

Analysis of Constant Time Error Accumulation and Budgeting in an IEC/IEEE 60802 Network

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Outline

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- ❑ Next steps

Introduction - 1

- Reference [1] is the most recent of a set of presentations that presented simulation results for $\max|dTE|$ (maximum absolute dynamic time error) for time synchronization transport over an IEC/IEEE 60802 network
- In the discussion of this presentation, it was indicated that the results and assumptions were acceptable for helping to decide requirements needed to meet the time synchronization objectives for an IEC/IEEE 60802 network
 - $\max|TE|$ of 1 μs over 64 hops, and over 100 hops if possible
- One of the next steps indicated in [1] is the consideration of constant time error (cTE)
 - $\max|TE|$ includes the effect of both cTE and dTE
 - The $\max|dTE|$ results in [1] were less than 1 μs ; however, it must be determined whether there is sufficient margin, i.e., budget, for cTE

Introduction - 2

- ❑ The current presentation considers analysis of the accumulation of cTE and budget needed, and implications of this for the cTE requirements for individual PTP Instances and PTP Links
- ❑ An initial analysis of the accumulation of cTE was given in [2]
- ❑ The following discussion of cTE accumulation is adapted from [2], with some updates
- ❑ Following this discussion, the dTE results are reviewed, followed by discussions of budgeting and equipment requirements

Analysis of cTE Accumulation - 1

- ❑ Prior to the presentation of [2], there were discussions of whether the accumulation of constant time error (cTE) should be linear or square-root-of-sum-of-squares (i.e., RMS)
- ❑ On the one hand, it was argued that if cTE is uniformly distributed, it is extremely unlikely that cTE will be at one extreme or another for all PTP instances and links in a hypothetical reference model (HRM)
 - With this argument, linear accumulation is overly conservative, and RMS accumulation should be used
- ❑ On the other hand, it was argued that, since cTE is fixed and constant, if a particular connection happens to have cTE at one extreme for all PTP Instances and links, it will remain this way permanently, or at least until the network is re-initialized
 - In the case where the cTE is permanently fixed, it is possible that cTE at most or all PTP Instances is similar and at one extreme if, for example, the same physical devices are used in most or all PTP Instances in the connection
 - With this argument, linear accumulation should be used

Analysis of cTE Accumulation - 2

- This presentation analyzes the accumulation of two types of cTE:
 - cTE that is constant and permanently fixed
 - cTE that is constant and fixed for the duration of the interval that the network is up and operating, i.e., the cTE can change when the network is re-initialized
- Two HRM cases are considered (both are of interest for IEC/IEEE 60802 networks)
 - 100 hops
 - 64 hops
- Delay variation due to very long term effects, e.g., di-urnal temperature variation effects in links or aging of oscillators, is considered to be part of dTE and not considered here
 - Note that these effects were not considered in the analysis of [1] and previous analyses
 - The effect of aging is negligible compared to the effect of temperature, which was considered
 - The effect of di-urnal temperature variation in links was considered small for the link lengths of interest

Analysis of cTE Accumulation - 3

□ There are three categories of cTE to consider

1) cTE associated with the physical medium (i.e., link)

- cTE in this category is fixed and constant for all time

- Denote this by $x_{l,1}$

2) cTE associated with the PTP Instance (i.e., node) that is fixed, and constant for all time

- Denote this by $x_{n,2}$

3) cTE associated with the PTP Instance (i.e., node) that is fixed while the node is up and operating, but changes when the node (or port if this cTE is associated only with the port) re-initializes

- Denote this by $x_{n,3}$

□ The category nomenclature is used here only for convenience; it is not standardized terminology, and can be replaced by other terminology if desired

Analysis of cTE Accumulation - 4

□ For simplicity, assume all the above components of cTE have zero mean and are uniformly distributed

