Analysis of Clock Synchronization Approaches for Residential Ethernet

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Outline

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- Synchronization model
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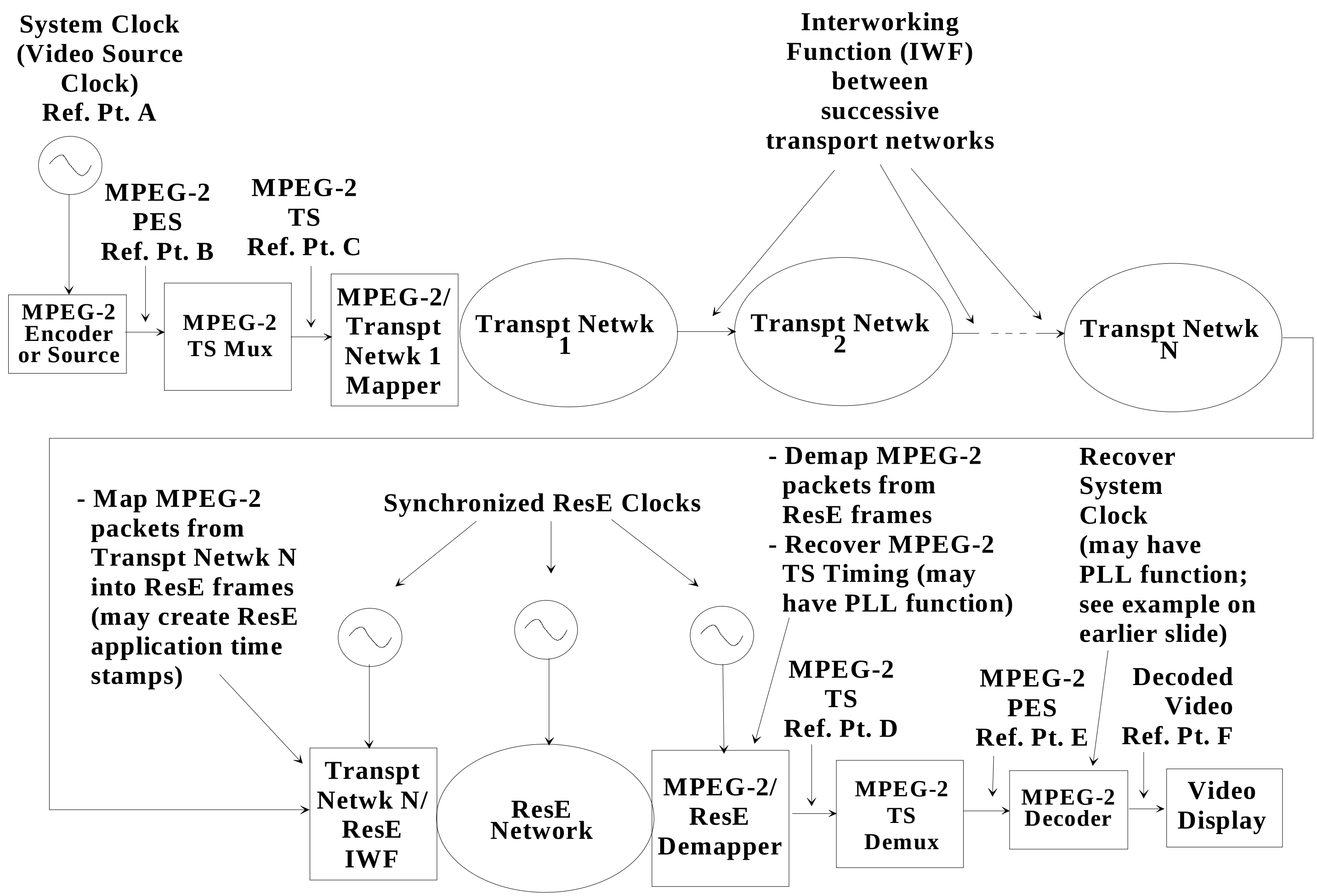
Introduction

- Residential Ethernet (ResE) is a new standardization activity in IEEE 802 that is considering extensions to Ethernet to allow the transport of time-sensitive traffic (e.g., high quality audio and video (A/V))
- A/V applications have tight jitter and wander requirements that must be met end-to-end
- To meet these requirements, synchronization is required at ResE ingress and egress points
- This analysis investigates if and how synchronization approaches based on IEEE 1588 can meet the ResE requirements

- This is a slightly modified version of a presentation to be delivered at the ***2005 Conference on IEEE 1588, Winterthur, Switzerland, October 10 – 12, 2005***

Application Reference Models

Example Reference Model for Transport of MPEG-2 Video over Service Provider Networks and Residential Ethernet [2]



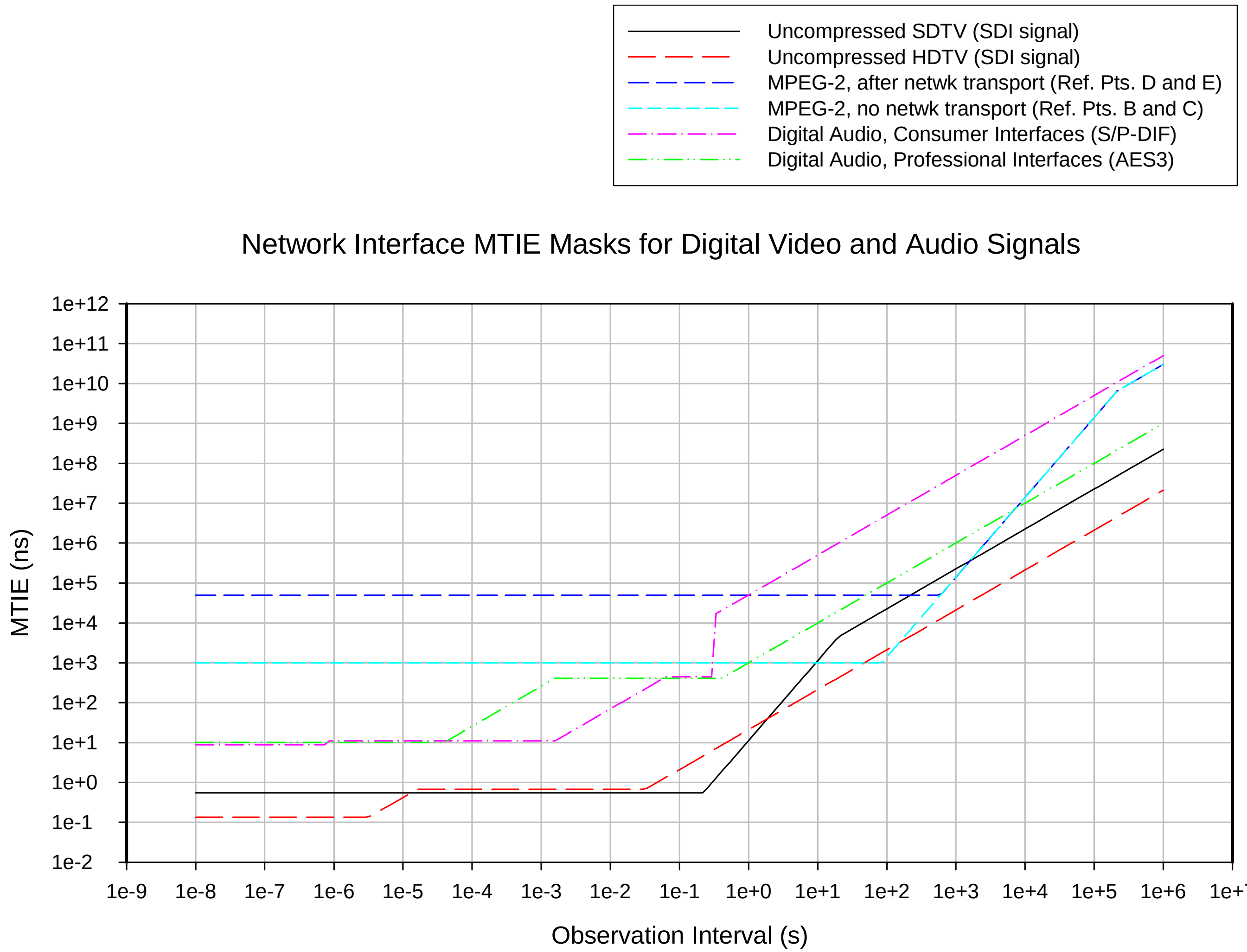
End-to-End Requirements

Summary of End-to-End Application Jitter and Wander Requirements (see[2] and references given there)

Requirement	Uncompressed SDTV	Uncompressed HDTV	MPEG-2, with network transport	MPEG-2, no network transport	Digital audio, consumer interface	Digital audio, professional interface
Wide-band jitter (UIpp)	0.2	1.0	50 μs peak-to-peak phase variation requirement (no measurement filter specified)	1000 ns peak-to-peak phase variation requirement (no measurement filter specified)	0.25	0.25
Wide-band jitter meas filt (Hz)	10	10			200	8000
High-band jitter (UIpp)	0.2	0.2			0.2	No requirement
High-band jitter meas filt (kHz)	1	100			400 (approx)	No requirement
Frequency offset (ppm)	±2.79365 (NTSC) ±0.225549 (PAL)	±10	±30	±30	±50 (Level 1) ±1000 (Level 2)	±1 (Grade 1) ±10 (Grade 2)
Frequency drift rate (ppm/s)	0.027937 (NTSC) 0.0225549 (PAL)	No requirement	0.000278	0.000278	No requirement	No requirement

