New Simulation Results for Time Error Performance for Transport over an IEC/IEEE 60802 Network Based on Updated Assumptions Revision 1

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Outline

- Introduction
- Assumptions for Simulation Cases
- Results
- Conclusion and Discussion of Next Steps

Changes relative to Revision 0 indicated by highlighting
In the July 2020 IEC/IEEE 60802 virtual meetings, further simulation results for dynamic time error for transport over an IEC/IEEE 60802 network were presented [1].

These were followup results, after initial results had been presented in the March 2020 and May 2020 virtual meetings [2], [3].

The assumptions for the July meeting simulations [1] were based on previous discussion at the January, March, and May 2020 802.1 meetings [2] – [4], and also on detailed discussion of the clock models used in 802.1AS, Annex B and the clock model assumptions for IEC/IEEE 60802 [5].

The simulation results in [2] (March meeting) indicated that the desired objective of max|TE| of 1 μs over 64 hops (and over 100 hops if possible) cannot be met using the assumptions for the 60802 local clock (± 100 ppm maximum frequency offset and 3 ppm/s maximum frequency drift rate), accumulation of neighborRateRatio to obtain grandmaster (GM) rateRatio, and other assumptions for the various 802.1AS parameters described in [3] (see results in slide 29 of [3]).

Based on discussion at the March meeting and subsequent email discussion between the March and May meetings, modified assumptions were suggested:

- Consider smaller maximum frequency drift rates (0.1, 0.3, and 1 ppm/s, in addition to the 3 ppm/s initially considered); this is equivalent to using a more stable oscillator.
- Consider measuring the rateRatio relative to the GM using successive Sync messages, rather than accumulating neighborRateRatio measured using Pdelay messages.
Simulations using each of these assumptions were run, and the results were presented at the May meeting [2].

In particular, for the measurement of GM rateRatio using successive Sync messages, the following were considered:

- Measure a new GM rateRatio on receipt of each Sync message, using the current and previous Sync message.
- Measure a new GM rateRatio on receipt of every 10\textsuperscript{th} Sync message, using that message and the 10\textsuperscript{th} previous Sync message (i.e., jumping window of size 10).

The May results [2] indicated the following:

- Depending on the timestamp granularity (i.e., 2 ns and 8 ns were considered), and mean Sync and mean Pdelay message rates (various combinations of 1 message/s and 32 messages/s were considered), it was possible to meet the objective of 1 $\mu$s max$|d\text{TE}|$ over 64 hops and over 100 hops if possible (leaving sufficient margin for other time error budget components, e.g., cTE) for maximum frequency drift rates of 0.1 ppm/s and 0.3 ppm/s.
  - It also was possible to meet this objective for 1 ppm/s, but only for Sync and Pdelay mean rates of 32 messages/s and timestamp granularity of 2 ns.
  - The method of measuring frequency offset relative to the GM using successive Sync messages resulted in max$|d\text{TE}|$ that exceeded 1 $\mu$s over 64 hops in all the cases considered.
In the discussions in the May 2020 IEC/IEEE 60802 meeting, and in subsequent email discussion, it was indicated that while it might be possible to improve the oscillator stability for some applications, there are other applications where this is not possible (i.e., the resulting cost would be too large).

It also was indicated that, in measuring GM rateRatio using successive Sync messages, a sliding window should be used rather than a jumping window. This would result in better time error performance.

One participant, who had been providing assumptions and requirements for the work so far, indicated he could provide revised assumptions in a presentation, which could then be discussed on the 802.1 email reflector prior to running new simulations.

The presentation was provided, and then revised after subsequent discussion on the reflector; the latest version of this presentation is [6] (note that this presentation is now superseded by a subsequent presentation, which is described shortly).

New simulations were subsequently run, based on the assumptions of [6] and subsequent email discussion on the 802.1 reflector.

These assumptions and simulation results are documented in [1] and were presented at the July 2020 IEC/IEEE 60802 meeting.
Since the March and May results for cases where GM rateRatio is measured by accumulating neighborRateRatio did not meet the desired objectives when the oscillator maximum drift rate was 3 ppm/s, and since the cases where GM rateRatio is measured using successive Sync messages also did not meet the desired objective with 3 ppm/s drift rate using a jumping window, it was decided to next focus on the method using successive Sync messages using a sliding window and taking the median over the current and most recent 7 computed GM rateRatio values.

- The simulation cases of [1] focused on this method.

An important consideration in this method is the residence time; 3 different assumptions for residence time were considered:

- 1 ms
- 4 ms
- 1 ms, 4 ms, 10 ms, each with equal probability
With the assumptions of [1] (these include a Sync interval of 0.03 s and a Pdelay interval of 1 s), it appears possible to meet the 1 μs objective for max|TE| over 100 hops (and therefore over 64 hops) if the residence time does not exceed 1 ms.

If the residence time is 4 ms (or less), it appears possible (with the current assumptions) to meet the 1 μs objective for max|TE| over 64 hops.

- For 100 hops, it appears there is sufficient margin to meet the objective if the median is not used in the GM rateRatio measurement.
  - If the median is used, there might not be sufficient margin (further analysis is needed of the gain peaking inherent in the rateRatio measurement, as described below).

If the residence time is allowed to take on the values 1 ms, 4 ms, and 10 ms, each independently (for each Sync message) with probability of 1/3, there is not sufficient margin to meet the 1 μs objective for max|TE| over either 64 hops or 100 hops.

The results clearly show the effect of gain peaking inherent in the estimation of rateRatio using successive Sync messages, i.e., the large increase in accumulated max|dTE| if residence time is sufficiently large compared to the interval over which GM rate ratio is measured.

