New Simulation results for Base Case and Case 1 of the Time Sync Breakout Held during the IEC/IEEE 60802 Ad Hoc Session

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One of the topics discussed in the Time Synchronization Breakout held during the June 2022 IEC/IEEE 60802 ad-hoc session was the assumed temperature profile used to model the frequency stability of the free-running LocalClock.

The current temperature profile consists of constant rate temperature ramps between -40°C and +85°C, where the rate of change of temperature is 1 deg C/s, and periods of constant temperature at either +85°C or -40°C in between the ramps.

The sudden change from a temperature that in varying linearly with time to a constant temperature results in a sudden change in the frequency drift rate.

- The sudden change in frequency drift rate resulted in potential compensation schemes for neighborRateRatio (nRR) and rateRatio (RR) relative to the Grandmaster (GM) to not perform as well as desired (see the simulation results in [2]).

It was pointed out that the abrupt transition from linearly varying temperature to constant temperature is not realistic, and it was suggested that a temperature profile where the temperature variation is smoother be used.
It was decided to use a temperature profile where the linearly varying temperature is replaced by a half-sinusoid of the same peak-to-peak amplitude as the current profile, and period chosen such that the time required for the current linear variation to increase from -40°C to +85°C, i.e., 125 s at the rate of 1 deg C/s is equal to the half-period of the sinusoid (and similarly for the temperature decrease)

Two simulation cases were identified:

- Base case: This is the same as case 16, subcase 4, of [3] (see slide 26 of [3]), except that the temperature profile is changed as described above (and will be described in more detail shortly)

- Case 1: This has the same assumptions as the Base case, except that the dynamic timestamp is has a uniform probability distribution over the range [-8 ns, +8 ns] (in the base case, the distribution is -8 ns with probability 0.5 and +8 ns with probability 0.5)

The current presentation gives single-replication simulation results for these two cases, and compares the new results with the previous results for case 16, subcase 4 of [3]
The temperature history is assumed to vary between –40°C and +85°C, at a rate of 1°C/s; this takes 125 s.

When the temperature is increasing and reaches +85°C, it remains at +85°C for 30 s.

The temperature then decreases from +85°C to –40°C at a rate of 1°C/s; this takes 125 s.

The temperature then remains at –40°C for 30 s.

The temperature then increases to +85°C at a rate of 1°C/s; this takes 125 s.

The duration of the entire cycle (i.e., the period) is therefore 310 s.

- The initial upward variation extends from 0 to 125 s.
- The subsequent constant temperature extends from 125 – 155 s.
- The subsequent downward variation extends from 155 – 280 s.
- The subsequent constant temperature extends from 280 – 310 s.
- The cycle then repeats.
The temperature history when increasing from \(-40^\circ\text{C}\) and \(+85^\circ\text{C}\) is assumed to be one-half of a sinusoid, varying from the trough (negative peak) to the positive peak.

The variation for the initial increase in the first cycle is therefore:

$$T(t) = -A \cos(\omega t + \phi) + B$$

where:

- \(T\) = temperature in deg C
- \(t\) = time in s
- \(A\) = 62.5 deg C
- \(B\) = 22.5 deg C
- \(\omega\) = \(\frac{2\pi}{250\ s} = \frac{\pi}{125}\) rad/s

\(\phi\) = phase of the temperature variation (in rad) = \(\frac{(\text{phase in s})\pi}{310\ s}\) rad

The variation for the subsequent decrease in the first cycle is:

$$T(t) = B + A \cos[\omega(t - 155) + \phi]$$
As a check, compute $T(280 \text{ s})$ for the case $\phi = 0$

$$T(t) = B + A \cos[\omega(t - 155) + \phi]$$

For $\phi = 0$:

$$T(280) = 22.5 + 62.5 \cos\left[\frac{\pi}{125}(280 - 155)\right]$$

$$= 22.5 + 62.5 \cos \pi = -40 \text{ as desired}$$
Temperature Profile - Sinusoidal temperature variation between -40 C and +85 C - 2 cycles

![Temperature Profile Graph]
Assumptions for Frequency Stability due to Temperature Variation

  - Specifically, the values $a_0$, $a_1$, $a_2$, and $a_3$ computed in [5] of Reference [5] will be used in the cubic polynomial fit, and the resulting frequency offset will be multiplied by 1.1 (i.e., a margin of 10% will be used).