1) $x_{l,1}$ is uniformly distributed over $[-D_{l,1}, D_{l,1}]$

2) $x_{n,2}$ is uniformly distributed over $[-D_{n,2}, D_{n,2}]$

3) $x_{n,3}$ is uniformly distributed over $[-D_{n,3}, D_{n,3}]$

□ In saying that the cTE components are uniformly distributed, we mean

- For components 1 and 2, a random cTE sample is permanently associated with each link and PTP Instance, respectively; this sample is taken from the respective uniform probability distribution
- For component 3, a random cTE sample is taken from the respective uniform probability distribution whenever the PTP Instance is re-initialized, and this value (sample) is associated with that PTP Instance until the next time it is initialized

Analysis of cTE Accumulation - 5

- As indicated above, cTE has zero mean; the cTE probability density functions are

$$p_{x_k}(u) = \frac{1}{2D_k} \quad \text{for } -D_k \leq u \leq D_k \quad \text{and } k = l, 1; n, 2; n, 3$$

- The variances of the cTE components are

$$\sigma_k^2 = \int_{-D_k}^{D_k} \frac{u^2}{2D_k} du = \frac{D_k^2}{3} \quad \text{for } k = l, 1; n, 2; n, 3$$

- In the following slides, consider an HRM consisting of $N+1$ PTP Instances and therefore N links (i.e., N hops)

Analysis of cTE Accumulation - 6

- Consider first cTE categories 1 and 2
- For these categories, cTE is fixed and permanent, for all time (i.e., for as long as the respective links and PTP Instances are in service.
 - This means that, if a particular path happens to have large cTE, the cTE will always be large for that path
 - In addition, category 2 cTE is due to components used in the respective PTP Instances. If the components in different PTP Instances are from the same vendor, the cTE in the different PTP Instances could be similar (or correlated). Also, there could be bias in the compensation procedures for link and node cTE.
 - It is true that category 1 and 2 cTE can be compensated. The cTE of interest here (i.e., the values of $D_{l,1}$ and $D_{n,2}$) of interest here are the values after any compensation
 - **In other words, the category 1 and 2 cTE of interest is the whatever cTE remains after compensation). This remaining cTE, while it might be small, will not be identically zero**

Analysis of cTE Accumulation - 7

- Given the above, the accumulation of category 1 and 2 cTE should be linear. The worst-case accumulated category 1 and 2 cTE is

$$\begin{aligned}\max|\text{cTE}(\text{accumulated, category 1 and 2})| &= ND_{n,2} + ND_{l,1} \\ &= N(D_{n,2} + D_{l,1})\end{aligned}$$

- In the above equation, it is assumed that the first and last PTP Instance contribute one-half the total $D_{n,2}$, as the first is the GM and has no ingress, and the last is the PTP End Instance or end application and has no egress

Analysis of cTE Accumulation - 8

- Next, consider cTE category 3
- For this category, cTE in each PTP Instance:
 - is independent of cTE in other PTP Instances
 - is different each time the PTP Instance is initialized
- With the above assumptions, the accumulated category 3 cTE is the sum of N random variables that are independent and identically distributed (and, actually, uniformly distributed)
- Then, by the Central Limit Theorem (which is a good approximation for $N = 64$ (and therefore also for $N = 100$)), the distribution of the accumulated cTE is approximately Gaussian (i.e., normal)

Analysis of cTE Accumulation - 9

- The accumulated category 3 cTE has zero mean and variance equal to

$$\sigma_{\text{accum, category 3}}^2 = \frac{ND_{n,3}^2}{3}$$

- The maximum absolute value of accumulated category 3 cTE can be taken to be an upper quantile of the Gaussian distribution for the accumulated category 3 cTE that corresponds to a chosen number of standard deviations from the mean. Below are exceedance probabilities for several different numbers of standard deviations from the mean, for a Gaussian distribution

$$\Pr\{|x| > 6\sigma\} = \frac{2}{\sigma\sqrt{2\pi}} \int_{6\sigma}^{\infty} e^{-u^2/2\sigma^2} du = 1.9732 \times 10^{-9}$$

$$\Pr\{|x| > 5\sigma\} = 5.7330 \times 10^{-7}$$

$$\Pr\{|x| > 4\sigma\} = 6.3342 \times 10^{-5}$$

$$\Pr\{|x| > 7\sigma\} = 2.5596 \times 10^{-12}$$

Analysis of cTE Accumulation - 10

- In the quality control area, a 6-sigma criterion is often used, and that will be used here, i.e., we will take the maximum absolute value of accumulated cTE to be 6 standard deviations from the mean
- Then $\max|\text{accumulated category 3 cTE}|$ is equal to

$$\max|\text{cTE(accumulated category 3)}| = 6D_{n,3}\sqrt{\frac{N}{3}} = 3.464D_{n,3}\sqrt{N}$$