- The effect is less pronounced using a sliding window, as in the current presentation (compared to the jumping window used in [2]); however, the effect is still present.
The discussion in the July 2020 meeting and in subsequent emails indicated the following points should be considered for the next steps:

- If the 1 μs objective for max|TE| over 64 hops (and over 100 hops if possible) cannot be met for maximum frequency drift rate of 3 ppm/s and assumptions for residence time, pdelay turnaround time, mean sync rate, and mean pdelay rate, this needs to be established
  - The measurement of GM rateRatio using accumulation of neighborRateRatio and using successive Sync messages should each be considered in establishing this

- The effect of phase and frequency variation at the GM needs to be considered in the simulations (so far, for simplicity, the GM has been assumed to have zero phase error, rather than ±50 ppm frequency offset and 3 ppm/s maximum frequency drift rate, with max|dTE_R| computed relative to the GM

- The simulations should be based on multiple independent replications, rather than a single replication, to lessen the effect of statistical variability

It was decided to discuss a revised set of assumptions on the 802.1 reflector, which could be used for subsequent simulations

- Base on this, the presentation [7] was prepared as a revision of [6], and [8] was prepared as a convenient summary of [7]
Summary of Assumptions for Simulations - 1

- In the following slides, the new assumptions are summarized, mainly by repeating the material of [8] (correcting one typo related to mean sync interval for cases involving accumulation of neighborRateRatio, see the following slides)

- Detailed background on the different assumptions are given in [1] – [7], but note the following points
  - Local clock phase and frequency variation is assumed to be sinusoidal
  - 300 multiple replications of each simulation case are performed, with random (independent) initial conditions for each replication; in particular
    - Initial phases of each Local Clock (including the GM in cases where the GM time and frequency error is modeled) are chosen randomly in [0, 2π]
    - Initial frequencies of each Local Clock (including the GM in cases where the GM time and frequency error is modeled) are chosen randomly in the range [50 - ε, 50] ppm, with ε = 5 ppm and maximum frequency drift rate of 3 ppm/s
      - This allows the modulation frequency (i.e., the frequency of the phase and frequency variation waveform to vary over a 10% range (i.e., (5 ppm/50 ppm) )

- For each of six simulation cases (described shortly), 2 subcases are considered
  - Source of GM time is assumed to be zero (though GM still has timestamp granularity), and max|dTE| is simulated
  - Source of GM time has same error as Local Clocks, and max|dTE_R| relative to GM is simulated
Summary of Assumptions for Simulations - 2

- For cases where source of GM time has non-zero error, \( \max|dTE_R| \) is computed using linear interpolation, because Sync message transmission times at the successive clocks (and therefore times at which time errors are computed at the successive clocks) are, in general, not the same.

- Note that \( dTE_R \) relative the GM is actually relative to the PTP output of the GM, and therefore does not include timestamp granularity at the GM output.
  - Possibly \( dTE_R \) should have included timestamp granularity at the GM output; in any case, it will be seen that timestamp granularity (2 ns) is negligible compared to \( \max|dTE_R| \) results (larger than 4 \( \mu \)s).

- The following slides repeat the tables of assumptions from [8], and then summarize some of the details of the assumptions that were described in [1].

- Following that, we first present results, i.e., \( \max|dTE| \), for each simulation case assuming the error in the source of GM time is zero.

- We then present approximate, analytic, results for \( \max|dTE_R| \) for each simulation case assuming the error in the source of GM time is as indicated in the tables of assumptions.
  - For now, only approximate analytic results are presented for this case because, in the course of doing the simulations, numerical difficulties were encountered.
    - The details of this will be described in subsequent slides, when the results are presented.
## Assumptions Common to All Simulation Cases - 1

<table>
<thead>
<tr>
<th>Assumption/Parameter</th>
<th>Description/Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hypothetical Reference Model (HRM), see note following the tables</td>
<td>100 PTP Instances (99 hops; GM, followed by 98 PTP Relay Instances, followed by PTP End Instance)</td>
</tr>
<tr>
<td>Timestamp granularity</td>
<td>2 ns</td>
</tr>
<tr>
<td>GM maximum frequency offset</td>
<td>± 50 ppm or zero (to compute dTE(_R) or dTE, respectively)</td>
</tr>
<tr>
<td>GM maximum frequency drift rate</td>
<td>3 ppm/s or zero (to compute dTE(_R) or dTE, respectively)</td>
</tr>
<tr>
<td>PTP End/Relay Instance maximum frequency offset (Local Clock)</td>
<td>± 50 ppm</td>
</tr>
<tr>
<td>PTP End/Relay Instance maximum frequency drift rate (Local Clock)</td>
<td>3 ppm/s</td>
</tr>
<tr>
<td>GM and Local Clock frequency variation</td>
<td>sinusoidal</td>
</tr>
<tr>
<td>Relative phases of GM and Local Clock frequency waveforms</td>
<td>Chosen randomly from a uniform distribution over [0, 2(\pi)] rad at initialization</td>
</tr>
<tr>
<td>Relative frequencies of Local Clock frequency waveforms</td>
<td>Choose randomly at initialization by allowing waveform amplitude to be random over a range [50 - (\varepsilon), 50] ppm; choose (\varepsilon = 5) ppm, so that the waveform frequency varies over a 10% range</td>
</tr>
<tr>
<td>Computed performance results</td>
<td>(\max</td>
</tr>
</tbody>
</table>
## Assumptions Common to All Simulation Cases - 2

<table>
<thead>
<tr>
<th>Assumption/Parameter</th>
<th>Description/Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Use syncLocked mode for PTP Instances downstream of GM</td>
<td>Yes</td>
</tr>
<tr>
<td>Window size for successive Sync messages method, when used</td>
<td>7 (take difference between respective timestamps of current Sync message and 7\textsuperscript{th} previous message)</td>
</tr>
<tr>
<td>Compute median for successive Sync messages method, when used</td>
<td>Yes</td>
</tr>
<tr>
<td>Endpoint filter parameters*</td>
<td>$K_pK_o = 11, K_iK_o = 65$ ($f_{3dB} = 2.6$ Hz, $1.288$ dB gain peaking, $\zeta = 0.68219$)</td>
</tr>
<tr>
<td>Simulation time</td>
<td>1050 s; discard first 50 s to eliminate any startup transient before computing $\max</td>
</tr>
<tr>
<td>Number of independent replications, for each simulation case</td>
<td>300</td>
</tr>
<tr>
<td>GM rateRatio and neighborRateRatio computation granularity</td>
<td>0</td>
</tr>
<tr>
<td>Mean link delay</td>
<td>500 ns</td>
</tr>
<tr>
<td>Link asymmetry</td>
<td>0</td>
</tr>
</tbody>
</table>

