Multi-Pair and Multi-Access Capacity:
Exploring the Potential

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Motivation

- **Needs**
  - Capacity: 3 -10 Mbps symmetric per pair
  - 3-6 pairs
  - Compatibility with legacy and basis systems –T1E1 DSM SG
  - Rate stability
  - CSA Range
  - Realistic impairments – e.g. PN4254
- **Multi-Pair and Multi-Access Capacities show potential for 2x-5x increase in rates**
  - Capacity computations provide bounds independent of implementation
  - Moore's law permits new approaches: 100x change in price-performance from 1995-2005
  - Research: Stanford, Princeton, industry, …

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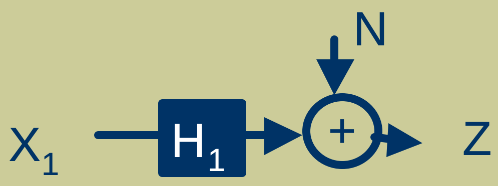
Different Formulations of Capacity Problems

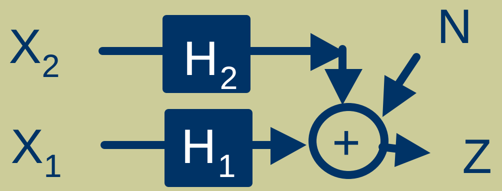
Gaussian disturber formulation (well researched)

- Transmit power constraint → water-filling



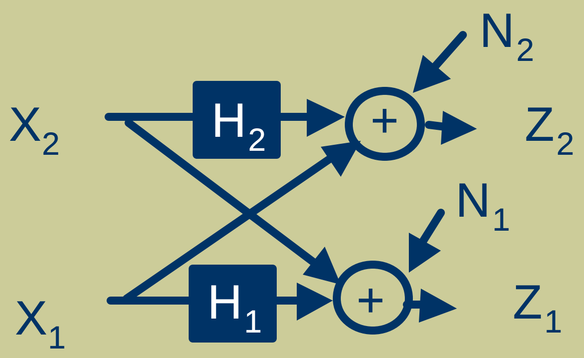
Multi-Access formulation (on-going research)

- Spectral and coding coordination, independent transmitters



Interference formulation (partial results)

- Green field versus Legacy



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Gaussian Channel Capacity

Single Pair (Scalar)

- *All* disturbers (including internal transmitters) in to a receiver are aggregated into a spectrally equivalent *scalar* Gaussian disturber

$$C = \sum_i \frac{1}{2}\int \log_2\left(1+\frac{\|H_{ii}\|^2 S_{xx_i}}{s^2+\sum_{j\neq i}\|H_{ij}\|^2 S_{xx_j}}\right)df$$

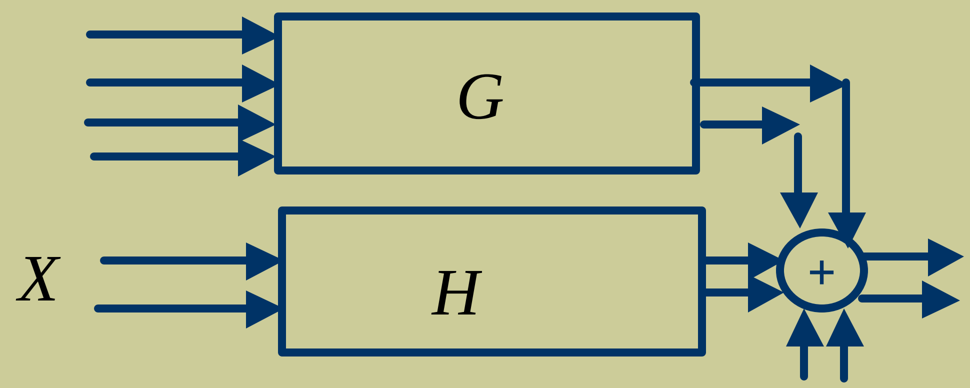
Multi-Pair (Vector)

- *Internal* transmitters are vector inputs
- *External* disturbers are aggregated into a spectrally equivalent *vector* Gaussian process

$$C = \frac{1}{2}\int \log_2 \det(I + HS_{xx}H'R^{-1})df$$

External Transmitters

Internal transmitters



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Gaussian Channel Capacities

Simulation Conditions

- 4-20 pairs at 9kft 26 AWG cable, *G.shdsl transmit mask*
- AWGN –140 dBm / Hz
- Benefits of vector processing over scalar increase with Stronger Cross couplings
  - Greater benefits with all couplings at Unger Mask
- *Joint* distribution of N x N couplings becomes important
- Monte Carlo assignment of channels on *measured coupling matrices* from a 25 pair cable
- Maxima of measured couplings are at the Unger Mask
- Look at 99 percentile rates

