New Multiple Replication 60802 Time Domain Simulation Results for Cases with Drift Tracking Algorithms and PLL Noise Generation

Revision 1

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Recent time sync work in 60802 has focused on developing clock drift tracking and compensation algorithms that would enable the objective of 1 $\mu$s maximum absolute value of time error relative to the grandmaster PTP Instance ($\max|TE_R|$) to be met over 64 hops, and over 100 hops if possible.

The algorithms are described in [1], and extensive Monte Carlo simulations are documented in [2] and in references cited in [1] and [2] (and in references cited in those references).

Initial time-domain simulation results, based on single replications of various simulation cases, are given in [10].

- Time domain simulations are needed because they more precisely model the time-dependent effects present in the algorithms.
- Time domain simulations also model endpoint filtering (e.g., PLL filtering) and noise generation at these filters (the Monte Carlo simulations do not model these effects).
- The Monte Carlo simulations were used to develop the algorithms because they run several orders of magnitude faster than the time-domain simulations.
The single-replication simulation cases in [10] included

- The drift tracking and compensation algorithms described in [1]
- Endpoint filter (e.g., PLL) noise generation, based on the stated assumptions for local clock stability (i.e., frequency drift behavior for the assumed oscillator type and temperature profile
  - Various PLL filter 3dB bandwidths are considered

Note that while the objective for $\max|TE_R|$ is 1 $\mu$s, the budget for relative dynamic time error ($dTE_R$) is 500 ns

- Reference [3] indicates 600 ns budget for $dTE_R$; however, it was indicated that 100 ns of this is budgeted for the end application
- This is relevant because the simulations model $dTE_R$ for the network transport, i.e., they do not model constant time error ($cTE$) nor the error in the end application
- Therefore, this presentation takes 500 ns as the objective for $\max|dTE_R|$

In this Rev 1, corrections are made on slides 4, 14, 15, and 17, and to the affiliation on the title slide
The results in [10] indicate that the $\text{max}|dTE_R|$ objective of 500 ns over 100 hops can be met with:

- the drift compensation and tracking, and mNRR smoothing, algorithms described in [1]
- Endpoint filter 3dB bandwidth and gain peaking of 1.8 Hz and $1.288 \times 2.1985$ dB, respectively
- Temperature profile, XO frequency stability, and other system parameters as described earlier (with the GM frequency stability equal to one-half the frequency stability at other PTP Instances)

The results in [10] also suggest that the $\text{max}|dTE_R|$ objective can be met with narrower endpoint filter bandwidth, e.g., 1.5 Hz or 1 Hz

Since it is necessary to run multiple replications of the simulation cases of interest, to obtain confidence intervals for the 0.95 quantile of $\text{max}|dTE_R|$ and also the maximum, it was decided to run multiple replications of simulations for various endpoint filter bandwidths of 1.5 Hz and less
The following slides, mostly taken from or adapted from [10], summarize the assumptions for the simulation cases (these assumptions were taken from slides 66 to 74 of [2])

- Except for the specific endpoint filter bandwidths, the assumptions are the same as for the single-replication cases of [10]

The third major bullet item of slide 68 of [2] documented 3 sets of assumptions for the drift tracking and error compensation algorithms

- These will be described later; however, it was decided that all the multiple replication simulations should use the first set of assumptions

The effect of endpoint filter noise generation is modeled as described in [10] and summarized here later

The summary of assumptions is followed by a summary of the simulation cases, simulation results, and conclusions
The temperature profile of [2] is a half-sinusoid with dwell time; it is similar to the temperature profile of [4], except that the periods of the sinusoidal increase and decrease are 95 s instead of 125 s.

