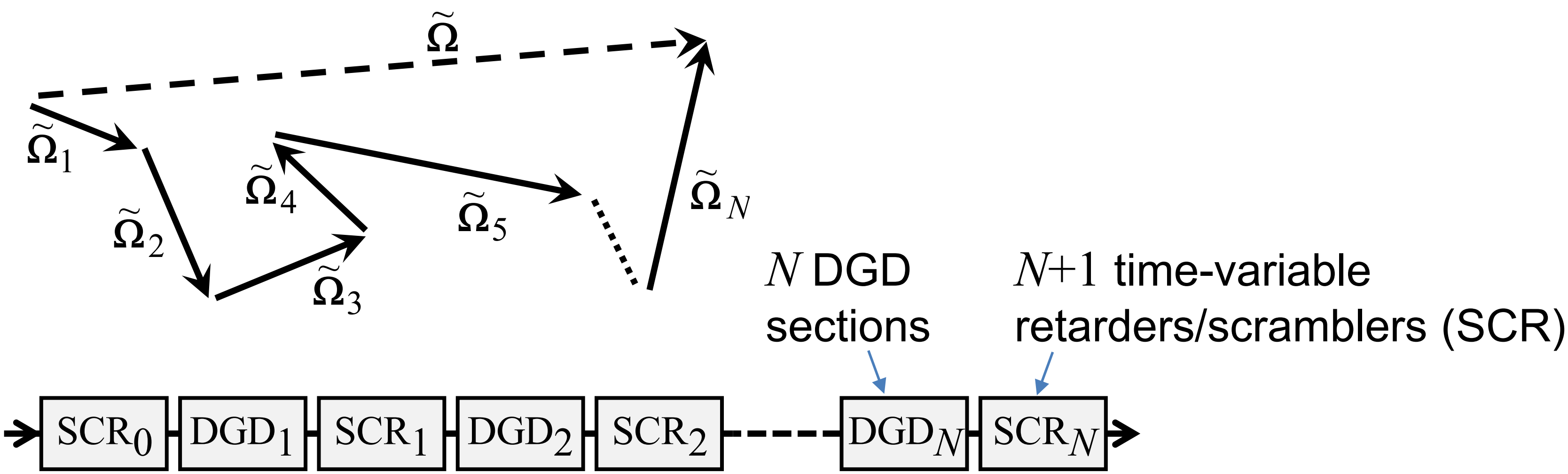
- Fourier Expansion of Mode Coupling (Noe et al. 2005) means a bent DGD profile.
- Its higher-order PMD modeling is far better („>2·dB“) than for the Taylor expansion of the PMD vector.
- When discretized, it becomes a sequence of cascaded DGD sections. This technology is available! We propose it for a PMD emulator (PMDE)! ⇒

Proposed PMD emulator structure



Number of retarders and DGD sections

- N DGD sections and N+1 time-variable retarders / polarization scramblers (SCR) with $N \to \infty$ are generally considered as optimum. 1st-order PMD gets a Maxwellian distribution.
- But $N \to \infty$ is too costly for a PMDE. Strong/frequent/fast polarization fluctuations can sometimes be generated at specific places (such as bridges, exchange offices). DGD profile (Noe et al. 1999) is also called hinge model (M. Brodsky et al., 2004, M. Boroditsky et al. 2005, H. Kogelnik et al. 2005). $\Rightarrow$ Small N seems permissible.
- Symmetric DGD profile permits setting zero PMD of all orders (neutral PMDE). $\Rightarrow$ Even N
- We think $N = 4$ would be better but $N = 2$ seems acceptable. $N = 1$ is of course possible but covers only 1st-order PMD.



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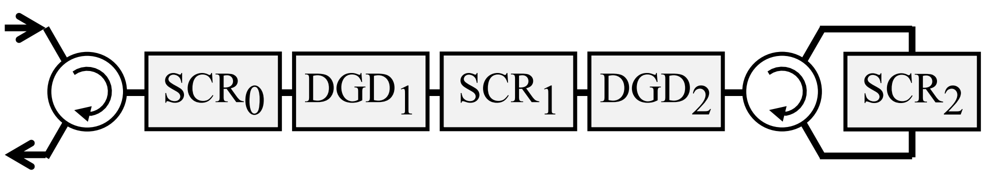
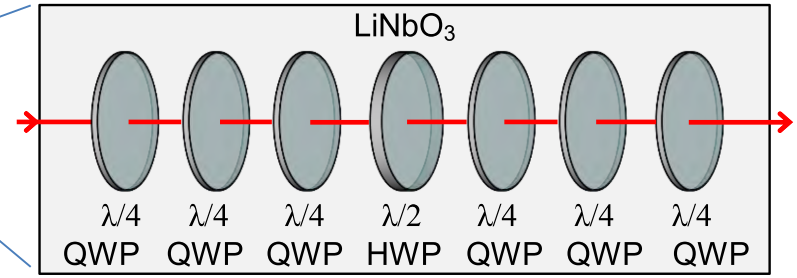
vith $N = 2$



Bidirectional structure with circulators could be advantageous. Not yet assessed!

Type of time-variable retarders

- All time-variable retarders of a link can be expected to behave as general endless elliptical retarders with 3 degrees-of-freedom (DOF).
- It was shown (Noe et al. 1999) that only one of them needs to be elliptical. The others only need to be able to endlessly transform any input polarization into a PSP of the subsequent DGD section, which needs 2 DOF.
- In practice, the SCR will not be aligned to the subsequent section PSPs. Hence, all SCR should be endless elliptical retarders with 3 DOF.
- Example of elliptical retarder: Rotating electrooptic waveplates in X-cut, Z-prop. $LiNbO_3$
- Due to unwanted waveplate ellipticities, >3 waveplates are practically needed for reliable operation.

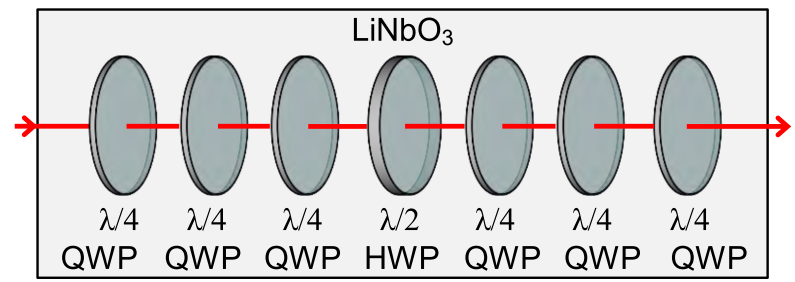


- Similar, much stronger constraints hold on fiber squeezers or liquid crystals with constant eigenmodes: 4 are needed in principle, but in practice, endless polarization control speed falls back several orders of magnitude compared to that of state-of-the-art $LiNbO_3$-based polarization controllers. >4, maybe 6 devices would be recommended/needed in each SCR.
- But the biggest practical challenge for fiber squeezers could be ageing: Will efficiency of piezo squeezers stay sufficiently large?

⇒ I (R. Noe) have given up my successful pioneering work with fiber squeezers ~30 years ago.

Speed of time-variable retarders

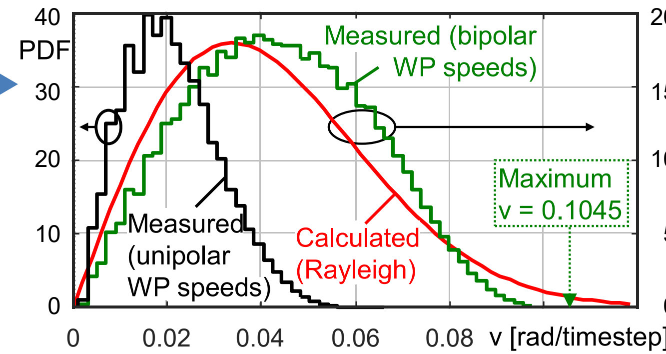
- To prove endless polarization transformation capability of a polarization transformer one must be able to operate it as an endless controller/tracker with quantifiable low outage probability at the highest possible speed. Example (Koch et al. 2011): Relative intensity error 0.45% is surpassed with probability of 10^{-10} when tracking 40 krad/s.
- All the same, scrambling speed can be 2…3 orders of magnitude higher, mainly because no searching/feedback is needed.
- Fiber polarization fluctuations due to lightning strikes with speeds up to 5.1 Mrad/s have been observed (D. Charlton et al. 2017). No upper speed bound is known!
- Time-variable retarders should be able to replicate the fluctuations caused by lightning strikes. These are polarization rotations due to the Faraday effect.



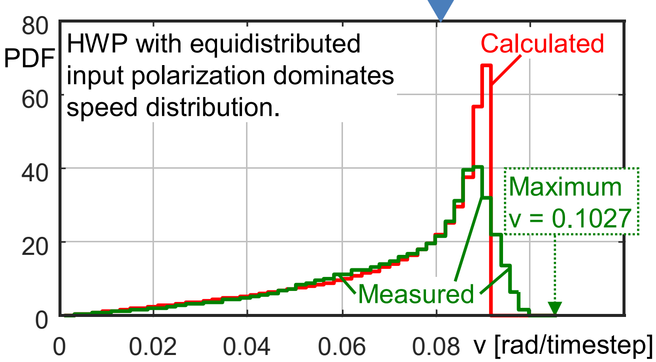
- A fast rotating halfwave plate (HWP) can do this. When it is preceded and followed by several quarterwave plates (QWP) the HWP rotations can be oriented and sized arbitrarily.
- To cover lightning strikes with some headroom, a maximum scrambling speed of 20 Mrad/s is recommended. Even 50 Mrad/s devices are on the market today (Novoptel).
- What does this have to do with 80 km ZR links? A 20 Mrad/s PMDE will be usable for long-haul, too! Procuring it avoids the need to buy an additional slow PMDE.