*Update: Due to error in input, actually had $K_pK_o = \omega_n^2 = (15.78 \text{ rad/s})^2 = 249$ With this, $f_{3dB} = 5.089$ Hz. Other parameters are as given above*
## Assumptions Common to All Simulation Cases - 3

<table>
<thead>
<tr>
<th>Assumption/Parameter</th>
<th>Description/Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dynamic timestamp error for event messages (Sync, Pdelay-Req, Pdelay_Resp) due to variable delays within the PHY</td>
<td>±8 ns; for each timestamp taken, a random error is generated. The error is + 8 ns with probability 0.5, and – 8 ns with probability 0.5. The errors are independent for different timestamps and different PTP Instances.</td>
</tr>
<tr>
<td>Window Size for mean link delay averaging (i.e., how many mean link delay samples are averaged over, assuming a sliding window)</td>
<td>16</td>
</tr>
</tbody>
</table>
Summary of Simulation Cases (parameters that are different for each case)

<table>
<thead>
<tr>
<th>Case</th>
<th>Method of computing GM rateRatio</th>
<th>Residence time (ms)</th>
<th>Pdelay turnaround time (ms)</th>
<th>Mean Sync Interval (ms)</th>
<th>Mean Pdelay Interval (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Accumulate neighborRateRatio</td>
<td>1</td>
<td>1</td>
<td>125</td>
<td>31.25</td>
</tr>
<tr>
<td>2</td>
<td>Accumulate neighborRateRatio</td>
<td>4</td>
<td>4</td>
<td>125</td>
<td>31.25</td>
</tr>
<tr>
<td>3</td>
<td>Accumulate neighborRateRatio</td>
<td>10</td>
<td>10</td>
<td>125</td>
<td>31.25</td>
</tr>
<tr>
<td>4</td>
<td>Use successive Sync messages</td>
<td>1</td>
<td>10</td>
<td>31.25</td>
<td>1000</td>
</tr>
<tr>
<td>5</td>
<td>Use successive Sync messages</td>
<td>4</td>
<td>10</td>
<td>31.25</td>
<td>1000</td>
</tr>
<tr>
<td>6</td>
<td>Use successive Sync messages</td>
<td>10</td>
<td>10</td>
<td>31.25</td>
<td>1000</td>
</tr>
</tbody>
</table>

Note that the mean Sync interval in cases 1 – 3 was mistakenly indicated as 0.125 ms in [8]; this was an error (typo)
As in previous simulations, the HRM is a linear chain that consists of 100 PTP Instances, and therefore with 99 PTP links connecting each successive pair of PTP Instance

- The first PTP Instance in the chain is the Grandmaster PTP Instance
- The next 98 PTP Instances are PTP Relay Instances
- The last PTP Instance is a PTP End Instance
- The PTP End Instance contains an endpoint filter, through which the transported time is computed

Actually, in [7] and [8] it was indicated that there should have been 101 PTP Instances (i.e., one additional PTP Relay Instance)

- Unfortunately, due to an oversight the number of PTP instances was not increased by 1 when preparing the simulation inputs
- However, it will be seen that the effect on the results should be negligible
Assumptions for HRM - 2

- As in previous simulations, the GM and each PTP Relay Instance do not filter the timestamps with an endpoint filter when computing the value of the originTimestamp and correctionField of each transmitted Sync message.
  - Rather, these fields are computed using the same fields of the most recently received Sync message, the <syncEventIngressTimestamp> of the most recently received Sync message, the <syncEventEgressTimestamp> of the Sync message being transmitted, and the current value of rateRatio (i.e., cumulative rateRatio).

- However, the information at each PTP Relay Instance is used to separately compute a filtered (recovered) time, which could be used, e.g., by a co-located end application.
  - This is equivalent to having a PTP End Instance collocated with the PTP Relay Instance.
Review of Endpoint Filter Model and Assumptions

- $K_p = \text{proportional gain}$
- $K_i = \text{integral gain}$
- $K_o = \text{VCO/DCO gain}$

Transfer function:

$$H(s) = \frac{K_p K_o s + K_i K_o}{s^2 + K_p K_o s + K_i K_o} = \frac{2\zeta \omega_n s + \omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2}$$

with

$$\omega_n = \sqrt{K_i K_o} \quad \zeta = \frac{K_p}{2} \sqrt{\frac{K_o}{K_i}}$$
Often the filter parameters (and requirements) are expressed in terms of 3 dB bandwidth \( (f_{3\text{dB}}) \) and gain peaking \( (H_p) \)

- These are related to damping ration \( (\zeta) \) and undamped natural frequency \( (\omega_n) \) by (see [6] and [7] of reference [2] here):

\[
f_{3\text{dB}} = \frac{\omega_n}{2\pi} \left[ 1 + 2\zeta^2 + \sqrt{(1 + 2\zeta^2)^2 + 1} \right]^{1/2}
\]

\[
H_p \text{ (dB)} = 20 \log_{10} \left\{ \left[ 1 - 2\alpha - 2\alpha^2 + 2\alpha\sqrt{2\alpha + \alpha^2} \right]^{-1/2} \right\}
\]

where

\[
\alpha = \frac{1}{4\zeta^2} = \frac{K_i}{K_p^2 K_o}
\]
As in previous simulation models, the VCO gain was folded into the proportional gain and integral gain (this is equivalent to setting the VCO gain to 1)

Filter assumption:

- $K_p K_o = 11$, $K_i K_o = 249$ (should have been 65)
- Using the equations on the previous slides, we obtain:
  - $\zeta = 0.68219$
  - $\omega_n = 15.78\,\text{rad/s}$
  - $H_p$ (gain peaking) = 1.28803 dB = (approx) 1.3 dB
  - $f_{3\text{dB}} = 5.089\,\text{Hz}$ (should have been 1.33 Hz; previously given as 2.6 Hz)
  - Note that the $\zeta$ and $\omega_n$ above are what were used in the simulations