- The temperature history is assumed to vary between – 40°C and +85°C, as a half sinusoid over 95 s.
- The dwell times are still 30 s, which means that the period of the temperature variation is 250 s instead of 310 s.
Temperature Profile - 2

The variation for the initial increase in the first cycle is therefore

\[ T(t) = -A \cos(\omega t + \phi) + B \]

where
\[ T = \text{temperature in deg C} \]
\[ t = \text{time in s} \]
\[ A = 62.5 \text{ deg C} \]
\[ B = 22.5 \text{ deg C} \]
\[ \omega = \frac{2\pi}{190 \text{ s}} = \frac{\pi}{95} \text{ rad/s} \]
\[ \phi = \text{phase of the temperature variation (in rad)} = \frac{(\text{phase in s})\pi}{250 \text{ s}} \text{ rad} \]

The variation for the subsequent decrease in the first cycle is

\[ T(t) = B + A \cos[\omega(t - 125) + \phi] \]

Specifically, the values $a_0$, $a_1$, $a_2$, and $a_3$ computed in [5] of Reference [5] will be used in the cubic polynomial fit, and the resulting frequency offset will be multiplied by 1.1 (i.e., a margin of 10% will be used).

The frequency stability data that this polynomial fit is based on is contained in the Excel spreadsheet attached to [4] of Reference [5] here.

This data was provided by the author of [4] of Reference [5] here.

The time variation of frequency offset is obtained from the cubic polynomial frequency dependence on temperature, and the temperature dependence on time described in the previous slide.

The time variation of phase/time error at the LocalClock entity is obtained by integrating the above frequency versus time waveform.

The time variation of frequency drift rate at the LocalClock entity is obtained by differentiating the above frequency versus time waveform.
The above gives the frequency stability for non-GM PTP Instances, as indicated slide 68 of [2]

For the GM, the frequency offset at a given temperature is one-half the frequency offset at the same temperature for non-GM PTP Instances, i.e., the coefficients $a_0$, $a_1$, $a_2$, and $a_3$ should be multiplied by 0.5 for the GM (after being increased by the factor of 1.1)

The phase offset, frequency offset, and frequency drift rate time history plots given in [4] show the qualitative form of the plots; the only difference here is that the period is 250 s instead of 310 s
The phase of the LocalClock time error waveform at each node is chosen randomly in the range [0, T], at initialization, where T is the period of the phase and frequency variation waveforms (i.e., 250 s).
Other Assumptions - 1

- Some of these slides documenting Other Assumptions are adapted from [2]
- The timestamp granularity is assumed to be 8 ns, based on a 125 MHz clock
  - The timestamp is truncated to the next lower multiple of 8 ns
  - At the GM, 4 ns is added
- The dynamic timestamp error is assumed to be uniformly distributed over [-6 ns, +6 ns]
- When GM noise is modeled, interpolation is used to compute dTE_R (relative to the GM), because the dTE samples at the GM and at subsequent PTP instances are not necessarily computed at the same time
- The simulation time is 1300 s, with the first 50 s discarded when computing max|dTE_R| to eliminate the effect of any startup transient
- 300 multiple replications of each simulation case are run
  - A 99% confidence interval for the 0.95 quantile of max|dTE_R| is obtained by placing the 300 results in ascending order; the interval extends from the 275th smallest to 294th smallest value, and a point estimate is taken as the 285th smallest value
Other Assumptions - 2

❑ Pdelay Interval
  ▪ Pdelay is used only to compute meanLinkDelay, and not neighborRateRatio (NRR)
  ▪ NRR is computed using successive Sync message (using the syncEgressTimestamp)
  ▪ The nominal Pdelay interval is 125 ms
  ▪ The actual Pdelay interval is assumed to be uniformly distributed in the range 
    \([0.9)(125 \text{ ms}), (1.3)(125 \text{ ms})\] = [112.5 ms, 162.5 ms]

❑ Sync Interval
  ▪ The Sync interval is assumed to be uniformly distributed in the range [119 ms, 131 ms]