- While the above accumulation of category 3 cTE goes asymptotically like the square root of N , note the presence of the multiplier 3.464
 - For $N \leq 12$, $3.464 \geq N$, and the above result gives a larger answer than the result of simply multiplying $D_{n,3}$ by N .
 - For this case, linear accumulation is correct; the central limit theorem does not give a valid result for 6 standard deviations
 - For N moderately larger than 12, linear accumulation gives a reasonable conservative approximation.
 - For N much larger than 12 (e.g., 64), the above equation should be used

Analysis of cTE Accumulation - 11

- For $N=100$, $\max|cTE|$ for the accumulated category 1 plus category 2 cTE is

$$\max|cTE(\text{accumulated, category 1 and 2})| = 100(D_{n,2} + D_{l,1})$$

- In the above

- $D_{n,2} = \max|cTE|$ for a single PTP Instance for category 2, **after any compensation has been performed**
- $D_{l,1} = \max|cTE|$ for a single link (category 1), **after any compensation has been performed**

- **While it is expected that $\max|cTE|$ will be small after compensation, it will not be identically zero (i.e., a requirement must be specified)**

Analysis of cTE Accumulation - 12

□ For $N=100$, $\max|\text{cTE}|$ for the accumulated category 3 cTE is

$$\max|\text{cTE}(\text{accumulated category 3})| = 6D_{n,3} \sqrt{\frac{100}{3}} = 34.64D_{n,3}$$

□ In the above

▪ $D_{n,3} = \max|\text{cTE}|$ for a single PTP Instance for category 3

□ **Note that while the above expression gives $\max|\text{cTE}|$ for the accumulated category 3 cTE that is less than linear (i.e., less than $100D_{n,3}$), it is larger than the square root of 100, i.e., 10, multiplied by $D_{n,3}$**

Analysis of cTE Accumulation - 13

- For $N=64$, $\max|cTE|$ for the accumulated category 1 plus category 2 cTE is

$$\max|cTE(\text{accumulated, category 1 and 2})| = 64(D_{n,2} + D_{l,1})$$

- In the above

- $D_{n,2} = \max|cTE|$ for a single PTP Instance for category 2, **after any compensation has been performed**
- $D_{l,1} = \max|cTE|$ for a single link (category 1), **after any compensation has been performed**

- **While it is expected that $\max|cTE|$ will be small after compensation, it will not be identically zero (i.e., a requirement must be specified)**

Analysis of cTE Accumulation - 14

□ For $N=64$, $\max|cTE|$ for the accumulated category 3 cTE is

$$\max|cTE(\text{accumulated category 3})| = 6D_{n,3} \sqrt{\frac{64}{3}} = 27.71D_{n,3}$$

□ In the above

▪ $D_{n,3} = \max|cTE|$ for a single PTP Instance for category 3

□ **Note that while the above expression gives $\max|cTE|$ for the accumulated category 3 cTE that is less than linear (i.e., less than $64D_{n,3}$), it is larger than the square root of 64, i.e., 8, multiplied by $D_{n,3}$**

Review of max |dTE| Results from [1]

- The following six slides are taken from [1], and summarize the max|dTE_R| results obtained in the simulations presented there
- References in those slides are references of [1]