- The frequency stability data that this polynomial fit is based on is contained in the Excel spreadsheet attached to [4] of Reference [5] here.
  - This data was provided by the author of [4] of Reference [5] here.

- The time variation of frequency offset is obtained from the cubic polynomial frequency dependence on temperature, and the temperature dependence on time described in the previous slide.
  - The time variation of phase/time error at the LocalClock entity is obtained by integrating the above frequency versus time waveform.
  - The time variation of frequency drift rate at the LocalClock entity is obtained by differentiating the above frequency versus time waveform.
The cubic dependence of frequency offset, $y$, as a function of temperature, $T$, is given by

$$y(T) = a_3 T^3 + a_2 T^2 + a_1 T + a_0$$

where

$T = \text{temperature in } ^\circ \text{C}$

$y = \text{frequency offset in ppm}$

$a_3, a_2, a_1, \text{and } a_0$ are coefficients to be determined from a least-squares fit

Let the time dependence of temperature (on the previous slides) be represented by $T(t)$; then the time-dependence of frequency offset is given by

$$y(t) = a_3 [T(t)]^3 + a_2 [T(t)]^2 + a_1 T(t) + a_0$$

Two cycles of the time history of frequency offset is plotted on the next two slides for the previous and new temperature profiles
Review - Previous Frequency Offset Versus Time

Frequency Offset History

Time (s) vs. Frequency Offset (ppm)

- Time values: 0, 200, 400, 600, 800
- Frequency Offset values: -20, -15, -10, -5, 0, 5, 10
Frequency Offset Versus Time for New Temperature Profile

Frequency Offset - Sinusoidal temperature variation between -40 C and +85 C - 2 cycles
The flat portions of the previous frequency offset curve, between 4.5 and 5 ppm and between -15 and -20 ppm, correspond to the periods when the temperature is constant at +85°C and -40°C, respectively.

In the new frequency offset curve, for the new temperature profile, the transitions between the varying and flat portions is more gradual.

While the curve appears to show that the slope is not continuous at the transition points, this is just an artifact of the plot and it actually is continuous (this will be seen in the following slides that consider the rate of change of frequency offset with time).
The frequency drift rate is obtained by differentiating $y(t)$ with respect to time

$$y(t) = A\{a_3[T(t)]^3 + a_2[T(t)]^2 + a_1T(t) + a_0\}$$

$$\frac{dy}{dt} = \frac{dy}{dT} \cdot \frac{dT}{dt} = A\{3a_3[T(t)]^2 + 2a_2[T(t)] + a_1\} \cdot \frac{dT}{dt}$$

For the temperature profiles above, $dT/dt$ is 0 during the periods when temperature is constant at $-40^\circ C$ or $+85^\circ C$

Two cycles of the time history of frequency drift rate is plotted on the next two slides for the previous and new temperature profiles
Frequency Drift Rate History

Time (s)
0 100 200 300 400 500 600

Frequency Drift Rate (ppm/s)
-1.5 -1.0 -0.5 0.0 0.5 1.0 1.5
Frequency Drift Rate Versus Time for New Temperature Profile

Frequency Drift Rate - Sinusoidal temperature variation between -40 C and +85 C - 2 cycles

Note that there are no jumps in frequency drift rate versus time.
The phase of the LocalClock time error waveform at each node is chosen randomly in the range \([0, T]\), at initialization, where \(T\) is the period of the phase and frequency variation waveforms (i.e., 310 s).
The Base Case assumptions are the same as those for case 16, subcase 4, of [3], and are summarized on the following slides. Some of the slides are adapted from previous presentation.

The Case 1 assumptions are the same as for the Base Case assumptions, except that the distribution of dynamic timestamp error is uniform over [-8 ns, +8 ns], instead of being +8 ns with probability 0.5 and -8 ns with probability 0.5.

The GM is assumed to have the same frequency stability (i.e., time error) as the subsequent PTP Instances.

- $dT_E_R$ and $\max|dT_E_R|$, relative to the GM, are the results computed.
- Since the $dT_E$ samples at the GM and at subsequent PTP Instances are not necessarily computed at the same time, interpolation is used to compute $dT_E_R$. 

