Frequency Hopping pattern selection

Arvind Krishna

IBM Thomas J. Watson Research Center
P.O. Box 704, Yorktown Heights, NY 105'3

Francois Le Maut

CER IBM La Gaude
06610 La Gaude, France

Abstract

In this submission we propose a set of frequency hopping patterns that can be used by a 802.11 compliant Slow-Frequency-Hopping Spread Spectrum physical layer. This set of FHP is specifically dedicated for the US 2.4 GHz ISM band but similar sets can be proposed for any other band. This set of FHP is based on the criteria which were defined in reference 1.

Introduction

To make an efficient use of the spectrum and to keep the overall performance of each co-located network at its ultimate capability, it is of prime importance to avoid or at least minimize interference between networks or cells of a same network. Therefore the cells have to be isolated from each others similarly to what exist naturally on a wired LAN where two networks or two lobes of the same network can overlap without interference. The purpose of a carefully selected Frequency Hopping Patterns is to achieve similar isolatior between cells.

A previous submission [1] described a set of criteria that have to be taken into account while selecting FHPs:

1. The system shall hop to channel frequencies that are selected at the hopping rate from a pseudorandomly ordered list of hopping frequencies. Each frequency must be used equally often on the average by each transmitter. At least 75 frequency channels should be visited pseudorandomly.

   This is an FCC 15.247 requirement.

2. Any two patterns should be on the same frequency channel as little as possible in order to avoid collision between packets of different co-located cells.

3. Any two FHPs should be on a nearby frequency channels at the same time as little as possible. This is due to imperfection in the filtering of the out of band signal which allow a significant amount of the energy to spread on the adjacent frequency channel. This also true for the receiver which is unable to filter all the energy transmitted on the adjacent channel.

4. Interference avoidance through temporal frequency diversity by jumping from one interfered area of the spectrum to a completely different set of frequencies. This will allow to minimize the effect of some kind of interference (microwave ovens) that affects a range of adjacent channels.
   An other important parameter is selection of adjacent channels with uncorrelated fading, this could be achieved by selecting channel spacing large enough so they don't fall in the coherent bandwidth. The coherence bandwidth Bc and the delay spread τ are relate as:
   $Bc = 1 / k\tau$

   Theoretically k is 2π when the power delay profile which gives the distribution of the received signal power in time has an exponential distribution. However in practical situation the value of k is not constant and can be between 2 and 15|

   For in-building transmission the multipath spread at 2.4 GHz is approximately 10 to 150ns depending of the structure of the building. This translates to a coherent bandwidth of approximately 6 to 20 MHz.

5. In order to avoid the case where the system has to re-acquire hop synchronization it is important to have hopping patterns in which there is no adjacent channel interference in contiguous hops.

Frequency Hopping pattern definition

On top of all the above requirements there are some some additional practical constrains which have to be dealt with the out of band characteristics, therefore at each end of the ISM band one channel should be left free to ease radio design and so on 83 channels offered only 81 are usable. For the patterns we construct, the pattern length ***p*** has to be a prime number. Hence, the length of the hopping sequence will be 79. Same rule apply for any frequency band that will be used for 802.11 compliant system in each country, for example ***p*** can be as low as 23 for Japan where the total bandwidth is limited to 27 MHz, and p can be as large as 89 for those European countries following the ETSI standard.

So in the case of FCC rule it is possible to derive a family of $p-1$ patterns, each of length p , with pattern F_i given by:

$F_i = f_i(0)\ f_i(1)\ f_i(2)\ .....\ f_i(p-1)$ with $f_i(j) = i * j \bmod p$

Each of the F_i contains each ***p*** frequency channels equally often, as each F_i is a permutation of ***0, 1 2, ..., p-1***

With a pattern length of ***p*** the number of FHP that can be derived is ***p-1***.