Summary of Subcases of Case 16 Simulated

Subcase	Sync Interval variation (%)	Pdelay Interval variation (%)	Window Size for neighborRateRatio measurement
1	10	0	11
2	20	0	11
3	30	0	11
4	10	30	11
5	20	30	11
6	30	30	11
7	10	0	7
8	20	0	7
9	30	0	7
10	10	30	7
11	20	30	7
12	30	30	7

max |dTE_R| Results - 3

Case 16 - single replication results

Base case: no Sync or Pdelay interval variation

Subcases 1-3: Sync var (+/- 10, 20, 30%)

Subcases 4-6: Sync (+/- 10, 20, 30%) and Pdelay var (0-30%)

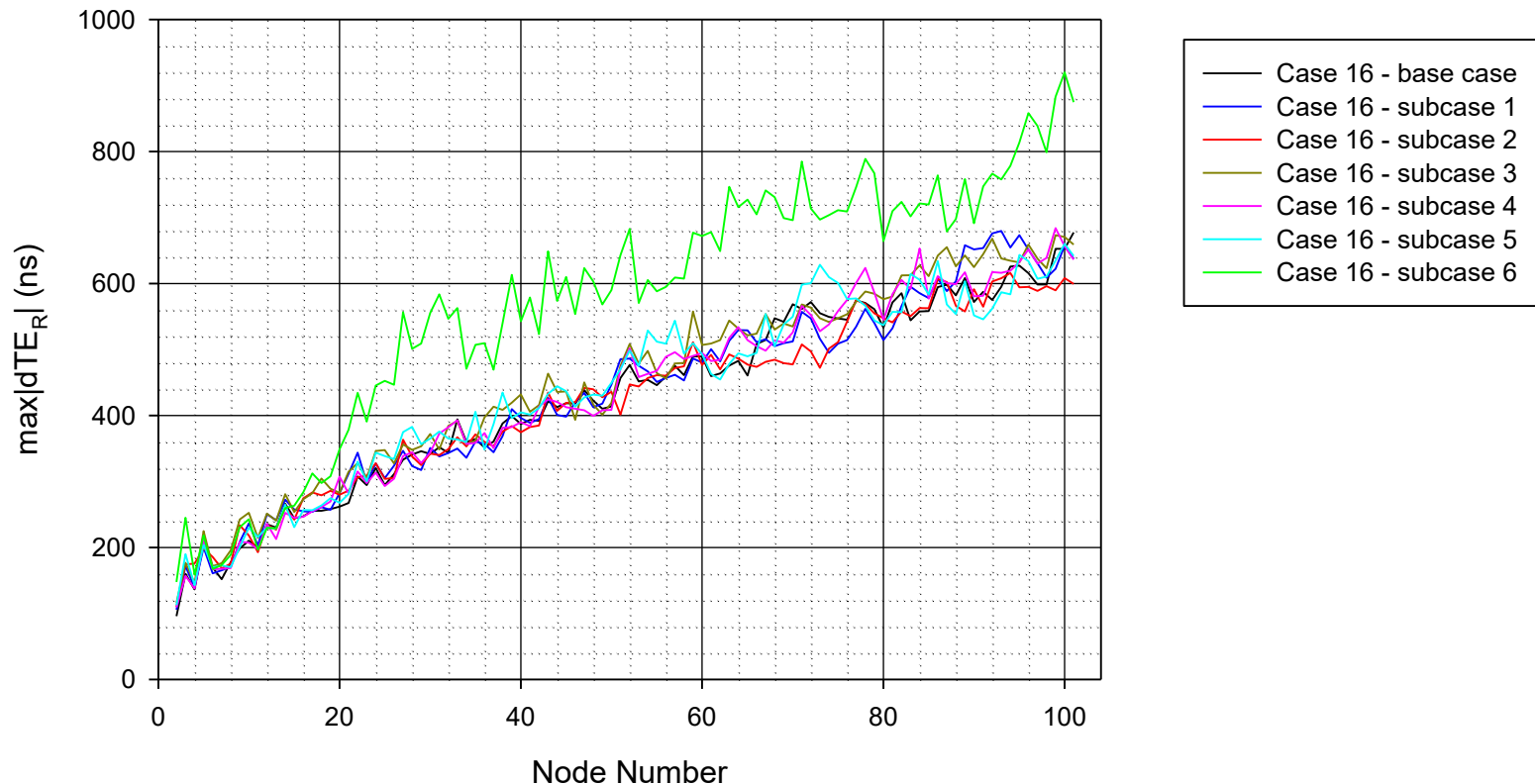
GM time error modeled

GM labeled node 1

Clock Model (all clocks): Frequency vs temperature stability and temperature vs time profile from [2]