End-to-End Requirements

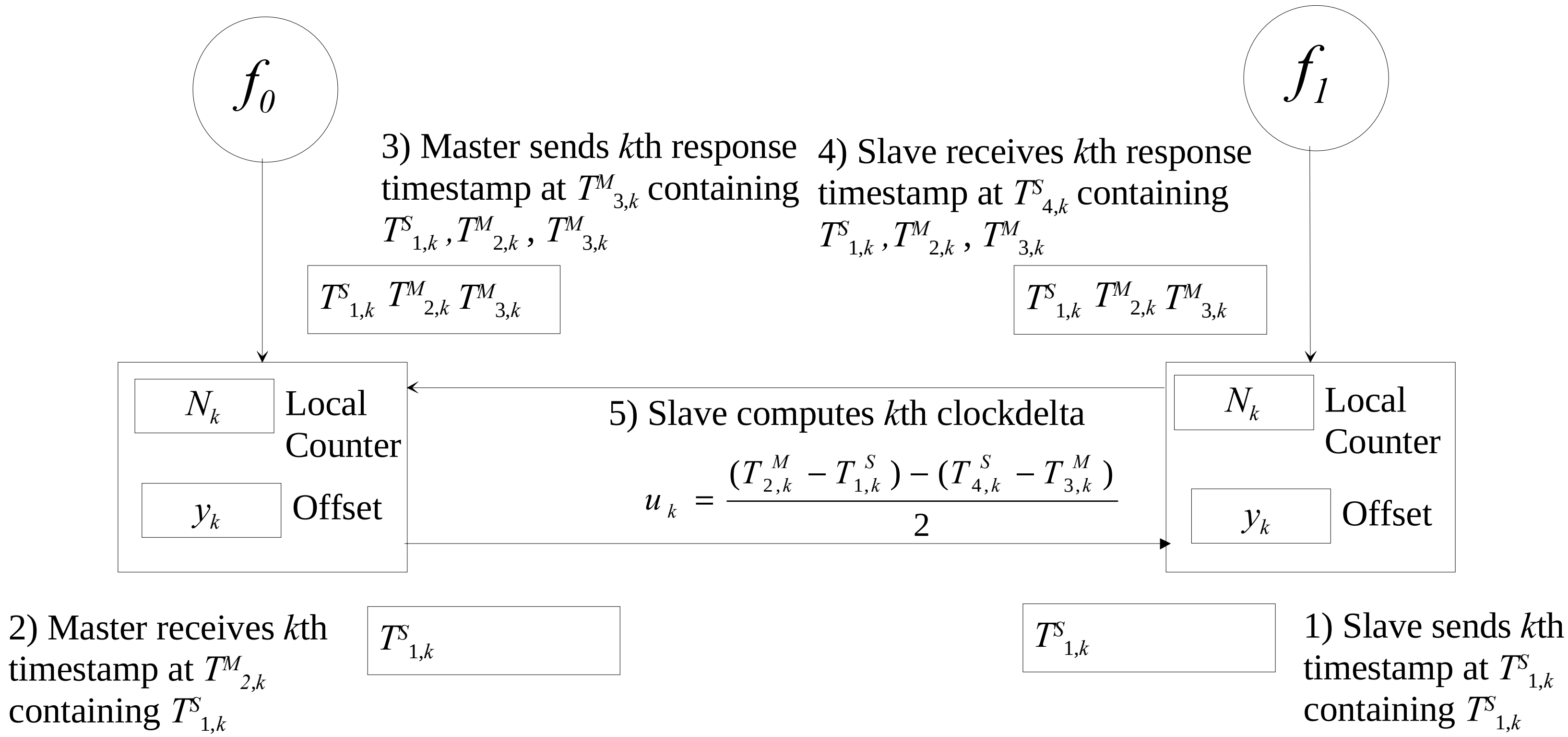
End-to-End Application Jitter and Wander Requirements Expressed as MTIE Masks [2] (see Appendix II for MTIE definition)



Synchronization Approaches

Basic 2-Way Time Stamp Approach used in IEEE 1588

- ResE will use this basic approach; however, a number of variations are possible
- Generally assumed a filtering function will be present at the endpoint
  - May be present at intermediate nodes (i.e., in some variations)



6) Slave computes current offset y_k in terms of current and possibly past clockdelta's u_k

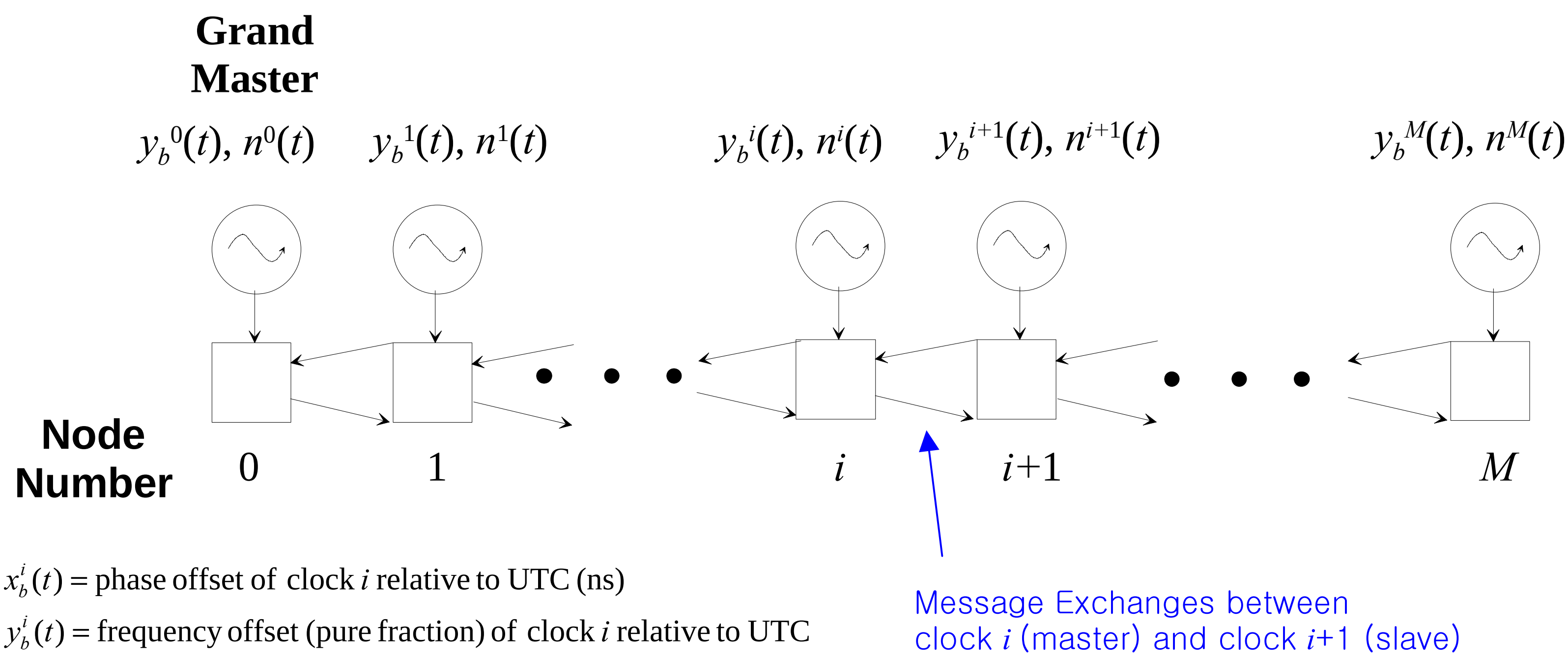
Synchronization Approaches

Variations/Choices

- This is ongoing work; in this paper we focus on (2) – (4)
- plan to look at all the approaches

1) Use one-way time stamp scheme with less frequent two-way exchange; obtain delay from two-way exchange and assume delay is fixed until next two-way exchange
2) **Instantaneous phase adjustments at intermediate nodes**
3) **Instantaneous phase and frequency adjustments at intermediate nodes (with instantaneous frequency adjustments possibly less frequent)**
   - **Described in [4]**
4) **Filtered phase adjustments at intermediate nodes, using digital filter running at local clock rate (with or without instantaneous frequency adjustments)**
5) Full phase-locked loops (PLLs) at intermediate nodes (i.e., filtered phase and frequency adjustments)
6) Use of transparent clock nodes
   a) End-to-end versus peer-to-peer
   b) Whether or not to adjust rate of local oscillator in transparent clock and, if so, whether to do filtering
7) Time stamp reflects current time versus delay by some number of frames
8) Time stamp reflects local free-running clock time versus latest corrected time based on most recent time stamps and possible filtering)

Synchronization Model



$x_b^i(t) =$ phase offset of clock i relative to UTC (ns)

$y_b^i(t) =$ frequency offset (pure fraction) of clock i relative to UTC

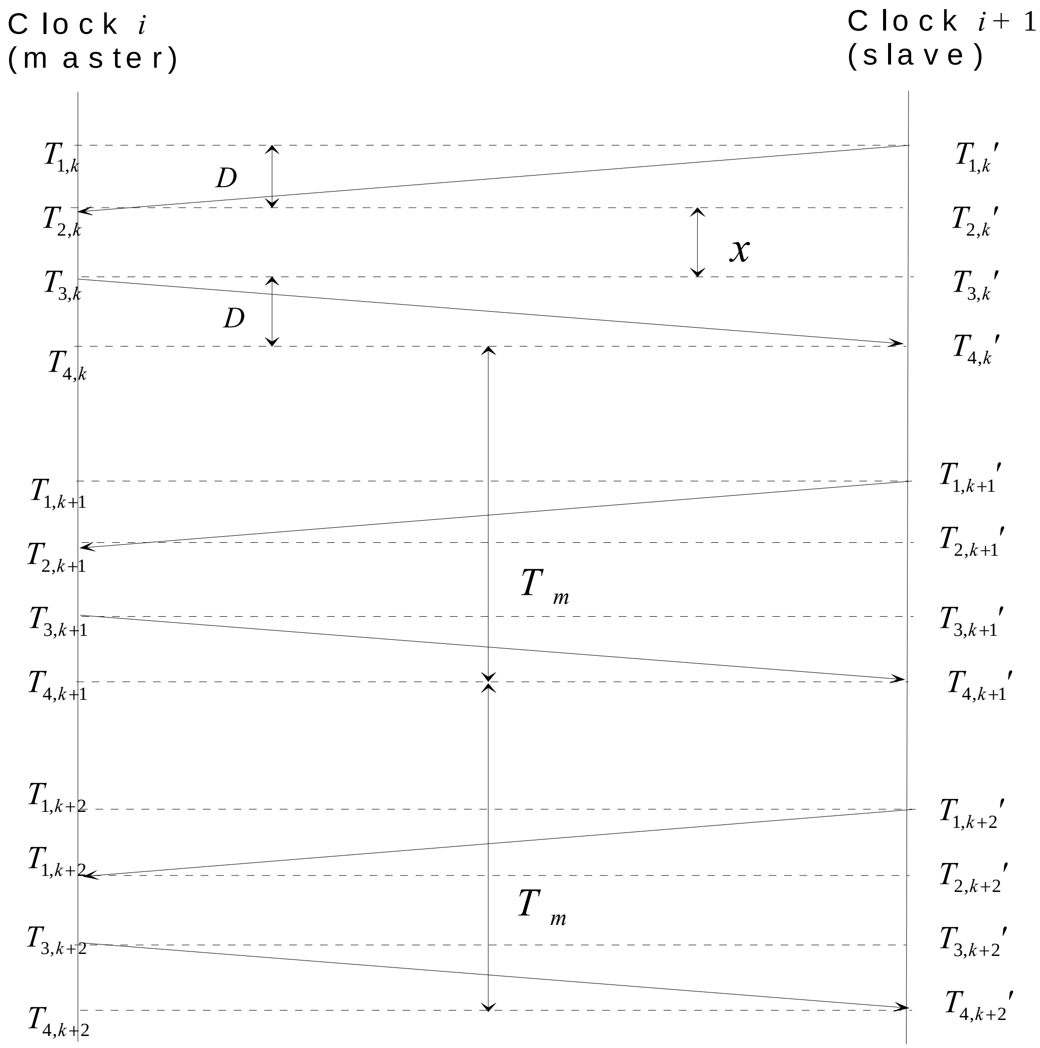
$n^i(t) =$ phase noise of clock i (ns)

$$x_b^i(i) = \int_0^t y_b^i(t)\,dt + n^i(t) = y_b^i t + n^i(t)$$

- Assumes the frequency offset is constant over time
- Assumes the phase offsets are zero at $t = 0$
- We are interested in timing relative to the GM; therefore, can set $y_b^0(t) = n^0(t) = 0$