This leads to a list of 78 Frequency Hopping Patterns

However not all those ***p-1*** patterns satisfy all the requirements simultaneously. Therefore starting from those 78 initial patterns, only $(p-1)-(2*F)=p-13=$ 66 satisfy criteria 4, for the one channel interference criteria (where F is the minimum amount that adjacent channel must be separated by). Out of those 66 only $p-13/\ 2k+1.=$ 22 patterns meet the criteria of one channel and one adjacent channel interference on each side, leaving therefore only 22 "good" patterns, assuming $k=1$ for the number of adjacent channel interference, to satisfy criteria 5, and considering that there is no topology knowledge.
Therefore with respect to criteria 2 and 3 those patterns are the best patterns.

Frequency pattern ordering

In the absence of any topology information concerning how the frequency hopping pattern are assigned, which will almost be always the case the number of "good" pattern is a family of 22, and those 22 must be assigned first before starting the assignment of the second or third set.

Most of the interference will come from adjacent channels as can be seen on the following where pattern 34 will get hit by pattern 33 and 35, but those hits do not occur in raw. However, if the FHPs are selected from three different families then contiguous hops will be subject to interference which is not desirable from a user perspective and contradict criteria 5.

```
                  Pattern 33:
 2   36     70  25  59  14  48   3  37    71    26  60  15  >49   4    38  72
27   61     16  50   5  39  73  28  62   >17 :.>51   6  40   74  29    63  18
52    7     41  75  30  64  19  53   8    42 :  76  31  65   20  54     9  43
77 :.>32   >66  21  55  10  44  78  33    67 :  22  56  11   45  79   >34  68
23 :  57    12  46  80  35  69  24  58    13 :  47
   :              Pattern 34:
 2 :  39    76  34  71  29  66  24  61    19 :  56  14  51    9  46     4  41
78 :  36    73  31  68  26  63  21  58   >16 :.>53  11  48    6  43    80  38
75 :  33    70  28  65  23  60  18  55    13 :  50   8  45    3  40    77  35
72 :.>30   >67  25  62  20  57  15  52    10 :  47   5  42   79  37    74  32
69 :  27    64  22  59  17  54  12  49     7 :  44
   :             Pattern 35:
 2 :  42     3  43   4  44   5  45   6    46 :   7  47   8  >48   9    49  10
50 :  11    51  12  52  13  53  14  54   >15 :.>55  16  56   17  57    18  58
19 :  59    20  60  21  61  22  62  23    63    24  64  25   65  26    66  27
67 :.>28   >68  29  69  30  70  31  71    32    72  33  73   34  74   >35  75
36    76    37  77  38  78  39  79  40    80    41
```

Figure 1. Example of interference between contiguous cells with good FHPs

```
                    Pattern 2:
 2  .>11 :.>20  29  38  47  56    65  74   4  13  22  31     40     49  58
67    76 :   6  15  24  33  42    51  60  69  78   8  17     26     35  44
53    62 :  71  80  10  19  28   >37  >46  55  64  73   3    12     21  30
39    48 :  57  66  75   5  14    23    32  41  50  59  68    77      7  16
25    34 :  43  52  61  70  79     9    18  27  36  45  54 :.>63   >72
         :                                                 :
         :          Pattern 24:                            :
 2   > 9 :.>16  23  30  37  44    51    58  65  72  79   7 :  14     21  28
35    42 :  49  56  63  70  77     5    12  19  26  33  40 :  47     54  61
68    75 :   3  10  17  24  31   >38   >45  52  59  66  73 :  80      8  15
22    29 :  36  43  50  57  64    71    78   6  13  20  27 :  34     41  48
55    62 :  69  76   4  11  18    25    32  39  46  53  60 :.>67   >74
                    Pattern 47:                            :
 2   >10 :.>18  26  34  42  50    58    66  74   3  11  19 :  27     35  43
51    59    67  75   4  12  20    28    36  44  52  60  68 :  76      5  13
21    29    37  45  53  61  69    77     6  14  22  30  38 :  46     54  62
70    78     7  15  23  31  39    47    55  63  71  79   8 :  16     24  32
40    48    56  64  72  80   9    17    25  33  41  49  57 :.>65   >73
```