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IEEE Std 802.1AS-2020 requires in 10.7.2.3 (an analogous requirement is in 9.5.9.2 of IEEE Std 1588-2019):

When the value of syncLocked is FALSE, time-synchronization messages shall be transmitted such that the value of the arithmetic mean of the intervals, in seconds, between message transmissions is within $\pm$ 30% of $2^{\text{currentLogSyncInterval}}$. In addition, a PTP Port shall transmit time-synchronization messages such that at least 90% of the inter-message intervals are within $\pm$ 30% of the value of $2^{\text{currentLogSyncInterval}}$. The interval between successive time-synchronization messages should not exceed twice the value of $2^{\text{portDS.logSyncInterval}}$ in order to prevent causing a syncReceiptTimeout event. The PortSyncSyncSend state machine (see 10.2.12) is consistent with these requirements, i.e., the requirements here and the requirements of the PortSyncSyncSend state machine can be met simultaneously.

NOTE 1—A minimum number of inter-message intervals is necessary in order to verify that a PTP Port meets these requirements. The arithmetic mean is the sum of the inter-message interval samples divided by the number of samples. For more detailed discussion of statistical analyses, see Papoulis [B25].
The above requirements do not specify the actual probability distribution; however, it was decided to model the Sync Intervals as being gamma-distributed.

- The gamma distribution is often used to model inter-message times in networks.
- The same model was used in simulations for the PTP Telecom Time Profile with full timing support from the network (ITU-T Rec. G.8275.1).

While both 802.1AS-2020 and 1588-2019 both allow variation in the duration of the Sync intervals up to ± 30% of the mean Sync interval, case 16, subcase 4 of [3] considers variations of ± β, with β = 10%.

The shape and scale parameters of the gamma distribution are chosen such that the distribution has the desired mean and that 90% of the probability mass is within β of the mean.

The resulting gamma distribution has a shape parameter of 270.5532; the details of how this parameter is obtained and how the samples of the gamma distribution are generated are given in [3].
IEEE Std 802.1AS-2020 has the following NOTE in 11.5.2.2 (it refers to the requirement in 9.5.13.2 of IEEE Std 1588-2019):

NOTE 3—The MDPdelayReq state machine ensures that the times between transmission of successive Pdelay_Request messages, in seconds, are not smaller than 2^{currentLogPdelayReqInterval}. This is consistent with IEEE Std 1588-2019, which requires that the logarithm to the base 2 of the mean value of the interval, in seconds, between Pdelay_Request message transmissions is no smaller than the interval computed from the value of the portDS.logMinPdelayReqInterval member of the data set of the transmitting PTP Instance. The sending of Pdelay_Request messages is governed by the LocalClock and not the synchronized time (i.e., the estimate of the Grandmaster Clock time). Since the LocalClock frequency can be slightly larger than the Grandmaster Clock frequency (e.g., by 100 ppm, which is the specified frequency accuracy of the LocalClock; see B.1.1), it is possible for the time intervals between successive Pdelay_Request messages to be slightly less than 2^{currentLogPdelayReqInterval} when measured relative to the synchronized time.

However, the actual requirement in 9.5.13.2 of IEEE 1588 is:

Subsequent Pdelay_Request messages shall be transmitted such that the value of the arithmetic mean of the intervals, in seconds, between Pdelay_Request message transmissions is not less than the value of 0.9 × 2^{portDS.logMinPdelayReqInterval}.

This requirement will be satisfied even if the LocalClock is 100 ppm fast due to the factor of 0.9 (frequency offsets resulting from the temperature profile and frequency stability models are less than 100 ppm).
IEEE 802.1AS and IEEE 1588-2019 do not specify the distribution for the Pdelay interval, nor do they specify the maximum amount that the actual intervals can exceed $2^{\text{portDS.logMinPdelayReqInterval}}$.

For the simulations, it was decided to use a uniform distribution over the range $[P, 1.3P]$, where $P$ is $2^{\text{portDS.logMinPdelayReqInterval}}$. 