Accumulate neighborRateRatio, which is measured with window of size 11 and median

KpKo = 11, KiKo = 65 (f3dB = 2.6 Hz, gain pk = 1.288 dB, zeta = 0.68219)



max |dTE_R| Results - 4

Case 16 - single replication results

Base case: no Sync or Pdelay interval variation

Subcases 7-9: Sync var (+/- 10, 20, 30%)

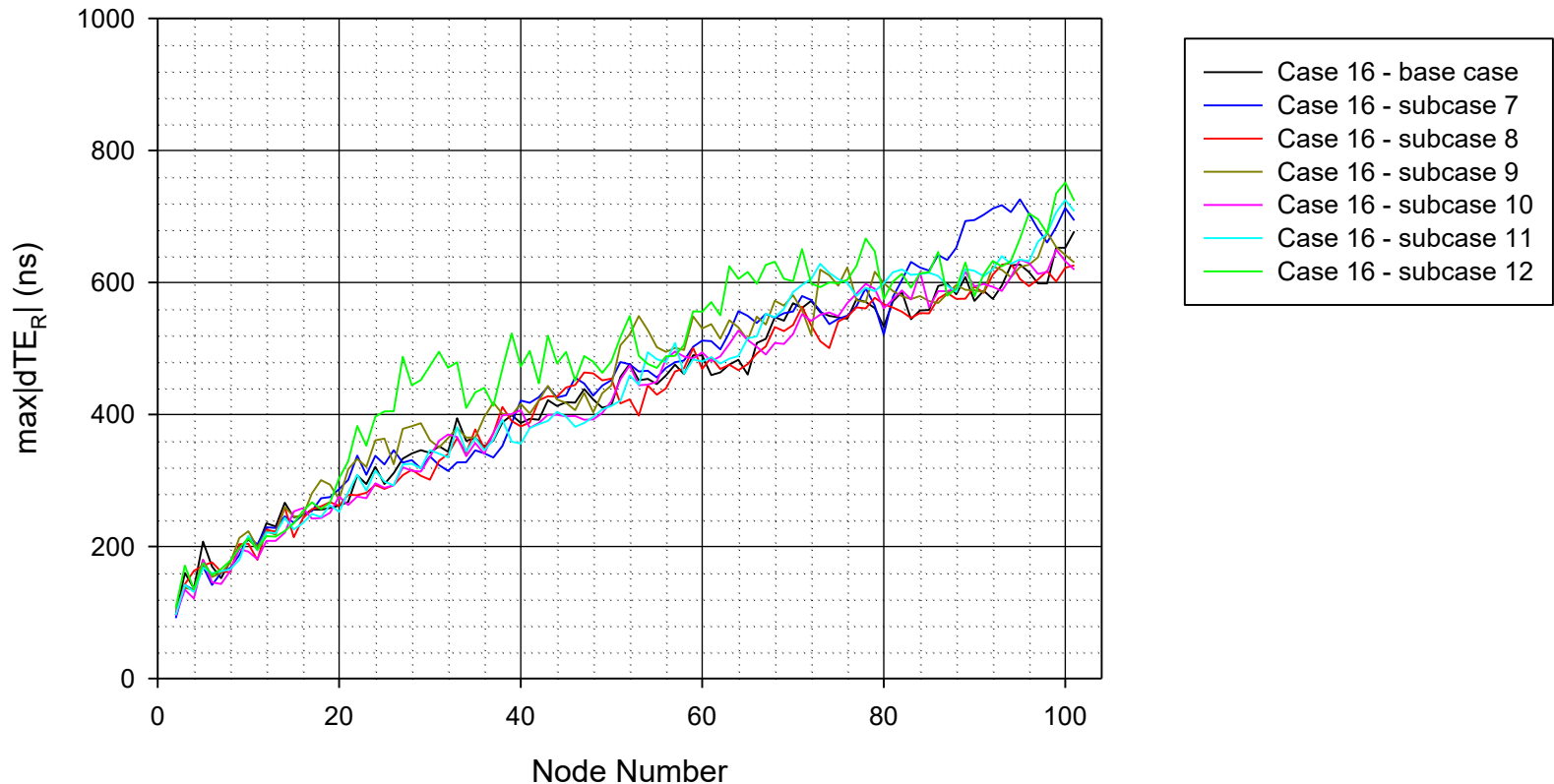
Subcases 10-12: Sync (+/- 10, 20, 30%) and Pdelay var (0-30%)