Figure 2. Example of interference between FHPs chosen from three different sets

Even if the computed hopping sequences agree with the above criteria for 7 MHz between adjacent hops not all of them satisfy some other criteria which deal more with practical situations associated with the environment such as the presence of any microwave oven with a frequency span of 10 MHz or any direct sequence with an overall channel width of 22 MHz. Therefore some Hopping Patterns may appear better than others for example patterns 26, 27, 42 and 43 may seem more appropriate to cope with the above problems.

However if there is topology knowledge frequency hopping pattern assignment similar to what is done for cellular system can be performed, as shown hereunder or with a different reuse factor.

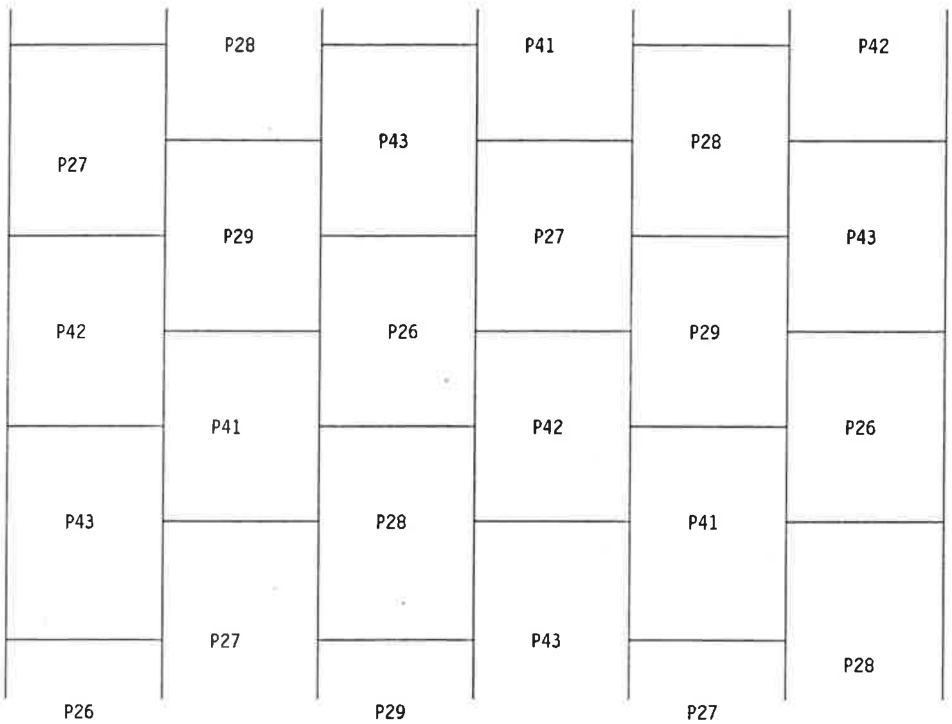

Figure 3. Example of FHP assignment for adjacent cells in case of topology knowledge

Conclusion

By assigning distinct hopping sequences to each cell several asynchronous networks, located in the same physical area, can share common frequency space. Nevertheless from time to time, two hopping sequences might hit the same channel at the same time, causing some throughput degradation, although not to serious extent as most probably a specific channel will not suffer from interference if only few channels are in use. Anyway based on the use of "good" Frequency Hopping Patterns the extend of interference become predictable as a function of the number of overlapping cells.