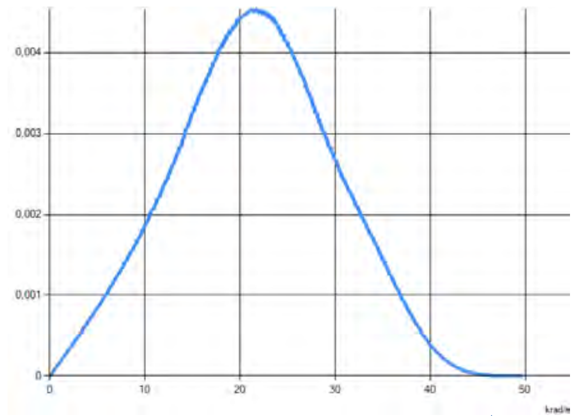
Speed of time-variable retarders (waveplates in $LiNbO_3$)

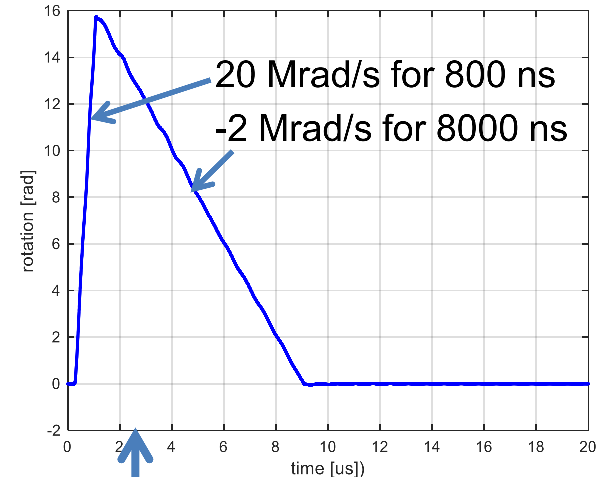
- The 7 rotating waveplates in $LiNbO_3$ allow generating a relatively nice Rayleigh distribution.



- The fast rotating HWP between slower rotating QWPs generates and dominates a peaked speed distribution (Koch et al. 2016). This permits accelerated testing (say, 50 times) compared to a truncated Rayleigh speed distribution with the same maximum speed.





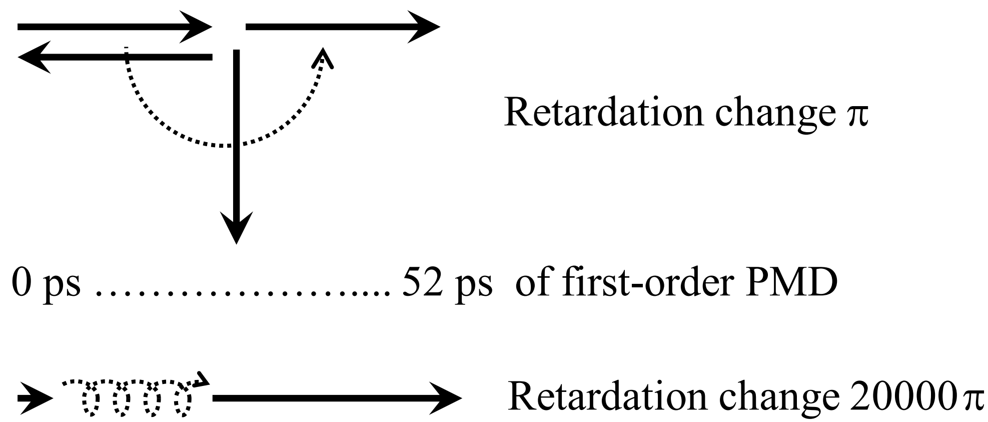


- Superposition of various peaked speed distribution permits generating a broad variety of speed distributions, including triangular (though I see no particular value in this).

- Emulation of polarization rotation due to lightning strikes by programmed HWP rotations
- Switching: **80 ns** (Novoptel)

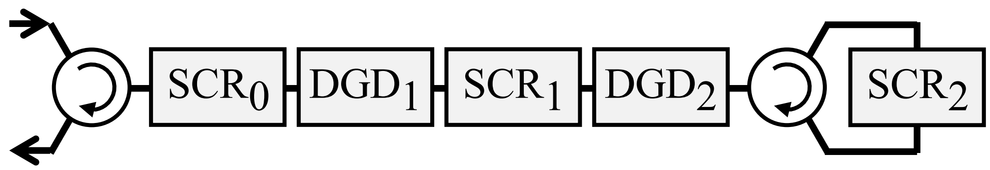
Type and DGD range of DGD sections

- In this example, the total 1st-order PMD can be changed from 0 ps to 52 ps by eithe a retardation change of π or a retardation change of 20000π.
- A retardation change by π is many orders of magnitude more likely than a change by 20000π !
- This shows that a section model with fixed
- Hence, variable DGD sections are unphysical.
- They can be tolerated all the same, provided that their drift or vibration is slow against polarization changes caused by the time-variable retarders.
- To change total DGD it is expected to be much cheaper to switch fixed DGD sections by optomechanical switches than to provide variable DGD sections.
- Choices to change section DGDs: Plug PMFs (not automatic), switch PMFs (automatic and cheap), or use variable DGD units (could add unwanted PDL)



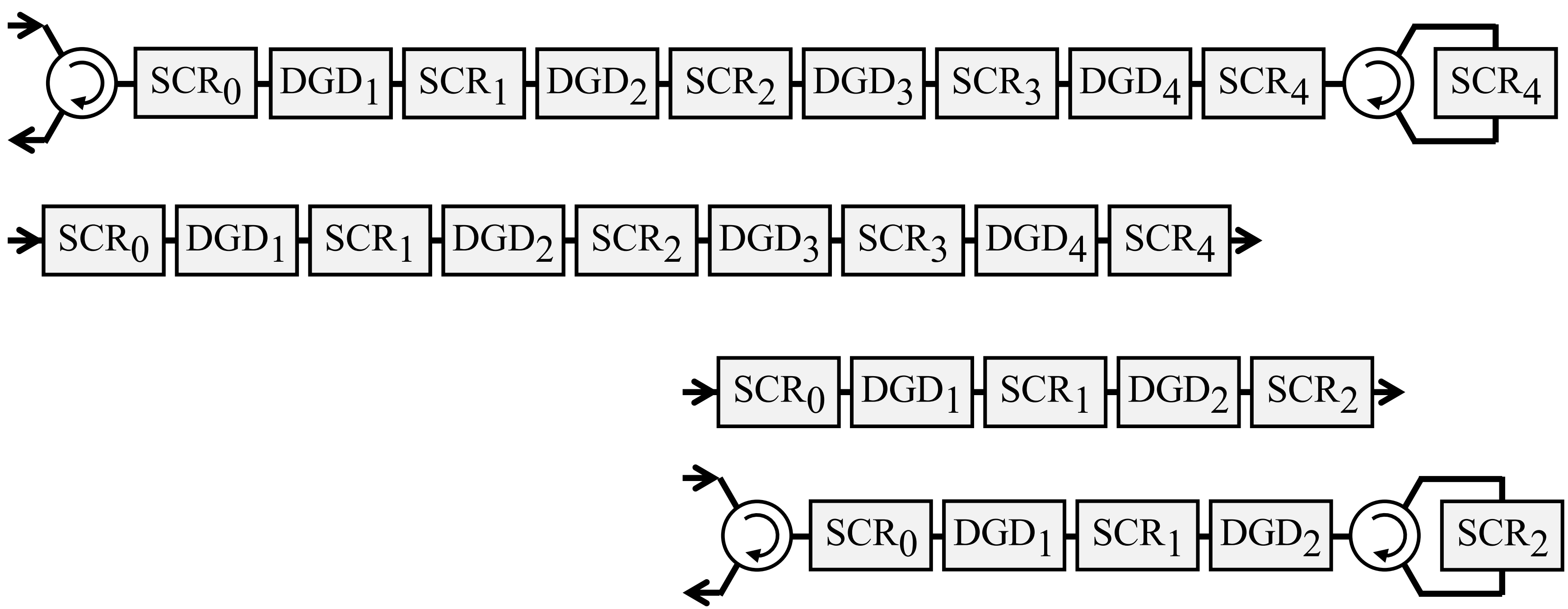
PMDE proposal for coherent 80 km ZR link

- **2 DGD sections and 3 time-variable retarders / polarization scramblers**
- Maximum DGD = $\tau_{\max}$ = 33 ps $\Rightarrow$ 16.5 ps of DGD per section
- Maximum SOPMD = 500 (ps)^2 $\Rightarrow$ 22.4 ps of DGD per section (or 39.6 ps in total for $N \to \infty$)
- But this discrepancy is typical, due to $\left|\tilde{\Omega}'(\omega)\right|_{\max} \leq 0.32 \cdot \left|\tilde{\Omega}(\omega)\right|^2_{\max}$.
- So, if the equalizer can generate 500 (ps)^2 of SOPMD it is able to generate a DGD of 45 ps. If it can generate a DGD of 33 ps it is able to generate a SOPMD of 272 (ps)^2. Smaller discrepancies for larger N.
- **Choose total DGD section length to match MIMO equalizer length!** (33 ps? 45 ps?)
- Caution: If few fixed equal PMDE section DGDs are known (e.g. 2 with 22.4 ps) then one could design the equalizer so that it can handle impulse responses which are nonzero only at a few places (e.g., –22.4 ps, 0 ps, +22.4 ps). This can be overcome by larger N (more nonzero portions of impulse response) or switchable or variable DGD sections (variable nonzero positions in impulse response).
- Retarders/scramblers should be **elliptical, based on 5...7 waveplates in $LiNbO_3$**.
- **Bidirectional structure may be advantageous.**



PMDE for long-range, high-end coherent transmission

- Same as for 80 km ZR, but,
- switchable (or variable) DGD sections,
- maybe more DGD sections, e.g. N = 4,
- much faster retarders/scramblers, e.g. 3 QWP + 1 HWP + 3 QWP, the HWP running at up to 20 Mrad/s.
- Re-use of PMDE for 80 km ZR is possible!