Model for Variable Pdelay Interval - 2
### Other Assumptions - 1

<table>
<thead>
<tr>
<th>Assumption/Parameter</th>
<th>Description/Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hypothetical Reference Model (HRM), see note following the tables</td>
<td>101 PTP Instances (100 hops; GM, followed by 99 PTP Relay Instances, followed by PTP End Instance)</td>
</tr>
<tr>
<td>Computed performance results</td>
<td>(a) ( \max</td>
</tr>
<tr>
<td>Use syncLocked mode for PTP Instances downstream of GM</td>
<td>Yes</td>
</tr>
<tr>
<td>Endpoint filter parameters</td>
<td>( K_pK_o = 11, K_iK_o = 65 ) (( f_{3dB} = 2.5998 ) Hz, 1.288 dB gain peaking, ( \zeta = 0.68219 ))</td>
</tr>
<tr>
<td>Timestamp granularity</td>
<td>8 ns</td>
</tr>
<tr>
<td>Dynamic timestamp error</td>
<td>Base case: +8 ns with probability 0.5, -8 ns with probability 0.5</td>
</tr>
<tr>
<td></td>
<td>Case 1: Uniformly distributed over ([-8 \text{ ns}, +8 \text{ ns}])</td>
</tr>
<tr>
<td>Simulation time</td>
<td>3150 s; discard first 50 s to eliminate any startup transient before computing ( \max</td>
</tr>
<tr>
<td>Residence time</td>
<td>1 ms</td>
</tr>
<tr>
<td>Pdelay turnaround time</td>
<td>10 ms</td>
</tr>
</tbody>
</table>
Other Assumptions - 2

<table>
<thead>
<tr>
<th>Assumption/Parameter</th>
<th>Description/Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of independent replications, for each simulation case</td>
<td>300</td>
</tr>
<tr>
<td>GM rateRatio and neighborRateRatio computation granularity</td>
<td>0 (i.e., we do not truncate when computing timestamp differences and ratios of differences, but use floating point arithmetic)</td>
</tr>
<tr>
<td>Mean link delay</td>
<td>500 ns</td>
</tr>
<tr>
<td>Link asymmetry</td>
<td>0</td>
</tr>
<tr>
<td>Any variable PHY delay in addition to the dynamic timestamp error described above is assumed to be zero</td>
<td>0</td>
</tr>
</tbody>
</table>
Assumptions for neighborRateRatio Computation

- neighborRateRatio is computed over a window of size 11, and the median of the most recent 11 values is used
Case 16, Subcase 4 of [3] assumptions, except as noted in previous slides
Single replication of simulations
+/-10% Sync Interval variation, 0-30% Pdelay Interval variation
GM time error modeled
GM labeled node 1
Accumulate neighborRateRatio, which is measured with window of size 7 and median
KpKo = 11, KiKo = 65 (f3dB = 2.6 Hz, gain pk = 1.288 dB, zeta = 0.68219)
Unfiltered max $|dTE_R|$, Single replication

Case 16, Subcase 4 of [3] assumptions, except as noted in previous slides
Single replication of simulations
+/-10% Sync Interval variation, 0-30% Pdelay Interval variation
GM time error modeled
GM labeled node 1
Accumulate neighborRateRatio, which is measured with window of size 7 and median
KpKo = 11, KiKo = 65 (f3dB = 2.6 Hz, gain pk = 1.288 dB, zeta = 0.68219)
### Summary of max|dTE<sub>R</sub>| Results at Last Node

#### Largest max|dTE<sub>R</sub>|, and corresponding node number

<table>
<thead>
<tr>
<th>Case</th>
<th>Filtered result (ns)</th>
<th>Node number</th>
<th>Unfiltered result (ns)</th>
<th>Node number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base case</td>
<td>555.7</td>
<td>91</td>
<td>638.3</td>
<td>101</td>
</tr>
<tr>
<td>Case 1</td>
<td>363.6</td>
<td>91</td>
<td>372.9</td>
<td>84</td>
</tr>
<tr>
<td>Base case with previous temperature profile</td>
<td>684.0</td>
<td>99</td>
<td>719.6</td>
<td>100</td>
</tr>
</tbody>
</table>
**Discussion of \( \max|dTE_R| \) Results**