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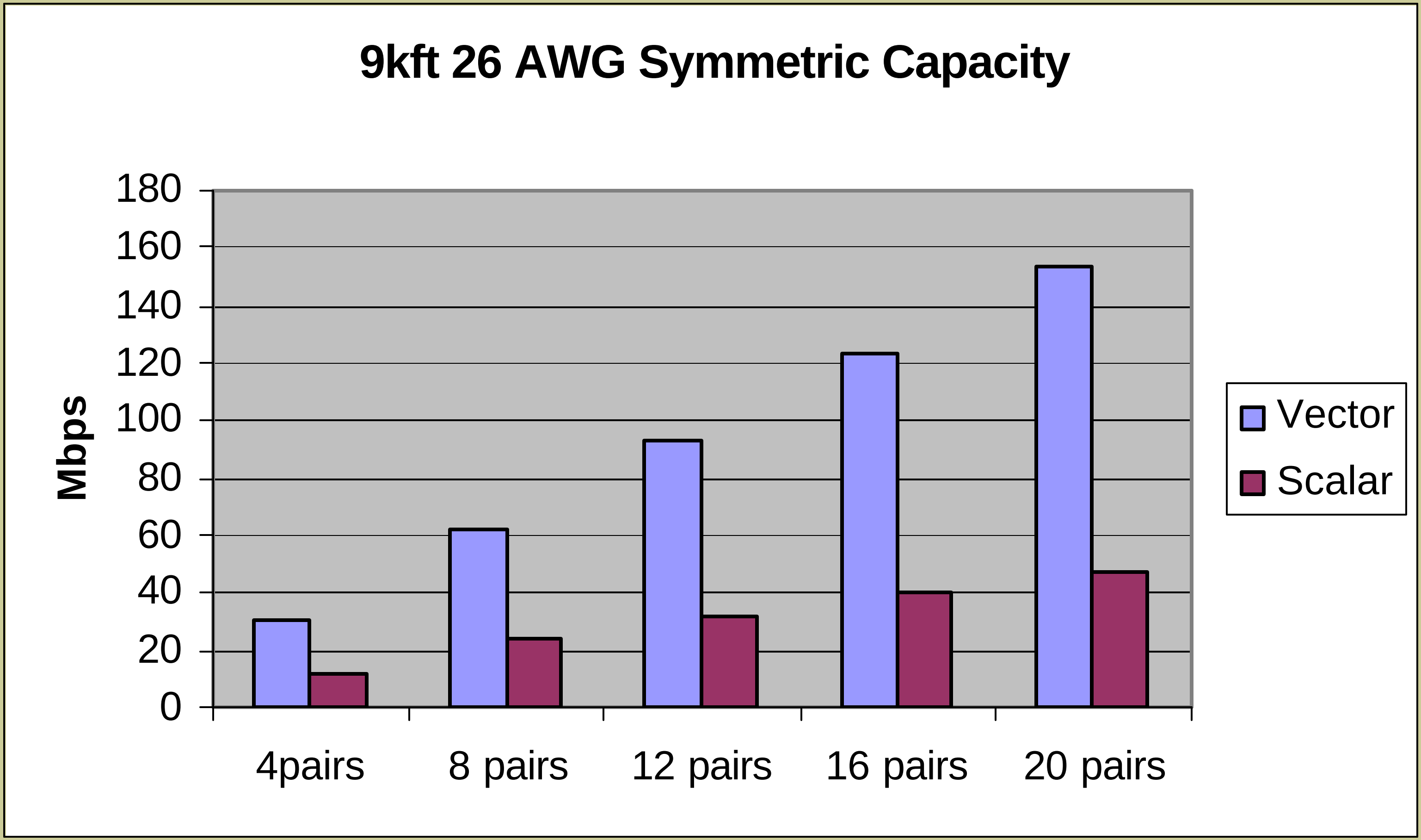