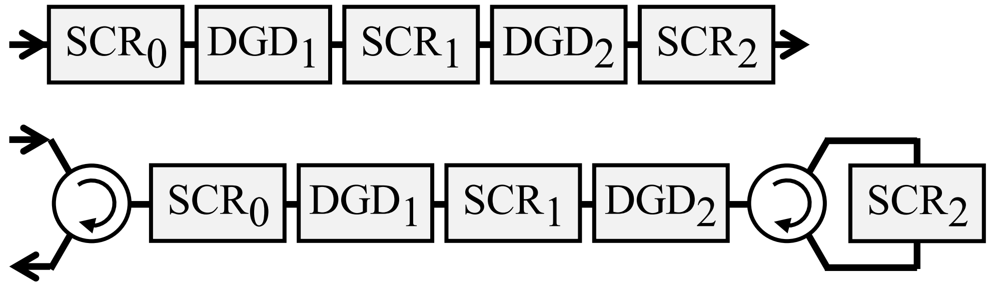
Availability of components and subsystems

- $LiNbO_3$ polarization transformers: EOSPACE, Fiberpro (arriving)
- Polarization scramblers based on $LiNbO_3$: Novoptel EPS1000, Newridge Technologies, Viavi, Keysight.
- Polarization scramblers based on fiber squeezers: Luna (General Photonics)

- Fixed DGD sections: Easy, cheap, off-the-shelf solution with PMF and possibly optomechanical switches
- Motorized variable DGD sections: Luna (General Photonics)

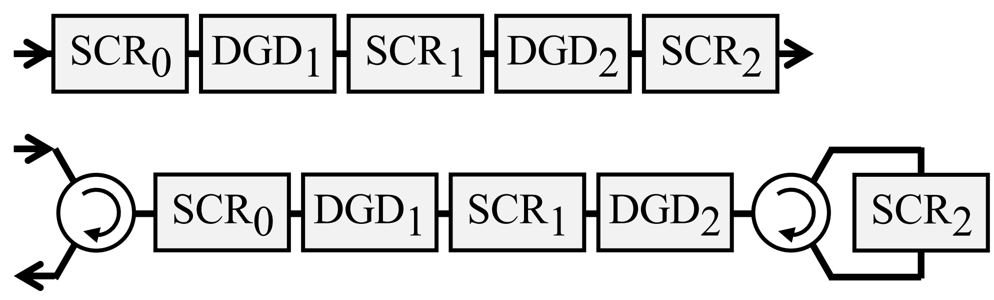
- Complete PMDEs: Novoptel (complying to the design proposal made here) https://www.novoptel.de/Scrambling/Scrambling_PMDE1000_en.php, presumably Newridge Technologies, Luna (General Photonics) and maybe others, at least in the near future.

- Additional info is welcome for enhancing the producer list.



Conclusions

- Natural PMD behaves like many fixed DGD sections with elliptical retarders in between.
- Truncated Taylor expansion of PMD vector doesn‘t make sense and becomes unnecessary once one decides on number of DGD sections.
- **PMD emulator should have N DGD sections between N+1 time-variable retarders.**
- ***N* = 2 or *N* = 4** seems to be a good choice.
- **DGD sections can be fixed, switchable or variable.**
- **Maximum PMD (≡ sum of DGD section lengths) should match MIMO equalizer length.**
- Time-variable retarders/scramblers should be **elliptical endless retarders**. A good choice are **7 waveplates in $LiNbO_3$. The faster, the better.**
- PMD testing is accelerated if the scrambler contains a fast rotating HWP between several slower rotating QWPs.
- 80 km ZR PMDE design is **re-usable** for high-end, long-range coherent transmission.
- Sufficient industrial basis and competition guarantees cost-effective PMDE supply.
- Bidirectional structure may be advantageous
- PMDEs are on the market.



Takeaway for coherent 80 km ZR link

- PMD emulator should contain **2 DGD sections placed between 3 time-variable retarders**.
- Given the 500 ps^2 2nd-order PMD spec, the coherent receiver will automatically tolerate impulse responses which are 40 ps long. $\Rightarrow$ **Each section should have a DGD of 20 ps.** This covers already the 33 ps 1st-order PMD spec.
- To check also for smaller PMD, **each DGD section should be switchable to 10 ps**, and to 2 external ports for connecting more DGD samples (if needed) or for bypassing section(s).
- **Time-variable retarders should be based on $LiNbO_3$ rotating waveplates, with 3 quarterwave plates, 1 halfwave plate and 3 more quarterwave plates.** Accelerated testing by fast rotating halfwave plate and sudden polarization jumps is recommended.
- **Maximum polarization change speed 50 krad/s. Scalability to 20 Mrad/s strongly recommended for re-usability in long-range coherent transmission.**
- At least one such PMD emulator is already commercially available.
- If the above appears too complicated, 1 DGD section that is switched 0 ps, 11 ps, 22 ps, 33 ps between 2 time-variable retarders is recommended.
- The above could be altered, depending for instance on coherent receiver hardware or spec changes. Alterations could encompass: Section DGDs, number N, bidirectional setup

$$\rightarrow \boxed{SCR_0} - \boxed{DGD_1} - \boxed{SCR_1} - \boxed{DGD_2} - \boxed{SCR_2} \rightarrow$$

Following pages contain spare viewgraphs for supplementary info if needed and do not belong to the foregoing presentation.

HWP works like circular retarder

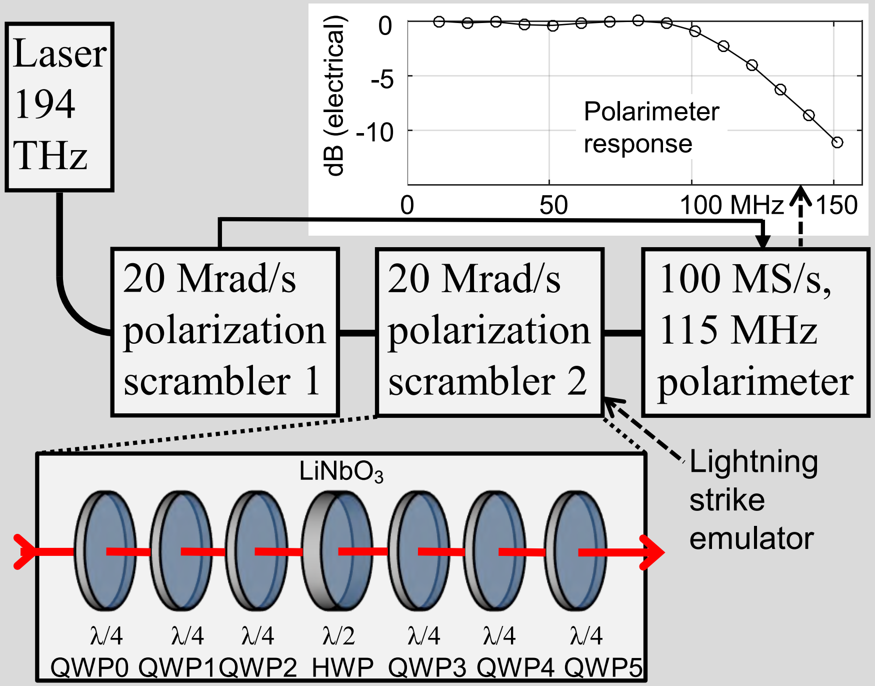
- Rotation matrix of HWP:

$$\mathbf{HWP}(\psi)=\begin{bmatrix}\cos(2\psi) & \sin(2\psi) & 0\\ \sin(2\psi) & -\cos(2\psi) & 0\\ 0 & 0 & -1\end{bmatrix}$$

- Polarization change inside fiber can be written as:

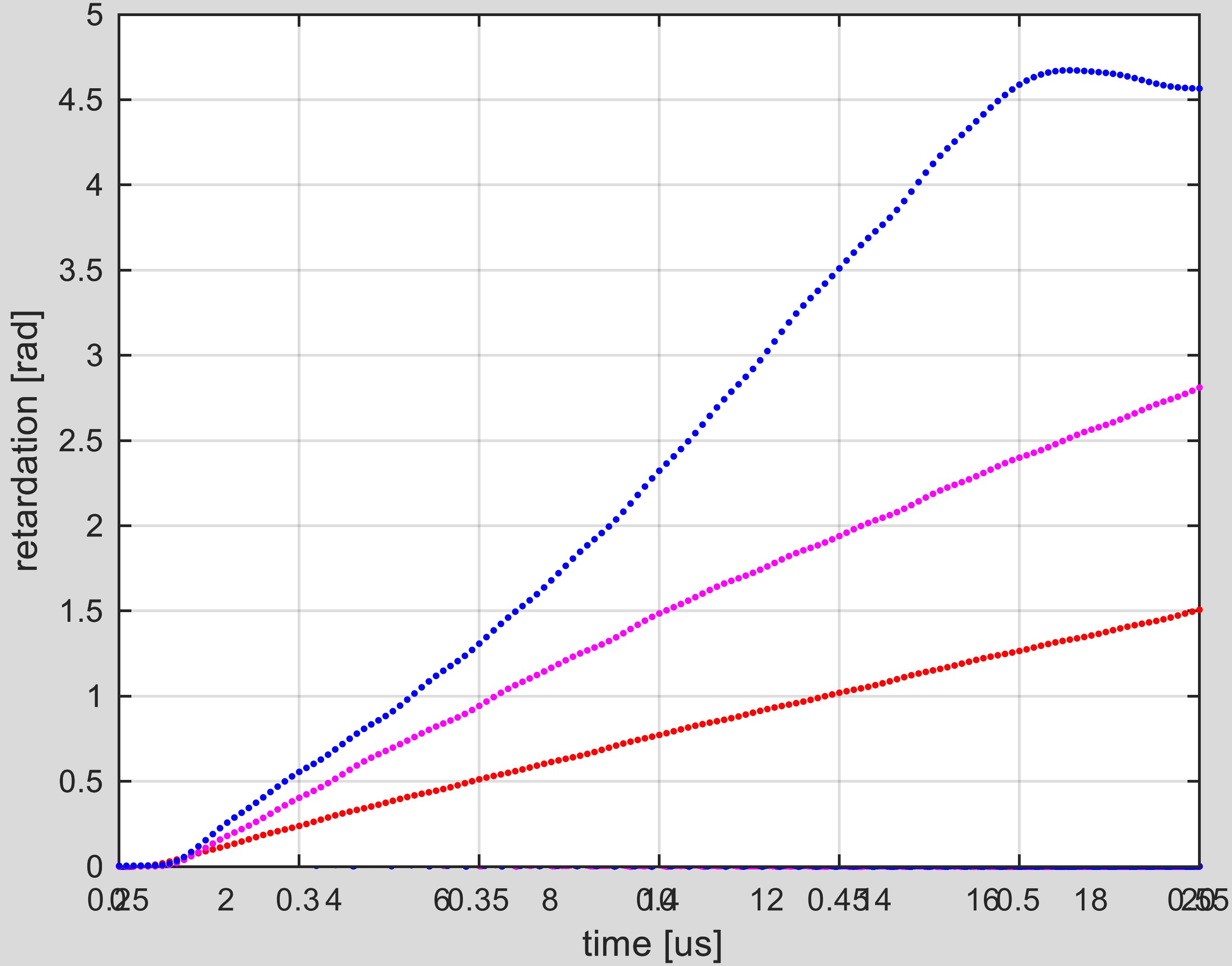
$$\mathbf{HWP}(\psi)\mathbf{HWP}(0)^T$$
$$=\begin{bmatrix}\cos(2\psi) & -\sin(2\psi) & 0\\ \sin(2\psi) & \cos(2\psi) & 0\\ 0 & 0 & 1\end{bmatrix}$$

- Total effect is like that of a circular retarder.
- This happens to be the same as caused by the Faraday effect during lightning strike.



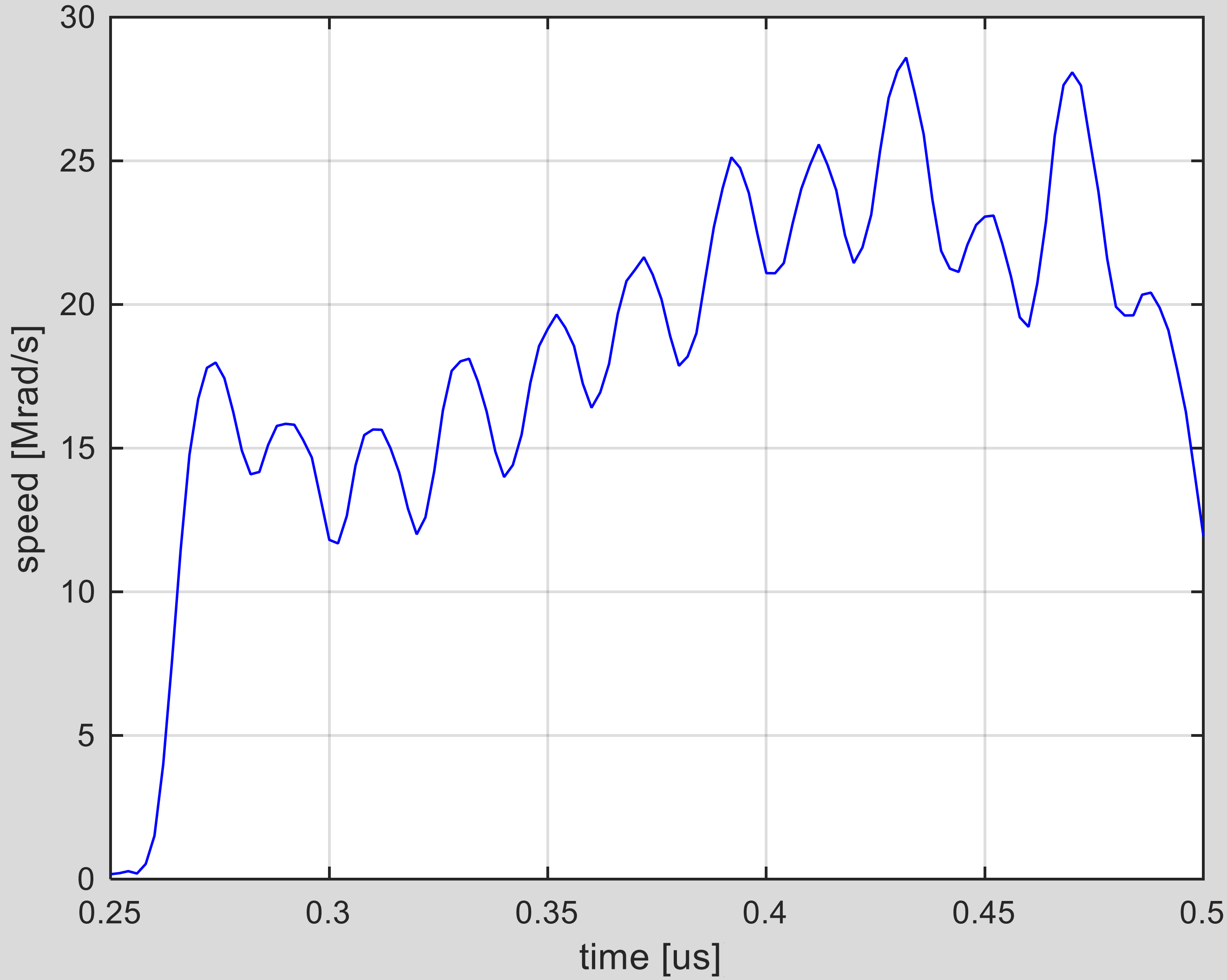
Perceived circular retardation

- 4.8 rad peak retardation
- Ascent with 5 / 10 / 20 Mrad/s
- Descent with 1/10 of ascent speed
- 2 ns temporal resolution



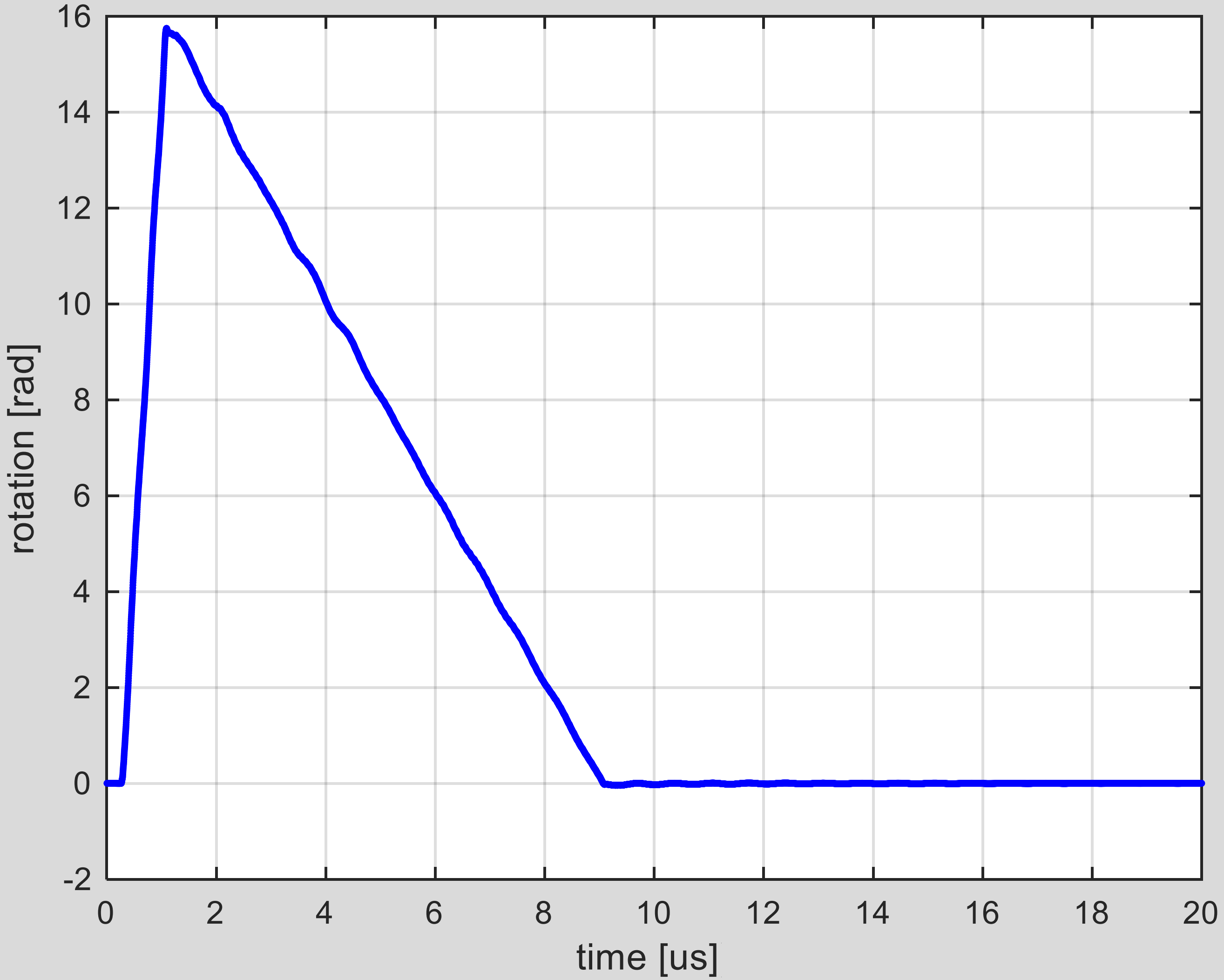
Temporal derivative of retardation

- Mean speed is 20 Mrad/s, as expected.
- 20 ns undulation is due to 20 ns sampling of rotating HWP.