Note that this filter is underdamped, and has appreciable gain peaking

- However, the damping ratio ($\zeta$) is close to $1/\sqrt{2} = (\text{approx}) 0.707$; this is often used to obtain a fast response with small overshoot, in cases where the filters are not cascaded (the endpoint filters are not cascaded)
Review of computation of GM rateRatio using successive Sync messages - 1

- Assume the computation is done every Sync message, using a window of size $n$ (i.e., a sliding window)
  - The computation is done on ingress of a Sync message at a PTP Instance
  - The window size $n$ includes the current Sync message (e.g., a window of size 8 consists of the current Sync message and the previous 7 Sync messages)
- Let $C_{kn}$ be the correctedMasterTime carried by Sync message $kn$
- Let $S_{kn}$ be the SyncEventIngressTimestamp for Sync message $kn$
- Then the initial computed rateRatio is

$$rateRatio_{kn} = \frac{C_{kn} - C_{(k-1)n}}{S_{kn} - S_{(k-1)n}}$$

- Note that frequency offset is equal to $rateRatio - 1$
- The above computation is performed for every Sync message that arrives at a PTP Instance
Finally, the median of the current and previous $n - 1$ computed values of initial GM rateRatio is obtained

- The median is computed by sorting the $n$ values from smallest to largest and taking the $p^{th}$ smallest value, where $p = \text{floor}(n) + 1$

For the simulations, we use the median
The following plots show:

- Max|dTE|, nodes 2 – 100, 99% confidence intervals for 0.95 quantile, and maximum over 300 replications
- Max|dTE|, nodes 2 – 100, 99% maximum over 300 replications (less cluttered than previous plot)
- Max|dTE|, nodes 2 – 65, 99% confidence intervals for 0.95 quantile, and maximum over 300 replications
- Max|dTE|, nodes 2 – 65, 99% maximum over 300 replications (less cluttered than previous plot)
Results for dTE, Zero Error in GM Time Source - 2

Simulation Cases 1 - 6
300 replications of simulation
Upper and lower 99% confidence intervals shown via short dashed lines
Clock Model: sinusoidal phase and frequency variation
  50 ppm max freq offset
  3 ppm/s maximum drift rate
  relative phases of modulation chosen randomly over [0,2*π] on initialization
  Actual modulation amplitude chosen randomly over [45 ppm, 50 ppm]
Cases 1 - 3: accumulate neighborRateRatio
Cases 4 - 6: measure GM rate ratio using successive Sync msgs

![Graph showing max|dTE| (ns) vs. Node Number]
Results for dTE, Zero Error in GM Time Source

Simulation Cases 1 - 6
300 replications of simulation
Clock Model: sinusoidal phase and frequency variation
  50 ppm max freq offset
  3 ppm/s maximum drift rate
  relative phases of modulation chosen randomly over [0, 2*pi] on initialization
  Actual modulation amplitude chosen randomly over [45 ppm, 50 ppm]
Cases 1 - 3: accumulate neighborRateRatio
Cases 4 - 6: measure GM rate ratio using successive Sync msgs

![Graph showing max|dTE| vs Node Number]
Results for dTE, Zero Error in GM Time Source - 4

Simulation Cases 1 - 6
300 replications of simulation
Upper and lower 99% confidence intervals shown via short dashed lines
Clock Model: sinusoidal phase and frequency variation
  50 ppm max freq offset
  3 ppm/s maximum drift rate
  relative phases of modulation chosen randomly over [0,2*pi] on initialization
    Actual modulation amplitude chosen randomly over [45 ppm, 50 ppm]
Cases 1 - 3: accumulate neighborRateRatio
Cases 4 - 6: measure GM rate ratio using successive Sync msgs
Results for dTE, Zero Error in GM Time Source - 5

Simulation Cases 1 - 6
300 replications of simulation
Clock Model: sinusoidal phase and frequency variation
   50 ppm max freq offset
   3 ppm/s maximum drift rate
   relative phases of modulation chosen randomly over [0,2*π] on initialization
   Actual modulation amplitude chosen randomly over [45 ppm, 50 ppm]
Cases 1 - 3: accumulate neighborRateRatio
Cases 4 - 6: measure GM rate ratio using successive Sync msgs

![Graph showing max|dTE| vs Node Number for different cases]
For cases 1 – 3 (accumulate neighborRateRatio) and 100 nodes, max|dTE| is approximately 300 ns for 1 ms residence and Pdelay turnaround time, and 500 ns for 4 ms residence and Pdelay turnaround time

- It is likely the 1000 ns max|TE| objective can be met for these cases (i.e., there is sufficient margin for cTE and other budget components)
- However, max|dTE| is approximately 850 ns for 10 ms residence and Pdelay turnaround time; this likely does not leave sufficient margin, and the max|TE| objective likely cannot be met in this case with the current parameters
  - However, if the endpoint filter bandwidth is lowered from 5.089 Hz to the desired 1.33 Hz, max|dTE| for this case will be lower, and there may be sufficient margin (but this needs to be confirmed)

For cases 1 – 3 and 65 nodes (64 hops), max|dTE| is approximately 250 ns, 420 ns, and 680 ns for 1 ms, 4 ms, and 10 ms residence and Pdelay turnaround times, respectively

- It is likely the max|TE| objective can be met in the former two cases, and in the third case if the remaining 320 ns is sufficient margin for cTE and other budget components
Results for dTE, Zero Error in GM Time Source - 7

- For cases 4 – 6 (measure GM rateRatio using successive Sync messages) and 100 nodes, max|dTE| is approximately 100 ns and 200 ns for 1 ms and 4 ms residence time, respectively
  - It is likely the 1000 ns max|TE| objective can be met for these cases (i.e., there is sufficient margin for cTE and other budget components)
  - However, max|dTE| is approximately 5700 ns (5.7 μs) for 10 ms residence time, which exceeds the 1 μs max|TE| objective by a large margin
  - The large increase in max|dTE| between 60 and 100 nodes is due to instability caused by the large residence time (10 ms) relative to the mean Sync interval of 31.25 ms (i.e., interval over which GM rateRatio is computed)