❑ Residence time
  ▪ The residence time is assumed to be a truncated normal distribution with 
    mean of 5 ms and standard deviation of 1.8 ms, truncated at 1 ms and 15 ms
  ▪ Probability mass greater than 15 ms and less than 1 ms is assumed to be 
    concentrated at 15 ms and 1 ms, respectively (i.e., truncated values are 
    converted to 15 ms or 1 ms, respectively)
Other Assumptions - 3

- **Pdelay Turnaround Time**
  - The Pdelay turnaround time is assumed to be a truncated normal distribution with mean of 10 ms and standard deviation of 1.8 ms, truncated at 1 ms and 15 ms
  - Probability mass greater than 15 ms and less than 1 ms is assumed to be concentrated at 15 ms and 1 ms, respectively (i.e., truncated values are converted to 15 ms or 1 ms, respectively)

- **Link Delay**
  - Link delay is assumed to be uniformly distributed between 5 ns and 500 ns
  - Link delays are generated randomly at initialization and kept at those values for the entire simulation
  - Link asymmetry is not modeled
Mean Link Delay Averaging

- The averaging function is assumed to be an IIR filter that uses 0.99 of the previously computed value and 0.01 of the most recent measurement.
- This is equivalent to the filter of the NOTE of B.4 of 802.1AS-2020, taken as a first-order filter, i.e.,

\[ y_k = a_1 y_{k-1} + b_0 x_k \]

- where \( y_k \) is the \( k \)th filter output, \( x_k \) is the \( k \)th measurement, \( a_1 = 0.99 \), and \( b_0 = 0.01 \) (Note that these values differ from those in Appendix D of [11], where \( a_1 = 0.999 \), and \( b_0 = 0.001 \); however, the longer averaging period in [11] should result in smaller \( \max|dT_{ER}| \) )
In previous simulations (i.e., prior to the simulations of [10] and the simulation cases here), the following were used for the endpoint PLL parameters $K_p$ (proportional gain), $K_i$ (integral gain), $K_o$ (VCO/DCO gain):

- $K_pK_o = 11$, $K_iK_o = 65$

This corresponds to the following 3dB bandwidth ($f_{3\text{dB}}$), gain peaking, and damping ratio ($\zeta$)

- $f_{3\text{dB}} = 2.5998$ Hz, $1.288 \times 2.1985$ dB gain peaking, $\zeta = 0.68219$

In addition, VCO/DCO noise generation was neglected.

The PLL model used in the simulator is second-order and linear, with 20 dB/decade roll-off

- It is based on a discretization that uses an analytically exact integrating factor to integrate the second-order system
- As a result, the PLL model in the simulator is stable regardless of the time step, i.e., sampling rate (though aliasing of the input or noise is possible)
- Details are given in Appendix VIII.2.2 of [7] (except that the relation between gain peaking and damping ratio is based on the exact result in 8.2.3 of [8] (see Eqs. (8-13 – 8-15 there)
However, many practical PLL implementations are based on a discrete time model where the integral block and VCO block of the PLL are modeled based on z-transforms.

- Depending on the details, this is mathematically equivalent to replacing derivatives by forward or backward differences.
- See Appendix I (Figure I-1 and Eq. (I-6) of [8] and 3.5 of [9] for examples.
- As a result, the model becomes unstable if the sampling rate is not large enough compared to the PLL 3dB bandwidth.
- A common rule of thumb is that the sampling rate should be at least ten times the PLL bandwidth.
- The analysis in 3.5 of [9] shows that, for the example there, the theoretical limit for stability is approximately $\pi$ times the 3dB bandwidth (i.e., the sampling rate must be at least $\pi$ times the 3dB bandwidth for the PLL to be stable).
- The PLL 3dB bandwidth above (used in simulations before those in [10]) of 2.5998 Hz implies that the sampling rate should be at least 25.998 Hz $\approx$ 26 Hz.
- However, the sampling rate here is the Sync rate, and the minimum Sync rate corresponds to the maximum Sync interval, which is 131 ms.
- The minimum Sync rate is therefore $1/(0.131 \text{ s}) = 7.634 \text{ Hz}$, which is too small.
- The theoretical limit of $\pi:1$ implies a Sync rate of at least $(\pi)(2.6 \text{ Hz}) = 8.17 \text{ Hz}$, which still exceeds the 7.634 Hz minimum Sync rate.
To begin to address this, additional simulation cases were run in [10] with various PLL bandwidths smaller than 2.6 Hz (gain peaking was kept at $1.288 \pm 2.1985$ dB)