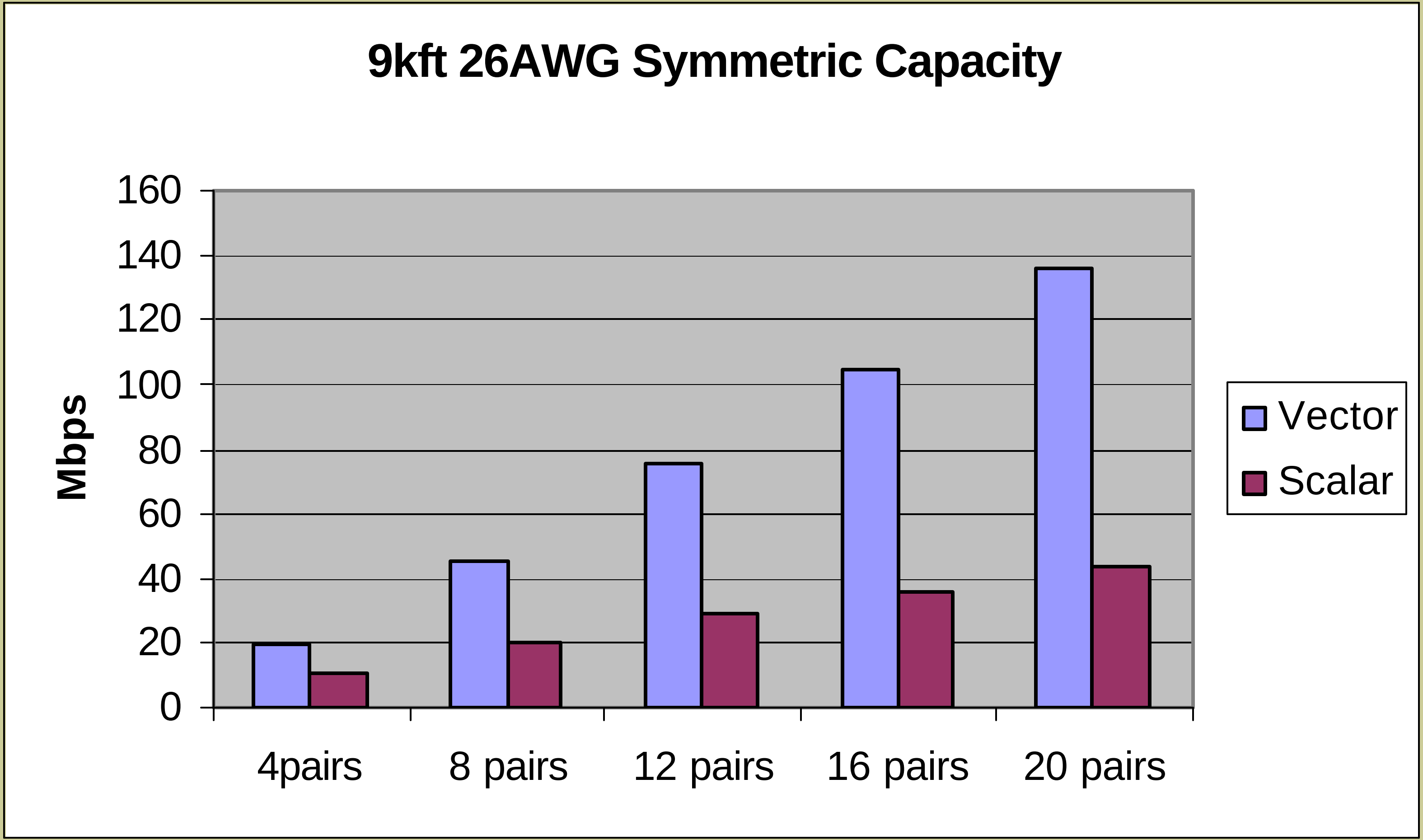
Typical Vector Capacity with Measured Cable Couplings