- Comparison of the Base Case (black curve on slides 26 and 27) and the Base Case with the previous temperature profile shows the effect of the new temperature profile
  - \( \max|dTE_R| \) with the new temperature profile is smaller than with the previous temperature profile by approximately 8% for the filtered results and 9% for the unfiltered results
  - This is due to the maximum LocalClock frequency drift rate being reduced from approximately 1.3 ppm/s to 0.76 ppm/s

- \( \max|dTE_R| \) is further reduced in Case 1, due to the dynamic timestamp error distribution being changed from always being at the maximum absolute value of error (i.e., 8 ns) to being uniformly distributed over \([-8 \text{ ns}, +8 \text{ ns}]\)
  - \( \max|dTE_R| \) is reduced to 364 ns with endpoint filtering and 373 ns without endpoint filtering

- The above results for Case 1 are for a chain of 101 nodes
  - This case would likely allow sufficient margin for cTE and other error budget components that the 1 \( \mu \)s objective could be met
  - But note that the residence time is 1 ms

- Non-uniform increase of \( \max|dTE_R| \) with node number is likely due to statistical variability, as these results are for a single replication
Reference [6] computed an equivalent frequency drift rate probability for the previous frequency stability model (i.e., based on the previous temperature profile), for use with Monte Carlo simulations (i.e., simulations not using a time domain model).

An equivalent frequency drift rate probability distribution is now computed for the new frequency stability model, based on the new temperature profile, and compared with the probability distribution for the previous frequency stability model.

Some of the following slides are adapted from [6].
The Monte Carlo simulations (see the references cited in [6]) used the following probabilistic model for Cases A – E

- Frequency drift rate is 0 ppm/s with 20% probability
- Frequency drift rate is chosen randomly from a uniform distribution in the range [-1.5 ppm/s, +1.5 ppm/s], with 20% probability
- Then, if $R$ is a random variable representing the drift rate in ppm/s, the drift rate probability density function (pdf) can be written

$$p_R(x) = 0.2\delta(x) + \frac{0.8}{3}$$

for $-1.5 \text{ ppm/s} \leq x \leq +1.5 \text{ ppm/s}$

One approach to deriving an equivalent probability density function for the frequency versus temperature stability model described above is to assume that a value of time is chosen randomly from a uniform distribution $p_t(x)$ over the 310 second period

- With this approach, the plots on slides 15 and 16 can be considered to represent the random variable $R$ (frequency drift rate) as a function of the random variable $t$ (time)
The functions given by the curves $R(t)$ on slides 15 and 16 can be used to derive $p_R(x)$ from $p_t(x)$

However, the functions $R(t)$ on slides 15 and 16 are not one-to-one, and the analytical derivation of $p_R(x)$ from the uniform distribution $p_t(x)$ will be complicated

- For each value of frequency drift rate in the respective range $[-1.35 \text{ ppm/s}, +1.35 \text{ ppm/s}]$ for the previous temperature profile and $[-0.76 \text{ ppm/s}, +0.76 \text{ ppm/s}]$ for the new temperature profile, the corresponding values of time must be found, and $dR/dt$ must be computed at each of these points

- This is complicated by the fact that the number of values of $t$ corresponding to a given value of $R$ is different depending on where $R$ is in the respective range

- In addition, the period between 125 s and 155 s, and between 280 s and 310 s, where the frequency drift rate is zero, give rise to a delta function at zero (in the pdf) of strength (i.e., amplitude) $60/310 = 0.19355$
Fortunately, a Monte Carlo approach can be used, which is approximate but much simpler

- Generate a random sample of time in the range \([0, 310]\) s
- Compute the corresponding frequency drift rate
- Repeat this a large number of times; the resulting values can be considered samples of the random variable \(R\)
- Use these samples to construct estimates of the pdf, probability function (histogram), or any other statistics of interest

Note that this is the technique that would be used if this equivalent pdf were used to generate random samples of frequency drift rate in the Monte Carlo simulations for 60802 network dTE

This approach was used to construct the pdf and probability function (histogram) on the following slides

- \(10^8\) samples of frequency drift rate were generated
Slides 36 and 37 show the simulated (Monte Carlo approach) probability histograms for the frequency drift rate, for both the previous and new temperature profiles, followed by a slide comparing the two probability histograms (slide 38).

- The histograms are based on $10^8$ samples, assuming the periodic time dependences of the frequency drift rate described previously and time chosen randomly from a uniform distribution over one period.