- The simulation cases of the current presentation consider PLL bandwidths in the range 0.5 Hz to 1.5 Hz

However, as the PLL bandwidth becomes narrower, noise generation can become appreciable if the same oscillator is used, because the transfer function from the noise to the output is a high-pass filter with corner frequency and damping ratio the same as for the low-pass transfer function from the PLL input to output

In the case here, it was indicated in one of the July 2023 60802 meetings that the same XO is used for the endpoint PLL filter as for the timestamping function
Therefore, noise generation was modeled, using the same local oscillator phase variation model used for the LocalClock.

- The noise was computed by passing the XO phase noise through a high-pass filter with the same 3dB bandwidth and damping ratio as the low-pass PLL filter, and adding the result to the PLL output that was computed from the input.
As indicated earlier, the drift tracking and compensation algorithms used here are described in detail in [1].

In the notation below, $m_{NRR_{\text{smoothingNA}}}$ is the number of Sync Intervals over which $n_{RR}$ is both computed and averaged, e.g., if $m_{NRR_{\text{smoothingNA}}} = 4$, we compute $n_{RR}$ over 4 Sync intervals and average the 4 most recently computed values.

In the notation below, $m_{NRR_{\text{compNAP}}}$ is the number of Sync Intervals over which the frequency drift rate estimate is computed.

All the simulation cases here use:

- $m_{NRR_{\text{compNAP}}} = 8$; $m_{NRR_{\text{smoothingNA}}} = 4$

The simulation cases area summarized on the following slide:

- The numbering of the simulation cases here follows the numbering in [10].
- The simulation cases of [10] are numbered 1 through 28, with additional cases 5a, 6a, 7a, 8a, 25a, 26a, 27a, 28a.
- The simulation cases here are numbered 29 through 35.
## Summary of Simulation Cases - 1

<table>
<thead>
<tr>
<th>Case</th>
<th>Drift Tracking and Compensation (mNRRcompNAP, mNRRsmoothingNA)</th>
<th>PLL 3dB Bandwidth (Hz)</th>
<th>PLL noise generation present (yes/no)</th>
<th>GM noise magnitude relative to non-GM PTP Instances</th>
</tr>
</thead>
<tbody>
<tr>
<td>29</td>
<td>(8, 4)</td>
<td>1.0</td>
<td>yes</td>
<td>0.5</td>
</tr>
<tr>
<td>30</td>
<td>(8, 4)</td>
<td>1.5</td>
<td>yes</td>
<td>0.5</td>
</tr>
<tr>
<td>31</td>
<td>(8, 4)</td>
<td>0.9</td>
<td>yes</td>
<td>0.5</td>
</tr>
<tr>
<td>32</td>
<td>(8, 4)</td>
<td>0.8</td>
<td>yes</td>
<td>0.5</td>
</tr>
<tr>
<td>33</td>
<td>(8, 4)</td>
<td>0.7</td>
<td>yes</td>
<td>0.5</td>
</tr>
<tr>
<td>34</td>
<td>(8, 4)</td>
<td>0.6</td>
<td>yes</td>
<td>0.5</td>
</tr>
<tr>
<td>35</td>
<td>(8, 4)</td>
<td>0.5</td>
<td>yes</td>
<td>0.5</td>
</tr>
</tbody>
</table>
Max $|dTE_R|$ Simulation Results

- Plots of $|dTE_R|$ are presented on the following slides (22 – 31) for $|dTE_R|$ before and after endpoint PLL filtering.