- Self FEXT and Self NEXT disturbance only, no external disturbers
- Monte Carlo results fall within 10% of typical

Typical Vector Capacity with Measured Cable Couplings



- Self FEXT, Self NEXT and 4 external disturbers (1 ADSL, 2 HDSL, 1 HDSL2)
- Relative benefits increase with number of pairs

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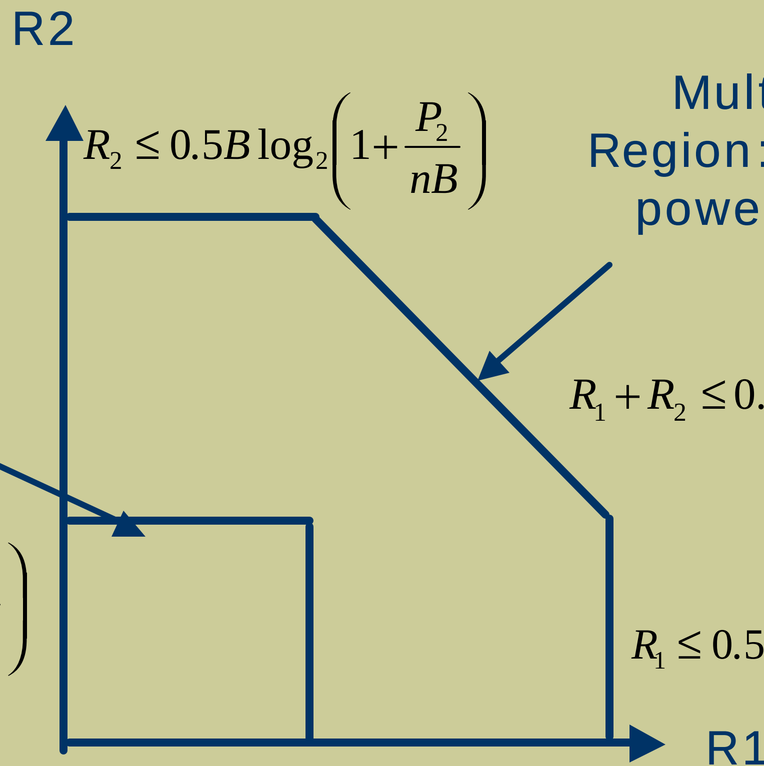
Multi-Access Formulation

Simplest Case (equal channel case)

$Y = X_1 + X_2 + N$

Achievable rate regions



$$R_2 \le 0.5B\log_2\left(1+\frac{P_2}{nB}\right)$$

Multi-Access Rate Region: The other user's power adds to signal power!

$$R_1 + R_2 \le 0.5B\log_2\left(1+\frac{P_1+P_2}{nB}\right)$$

$$R_1 \le 0.5B\log_2\left(1+\frac{P_1}{nB}\right)$$

Single User Rate Region:

The other user's power adds to noise power

$$R_2^* \le 0.5B\log_2\left(1+\frac{P_2}{P_1+nB}\right)$$

$$R_1^* \le 0.5B\log_2\left(1+\frac{P_1}{P_2+nB}\right)$$

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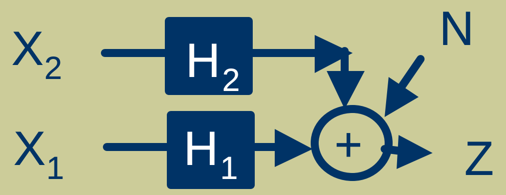
Multi-Access Formulation

General Multi-user case

- For unequal channels with ISI (*Cheng and Verdu, 1993*) showed rate regions with power constraints
- Iterative water-filling converges to optimal for power constraints (*Yu, Rhee, Boyd, Cioffi, 2001*)
- *G.dmt.bis/G.lite.bis, T1E1.4/2001-200R5, T1E1.4/2001-278, T1E1.4/2001-284* address mechanisms to enable optimization



Multi-User Detection

Multi-User detection exploits structure of disturbers

- **Excess bandwidth**
- **Finite alphabet**
- **Receiver-only technology**

Multi-User receivers can increase achievable rates beyond single-user capacities in DSL

T1E1.4/2000-251, Zeng and Cioffi IEEE JSAC 2001

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Interference Problems

Interference limits

- Static capacity limits due to legacy and basis system
- Probabilistic future scenarios cover 95%-99% of cases
  - Useful for public-private combination - PN4254

Dynamic stability issues for further study

- Capacity calculations assume steady state
- Steady state requires convergence of adaptive schemes (e.g. bit swapping, TEQ/FEQ, DFE coefficients) in basis systems
- Convergence of adaptation needs to be ensured but is adversely affected by non-stationary noise
- Increased non-stationarity directly reduces the achievable rates compared to the steady state rates

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Summary

Benefits of Physical Layer Processing are in addition to those from higher layer aggregation or bonding

- Multi-Pair (vector) processing
- Multi-Access Methods

Future work needed to address issues

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