- As indicated previously, the range of frequency drift rate is $[-1.35 \text{ ppm/s}, +1.35 \text{ ppm/s}]$ for the previous temperature profile and $[-0.76 \text{ ppm/s}, +0.76 \text{ ppm/s}]$ for the new temperature profile.

- For the previous temperature profile, the total range of 2.7 ppm/s is divided into 27 bins of size 0.1 ppm/s each.

- For the new temperature profile, the total range of 1.52 ppm/s is divided into 16 bins of size approximately 0.1 ppm/s each.

- Each of the $10^8$ samples is placed on one of the bins, and on completion the number of samples in each bin is divided by $10^8$.

- The height of each bar is the simulated probability that a frequency drift rate sample will be in the respective bin (range).
The large peak at 0 ppm/s represents the probability that the frequency drift rate is zero; ideally, this would have height approximately 0.2 (i.e., the ratio of the time the frequency drift rate is zero (i.e., 60 s) divided by the total cycle time (i.e., 310 s)), but since the bin of nonzero width, it include the probability of being in a range of width 0.1 ppm/s or 0.061 ppm/s about zero.

With this added probability, the height is 0.26 for the previous temperature profile and 0.24 for the new temperature profile.

The side-peaks around 0.3 ppm/s and -0.3 ppm/s for the previous temperature profile and 0.5 and -0.5 ppm/s for the new temperature profile represent increased probability of these (and smaller absolute value) frequency drift rates.

In addition, and most importantly, the range of frequency drift rates is reduced for the new temperature profile from [-1.35 ppm/s, +1.35 ppm/s] to and [-0.76 ppm/s, +0.76 ppm/s] for the new temperature profile; this reduces the accumulated max|dTE_R|.
Equivalent Drift Rate Probability - 3

Probability Histogram for Frequency Drift Rate

Previous Temperature Profile
26 bins
Probability Histogram for Frequency Drift Rate

New Temperature Profile
26 bins

Frequency Drift Rate (ppm/s)

Probability
Equivalent Drift Rate Probability - 5

Probability Histogram for Frequency Drift Rate

Previous Temperature Profile
26 bins

New Temperature Profile
16 bins
Slide 40 shows probability histograms, but with 10,001 bins (and $10^8$ samples) for the previous temperature profile and 5662 bins (and $10^8$ samples) for the new temperature profile.

- Now, the height (probability) for each bin is smaller than on slides 36-38, because the bins are narrower.
- However, the probabilities do still sum to 1.
Equivalent Drift Rate Probability - 7

Probability Density for Frequency Drift Rate

- Previous Temperature Profile
  10000 bins

- New Temperature Profile
  5662 bins
Slide 42 shows an estimate of the probability density function, based on 10,001 bins and $10^8$ samples for the previous temperature profile, and 5662 bins and $10^8$ samples for the new temperature profile.

- Each point is obtained by dividing the corresponding probability for each bin in slide 40 by the width of the bin.

- This results in the area under the curve summing to 1 (i.e., it is equal to the summation over bins of the height of each bin multiplied by the bin width, which is equal to the sum over the bins of the probabilities).
Equivalent Drift Rate Probability - 9

- Previous Temperature Profile
  - 10000 bins

- New Temperature Profile
  - 5662 bins
Conclusions and Future Work

- The new temperature profile produces a significantly smaller maximum frequency drift rate
  - The [-1.35 ppm/s, +1.35 ppm/s] range for the previous temperature profile is reduced to [-0.76, +0.76 ppm/s] for the new temperature profile

- This results in a reduction of max|dTE_R| of 8% for filtered results and 9% for unfiltered results

- max|dTE_R| is further reduced in Case 1 to 364 ns with endpoint filtering and 373 ns without endpoint filtering, due to the dynamic timestamp error distribution being changed from always being at the maximum absolute value of error (i.e., 8 ns) to being uniformly distributed over [-8 ns, +8 ns]
  - This case would likely allow sufficient margin for cTE and other error budget components that the 1 µs objective could be met
  - But note that the residence time is 1 ms

- It would be useful to run one or more Monte Carlo simulation cases with the drift rate probability distribution computed here (slides 37, 40, and 42) to help validate the Monte Carlo model
Thank you
References - 1