–Note that messages from master to slave and slave to master do not necessarily occur at the same times
–Note that messages from master to slave and slave to master may not occur at the same rates

Synchronization Model



T_m = time between successive messages from master to slave, measured relative to UTC

D = propation delay between master and slave

x = time offset between master and slave (will be initialized randomly between 0 and T_m and either kept constant or allowed to change by frequency offset between master and slave multiplied by T_m

- Assume $D << T_m$, and therefore probablity that messages from master to slave and slave to master overlap in time is negligible

- Unprimed quantities are relative to master clock

- Primed quantities are relative to slave clock

Then, can express the phase offset in discrete time (k = time index; UTC time at step $k = kT_m$)

$$x_{b,k}^{i} = y_{b}^{i} T_m k + n_k^{i}$$

Synchronization Model

❑Outline of model derivation

- For variations (3) and (4), express frequency offset estimate of slave relative to master over *P* time steps in terms of the ${x_{b,k}}^i$ and ${x_{b,k}}^{i+1}$ (tilde denotes relative frequency offset between current and previous node)
  - Compare time differences in free-running master and slave clocks over PT_m

$$\tilde{y}^i_{kP} = \frac{\left(PT_m + x^i_{b,kP} - x^i_{b,(k-1)P}\right) - \left(PT_m + x^{i+1}_{b,kP} - x^{i+1}_{b,(k-1)P}\right)}{PT_m + x^{i+1}_{b,kP} - x^{i+1}_{b,(k-1)P}}$$

$$\tilde{y}^i_{kP+1} = \tilde{y}^i_{kP+2} = \cdots = \tilde{y}^i_{kP+P-1} = \tilde{y}^i_{kP}$$

- For variations (3) and (4), calculate cumulative frequency offset of current node relative to GM $\quad y^i_k = \sum_{j=1}^{i} \tilde{y}^j_k$
- For variation (3) and (4), express corrected phase error estimate ${x_j}^i$ in terms cumulative frequency offset estimate and free-running clock phase error ${x_{b,j}}^i$
  - Choose phase error estimate at all time steps between frequency updates to be consistent with current frequency offset estimate

$$\frac{(j-k)PT_m + x^i_j - x^i_{kP}}{(j-k)PT_m + x^i_{b,j} - x^i_{b,kP}} = 1 + y^i_k$$

$$x^i_j = x^i_{kP} + \left(x^i_{b,j} - x^i_{b,kP}\right)\left(1 + y^i_k\right) + (j - kP)T_m y^i_k$$

Synchronization Model

- For variations (2) – (4), calculate clock delta in terms of either corrected phase error estimates (for cases where frequency adjustments are made) or free-running clock phase errors
  - Apply result for clock delta in step (5) on slide 7
  - Need phase error values at intermediate times $T_{1,k}', T_{2,k}, T_{3,k}$
  - Obtain these by interpolation; result depends on x and D
    - Take limit $D \to 0$
    - Note: assumption is being made that we can interpolate on the noise
      - Reasonable as long as the desired noise level is chosen for sampling rate T_m
  - See paper for details
- Calculate cumulative clock delta for all nodes up to the current one (GM clock delta is zero)
- Add cumulative clock delta to corrected or free-running clock phase error to obtain unfiltered phase estimate
- Filter the unfiltered phase estimate with a digital filter that runs at the local clock rate

Synchronization Model

- Since the filter is linear, the result is the same for the case where each clock delta is filtered at each respective intermediate node versus filtering the cumulative clock delta
  - If synchronization is needed at each node, the work is the same in either case
- Filter model is a digital implementation of standard 2nd order, linear filter with 20 dB/decade roll-off

$$H(s) = \frac{2\varsigma\omega_n s + \omega_n^2}{s^2 + 2\varsigma\omega_n s + \omega_n^2}$$

$$\omega_n = \text{undamped natural frequency}$$

$$\varsigma = \text{damping ratio}$$

$$f_n = 3\text{ dB bandwidth} = (\omega_n / 2\pi)\left[(2\varsigma^2 + 1) + \sqrt{(2\varsigma^2 + 1)^2 + 1}\right]$$

$$H_p = \text{gain peaking} = \left[1 - 2\alpha - 2\alpha^2 + 2\alpha\sqrt{2\alpha + \alpha^2}\right]^{-1/2}$$

$$\text{where } \alpha = 1/(4\varsigma^2)$$

- The digital implementation is obtained by expressing the filter in state variable form (See [6] and [7] for details)
  - State vector at current time step is written as convolution integral of input vector and impulse response matrix
  - Impulse response matrix is calculated exactly and integral is evaluated using trapezoidal approximation for input
  - Output is written in terms of states

Synchronization Model

- Additional aspects of model
  - Clock noise model is described in appendix
  - Simulation time step is a sub-multiple of the inter-message time T_m (cannot exceed T_m)
  - Time between frequency estimate updates is a multiple of T_m
  - Time offset between master$\rightarrow$slave and slave$\rightarrow$master messages may be initialized randomly or initialized with user-specified values
  - Time offset between master$\rightarrow$slave and slave$\rightarrow$master messages may remain constant over the simulation or vary over T_m by the relative frequency offset between master and slave, multiplied by T_m
    - Former requires that the master and slave send messages at the same rate
    - Latter corresponds to messages being sent at the free-running clock rates
  - Finite precision of clock is modeled
    - Granularity, in units of time, is supplied as input parameter

Parameters Common to All Simulation Cases

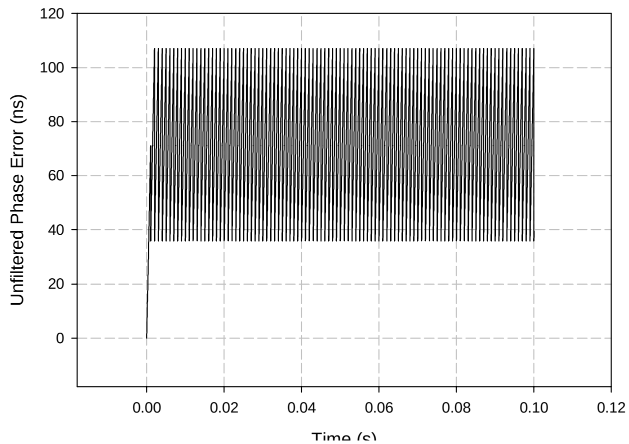
- 10 hops
  - GM followed by 10 slave clocks, in chain
- Slave clock frequency tolerance = ± 100 ppm
- Inter-message time (T_m) = 1 ms
- Time between frequency offset updates = 10 ms (if frequency offset is estimated)
- Filter bandwidth = 10 Hz
- Filter gain peaking = 0.1 dB
- Simulation time step = 0.01 ms
  - Used small time step to ensure phase peaks were captured

Simulation Cases 1 and 2

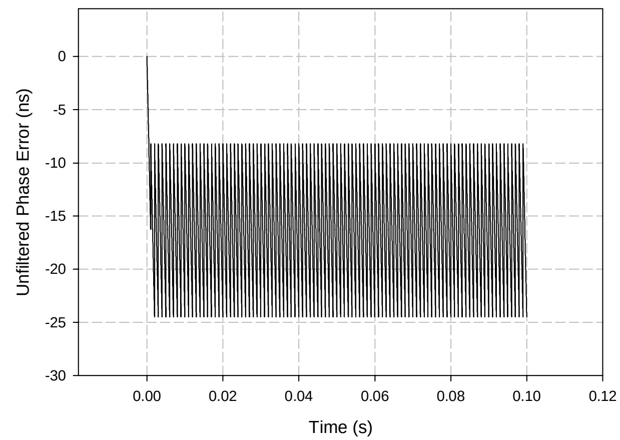
- Assumptions
  - No clock phase noise
  - Granularity of clock = 0
  - No frequency adjustments (Case 1); Instantaneous frequency adjustments (Case 2)
  - Offset between master→slave and slave→master messages set to T_m at each node (deterministic and constant)
- Results (see plots on next slide)
  - With instantaneous phase adjustments (no filtering) and no frequency adjustments, steady-state peak-to-peak phase error can be large (tens of ns) and depends on frequency offsets
    - With 10 Hz filter and no frequency adjustments, steady-state peak-to-peak phase error is reduced to a few tenths of a ns
  - With instantaneous frequency adjustments, steady state peak-to-peak phase error is very small
    - Approximately 0.07 ns with no filtering
    - Approximately 0.00055 ns (0.55 ps) with filtering
    - With no clock noise and zero phase granularity, frequency offsets can be measured very accurately
  - Phase variation does not increase monotonically with number of clocks in chain

Simulation Case 1

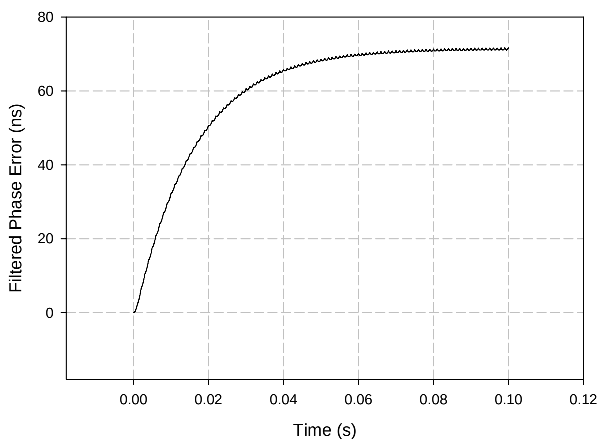
Case 1, Node 1
Instantaneous Phase Adjustments
No Frequency Adjustments