GM time error modeled

GM labeled node 1

Clock Model (all clocks): Frequency vs temperature stability and temperature vs time profile from [2]

Accumulate neighborRateRatio, which is measured with window of size 7 (11 for base case) and median KpKo = 11, KiKo = 65 (f3dB = 2.6 Hz, gain pk = 1.288 dB, zeta = 0.68219)



max |dTE_R| Results - 3

Subcase	max dTE _R , 64 hops (ns)	max dTE _R , 100 hops (ns)
Base case	460	677
1	529	637
2	477	599
3	521	659
4	514	636
5	490	642
6	727	875
7	549	694
8	476	626
9	513	630
10	513	619
11	515	708
12	616	724

64 hops results
are for node 65

100 hops results
are for node 101

Base case is case
16 of [1],
replication 1

Discussion of $\max |dTE_R|$ Results - 1

- ❑ The results for 64 hops range from 477 ns (subcase 2) to 727 ns (subcase 6), and all of them exceed the base case result of 460 ns
- ❑ The results for 100 hops range from 599 ns (subcase 2) to 875 ns (subcase 6); 8 of the subcases have results that are less than the base case result of 677 ns
- ❑ Case 6, which has 30% variation for both Sync and Pdelay intervals and uses a window of size 11, is the worst case (this is clearly indicated in the plot showing subcases 1 – 6)
 - This is likely because the larger variation in the Pdelay interval and the larger window size results in a less accurate neighborRateRatio measurement, and larger Sync intervals for some Sync messages results in greater time error
- ❑ It is difficult to discern trends from the results due to their statistical variability
 - Other than the fact that case 6 has larger $\max |dTE_R|$ than the other cases, and that in general the variability of the Sync and Pdelay intervals results in larger $\max |dTE_R|$, general trends are not evident
 - The plots of the multiple replication results in [1] (slides 14 and 15 of [1]) are much smoother)

Discussion of $\max |dTE_R|$ Results - 2

- In particular, it is not clear whether, in general, a window of size 7 gives better or worse results than a window of size 11
 - While the larger window averages (filters) more of the variability in the neighborRateRatio measurement, which improves the estimate, the actual frequency offset changes more during the duration of the larger window, which makes the estimate worse
- It appears possible to meet the 1 μ s objective over 64 hops, and over 100 hops if possible, but subject to the margin needed for cTE
 - Note that the results will increase when multiple replications of the simulations are run; the increase could be as much as 100 – 250 ns
 - This was observed previously for case 16, where the multiple replication results were 710 ns and 815 ns for nodes 64 and 100, respectively (see slide 20 of [1]), and the single replication results were 460 ns and 677 ns, respectively (see the base case in slide 31 above, which is for replication 1 but with GM time error modeled)

Budgeting and cTE Equipment Requirements - 64 Hops - 1

- From slide 23, max|dTE| for 64 hops, for the 12 cases simulated in [1], ranges from 477 – 727 ns for subcases 1 – 6 (window size of 11 for neighborRateRatio measurement) and 476 – 616 ns for subcases 7 – 12 (window size of 7 for neighborRateRatio measurement)
 - 477 ns: subcase 2
 - 727 ns: subcase 6
 - 476 ns: subcase 8
 - 616 ns: subcase 12
- The results indicate that subcases 7 – 12 (window size 7) give better results; these results will be used for the budgeting