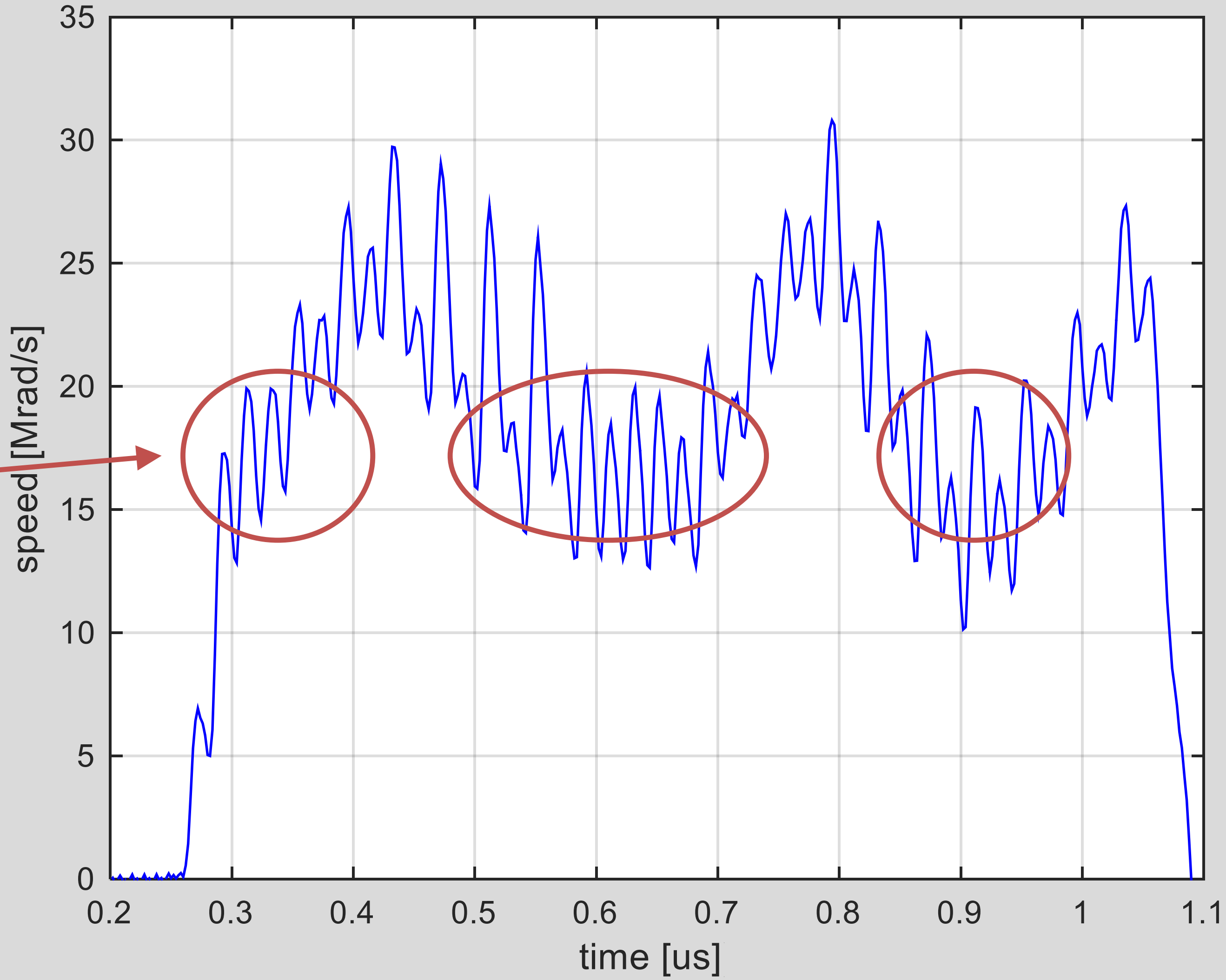
Retardation >2π ? Rotation around mean axis >2π !

- For a retarder with constant eigenmodes it holds: During a retardation change 0...2π... the rotating polarization needs to return exactly to the original value.
- Since this is not exactly found in practice, eigenmodes will change fairly abruptly and retardation will decrease below instead of increasing beyond 2π.
- So, simply the rotation angle around a mean axis is determined. This can go arbitrarily high during HWP rotation.



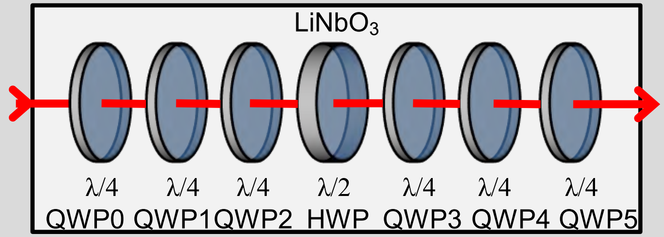
Temporal derivative of rotation around mean axis

- Mean speed is 20 Mrad/s, as expected.
- 20 ns undulation is due to 20 ns sampling of rotating HWP.
- 320 ns undulation is due to sluggish large-signal behavior of electrode voltage amplifiers.



Summary and ...

- Limited only in speed at currently 20 Mrad/s, any polarization rotation pattern can be generated. E.g., 20 Mrad/s · 5 µs = 100 rad.
- 20 ns and 320 ns speed undulations are believed to be uncritical because they hardly affect polarization rotation slopes.
- Larger speeds (40 Mrad/s?) and other eigenmode/retardation behavior could be implemented.
- 3 QWPs before and 3 QWPs behind HWP can and should randomize rotation axis. Currently 1 Mrad/s per QWP available; more is easily feasible.
- Can be repeated every, say, 20 µs.
- Several synchronized polarization scramblers with differential group delay elements in between them would permit emulation of more complicated scenarios (if needed).
- For instance, full conversion between circular polarizations lets Faraday effects counteract before and behind mode converter. PMD is therefore likely to reduce overall effect.



Lightning strike emulator

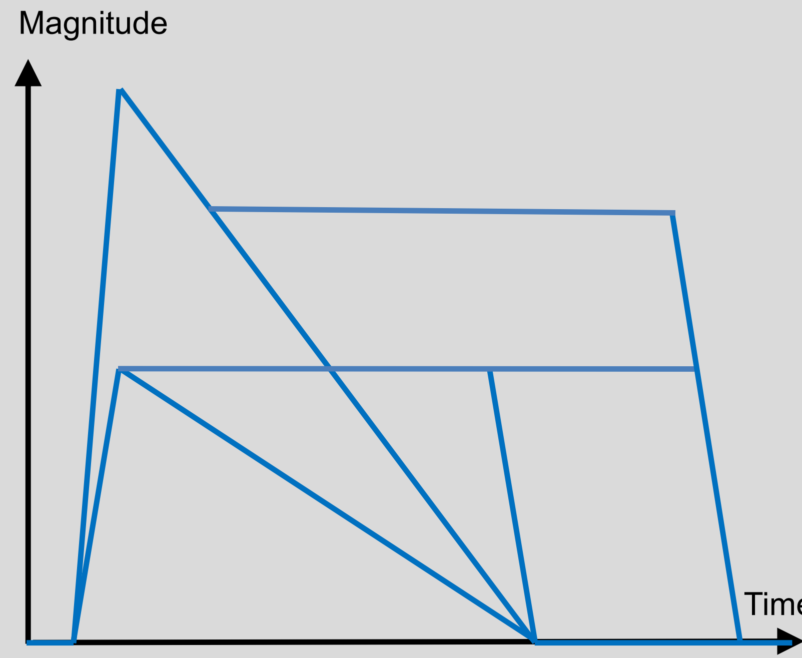
... discussion of „standard“ lightning strike emulation

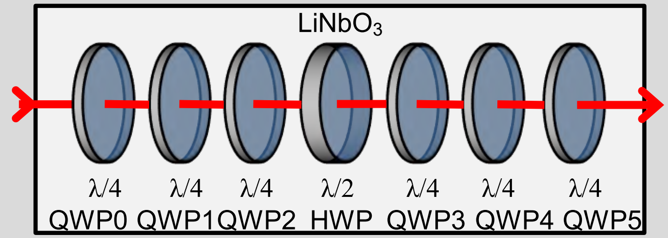
- Polarization rotation speed? >20 Mrad/s?
- Magnitude? 10 rad? 100 rad? Shape? Duration?
- Constant rotation axis? Even with PMD?
- Or are mostly speed and magnitude decisive?
- 3 QWPs before and 3 QWPs behind HWP should randomize rotation axis.
- **Parametrized sets of lightning strike emulation for transponder manufacturers and telecom carriers.**
- To be generated by polarization scramblers (such as 20 Mrad/s EPS1000) and verified by polarimeters (such as 100 MS/s PM1000).

⇒ **„Standard“ lightning strike emulation**

⇒ Open for further contributions. Input highly welcome!