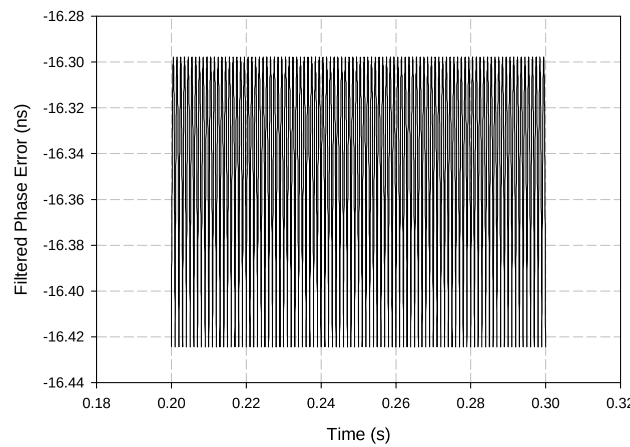
Case 1, Node 10
Instantaneous Phase Adjustments
No Frequency Adjustments



Case 1, Node 1
Filtered Phase Adjustments
No Frequency Adjustments
(Initial transient is included)

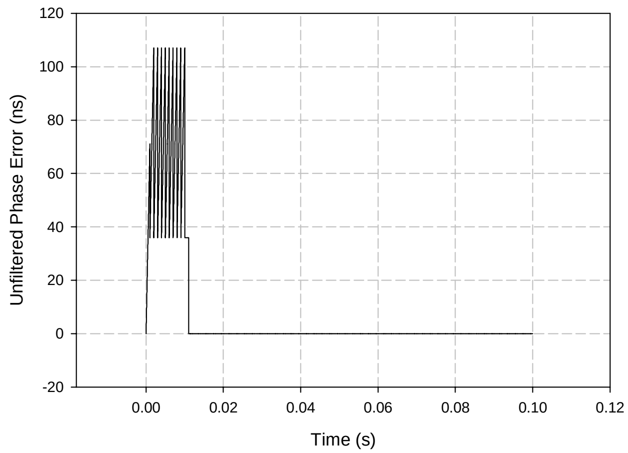


Case 1, Node 10
Filtered Phase Adjustments
No Frequency Adjustments
(Plot begins after initial transient has decayed)

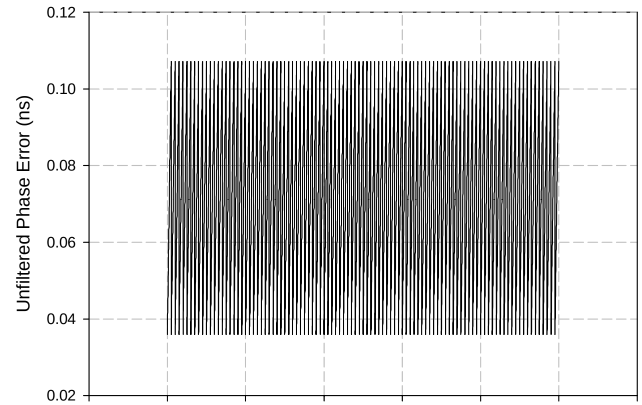


Simulation Case 2

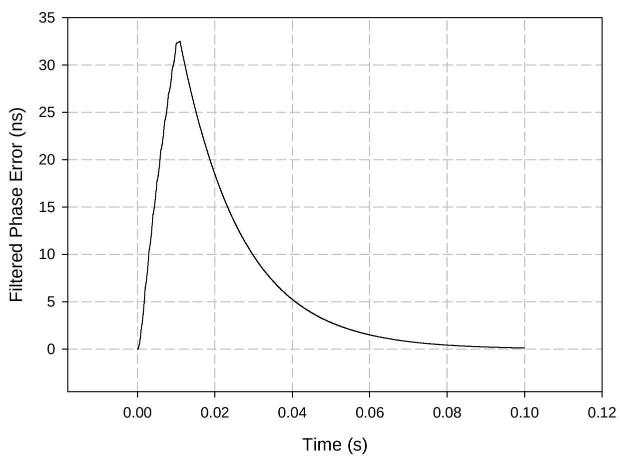
Case 2, Node 1
Instantaneous Phase Adjustments
Instantaneous Frequency Adjustments



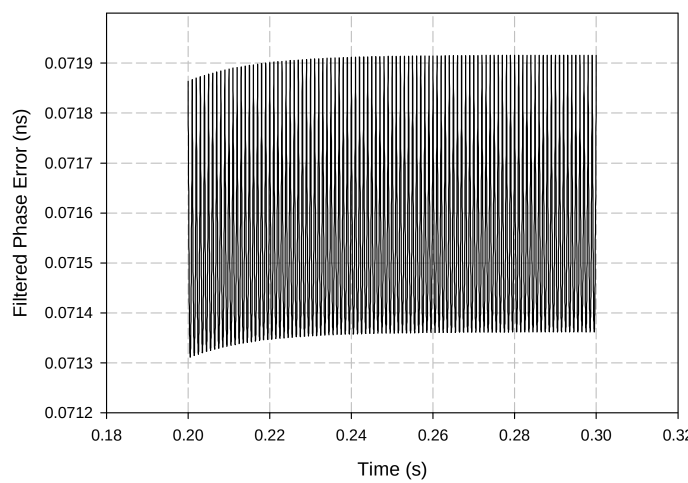
Case 2, Node 10
Instantaneous Phase Adjustments
Instantaneous Frequency Adjustments



Case 2, Node 1
Filtered Phase Adjustments
Instantaneous Frequency Adjustments



Case 2, Node 10
Filtered Phase Adjustments
Instantaneous Frequency Adjustments



Simulation Cases 3 and 4

❑Assumptions

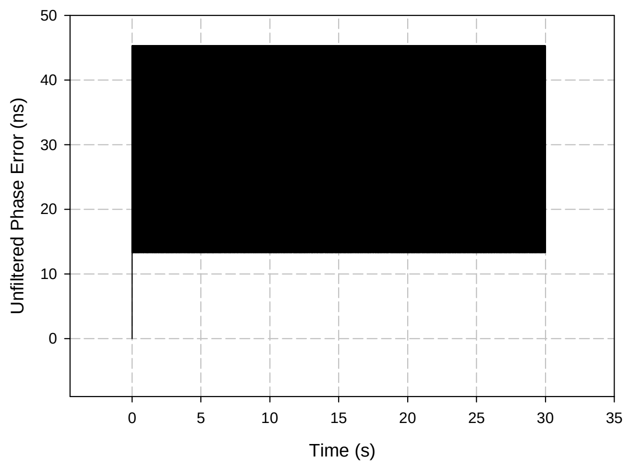
- With clock phase noise (model described in Appendix)
- Granularity of clock = 1 ns
- No frequency adjustments (Case 3); Instantaneous frequency adjustments (Case 4)
- Offset between master→slave and slave→master messages initialized randomly at each node
  - All nodes send messages at the same rate (offsets remain constant over simulation)

❑Results (see plots on next slide)

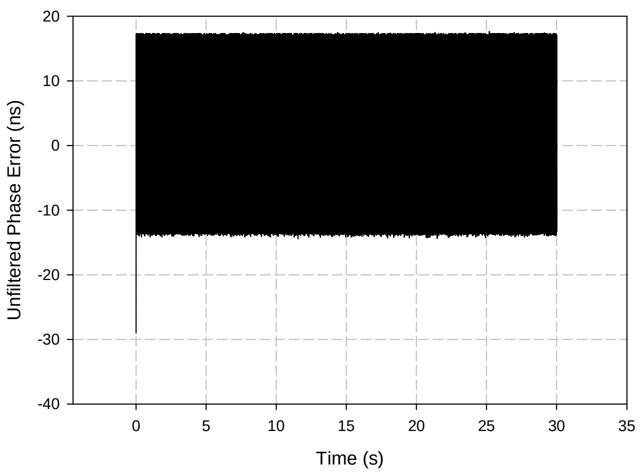
- With 10 Hz filter, MTIE is in roughly the same range with and without frequency adjustments at longer observation intervals
  - MTIE is slightly smaller with frequency
  - Maximum peak-to-peak phase variation is around 1 ns for both cases
- Without filtering, MTIE ranges from approximately 15 – 60 ns without frequency adjustments and 1 – 4 ns with frequency adjustments
- Phase variation does not increase monotonically with number of clocks in chain (in all cases)
- Note that the results exhibit large statistical variability
  - Must run multiple, independent replications of the simulations to obtain confidence intervals for the results

Simulation Case 3

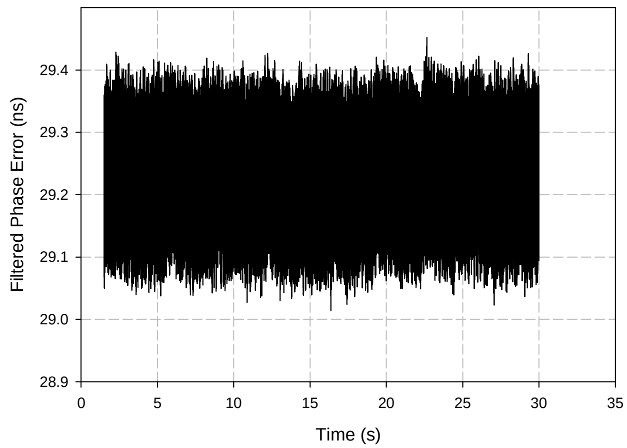
Case 3, Node 1
Instantaneous Phase Adjustments
No Frequency Adjustments



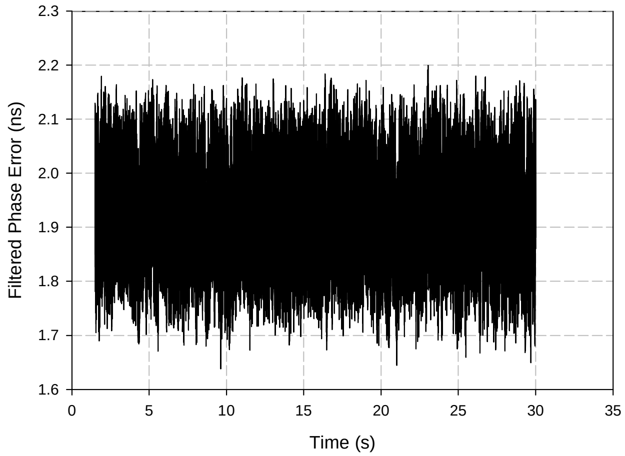
Case 3, Node 10
Instantaneous Phase Adjustments
No Frequency Adjustments