Budgeting and cTE Equipment Requirements - 64 Hops - 2

- Slide 25 indicates that, based on previous comparisons of single and multiple replication results, multiple replication results could be 100 – 250 ns larger
 - This was based on results for case 16 (of [1] and previous results referenced in [1]); the 250 ns increase was for 64 hops
 - Adding 250 ns to the above results gives a range of 726 – 866 ns
- Taking 866 ns as the multiple replication estimate of $\max|dTE_R|$ for 64 hops leaves 134 ns for cTE
- For 64 hops, the tentative estimate of the budget for cTE is 134 ns
- Using the results on slides 17 and 18, and noting that the cTE results for categories 1 and 2 and the results for category 3 can be added, produces

$$\begin{aligned}\max|cTE| &= 64(D_{n,2} + D_{l,1}) + 27.71D_{n,3} \\ &= 64(D_{n,2} + D_{l,1} + 0.4330D_{n,3})\end{aligned}$$

Budgeting and cTE Equipment Requirements - 64 Hops - 3

□ The above produces

$$D_{n,2} + D_{l,1} + 0.4330D_{n,3} \leq \frac{134 \text{ ns}}{64}$$

□ Or (and after rounding to 2 significant digits)

$$D_{n,2} + D_{l,1} + 0.4330D_{n,3} \leq 2.094 \text{ ns} \approx 2.1 \text{ ns}$$

□ The actual limit for a single PTP Instance plus link depends on the fraction of cTE that is category 1 and 2 versus category 3

- If all the cTE is category 1 and 2, the limit is 2.1 ns
- If all the cTE is category 3, the limit is $2.1 \text{ ns} / 0.4330 = 4.85 \text{ ns}$
- In reality, the limit is probably somewhere in between the two values

Budgeting and cTE Equipment Requirements - 100 Hops - 1

- From slide 23, $\max|dTE|$ for 100 hops, for the 12 cases simulated in [1], ranges from 599 – 875 ns for subcases 1 – 6 (window size of 11 for neighborRateRatio measurement) and 626 – 724 ns for subcases 7 – 12 (window size of 7 for neighborRateRatio measurement)
 - 599 ns: subcase 2
 - 875 ns: subcase 6
 - 626 ns: subcase 8
 - 724 ns: subcase 12

- As for 64 hops, the results for 100 hops indicate that subcases 7 – 12 (window size 7) give better results; these results will be used for the budgeting
 - The subcases corresponding to the upper and lower ends of each range are the same as for 64 hops

Budgeting and cTE Equipment Requirements - 100 Hops - 2

- ❑ As indicated in the budgeting for 64 hops, slide 25 indicates that, based on previous comparisons of single and multiple replication results, multiple replication results could be 100 – 250 ns larger
 - Adding 250 ns to the above results gives a range of 876 – 974 ns
- ❑ Taking 974 ns as the multiple replication estimate of $\max|dTE_R|$ for 64 hops leaves 26 ns for cTE
- ❑ For 100 hops, the tentative estimate of the budget for cTE is 26 ns
- ❑ Using the results on slides 17 and 18, and noting that the cTE results for categories 1 and 2 and the results for category 3 can be added, produces

$$\begin{aligned}\max|cTE| &= 100(D_{n,2} + D_{l,1}) + 34.64D_{n,3} \\ &= 100(D_{n,2} + D_{l,1} + 0.3464D_{n,3})\end{aligned}$$

Budgeting and cTE Equipment Requirements - 100 Hops - 3

□ The above produces

$$D_{n,2} + D_{l,1} + 0.3464D_{n,3} \leq \frac{26 \text{ ns}}{100}$$

□ Or (and after rounding to 2 significant digits)

$$D_{n,2} + D_{l,1} + 0.3464D_{n,3} \leq 0.26 \text{ ns}$$

□ The actual limit for a single PTP Instance plus link depends on the fraction of cTE that is category 1 and 2 versus category 3