- For cases 4 – 6 and 65 nodes (64 hops), max|dTE| is approximately 40 ns, 80 ns, and 630 ns for 1 ms, 4 ms, and 10 ms residence and Pdelay turnaround times, respectively
  - It is likely the max|TE| objective can be met in the former two cases, and in the third case if the remaining 370 ns is sufficient margin for cTE and other budget components
  - But if the 3dB bandwidth is lowered from 5.089 Hz to the desired 1.33 Hz, there will be more margin for cTE for case 6
The results indicate that the method of measuring GM rateRatio using successive Sync messages gives better time error performance than measuring by accumulating neighborRateRatio provided the ratio of residence time to mean Sync interval is sufficiently small.

- However, for larger ratio of residence time to mean Sync interval, this method becomes unstable with the number of hops.

If it is desired to use the method of measuring GM RateRatio with successive Sync messages, it might be useful to examine the instability in more detail, e.g., by updating the stability analysis of [10] and [11].

- The stability analysis could provide a rule of thumb on how many hops could be present before dTE begins to increase rapidly, for given mean Sync interval and residence time.
  - Recall also the split syntonization scheme [9] – [14]
The following slides show sample time error time histories for cases 1 – 6

- All results are for the replication #1 (of 300 replications)
- For each case, results are shown for nodes 2, 60, and 100
- Note that the time histories do show the first 50 s, even though this was eliminated when computing max|dTE|
  - A large transient is sometimes present during the first 20 s or less, though the horizontal and vertical scales have not been increased to show the transient

The time histories show an overall sinusoidal envelope that modulates the faster phase changes that occur when a Sync message arrives and is used to update the time

- This was seen in earlier results (see [1] – [3])
- The sinusoidal envelope becomes distorted with increasing node number, and the distortion is (a) somewhat more rapid with larger residence time, and (b) much more rapid in cases 4 – 6 compared to cases 1 -3

There also is a longer-term beating effect, which is more pronounced for larger residence times

- The beating effect is clearly shown in cases 2 and 3
- The beating effect is masked in cases 4 – 6 by the distortion of the sinusoidal envelope
- The 1050 s simulation time is not sufficient to show a complete beat cycle; future simulations should be run for longer times (e.g., simulate 2050 s)
Case 1, PTP Instance (node) 2
Filtered phase offset relative to GM (GM has zero time error)
Clock Model: Sinusoidal phase and frequency error variation,
  random phase and frequency offsets chosen over [0,2*π]
  and [45 ppm, 50 ppm], respectively, at initialization
measure GM rateRatio by accumulating neighborRateRatio
Residence Time: 1 ms
Sync interval: 125 ms
Pdelay Interval: 31.25 ms
Results for dTE, Zero Error in GM Time Source - 11

Case 1, PTP Instance (node) 2
detail of 220 - 280 s
Filtered phase offset relative to GM (GM has zero time error)
Clock Model: Sinusoidal phase and frequency error variation,
    random phase and frequency offsets chosen over [0,2*pi]
    and [45 ppm, 50 ppm], respectively, at initialization
measure GM rateRatio by accumulating neighborRateRatio
Residence Time: 1 ms
Sync interval: 125 ms
Pdelay Interval: 31.25 ms

Note: The relatively fast pattern of ramps and Jumps actually is more regular; the apparent irregularity is due to reduced granularity of the points plotted
Case 1, PTP Instance (node) 60
Filtered phase offset relative to GM (GM has zero time error)
Clock Model: Sinusoidal phase and frequency error variation,
random phase and frequency offsets chosen over $[0, 2\pi]$ and $[45 \text{ ppm}, 50 \text{ ppm}]$, respectively, at initialization
measure GM rateRatio by accumulating neighborRateRatio
Residence Time: 1 ms
Sync interval: 125 ms
Pdelay Interval: 31.25 ms
Case 1, PTP Instance (node) 100
Filtered phase offset relative to GM (GM has zero time error)
Clock Model: Sinusoidal phase and frequency error variation,
  random phase and frequency offsets chosen over [0,2*pi]
  and [45 ppm, 50 ppm], respectively, at initialization
measure GM rateRatio by accumulating neighborRateRatio
Residence Time: 1 ms
Sync interval: 125 ms
Pdelay Interval: 31.25 ms
Results for dTE, Zero Error in GM Time Source - 14

Case 2, PTP Instance (node) 2
Filtered phase offset relative to GM (GM has zero time error)
Clock Model: Sinusoidal phase and frequency error variation,
random phase and frequency offsets chosen over [0,2*pi]
and [45 ppm, 50 ppm], respectively, at initialization

measure GM rateRatio by accumulating neighborRateRatio

Residence Time: 4 ms
Sync interval: 125 ms
Pdelay Interval: 31.25 ms
Case 2, PTP Instance (node) 60
Filtered phase offset relative to GM (GM has zero time error)
Clock Model: Sinusoidal phase and frequency error variation,
random phase and frequency offsets chosen over [0,2*pi]
and [45 ppm, 50 ppm], respectively, at initialization
measure GM rateRatio by accumulating neighborRateRatio
Residence Time: 4 ms
Sync interval: 125 ms
Pdelay Interval: 31.25 ms
Case 2, PTP Instance (node) 100
Filtered phase offset relative to GM (GM has zero time error)
Clock Model: Sinusoidal phase and frequency error variation,
    random phase and frequency offsets chosen over [0,2*pi]
    and [45 ppm, 50 ppm], respectively, at initialization
measure GM rateRatio by accumulating neighborRateRatio
Residence Time: 4 ms
Sync interval: 125 ms
Pdelay Interval: 31.25 ms
Results for dTE, Zero Error in GM Time Source - 17