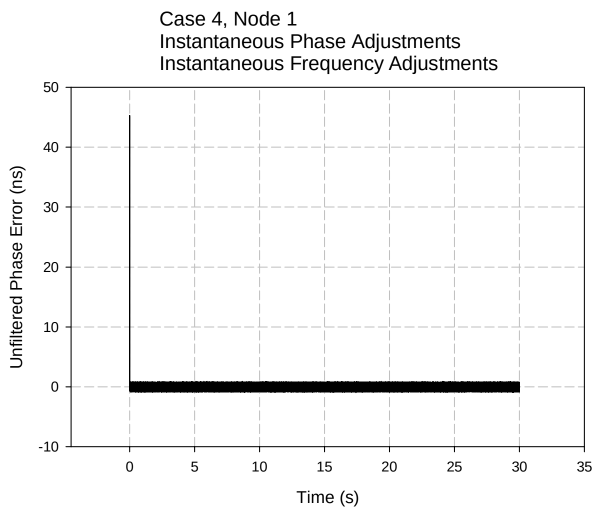
Case 3, Node 1
Filtered Phase Adjustments
No Frequency Adjustments
(plot begins after initial transient has decayed)

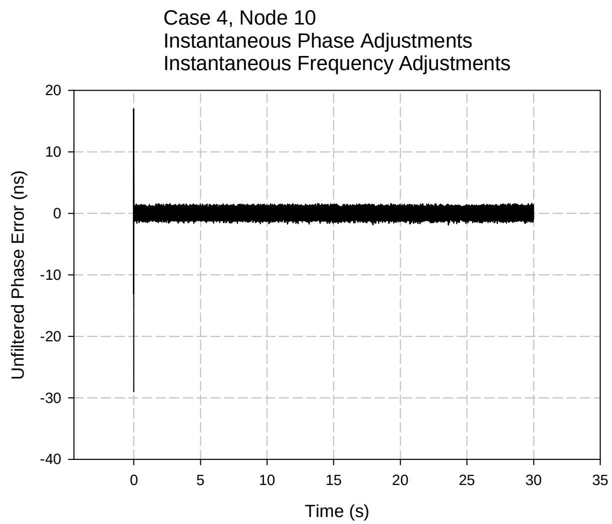


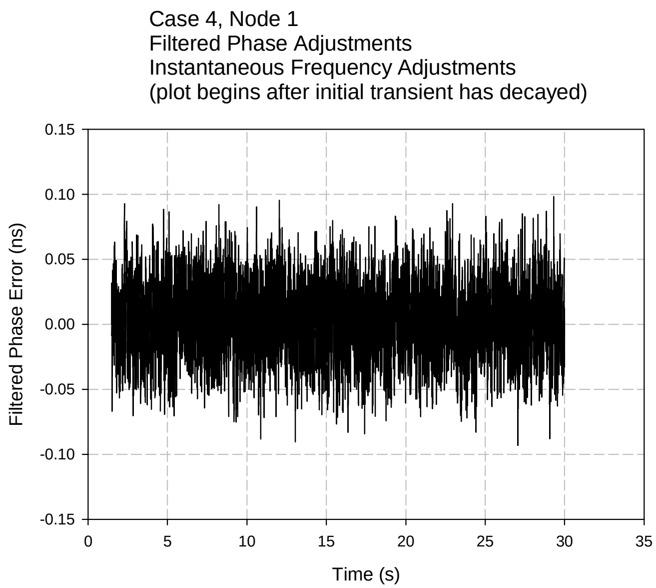
Case 3, Node 10
Filtered Phase Adjustments
No Frequency Adjustments
(Plot begins after initial transient has decayed)

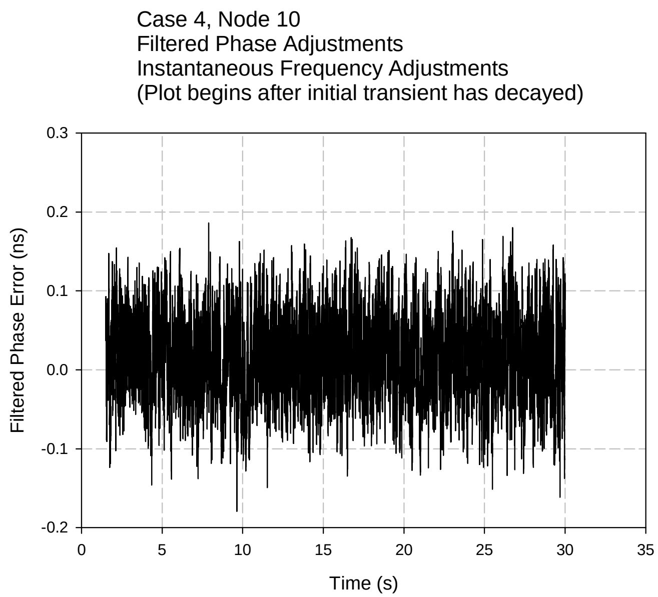


Simulation Case 4

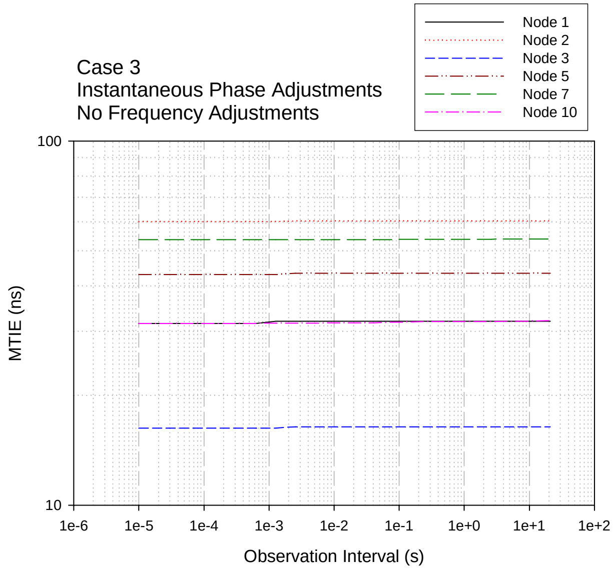


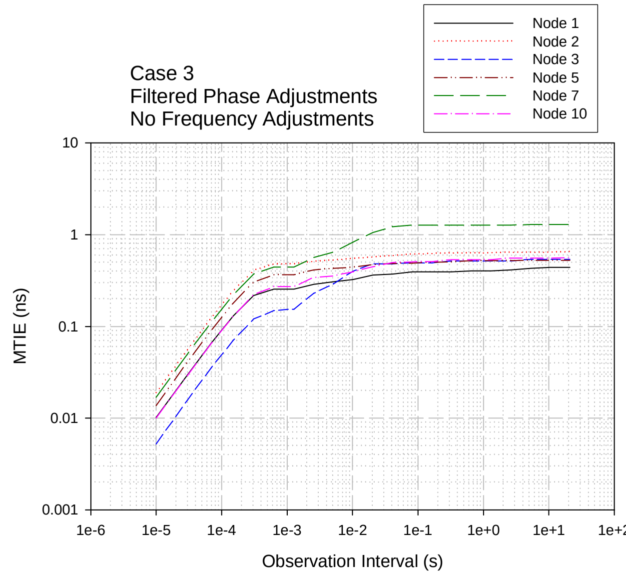


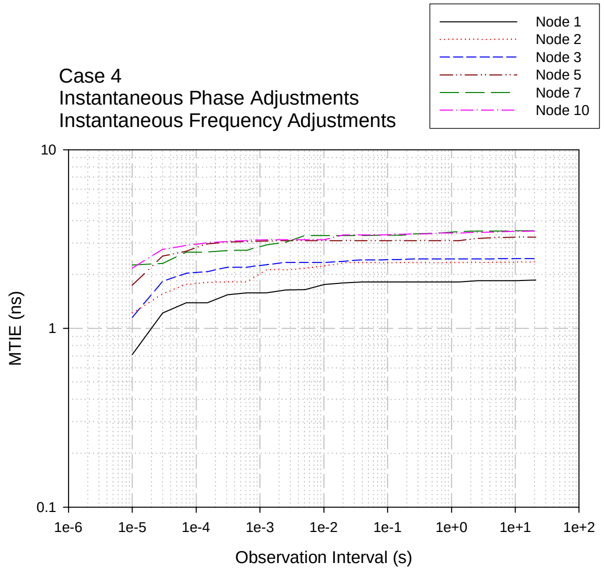


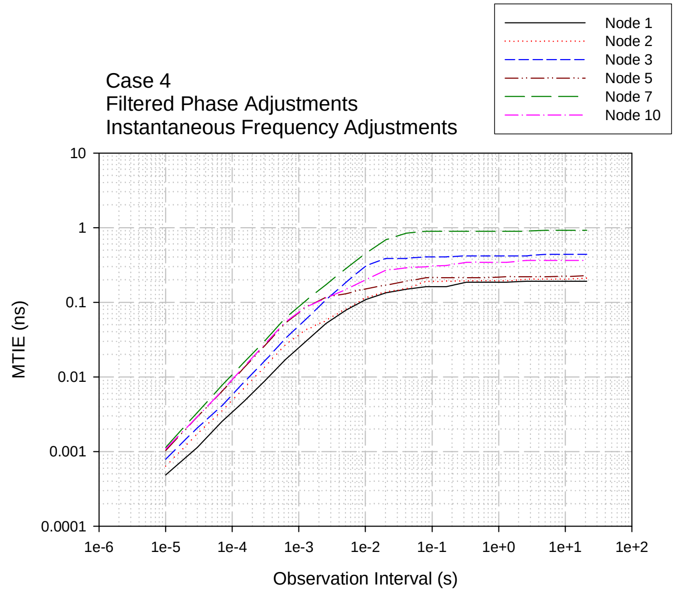


Simulation Cases 3 and 4









Simulation Cases 5 and 6

❑Assumptions

- With clock phase noise (model described in Appendix)
- Granularity of clock = 1 ns
- No frequency adjustments (Case 5); Instantaneous frequency adjustments (Case 6)
- Offset between master→slave and slave→master messages initialized randomly at each node
  - All nodes send messages at local free-running clock rate (offsets vary over simulation)

❑Results (see plots on following slides)

- If frequency adjustments are not made, phase steps occur due to variation in time offset between master→slave and slave →master messages
  - This time offset results in a phase error equal to the size of the offset (in units of time) multiplied by the fractional frequency difference between the free-running master and slave clocks
  - As the time offset increases from 0 to T_m (or decreases from T_m to 0, phase offset changes
  - When the time offset reaches T_m (or 0) it jumps to 0 (or T_m) as one message "walks past" the other
  - This produces a step change in phase error equal to yT_m , where y is the relative frequency offset between the master and slave
    - –E.g., for $T_m = 0.001$ s and $y = 100$ ppm, the phase error jump is 100 ns

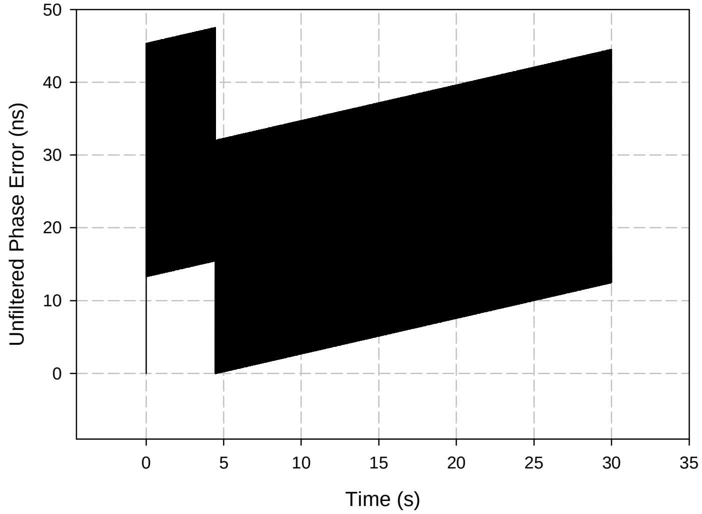
Simulation Cases 5 and 6

❑Results (Cont.)