- If all the cTE is category 1 and 2, the limit is 0.26 ns
- If all the cTE is category 3, the limit is $0.26 \text{ ns} / 0.3464 = 0.75 \text{ ns}$
- In reality, the limit is probably somewhere in between the two values

Budgeting and cTE Equipment Requirements - 100 Hops - 4

- ❑ The range 0.26 – 0.75 ns for the cTE limit, for a single piece of equipment and single link, for 100 hops is very tight
- ❑ Examining the max|dTE| results for subcases 7 – 12 for 100 hops indicates that cases 8 – 10 are best (619 – 630 ns), but cases 7, 11, and 12 are close to or exceed 700 ns
 - If the simulated max|dTE| could be kept to 600 – 650 ns, then the estimate of max|dTE| for multiple replications would be 850 – 900 ns.-
 - This would leave 100 – 150 ns for cTE
 - Using 100 ns, the range for the cTE requirement, would be 1 – 2.9 ns

Conclusion

- The budget for cTE, out of the total 1 μ s objective, is:
 - 134 ns (64 hop HRM, based on subcases 7 - 12)
 - 26 ns (100 hop HRM, based on subcases 7 - 12)
 - 100 ns (100 hop HRM, based on subcases 8 – 10)
- The resulting cTE requirement is in the range:
 - 2.1 ns – 4.85 ns (64 hop HRM, based on subcases 7 - 12)
 - 0.26 ns – 0.75 ns (100 hop HRM, based on subcases 7 - 12)
 - 1 ns – 2.9 ns (100 hop HRM, based on subcases 8 – 10)
- The reason the cTE requirement is stated as a range is that the requirement depends on what fraction of cTE is category 1 and 2, versus category 3
- Since it is not known, a priori, what this fraction is, the lower end of the range should be used
- **In all cases, the cTE requirement is extremely stringent**
 - **Note that these cTE limits are on remaining cTE after any compensation**

Next Steps - 1

- It should be decided what practical limit on cTE is achievable, for each category
 - cTE due to physical link asymmetry
 - cTE due to asymmetry within the PTP Instance (i.e., node) that is fixed and constant for all time
 - cTE associated with the PTP Instance (i.e., node) that is fixed while the node is up and operating, but changes when the node (or port if this cTE is associated only with the port) re-initializes
- **Note that these cTE limits are on remaining cTE after any compensation**
- If the practical limits exceed the limits on the previous slide, it might be worth running a few multiple replication simulations cases (for several of subcases 7 – 12 of case 16) to see if the actual increase in $\max|dTE|$ when considering multiple replications might be sufficiently smaller than 250 ns
 - For example, if the increase were 100 ns, there would be an additional 150 ns budget for cTE, which would allow the equipment limits to increase by 1.5 ns (100 hop HRM) and 2.34 ns (64 hop HRM)

Next Steps - 2

- Also, any changes to existing assumptions that would decrease dTE would result in more budget for cTE, for example:
 - Smaller residence time
 - more constrained temperature profile
 - More stable oscillator (with respect to temperature variation)
- Finally, it was stated that the desire is to meet the 1 μs objective for the 100 hop HRM case **if possible**
 - It might be decided that meeting the objective for 100 hops is not practical

References

- [1] Geoffrey M. Garner, *New Simulation Results for dTE for an IEC/IEEE 60802 Network, with Variable Inter-Message Intervals, Revision2*, IEC/IEEE 60802 presentation, July 1, 2021 (available at <https://www.ieee802.org/1/files/public/docs2021/60802-garner-single-replic-simul-results-variable-intermsg-intervals-0621-v02.pdf>)
- [2] Geoffrey M. Garner, *Analysis of the Accumulation of Constant Time Error in an IEC/IEEE 60802 Network*, IEC/IEEE 60802 presentation, March 16, 2020 (available at <https://www.ieee802.org/1/files/public/docs2020/60802-garner-analysis-of-accum-of-cTE-in-60802-network-0320-v00.pdf>)

Thank you