- **Proposal: Different shapes (32), speed (1, 2, 5, 10, 20 Mrad/s) and duration (1, 2, 5, 10, 20 μs) scalable, randomized axis (0.1, 0.2, 0.5, 1 Mrad/s)**

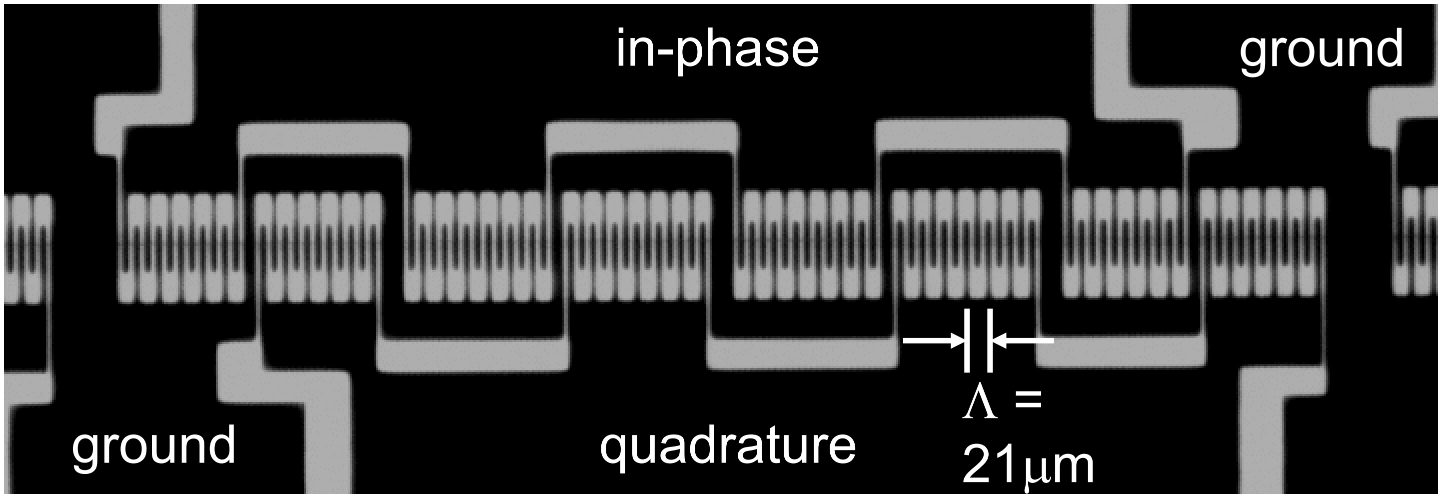




Lightning strike emulator

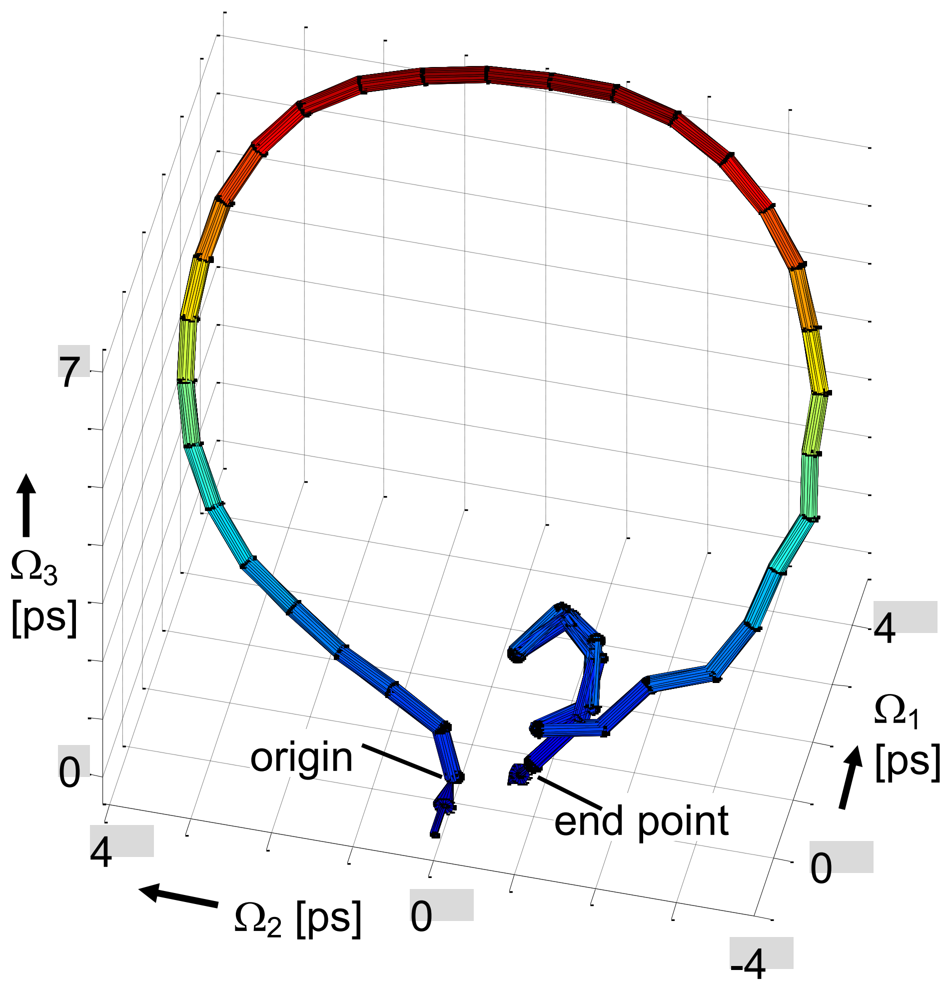
Fabricated by Prof. Sohler

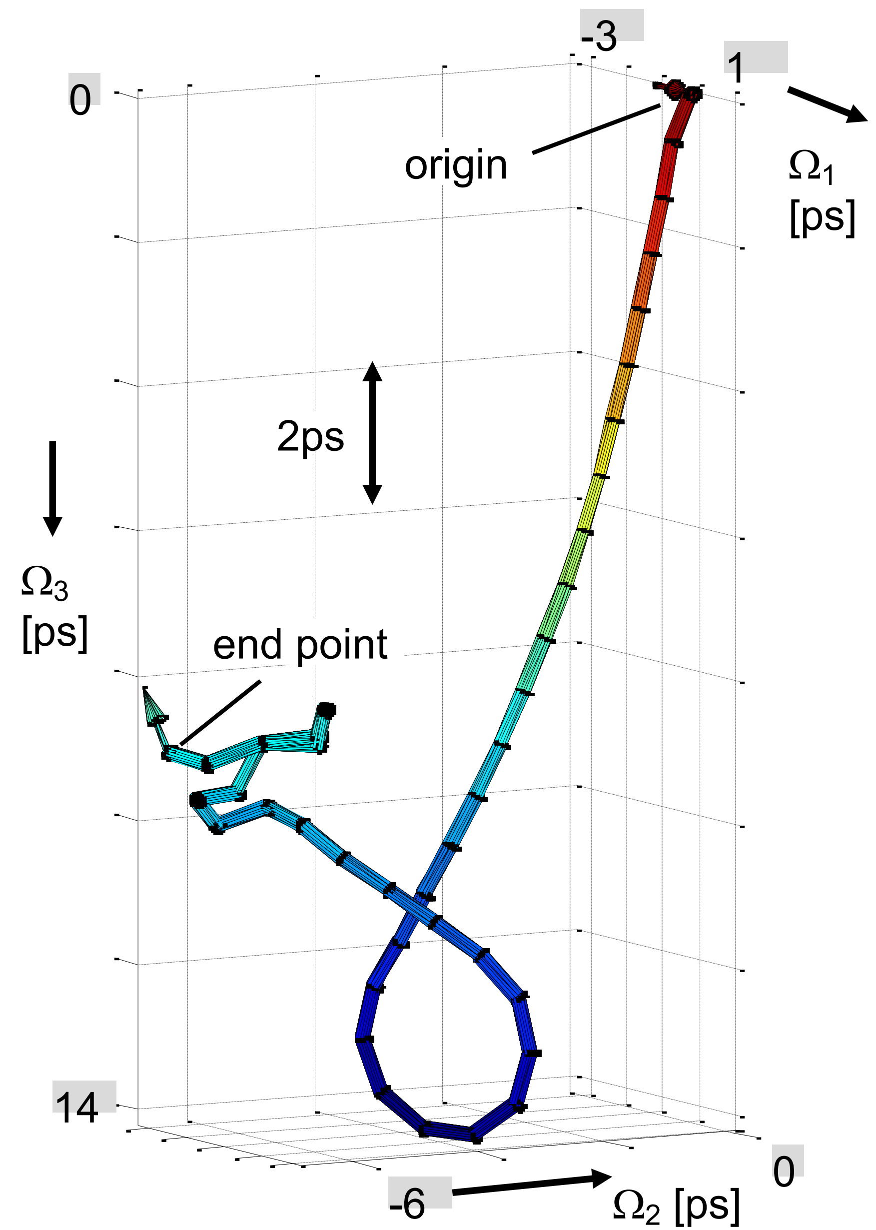
Distributed PMD compensator in X-cut, Y-propagation $LiNbO_3$



- Optical bandwidth ~3 THz
- Thermal tuning ~100 GHz/K
- Voltages <80V
- 73 electrode pairs (~1.25 mm) on 93 mm long substrate
- Combined differential group delay of 2 units: 43 ps