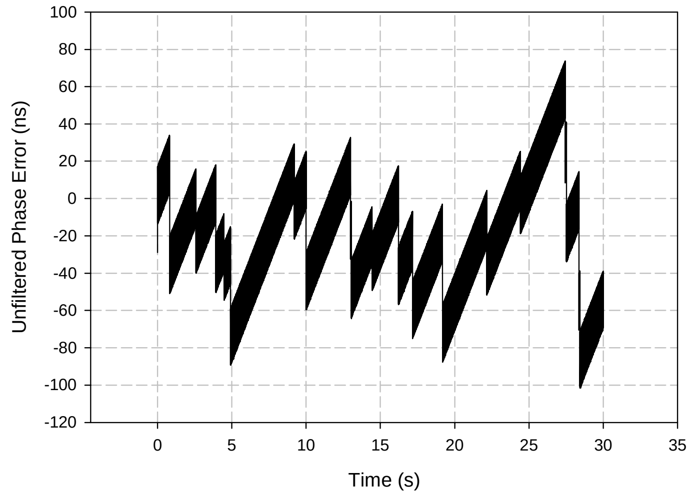
- The 10 Hz filter removes the fast phase variation due to instantaneous phase adjustments, clock phase noise, and non-zero granularity; however, it cannot remove the phase variation due to variation in the time offset between the master→slave and slave→master messages as this variation is much slower
- The effect does not occur when frequency adjustments are made because the error in phase correction due to the frequency offset between the nodes is corrected for
- MTIE for the case with frequency adjustments is roughly the same as in the corresponding case where the master→slave and slave→master message time offset does not vary (Case 4)
- Phase variation does not increase monotonically with number of clocks in chain (in all cases)
- Note that the results exhibit large statistical variability
  - Must run multiple, independent replications of the simulations to obtain confidence intervals for the results

Simulation Case 5

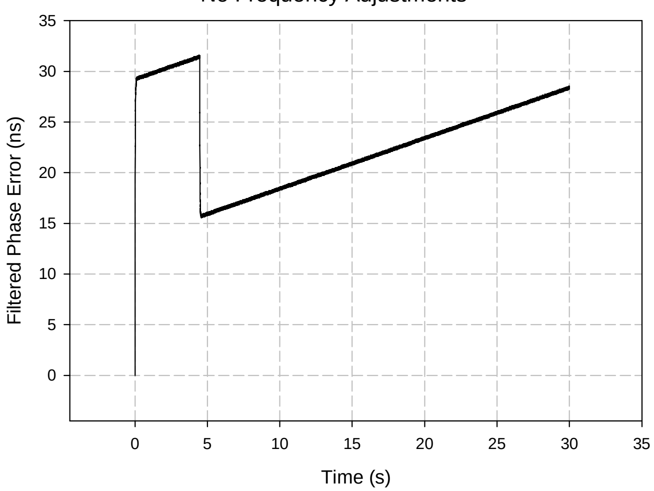
Case 5, Node 1
Instantaneous Phase Adjustments
No Frequency Adjustments



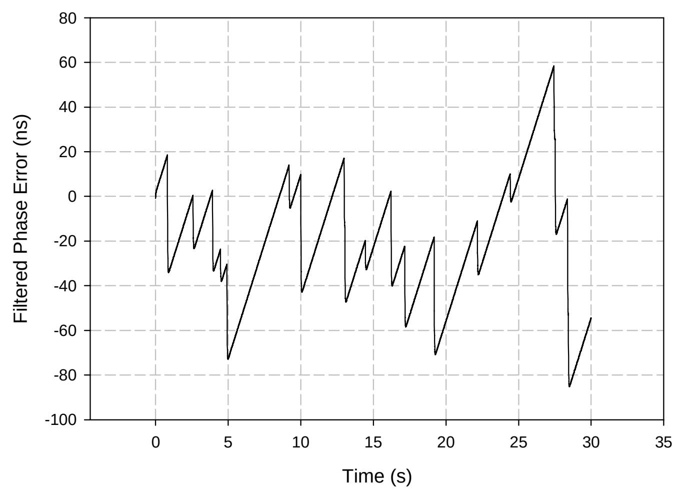
Case 5, Node 10
Instantaneous Phase Adjustments
No Frequency Adjustments



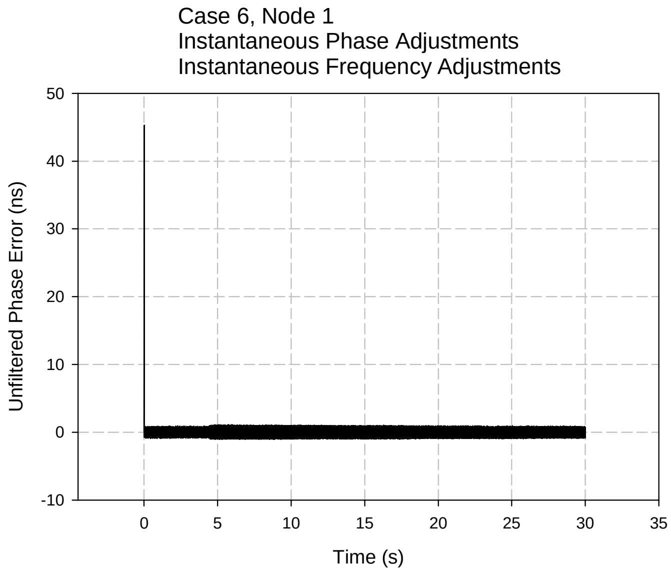
Case 5, Node 1
Filtered Phase Adjustments
No Frequency Adjustments

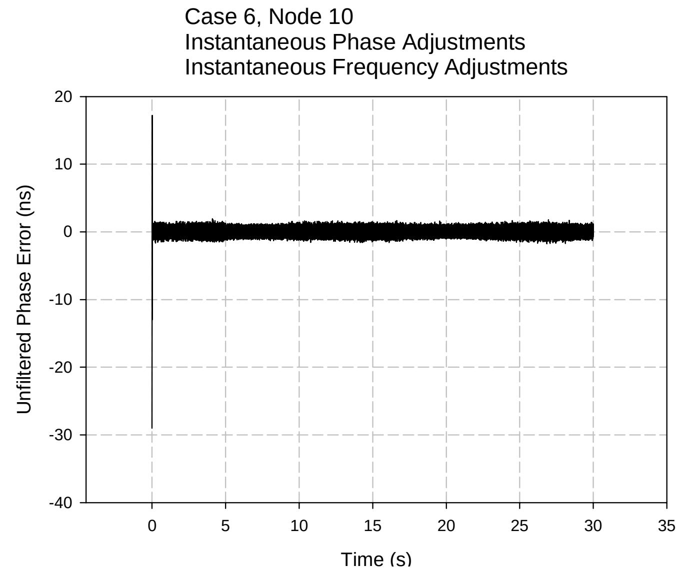


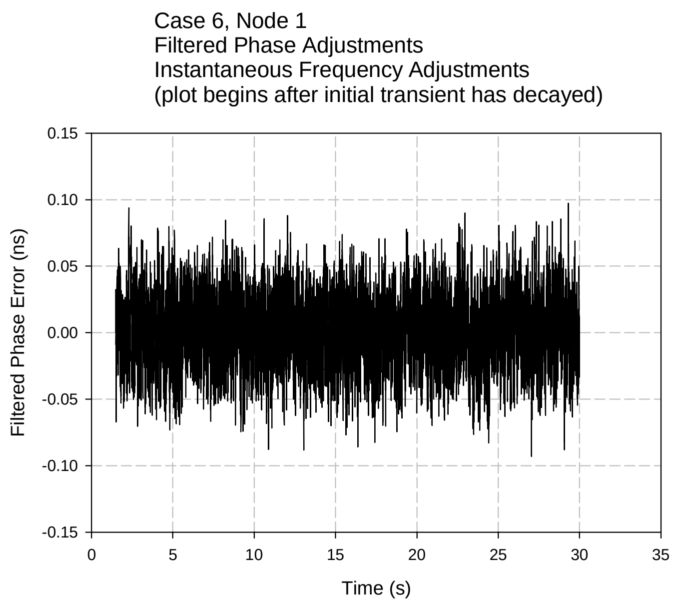
Case 5, Node 10
Filtered Phase Adjustments
No Frequency Adjustments