BIBLIOGRAPHY

[1] A. Krishna and K.S. Natarajan, *Selection Criteria for Frequency Hopping Pattern Set*, IEEE P802.11/92-84

First set of 22 patterns

Pattern 24:

2 9 16 23 30 37 44 51 58 65 72 79 7 14 21 28 35 42 49 56 63 70 77 5 12 19 26 33 40 47 54 61 68 75 3 10 17 24 31 38

45 52 59 66 73 80 8 15 22 29 36 43 50 57 64 71 78 6 13 20 27 34 41 48 55 62 69 76 4 11 18 25 32 39 46 53 60 67 74

Pattern 25:

2 12 22 32 42 52 62 72 3 13 23 33 43 53 63 73 4 14 24 34 44 54 64 74 5 15 25 35 45 55 65 75 6 16 26 36 46 56 66 76

7 17 27 37 47 57 67 77 8 18 28 38 48 58 68 78 9 19 29 39 49 59 69 79 10 20 30 40 50 60 70 80 11 21 31 41 51 61 71

Pattern 26:

2 15 28 41 54 67 80 14 27 40 53 66 79 13 26 39 52 65 78 12 25 38 51 64 77 11 24 37 50 63 76 10 23 36 49 62 75 9 22 35

48 61 74 8 21 34 47 60 73 7 20 33 46 59 72 6 19 32 45 58 71 5 18 31 44 57 70 4 17 30 43 56 69 3 16 29 42 55 68

Pattern 27:

2 18 34 50 66 3 19 35 51 67 4 20 36 52 68 5 21 37 53 69 6 22 38 54 70 7 23 39 55 71 8 24 40 56 72 9 25 41 57 73

10 26 42 58 74 11 27 43 59 75 12 28 44 60 76 13 29 45 61 77 14 30 46 62 78 15 31 47 63 79 16 32 48 64 80 17 33 49 65

Pattern 28:

2 21 40 59 78 18 37 56 75 15 34 53 72 12 31 50 69 9 28 47 66 6 25 44 63 3 22 41 60 79 19 38 57 76 16 35 54 73 13 32

51 70 10 29 48 67 7 26 45 64 4 23 42 61 80 20 39 58 77 17 36 55 74 14 33 52 71 11 30 49 68 8 27 46 65 5 24 43 62

Pattern 29:

2 24 46 68 11 33 55 77 20 42 64 7 29 51 73 16 38 60 3 25 47 69 12 34 56 78 21 43 65 8 30 52 74 17 39 61 4 26 48 70

13 35 57 79 22 44 66 9 31 53 75 18 40 62 5 27 49 71 14 36 58 80 23 45 67 10 32 54 76 19 41 63 6 28 50 72 15 37 59

Pattern 30:

2 27 52 77 23 48 73 19 44 69 15 40 65 11 36 61 7 32 57 3 28 53 78 24 49 74 20 45 70 16 41 66 12 37 62 8 33 58 4 29

54 79 25 50 75 21 46 71 17 42 67 13 38 63 9 34 59 5 30 55 80 26 51 76 22 47 72 18 43 68 14 39 64 10 35 60 6 31 56

Pattern 31:

2 30 58 7 35 63 12 40 68 17 45 73 22 50 78 27 55 4 32 60 9 37 65 14 42 70 19 47 75 24 52 80 29 57 6 34 62 11 39 67

16 44 72 21 49 77 26 54 3 31 59 8 36 64 13 41 69 18 46 74 23 51 79 28 56 5 33 61 10 38 66 15 43 71 20 48 76 25 53

Pattern 32:

2 33 64 16 47 78 30 61 13 44 75 27 58 10 41 72 24 55 7 38 69 21 52 4 35 66 18 49 80 32 63 15 46 77 29 60 12 43 74 26

57 9 40 71 23 54 6 37 68 20 51 3 34 65 17 48 79 31 62 14 45 76 28 59 11 42 73 25 56 8 39 70 22 53 5 36 67 19 50

Pattern 33:

2 36 70 25 59 14 48 3 37 71 26 60 15 49 4 38 72 27 61 16 50 5 39 73 28 62 17 51 6 40 74 29 63 18 52 7 41 75 30 64