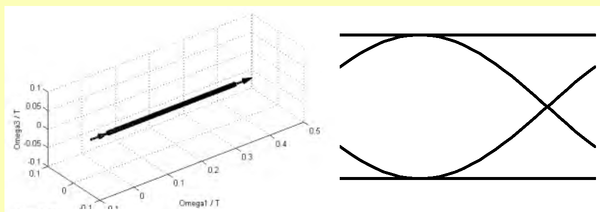
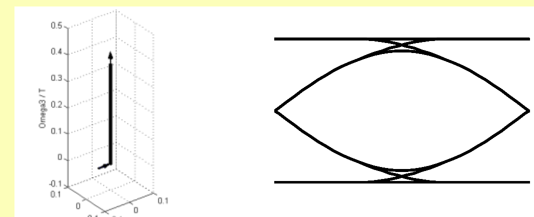
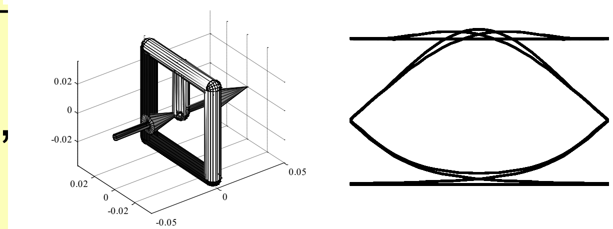
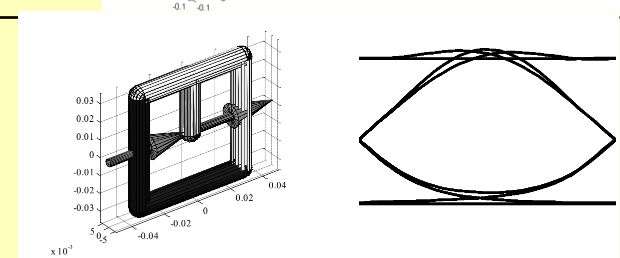
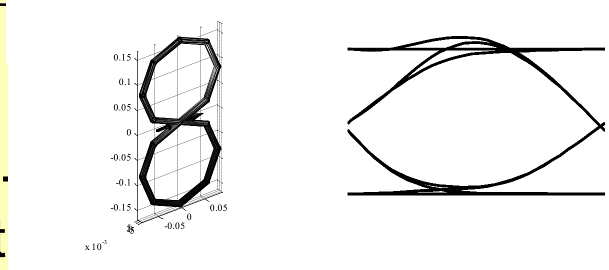
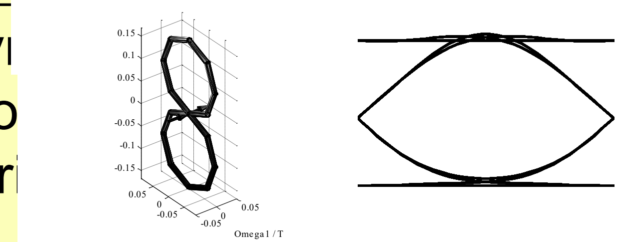
Measured differential group delay profiles of distributed PMD compensator





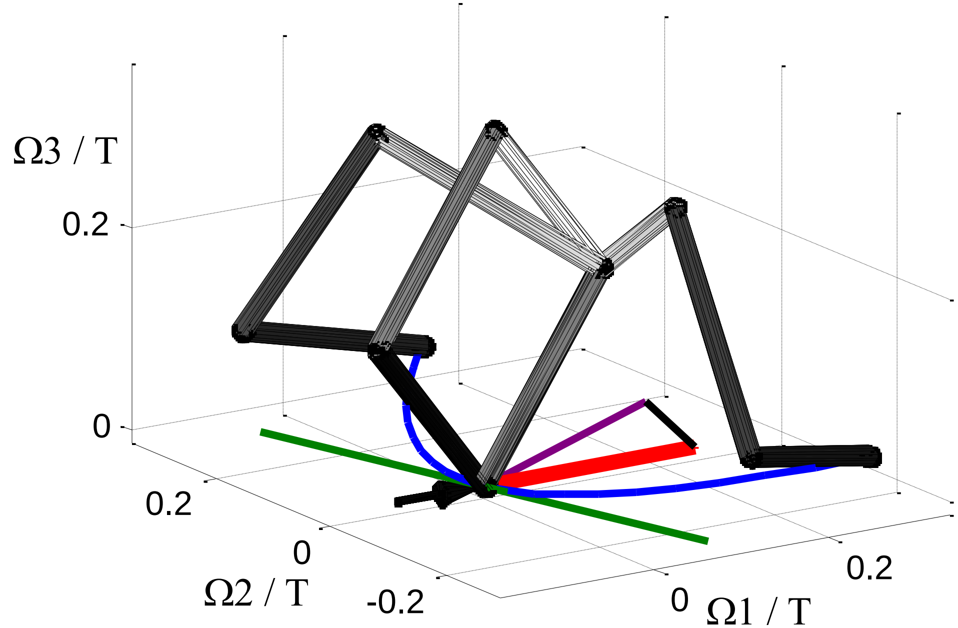
PMD definition and categorization

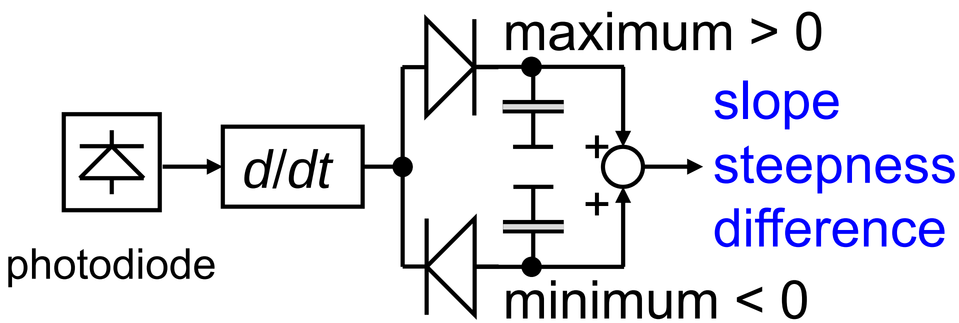
- Taylor series expansion of PMD vector is unphysical because PMD changes quasi periodically as a function of frequency.
- If Taylor series is used: Categorize various orders of PMD depending on their relation to the **input polarization**.

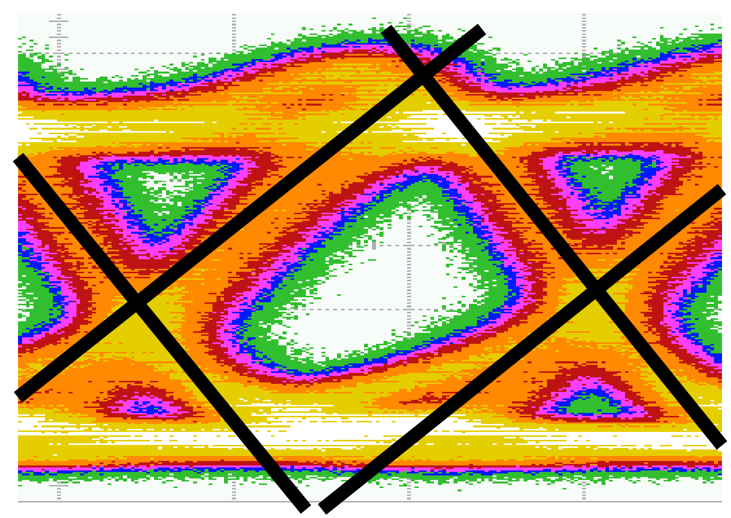
order	parallel to input polarization	perpendicular to input polarization: **mix of opposed parallel cases**
1	**delay** 	symmetric eye closure 
2	**(2nd-order) CD**, adds to fiber CD, symmetric overshoot, curva 	(depends on fiber CD) 
3	**3rd-order CD**, slope steepness difference, asym- metric overshoot 	vertically asy metric, horizo tally symmetri eye closure 

Slope steepness difference indicates higher-order PMD

- Assuming perfect arrival time detection, resulting DGD profile of fiber and PMD compensator will most likely form a loop.
- As a function of optical frequency, sections with given constant DGDs twist, thereby sliding loop endpoint on a parabola.
- Projection of parabola ordinate along input polarization causes eye diagram shear proportional to loop area.
- Slope steepness difference variations always exists due to scrambling.







Fourier expansion of mode coupling (FEMC)

- A frequency-independent mode conversion at the fiber input. This is described by 2 parameters, for example retardation and orientation of an SBA.
- A total DGD.
- A frequency-independent mode conversion at the fiber output. In the general case a mode conversion (2 parameters, as at the input) and a differential phase shift (one more parameter) are needed. In total this means that there is a frequency-independent elliptical retarder at the output.
- Complex Fourier coefficients F_k of mode coupling along the birefringent medium, which exhibits the above total DGD only in the absence of mode conversion.