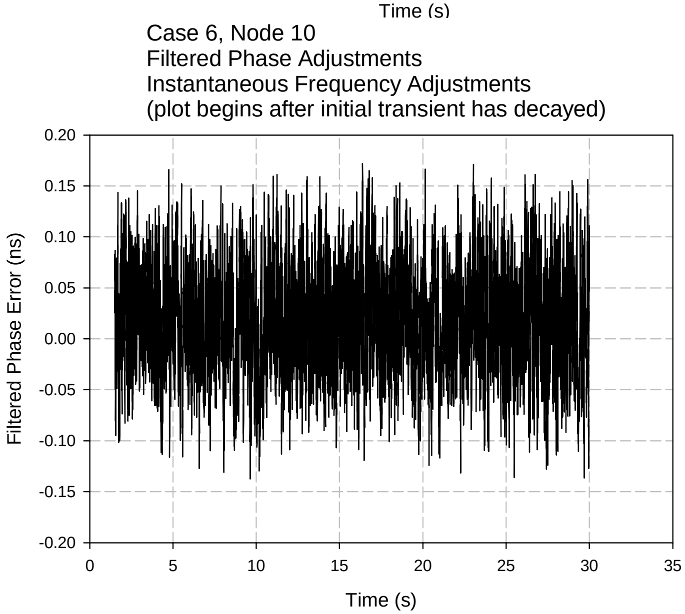


Simulation Case 6

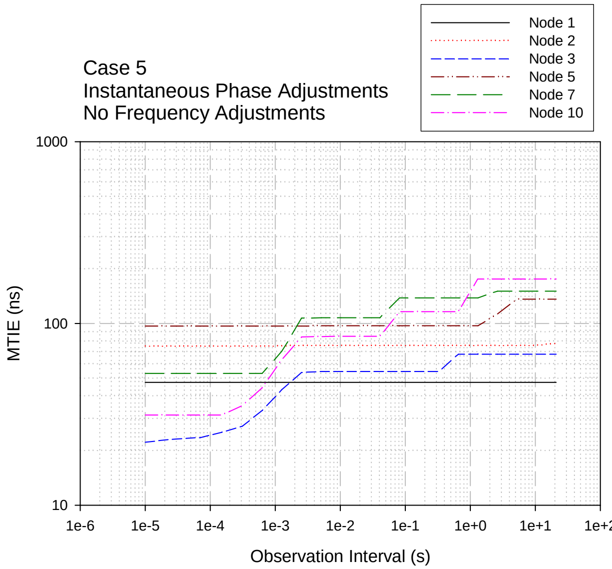


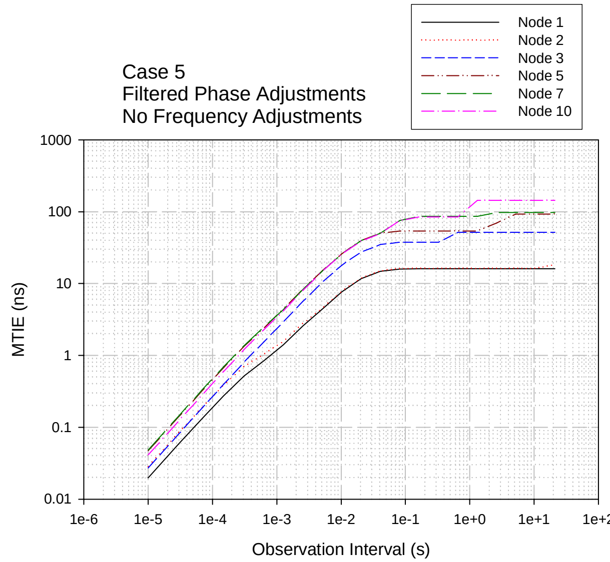


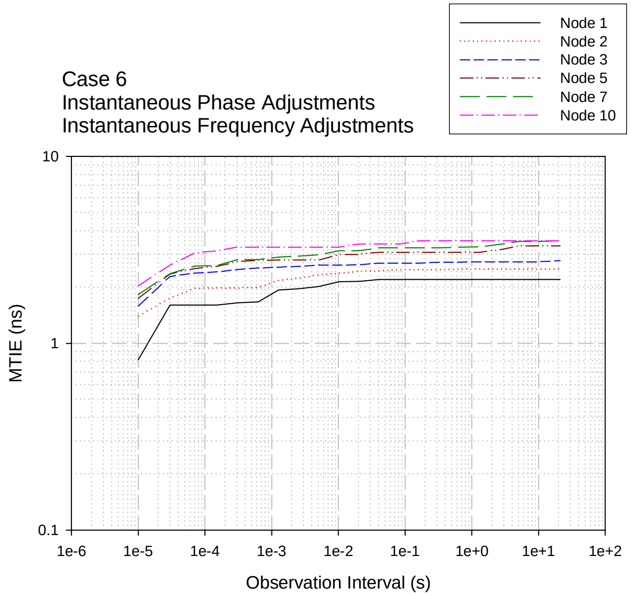


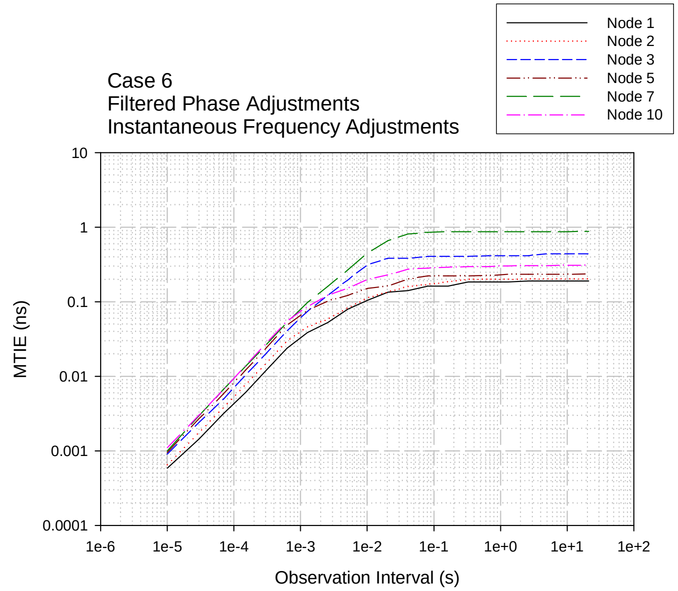


Simulation Cases 5 and 6









Conclusions

- In ideal case of no clock noise, zero phase granularity, and no variation in the time offset between the master→slave and slave→master messages , can achieve extremely small peak-to-peak phase variation in steady state
  - 0.07 ns with no filtering and frequency adjustments (Case 2, node 10)
  - 0.00055 ns with filtering and frequency adjustments (Case 2, node 10)
  - 0.12 ns with filtering and no frequency adjustments (Case 1, node 10)
- However, with clock noise (using the model of the appendix) and 1 ns phase granularity, peak-to-peak phase variation in steady state is larger
  - 1 – 4 ns with no filtering and frequency adjustments, whether or not time offset between the master→slave and slave→master messages vary
  - 0.2 – 1 ns with filtering and frequency adjustments, whether or not time offset between the master→slave and slave→master messages vary
  - 0.4 – 1.5 ns with filtering and no frequency adjustments if time offset between the master→slave and slave→master messages does not vary
  - 20 – 200 ns with filtering and no frequency adjustments if time offset between the master→slave and slave→master messages does vary

Conclusions

- The cases with clock noise and 1 ns phase granularity indicate that MTIE masks for uncompressed digital video are exceeded if filtering is not done
  - This indicates that filtering is necessary, whether or not instantaneous frequency adjustments are made
  - While end-to-end digital audio masks are met, note that ResE gets only a budget allocation of the total (see [15] for digital audio reference models); also must consider statistical variability of the results (future work)
- The uncompressed digital video masks are slightly exceeded with 10 Hz, 0.1 dB filtering
  - Note that the masks apply to the end-to-end application
    - ResE gets only a budget allocation of the total
    - Get some additional phase variation (likely small) due to the finite granularity of the application time stamps relative to the synchronization signals described here
  - This means it is likely that the filter must have BW that is somewhat narrower than 10 Hz
- Results show that if instantaneous frequency adjustments are not made, must ensure that master→slave and slave→master messages are sent at nominally the same rate, to avoid variation of their time offset and resulting large phase variation for this case
- Note that only variations (2) – (4) (see slide 8) have been addressed here

Future Work

- ❑ Analysis of additional parameter variations
  - ▪ Filter BW
  - ▪ Larger clock noise level
    - • Choose level that bounds noise in oscillators expected to be used in ResE
  - ▪ Clock phase granularity
- ❑ Determination of statistical confidence intervals for MTIE (and possibly TDEV) by running multiple, independent replications a simulation case
- ❑ Analysis of other variations/choices (slide 8)

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Appendix I – Clock Noise Model

- Clock phase noise may be modeled as a sum of random processes with power spectral density (PSD) of the form $Af^{-\alpha}$
  - In practice, the PSD has 3 terms (see [8] and [9])
    - α = 0, White Phase Modulation (WPM)
    - α = 1, Flicker Phase Modulation (FPM)
    - α = 3, Flicker Frequency Modulation (FFM)
  - Can write the PSD, $S_x(f)$ as

$$S_x(f) = \frac{A}{f^3} + \frac{B}{f} + C, \text{ where } S_x(f) \text{ has units of } \mathrm{ns}^2/\mathrm{Hz}$$

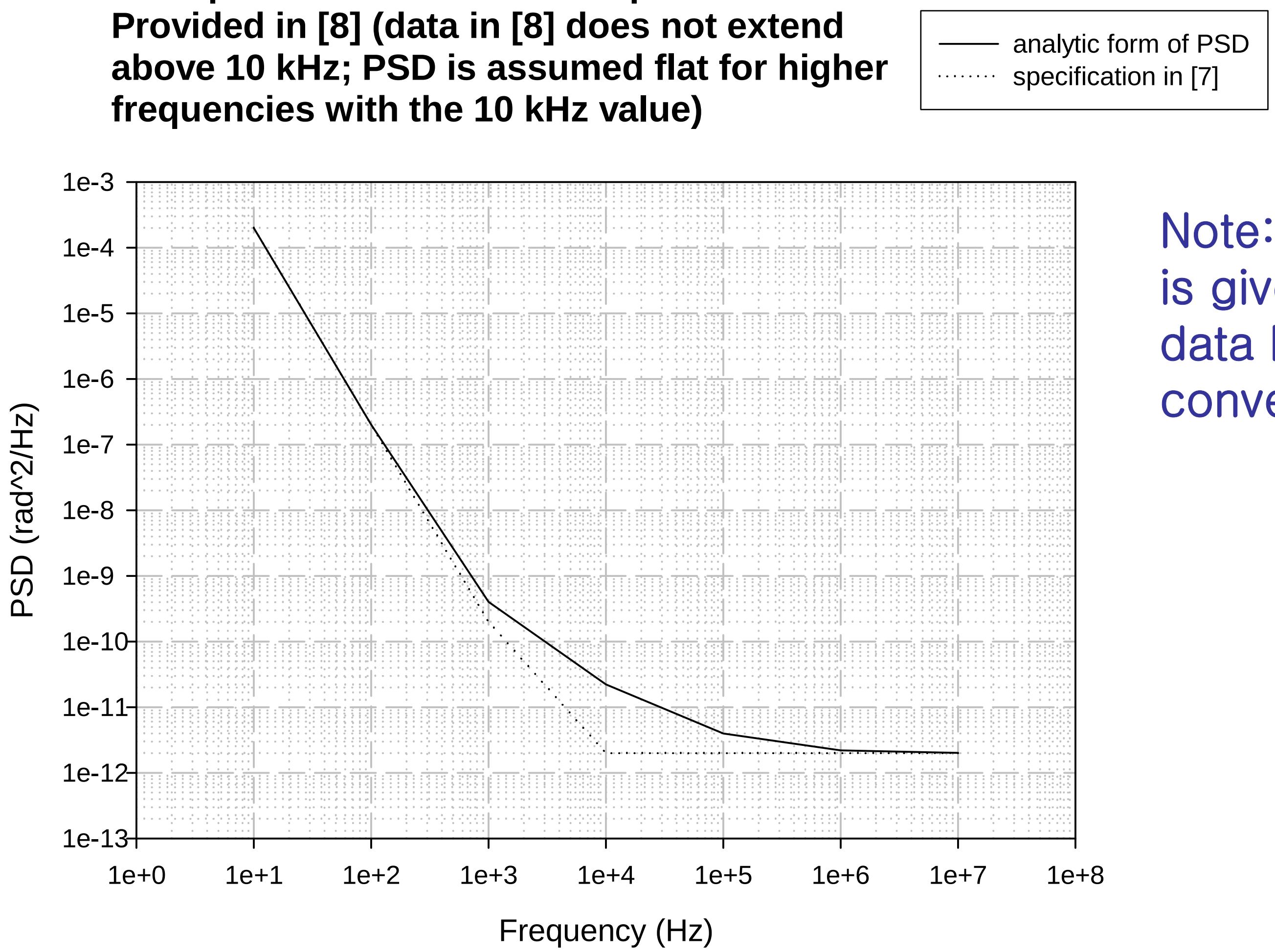
  - Often express as

$$S_\phi(f) = (2\pi\nu_0)^2 S_x(f), \text{ where units of } S_\phi(f) \text{ are } \mathrm{rad}^2/\mathrm{Hz}$$