Case 3, PTP Instance (node) 2
Filtered phase offset relative to GM (GM has zero time error)
Clock Model: Sinusoidal phase and frequency error variation,
random phase and frequency offsets chosen over [0, 2*pi]
and [45 ppm, 50 ppm], respectively, at initialization

measure GM rateRatio by accumulating neighborRateRatio
Residence Time: 10 ms
Sync interval: 125 ms
Pdelay Interval: 31.25 ms
Case 3, PTP Instance (node) 60
Filtered phase offset relative to GM (GM has zero time error)
Clock Model: Sinusoidal phase and frequency error variation,
    random phase and frequency offsets chosen over [0,2*pi]
    and [45 ppm, 50 ppm], respectively, at initialization
measure GM rateRatio by accumulating neighborRateRatio
Residence Time: 10 ms
Sync interval: 125 ms
Pdelay Interval: 31.25 ms
Case 3, PTP Instance (node) 100
Filtered phase offset relative to GM (GM has zero time error)
Clock Model: Sinusoidal phase and frequency error variation,
random phase and frequency offsets chosen over [0,2*pi]
and [45 ppm, 50 ppm], respectively, at initialization
measure GM rateRatio by accumulating neighborRateRatio
Residence Time: 10 ms
Sync interval: 125 ms
Pdelay Interval: 31.25 ms
Case 4, PTP Instance (node) 2
Filtered phase offset relative to GM (GM has zero time error)
Clock Model: Sinusoidal phase and frequency error variation,
    random phase and frequency offsets chosen over [0,2*π] and [45 ppm, 50 ppm], respectively, at initialization
measure GM rateRatio using successive Sync messages
Residence Time: 1 ms
Sync interval: 31.25 ms
Pdelay Interval: 1000 ms
Case 4, PTP Instance (node) 60
Filtered phase offset relative to GM (GM has zero time error)
Clock Model: Sinusoidal phase and frequency error variation,
random phase and frequency offsets chosen over \([0,2\pi]\) and \([45 \text{ ppm}, 50 \text{ ppm}]\), respectively, at initialization
measure GM rateRatio using successive Sync messages
Residence Time: 1 ms
Sync interval: 31.25 ms
Pdelay Interval: 1000 ms
Results for dTE, Zero Error in GM Time Source - 22

Case 4, PTP Instance (node) 100
Filtered phase offset relative to GM (GM has zero time error)
Clock Model: Sinusoidal phase and frequency error variation,
random phase and frequency offsets chosen over [0,2*\pi]
and [45 ppm, 50 ppm], respectively, at initialization
measure GM rateRatio using successive Sync messages
Residence Time: 1 ms
Sync interval: 31.25 ms
Pdelay Interval: 1000 ms
Case 5, PTP Instance (node) 2
Filtered phase offset relative to GM (GM has zero time error)
Clock Model: Sinusoidal phase and frequency error variation,
random phase and frequency offsets chosen over [0,2*pi]
and [45 ppm, 50 ppm], respectively, at initialization
measure GM rateRatio using successive Sync messages
Residence Time: 4 ms
Sync interval: 31.25 ms
Pdelay Interval: 1000 ms
Results for dTE, Zero Error in GM Time Source - 24

Case 5, PTP Instance (node) 60
Filtered phase offset relative to GM (GM has zero time error)
Clock Model: Sinusoidal phase and frequency error variation, random phase and frequency offsets chosen over [0,2*pi] and [45 ppm, 50 ppm], respectively, at initialization
measure GM rateRatio using successive Sync messages
Residence Time: 4 ms
Sync interval: 31.25 ms
Pdelay Interval: 1000 ms
Case 5, PTP Instance (node) 100
Filtered phase offset relative to GM (GM has zero time error)
Clock Model: Sinusoidal phase and frequency error variation,
random phase and frequency offsets chosen over [0,2\*pi]
and [45 ppm, 50 ppm], respectively, at initialization
measure GM rateRatio using successive Sync messages
Residence Time: 4 ms
Sync interval: 31.25 ms
Pdelay Interval: 1000 ms
Case 6, PTP Instance (node) 2
Filtered phase offset relative to GM (GM has zero time error)
Clock Model: Sinusoidal phase and frequency error variation,
  random phase and frequency offsets chosen over \([0,2\pi]\)
  and \([45 \text{ ppm}, 50 \text{ ppm}]\), respectively, at initialization
measure GM rateRatio using successive Sync messages
Residence Time: 10 ms
Sync interval: 31.25 ms
Pdelay Interval: 1000 ms
Results for dTE, Zero Error in GM Time Source - 27

Case 6, PTP Instance (node) 60
Filtered phase offset relative to GM (GM has zero time error)
Clock Model: Sinusoidal phase and frequency error variation,
random phase and frequency offsets chosen over \([0,2\pi]\)
and [45 ppm, 50 ppm], respectively, at initialization
measure GM rateRatio using successive Sync messages
Residence Time: 10 ms
Sync interval: 31.25 ms
Pdelay Interval: 1000 ms
Results for dTE, Zero Error in GM Time Source - 28

Case 6, PTP Instance (node) 100
Filtered phase offset relative to GM (GM has zero time error)
Clock Model: Sinusoidal phase and frequency error variation,
random phase and frequency offsets chosen over [0,2*pi]
and [45 ppm, 50 ppm], respectively, at initialization
measure GM rateRatio using successive Sync messages
Residence Time: 10 ms
Sync interval: 31.25 ms
Pdelay Interval: 1000 ms
As indicated earlier, dTE_R at a respective node relative to the GM is, in principle, computed by subtracting dTE at the output of the endpoint filter at that node from dTE at the output of the GM.

The values of dTE at the respective node and the GM must be taken at the same time instant; since the times at which dTE is computed at different nodes are, in general, not the same, interpolation must be performed.

- The interpolation was chosen to be linear here.

However, the GM (and PTP Relay Instance Local Clock) waveforms, for the periodic phase and frequency errors considered here, have relatively low frequency and large amplitude.