- Filtered and unfiltered $|dTE_R|$ for nodes 65 and 101 are summarized in the table on slide 32.

- Slide 22 shows 99% confidence intervals for the 0.95 quantile and maximum over 300 replications, for cases 29 – 35.

- Slide 23 shows maximum over 300 replications, for cases 29– 35.

- Slide 24 shows unfiltered $|dTE_R|$ (it is the same for all six cases because only the filter bandwidth varies for these cases).
  
  - The remaining plots show the 99% confidence intervals for the 0.95 quantile and maximum over 300 replications for each case individually; these plots are provided because the plots showing all the cases together are fairly cluttered.
Filtered $\text{max} |dT_E_R|$, Cases 29 - 35

Cases 29, 30, 31, 32, 33, 34, 35 - mult replic results - filt
GM time error modeled; $dT_{ER}$ is relative to GM

GM labeled node 1
Algorithms of Annex D
Gain peaking = 0.1 dB
3dB BWs for cases 29-33: 1.0, 1.5, 0.9, 0.8, 0.7, 0.6, 0.5 Hz
Temp profile: half-sinusoid with 95 s period and 30 s dwell, -40 to +85 C
XO freq stability

Node Number

0 20 40 60 80 100

max$|dT_{ER}|$ (ns), filtered

0 100 200 300 400 500 600

Case 29 - lower confid limit
Case 29 - point estim
Case 29 - upper confid limit
Case 29 - max over 300 runs
Case 30 - lower confid limit
Case 30 - point estim
Case 30 - upper confid limit
Case 30 - max over 300 runs
Case 31 - lower confid limit
Case 31 - point estim
Case 31 - upper confid limit
Case 31 - max over 300 runs
Case 32 - lower confid limit
Case 32 - point estim
Case 32 - upper confid limit
Case 32 - max over 300 runs
Case 33 - lower confid limit
Case 33 - point estim
Case 33 - upper confid limit
Case 33 - max over 300 runs
Case 34 - lower confid limit
Case 34 - point estim
Case 34 - upper confid limit
Case 34 - max over 300 runs
Case 35 - lower confid limit
Case 35 - point estim
Case 35 - upper confid limit
Case 35 - max over 300 runs
Filtered max |dTE_R|, Cases 29 - 35, max over 300 runs

Cases 29, 30, 31, 32, 33, 34, 35 - mult replic results - filt
GM time error modeled; dTE_R is relative to GM
GM labeled node 1
Algorithms of Annex D
Gain peaking = 0.1 dB
3dB BWs for cases 29-33: 1.0, 1.5, 0.9, 0.8, 0.7, 0.6, 0.5 Hz
Temp profile: half-sinusoid with 95 s period and 30 s dwell, -40 to +85 C
XO freq stability

Node Number

max|dTE_R| (ns), filtered

0 100 200 300 400 500 600
0 20 40 60 80 100

Case 29 - max over 300 runs
Case 30 - max over 300 runs
Case 31 - max over 300 runs
Case 32 - max over 300 runs
Case 33 - max over 300 runs
Case 34 - max over 300 runs
Case 35 - max over 300 runs
Unfiltered \( \max |dTE_R| \), Cases 29 - 35 (same for all cases)

Cases 29, 30, 31, 32, 33 - mult replic results - unfilt
Results are the same for all the cases since they differ only in filt BW
GM time error modeled; \(dTE_R\) is relative to GM; GM labeled node 1

Algorithms of Annex D
Temp profile: half-sinusoid with 95 s period and 30 s dwell, -40 to +85 C
XO freq stability
Sync Int: 119 - 131 ms unif; Pdelay Int: 112.5 - 162.5 ms unif
Res: mean 5ms, sig 1.8ms, 1ms min, 15ms max
Pdelay turn: same as Res, except 10ms mean; Link: 5 - 500ns unif
Filtered max |dTE_R| , Case 29