  - An example PSD specification is given in Figure 12 of [8], and reproduced on the next slide
    - Data in [8] is given in dBc/Hz; data has been converted to rad^2/Hz
    - Data in [8] is given only for frequencies below 10 kHz; here, we assume the PSD is flat above 10 kHz
    - Dotted curve on the next slide is the converted data of [8]; solid line is a conservative fit of the above power law sum
  - The specifications for the individual products of [7] and [8] are below this example, at least for those products where phase noise specifications are provided

Appendix I – Clock Noise Model

Example Clock Phase Noise Specification Provided in [8] (data in [8] does not extend above 10 kHz; PSD is assumed flat for higher frequencies with the 10 kHz value)



Note: Data in [8] is given in dBc/Hz; data has been converted to rad^2/Hz

Appendix I – Clock Noise Model

- Another measure for clock noise, which is more convenient because it is a time domain parameter, is Time Variance (TVAR)
  - Time Deviation (TDEV) is the square root of TVAR
- TVAR is 1/6 times the expectation of the square of the second difference of the phase errror averaged over an interval

$$\mathrm{TVAR}(\tau) = \frac{1}{6} E\left[\left(\Delta^2 \bar{x}\right)^2\right]$$

where $E[\cdot]$ denotes expectation,

$\bar{x}$ denotes average over the integration time τ,

and Δ^2 denotes second difference

- TVAR may be estimated from measured or simulated data using [5]

$$\mathrm{TVAR}(n\tau_0) = \frac{1}{6n^2(N-3n+1)} \sum_{j=1}^{N-3n+1} \left[\sum_{i=j}^{n+j-1} \left(x_{i+2n} - 2x_{i+n} + x_i\right) \right]^2, \quad \mathrm{n} = 1, 2, \ldots, \text{integer part}(N/3)$$

where τ_0 is the sampling interval and $\tau = N\tau_0$

Appendix I – Clock Noise Model

- TVAR is equal to $\tau^2/3$ multiplied by the Modified Allan Variance
- For power-law noises with PSD proportional to $f^{-\alpha}$, TVAR is proportional to τ^β, where $\beta = \alpha - 1$
- The magnitude of TVAR may be related to the magnitude of PSD for power-law noises; see [10] and [11] for details
  - FFM

$$S_x(f) = \frac{A}{f^3} \qquad \mathrm{TVAR}(\tau) = \frac{(2\pi)^2 9 \ln 2}{20} A \tau^2$$

  - FPM (result is from [10]; a more exact expression is given in [11])

$$S_x(f) = \frac{B}{f} \qquad \mathrm{TVAR}(\tau) = \frac{3.37}{3} B$$

  - WPM

$$S_x(f) = C \qquad \mathrm{TVAR}(\tau) = \frac{\tau_0 f_h}{\tau} C$$

f_h = noise bandwidth

Appendix I – Clock Noise Model

❑Simulation of WPM

- WPM is simulated as a sequence of independent, identically distributed random samples
- Noise distribution is taken as Gaussian with zero mean
- Variance and sampling time determine TDEV level
  - Choose variance such that, with given sampling time, the computed TDEV from a sample history is close to value obtained from above relation between TDEV and PSD
    - –Assume noise bandwidth is equal to line rate (100 MHz)

❑Simulation of FPM

- FPM is simulated by passing a sequence of independent, identically distributed random samples through a Barnes/Jarvis filter [12] – [14]
  - If white noise is input to a filter with frequency response $H(f) = f^{-1/2}$, the output is a random process with PSD proportional to $1/f$
  - The Barnes/Jarvis filter approximates an $f^{-1/2}$ frequency response using a bank of lead/lag filters
    - –The actual frequency response of this filter is a “staircase”
    - –The spacings of the poles and zeros are chosen such that the average slope is –10 dB/decade

Appendix I – Clock Noise Model

❑Simulation of FPM (Cont.)

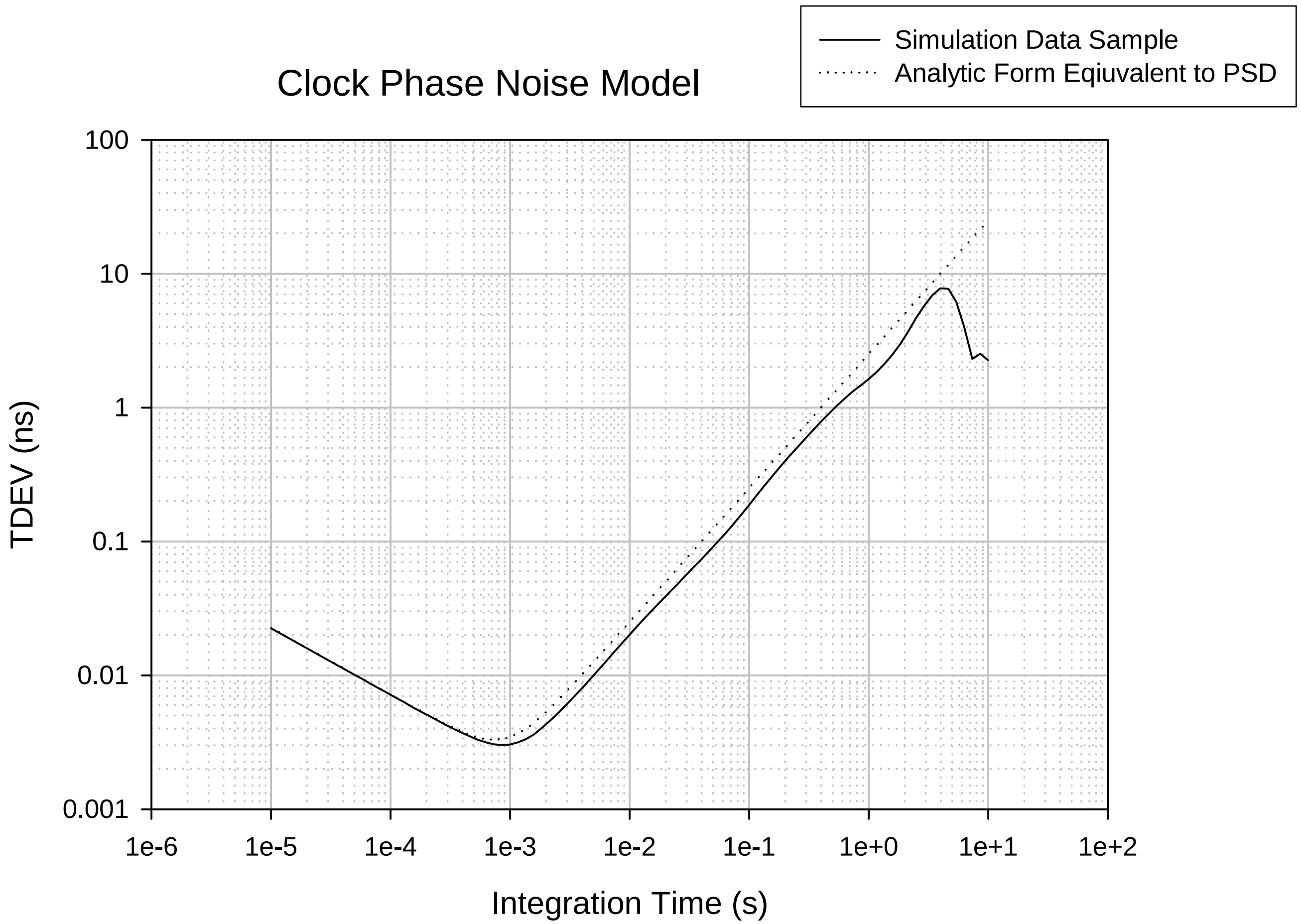
- Noise distribution is taken as Gaussian with zero mean
- Variance determines TDEV level
  - Choose variance such that the computed TDEV from a sample history is close to value obtained from above relation between TDEV and PSD

❑Simulation of FFM

- Input a sequence of independent, identically distributed random samples through a Barnes/Jarvis filter followed by an integrator (accumulator)
- Noise distribution is taken as Gaussian with zero mean
- Variance determines TDEV level
  - Choose variance such that the computed TDEV from a sample history is close to value obtained from above relation between TDEV and PSD

❑Next slide shows TDEV for simulated data sample (10^{-5} s time step) and analytic form equivalent to PSD (solid curve on slide 35)

Appendix I – Clock Noise Model



Appendix II - Definition of MTIE

- Jitter and wander requirements can be expressed in terms of Maximum Time Interval Error (MTIE) masks
- MTIE is peak-to-peak phase variation for a specified observation interval, expressed as a function of the observation interval
  - An estimate of MTIE may be computed by (see [5])

$$\mathrm{MTIE}(n\tau_0) \cong \max_{1 \le k \le N-n} \left(\max_{k \le i \le k+n} x(i) - \min_{k \le i \le k+n} x(i) \right), \quad n = 1, 2, \ldots, N-1$$

where τ_0 is the sampling interval, $n\tau_0$ is the observation interval, $x(i)$ is the i^{th} phase sample, and N is the number of phase samples ($N\tau_0$ is the measurement interval)

- The derivation of the MTIE masks on slide 6 from the jitter and wander requirements is given in [2]