- For example, as shown in [5] (slides 5 – 9 of [5]), for the case of 100 ppm maximum frequency offset, 3 ppm/s maximum frequency drift rate, and sinusoidal phase and frequency variation, the phase error waveform has amplitude of 3.33 ms (3.33 × 10^6 ns) and 4.7746 mHz.
  - Analogous values for the 50 ppm nominal amplitude waveform used here will be provided shortly.

But, the low-pass endpoint filters used here have 3 dB bandwidth of 5.089 Hz.

This means that the ratio of GM waveform frequency to filter bandwidth is approximately 0.001.
The endpoint filter acts on both the combined GM phase waveform and effect of the gPTP transport.

In addition, the accumulated time error due to the gPTP transport is, from the results above for cases with the zero error in the GM source, on the order of hundreds of ns.

- In other words, the error caused by the gPTP transport is on the order of $10^{-3}$ of the GM error.

The above ratio of GM phase waveform frequency to endpoint filter bandwidth results in extremely small attenuation and phase shift of the component of the endpoint filter output waveform due to the GM error.

The result of this is that if $d\text{TE}_R$ is computed by subtracting the interpolated output of the endpoint filter from the interpolated GM phase error, double precision arithmetic is not sufficient to give a valid result because the computation involves taking the difference between quantities that are extremely close to each other.

- Initial simulations using this approach produced $d\text{TE}_R$ results that were approximately an order of magnitude too large.

- It was realized that the results were too large by comparing with the analytical result of high-pass filtering the GM phase waveform; as will be shown below, this gives the result for the component of $d\text{TE}_R$ due only to the GM time error.
A better simulation approach is to compute $d\text{TE}_R,\text{unfiltered}$ (i.e., relative dynamic time error at a PTP Relay Instance before endpoint filtering) and then filter this quantity with the endpoint filter.

However, the present simulator is not configured to do this; in fact, it would be better to compute $d\text{TE}_R,\text{unfiltered}$ and filter it in a post processing operation rather than add interpolation to the simulator.

This will be done for future simulations.

For now, the effect of the GM time error can be estimated by high-pass filtering the GM time error waveform with a filter whose corner frequency and damping ratio are the same as those of the endpoint filter.

- This can be computed analytically, and is done on the following slides.
As a first step in the analytic computation, we obtain the GM phase and frequency variation waveform as follows:

\[ y(t) = A \sin(2\pi f_0 t) \]
\[ \dot{y}(t) = 2\pi f_0 A \cos(2\pi f_0 t) \]

where

\[ y(t) = \text{instantaneous frequency offset in ppm} \]
\[ A = \text{frequency offset amplitude in ppm} \]

Setting \( A \) to 50 ppm and the maximum frequency drift rate \( 2\pi f_0 A \) to 3 ppm/s produces

\[ 2\pi f_0 (50 \text{ ppm}) = 3 \text{ ppm/s} \]

\[ f_0 = \frac{3}{100\pi} \text{ Hz} = 9.549 \times 10^{-3} \text{ Hz} = 9.549 \text{ mHz} \]

Note that the maximum time offset of the GM is

\[ \frac{50}{2\pi (9.549 \times 10^{-3} \text{ Hz})} = 8.334 \times 10^{-4} \text{ s} = 0.8334 \text{ ms} \]

As was seen in previous presentations ([4], [5]), the GM error waveform is of relatively low frequency and large amplitude.
The endpoint filter bandwidth and damping ratio used for the dTE simulations for zero GM time error were 5.089 Hz and 0.68219, which corresponded to undamped natural frequency of $\omega_n = 15.78$ rad/s and $K_iK_o = 249$

However, the desired $K_iK_o$ is 65, which with the same damping ratio of 0.68219 (i.e., $K_pK_o = 11$) gives $\omega_n = 8.062$ rad/s and $f_{3dB} = 1.33$ Hz

These desired values are used in the calculations below

- Note that a narrower bandwidth endpoint filter results in more filtering of the GM time error and therefore larger dTE$_R$ due to the GM error
Results for dTE_R, Nonzero Error in GM Time Source - 6

As indicated above, while the 1.33 Hz endpoint filter essentially passes the much lower GM error waveform frequency of 9.55 mHz, there is a small attenuation and phase shift because the endpoint filter is not ideal (i.e., it has 20 dB/decade roll-off and non-zero gain peaking)

- The resulting difference between the output waveform from the endpoint filter and the GM error waveform, both of which have phase amplitudes on the order of 0.833 ms = 833 \( \mu \)s, is on the order of 46 ns (see the following slides), i.e., a lower bound for \( \max |dTE_R| \)

The analysis is as follows

The transfer function for the 2nd order endpoint filter, \( H(s) \), is

\[
H(s) = \frac{2\zeta \omega_n s + \omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2}
\]

where \( \omega_n \) is the undamped natural frequency in rad/s and \( \zeta \) is the damping ratio (see slides 17 and 18). The difference between the GM waveform, which to first approximation is the input to the input to the endpoint filter (ignoring the effect of the network, which is much smaller) and the output of the endpoint filter, is the high-pass error transfer function \( H_e(s) = 1 - H(s) \), i.e.

\[
H_e(s) = \frac{s^2}{s^2 + 2\zeta \omega_n s + \omega_n^2}.
\]

Setting \( s = j\omega = 2\pi jf \), \( \omega_n = 2\pi f_n \) produces

\[
H_e(j\omega) = \frac{-\omega^2}{-\omega^2 + 2\zeta \omega_n \omega j + \omega_n^2}.
\]
Results for dTE_R, Nonzero Error in GM Time Source - 7

Analysis (cont.)