Case 29 - mult replic results - filt
GM time error modeled; dTE_R is relative to GM
GM labeled node 1
Algorithms of Annex D
Gain peaking = 0.1 dB
3dB BW: 1.0 Hz
Temp profile: half-sinusoid with 95 s period and 30 s dwell, -40 to +85 C
XO freq stability

![Graph showing max |dTE_R| results for Case 29]
Filtered max $|dTE_R|$, Case 30

Case 30 - mult replic results - filt
GM time error modeled; $dTE_R$ is relative to GM
GM labeled node 1
Algorithms of Annex D
Gain peaking = 0.1 dB
3dB BW: 1.5 Hz
Temp profile: half-sinusoid with 95 s period and 30 s dwell, -40 to +85 C
XO freq stability
Case 31 - mult replic results - filt
GM time error modeled; dTE_R is relative to GM
GM labeled node 1
Algorithms of Annex D
Gain peaking = 0.1 dB
3dB BW: 0.9 Hz
Temp profile: half-sinusoid with 95 s period and 30 s dwell, -40 to +85 C
XO freq stability
Filtered $\max |dTE_R|$, Case 32

Case 32 - mult replic results - filt
GM time error modeled; $dTE_R$ is relative to GM
GM labeled node 1
Algorithms of Annex D
Gain peaking = 0.1 dB
3dB BW: 0.8 Hz
Temp profile: half-sinusoid with 95 s period and 30 s dwell, -40 to +85 C
XO freq stability

![Graph showing $\max |dTE_R|$ vs Node Number]
Filtered $\max |d\text{TE}_R|$, Case 33

Case 33 - mult replic results - filt
GM time error modeled; $d\text{TE}_R$ is relative to GM
GM labeled node 1
Algorithms of Annex D
Gain peaking = 0.1 dB
3dB BW: 0.7 Hz
Temp profile: half-sinusoid with 95 s period and 30 s dwell, -40 to +85 C
XO freq stability

![Graph showing max $|d\text{TE}_R|$ filtered for Case 33]
Filtered max $|d\text{TE}_R|$, Case 34

Case 34 - mult replic results - filt
GM time error modeled; $d\text{TE}_R$ is relative to GM

GM labeled node 1
Algorithms of Annex D
Gain peaking = 0.1 dB
3dB BW: 0.6 Hz
Temp profile: half-sinusoid with 95 s period and 30 s dwell, -40 to +85 C
XO freq stability
Filtered $\text{max} |d\text{TE}_R|$, Case 35

Case 35 - mult replic results - filt
GM time error modeled; $d\text{TE}_R$ is relative to GM

GM labeled node 1
Algorithms of Annex D
Gain peaking = 0.1 dB
3dB BW: 0.5 Hz
Temp profile: half-sinusoid with 95 s period and 30 s dwell, -40 to +85 C
XO freq stability
### Summary of Filtered max|dTE<sub>R</sub>| Results at Nodes 65 and 101

| Case | PLL 3dB Bandwidth (Hz) | Filtered max|dTE<sub>R</sub>| Node 65 (ns) | Filtered max|dTE<sub>R</sub>| Node 101 (ns) |
|------|------------------------|--------------|-----------------|-----------------|
| 30   | 1.5                    | 339.4        | 507.8           |
| 29   | 1.0                    | 337.4        | 498.7           |
| 31   | 0.9                    | 347.3        | 497.6           |
| 32   | 0.8                    | 358.7        | 497.2           |
| 33   | 0.7                    | 372.4        | 498.4           |
| 34   | 0.6                    | 394.1        | 504.6           |
| 35   | 0.5                    | 441.5        | 535.7           |
| unfiltered | -             | 363.5        | 548.9           |

**Note 1:** Unfiltered results are the same for all cases because the cases differ only in the filter bandwidth.

**Note 2:** Case 29 follows case 30 so that cases are in order of decreasing endpoint filter bandwidth.
The 500 ns objective for \(\max|dTE_R|\) is met after 100 hops (i.e., at node 101) for cases 29, 31, 32, and 33, i.e., for endpoint filter bandwidths of 1 Hz, 0.9 Hz, 0.8 Hz, and 0.7 Hz.