Soleil-Babinet analog (SBA)

retardation (= bend angle)

orientation (= bend orientation)

$$F_k = \int_0^L \left(\frac{d\varphi(z)}{dz} e^{j\psi(z)} \right) e^{-j2\pi k z/L} dz$$

Order and number of real parameters in higher-order PMD definition methods

Method (below) and its order (right)	1	2	3	4
Taylor expansion of PMD vector (TEPV, Jones matrix given by Heismann)	3	6	9	12
Exponential Jones matrix expansion (EMTY = Eyal, Marshall, Tur, Yariv)	3	6	9	12
Sequence of DGD sections (SDGD)	3	5	7	9
Fourier expansion of mode coupling (FEMC)	3	5	9	13

1st-order PMD, identical for all methods

F_0, uniform bending of DGD profile

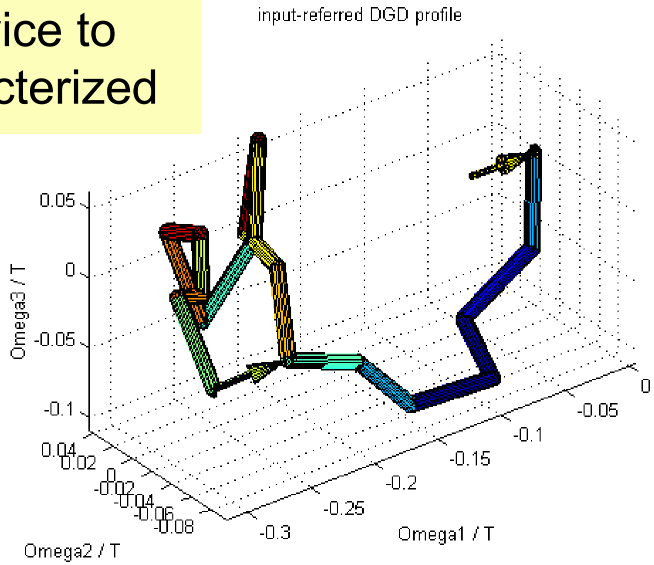
F_{-1}, F_0, F_1, more complicated bending of DGD profile

F_{-2}, F_{-1}, F_0, F_1, F_2

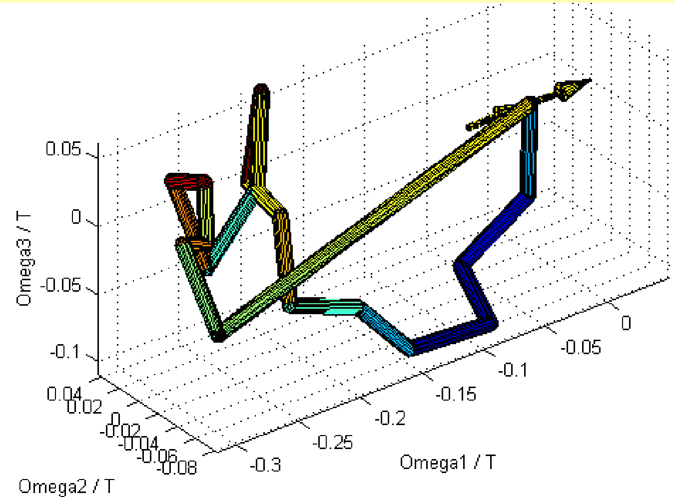
3 extra parameters are needed for all methods if frequency-independent output polarization transformation needs also to be described.

DGD profile of an exemplary PMD structure, cascaded with inverted FEMC structures

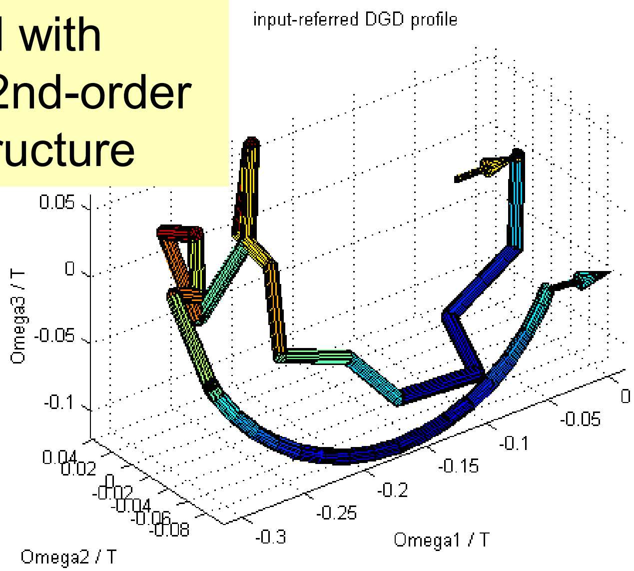
PMD device to be characterized



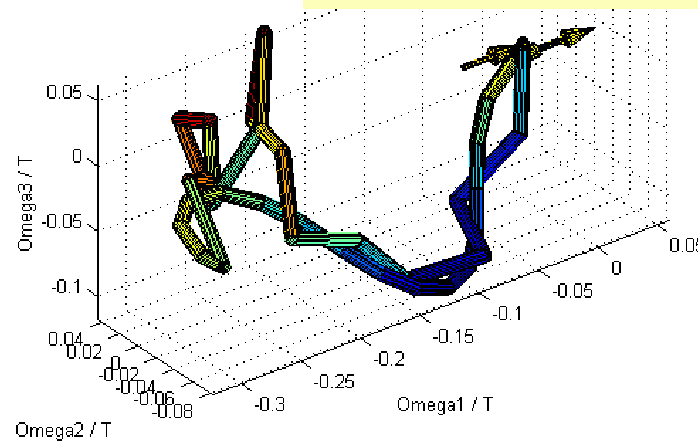
cascaded with inverted 1st-order structure



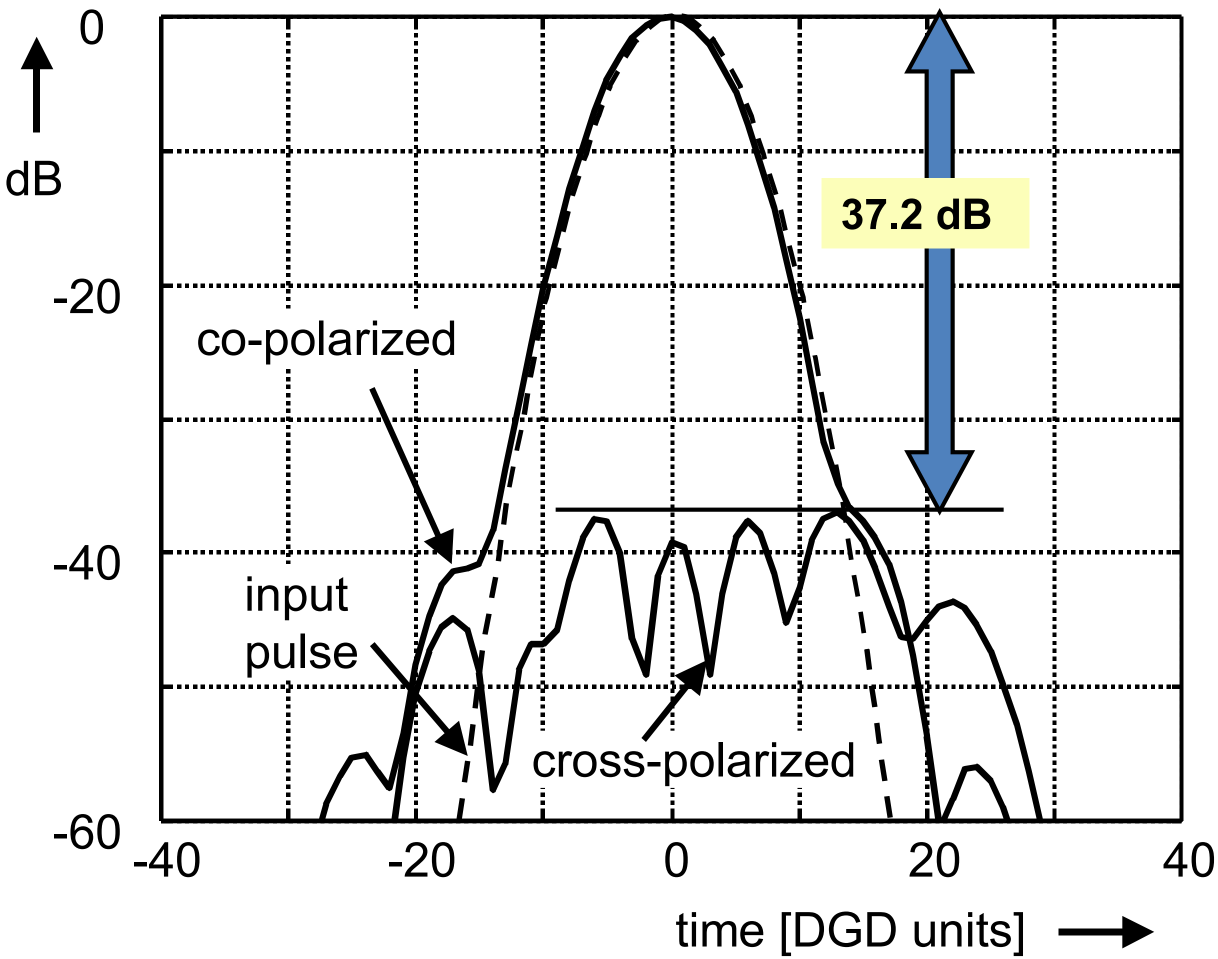
cascaded with inverted 2nd-order FEMC structure



cascaded with inverted 3rd-order FEMC structure



Extinction of cross polarization at output of PMD device cascaded with inverted 3rd-order FEMC structure



- Gaussian input pulse width is chosen equal to total DGD of FEMC structure after convergence of search algorithm.
- Search algorithm maximizes cross polarization extinction.
- Ideal PMD description would result in infinite cross polarization extinction.

Suppression of cross polarization by equalizers (= inverted structures) defined by higher-order PMD definition methods

Method order	1	2	3
Gaussian input pulse width [a.u.]	5.6	7.2	9.5
Taylor expansion of PMD vector (TEPV)	10.3 dB	14.8 dB	19.9 dB
Exponential Jones matrix expansion (EMTY)	10.3 dB	12.6 dB	16.1 dB
Fourier expansion of mode coupling (FEMC)	10.3 dB	21.6 dB	35.5 dB

- dB and pulse width values are averaged over 75 PMD examples.
- Pulse widths are chosen equal for all methods, using the value obtained after convergence of FEMC for one particular order. Part of extinction improvement of high method orders is due to broader pulses.
- Extinction improvement of higher-order FEMC over 1st-order PMD seems to be ≥2 times larger (in dB) than that of TEPV or EMTY !
- Reason: FEMC (and SDGD) are closely related to natural PMD, unlike higher-order TEPV and EMTY.
- Drawback: Finding FEMC coefficients is a numerical optimization process ⇒ more research is needed.