The magnitude of the error transfer function is

\[ |H_e(j\omega)| = \frac{\omega^2}{[(\omega_n^2 - \omega^2)^2 + 4\zeta^2 \omega_n^2 \omega^2]^{1/2}}. \]

Dividing numerator and denominator by \( \omega_n^2 \) and setting \( x = \omega / \omega_n \) produces

\[ |H_e(j\omega)| = \frac{x^2}{[(1-x^2)^2 + 4\zeta^2 x^2]^{1/2}}. \]

Substituting \( \omega = 2\pi f_0 = 2\pi (9.549 \times 10^{-3}) \) Hz = 0.06 rad/s (see slide 35), \( \omega_n = 8.062 \) rad/s (see slide 19), \( x = 7.442 \times 10^{-3} \), and \( \zeta = 0.68219 \) (see slide 19) produces

\[ |H_e(j(0.06 \text{ rad/s}))| = \frac{(7.442 \times 10^{-3})^2}{\{[1-(7.442 \times 10^{-3})^2]^2 + 4(0.68219)^2 (7.442 \times 10^{-3})^2\}^{1/2}} = 5.54 \times 10^{-5} \]

The component of \( \max|dTE_R| \) due to the GM time error is equal to the GM time error amplitude, i.e., 0.8334 ms (see slide 33), multiplied by the magnitude of the error transfer function evaluated at the GM time error waveform frequency

- The result is \((0.8334 \text{ ms})(5.54 \times 10^{-5}) = 4.62 \times 10^{-5} \text{ ms} = 46.2 \text{ ns}\)

The conclusion is that the GM time error sinusoidal variation causes an increase in \( \max|dTE_R| \) of approximately 46 ns

- This assumes the endpoint filter has bandwidth and damping ratio of 1.33 Hz and 0.68219, respectively
It also was indicated earlier that the phase shift of the GM time error waveform caused by the endpoint filter is extremely small. This can be seen as follows:

Setting \( s = j\omega \) in the transfer function for the 2nd order endpoint filter, \( H(s) \), produces

\[
H(j\omega) = \frac{2\zeta \omega_n \omega j + \omega_n^2}{2\zeta \omega_n \omega j + \omega_n^2 - \omega^2}.
\]

Dividing the numerator and denominator by \( \omega_n^2 \) and setting \( x = \omega / \omega_n \) produces

\[
H(j\omega) = \frac{1 + 2\zeta x j}{1 - x^2 + 2\zeta x j}.
\]

The phase shift, \( \phi_H(x) \), is given by

\[
\phi_H(x) = \tan^{-1}(2\zeta x) - \tan^{-1}\left(\frac{2\zeta x}{1-x^2}\right).
\]

Setting \( x = 7.442 \times 10^{-3} \) and \( \zeta = 0.68219 \) produces (note that for \( x \) small, \( \tan^{-1} x \approx x - x^3 / 3 \))

\[
\phi_H(x) = \tan^{-1}(1.01537160 \times 10^{-2}) - \tan^{-1}(1.01542783 \times 10^{-2}) = -5.6 \times 10^{-7} \text{ rad} = 8.9 \times 10^{-8} \text{ UI} = 3.2 \times 10^{-5} \text{ deg}
\]

The phase shift of \( 3.2 \times 10^{-5} \) degrees (\( 1.92 \times 10^{-3} \) arc minutes, or 0.1152 arc seconds) is extremely small.
We can approximate $\max|dTE_R|$ for each case by adding the approximate contribution of the GM time error to $\max|dTE|$ for the case where the GM time error is zero.

- In doing this, the 46 ns contribution of the GM time error is rounded to 50 ns.

- Adding the contributions linearly is somewhat conservative, as these are components of $dTE$.
  - But note that the time variation is sinusoidal (i.e., not random noise processes), with frequencies at each clock that are close to each other but not identical, which means that over time there is some chance that peaks of the sinusoids due to the different contributions will line up.
  - In any case, future simulations should be run that include the GM time error (as described above).

The results are summarized on the next slide.
The 1 μs objective for max|TEₚ| can likely be met for cases 1, 2, 4, and 5 (1 or 4 ms residence times), for 100 nodes and 65 nodes.

For cases 3 and 6 (10 ms residence time) the 1 μs objective for max|TEₚ| can possibly be met for 65 nodes; it must be checked whether 270 ns (case 3) and 320 ns (case 6) is enough margin for cTE.

The 1 μs objective for max|TEₚ| cannot be met for case 6 for 100 nodes; there is likely insufficient margin for cTE in case 3, and the objective is exceeded by dTEₚ in case 6 by a large margin.

| Case | Syntonization Method | Residence time (ms) | Mean Sync Interval (ms) | Mean Pdelay Interval (ms) | Max|dTEₚ|, 100 nodes (ns) | Max|dTEₚ|, 65 nodes (ns) |
|------|----------------------|---------------------|------------------------|--------------------------|----------------------------|--------------------------|
| 1    | Accumulate neighborRateRatio | 1                   | 125                    | 31.25                    | 350                        | 300                      |
| 2    |                      | 4                   | 125                    | 31.25                    | 550                        | 470                      |
| 3    |                      | 10                  | 125                    | 31.25                    | 900                        | 730                      |
| 4    | Use successive Sync messages | 1                   | 31.25                  | 1000                     | 150                        | 90                       |
| 5    |                      | 4                   | 31.25                  | 1000                     | 250                        | 130                      |
| 6    |                      | 10                  | 31.25                  | 1000                     | 5750                       | 680                      |
Assuming the assumptions on mean Sync and Pdelay intervals, and other assumptions of this presentation, are acceptable, the above results should be confirmed by performing the following simulations:

- For zero GM time error, simulate cases 1–6 for 101 nodes, and with the correct assumptions on endpoint filter bandwidth (i.e., undamped natural frequency).
- For nonzero GM time error, simulate cases 1–6 for 101 nodes, and with the correct assumptions on endpoint filter bandwidth (i.e., undamped natural frequency).
  - In these simulations, compute the dTE_{R,unfiltered} waveforms via linear interpolation, and then low-pass filter those results with the endpoint filter.

If it is desired to measure GM rateRatio using successive Sync messages, it would be desirable to update the stability analysis of [10] and [11]

- The result of this would include a rule of thumb indicating the relation between residence time, mean Sync interval, and number of hops after which dTE_{R} will grow rapidly.
  - As indicated in [1], the use of this method would require an amendment to 802.1AS.
  - If this work were done, the amendment could include an informative annex with the above stability analysis and rule of thumb.
Thank you
References - 1