- The objective is exceeded for bandwidths of 1.5 Hz, 0.6 Hz, and 0.5 Hz.

The overall trend of the results is as expected, i.e., \(\max|dTE_R|\) decreases as bandwidth is decreased from 1.5 Hz due to increased filtering of the time error; however, after reaching a minimum \(\max|dTE_R|\) begins to increase due to the increasing effect of endpoint filter noise generation as the bandwidth is decreased further.

For the cases here, \(\max|dTE_R|\) decreases to a minimum of 497.2 ns for a bandwidth of 0.8 Hz, and then increases.

- However, \(\max|dTE_R|\) remains below 500 ns for bandwidths in the range 0.7 Hz to 1.0 Hz.

  - Note that these results are for discrete values of bandwidth; the actual bandwidth range for which \(\max|dTE_R|\) is less than 500 ns likely extends from slightly less than 0.7 Hz to slightly greater than 1.0 Hz.
The results here are better than many of the results obtained in [10]; this is because only cases 5a – 8a and 25a – 28a of [10] correctly had the GM noise magnitude at \( \frac{1}{2} \) the level of the LocalClock noise at the other PTP Instances most of the results of [10] (except for cases 1 – 4, which had no GM noise, the other cases had the GM noise level the same as the LocalClock noise levels at the other nodes).

As indicated earlier, the maximum Sync interval of 0.131 s corresponds to a minimum Sync rate of \( \frac{1}{\text{0.131 s}} = 7.634 \text{ Hz} \). The ratio of this to the 1.0 Hz bandwidth, i.e., the upper end of the range, is 7.634. This is less than the 10:1 rule of thumb, but exceeds the theoretical limit of stability for one common PLL implementation. However, the ratio of this to the minimum bandwidth of 0.7 Hz is 10.91, which is greater than the 10:1 rule of thumb.

Therefore, the endpoint filter bandwidth range for which the 10:1 rule of thumb is met (0.7 Hz to 0.7634 Hz) at least partially covers the range for which the \( \max|d\text{TE}_R| \) objective is met (0.7 Hz to 1.0 Hz). The theoretical limit of \( \pi:1 \) is met for the entire 0.7 Hz to 1.0 Hz range.
The simulation results indicate that the $\max|d\text{TE}_R|$ objective of 500 ns can be met, under the assumptions described in slides 6 – 18 of this presentation, for endpoint filter bandwidths in the range 0.7 Hz to 1.0 Hz.

These assumptions are contained in [11], either as normative requirements or in the Informative Annex D.

In particular, the drift tracking and compensation algorithms used here and described in [1] are also described in Annex D of [11] as an example of algorithms that can be used to meet the $\max|d\text{TE}_R|$ objective.

Therefore, it can be stated as a conclusion:

Based on the simulation results here, the $\max|d\text{TE}_R|$ objective of 500 ns is met for endpoint filter bandwidths in the range 0.7 Hz to 1 Hz.
Based on the simulation results here, a minimum bandwidth requirement of 0.7 Hz can be added to Table 10 of IEC/IEEE 60802, to the row “ClockTimeReceiver (servo controller)”, i.e., in column 2, add (the comma is used instead of a period, per IEC convention):

- **Minimum Bandwidth (Hz): 0.7 Hz**

A minimum bandwidth requirement is needed because, if the bandwidth is too narrow (i.e., less than 0.7 Hz) the effect of noise generation will be appreciable and the max|dTE_R| objective will be exceeded.
Thank you