19 53 8 42 76 31 65 20 54 9 43 77 32 66 21 55 10 44 78 33 67 22 56 11 45 79 34 68 23 57 12 46 80 35 69 24 58 13 47

Pattern 34:

2 39 76 34 71 29 66 24 61 19 56 14 51 9 46 4 41 78 36 73 31 68 26 63 21 58 16 53 11 48 6 43 80 38 75 33 70 28 65 23

60 18 55 13 50 8 45 3 40 77 35 72 30 67 25 62 20 57 15 52 10 47 5 42 79 37 74 32 69 27 64 22 59 17 54 12 49 7 44

Pattern 35:

2 42 3 43 4 44 5 45 6 46 7 47 8 48 9 49 10 50 11 51 12 52 13 53 14 54 15 55 16 56 17 57 18 58 19 59 20 60 21 61

22 62 23 63 24 64 25 65 26 66 27 67 28 68 29 69 30 70 31 71 32 72 33 73 34 74 35 75 36 76 37 77 38 78 39 79 40 80 41

Pattern 36:

2 45 9 52 16 59 23 66 30 73 37 80 44 8 51 15 58 22 65 29 72 36 79 43 7 50 14 57 21 64 28 71 35 78 42 6 49 13 56 20

63 27 70 34 77 41 5 48 12 55 19 62 26 69 33 76 40 4 47 11 54 18 61 25 68 32 75 39 3 46 10 53 17 60 24 67 31 74 38

Pattern 37:

2 48 15 61 28 74 41 8 54 21 67 34 80 47 14 60 27 73 40 7 53 20 66 33 79 46 13 59 26 72 39 6 52 19 65 32 78 45 12 58

25 71 38 5 51 18 64 31 77 44 11 57 24 70 37 4 50 17 63 30 76 43 10 56 23 69 36 3 49 16 62 29 75 42 9 55 22 68 35

Pattern 38:

2 51 21 70 40 10 59 29 78 48 18 67 37 7 56 26 75 45 15 64 34 4 53 23 72 42 12 61 31 80 50 20 69 39 9 58 28 77 47 17

66 36 6 55 25 74 44 14 63 33 3 52 22 71 41 11 60 30 79 49 19 68 38 8 57 27 76 46 16 65 35 5 54 24 73 43 13 62 32

Pattern 39:

2 4 27 79 52 25 77 50 23 75 48 21 73 46 19 71 44 17 69 42 15 67 40 13 65 38 11 63 36 9 61 34 7 59 32 5 57 30 3 55

28 80 53 26 78 51 24 76 49 22 74 47 20 72 45 18 70 43 16 68 41 14 66 39 12 64 37 10 62 35 8 60 33 6 58 31 4 56 29

Pattern 40:

2 57 33 9 64 40 16 71 47 23 78 54 30 6 61 37 13 68 44 20 75 51 27 3 58 34 10 65 41 17 72 48 24 79 55 31 7 62 38 14

69 45 21 76 52 28 4 59 35 11 66 42 18 73 49 25 80 56 32 8 63 39 15 70 46 22 77 53 29 5 60 36 12 67 43 19 74 50 26

Pattern 41:

2 60 39 18 76 55 34 13 71 50 [illegible] 8 66 45 24 3 61 40 19 77 56 35 14 72 51 30 9 67 46 25 4 62 41 20 78 57 36 15 73 52

31 10 68 47 26 5 63 42 21 79 [illegible] 37 16 74 53 32 11 69 48 27 6 64 43 22 80 59 38 17 75 54 33 12 70 49 28 7 65 44 23

Pattern 42:

2 63 45 27 9 7 52 34 16 77 59 41 23 5 66 48 30 12 73 55 37 19 80 62 44 26 8 69 51 33 15 76 58 40 22 4 35 47 29 1

72 54 36 18 79 61 43 25 7 68 50 32 14 75 57 39 21 3 64 46 28 10 71 53 35 17 78 60 42 24 6 67 49 31 13 74 56 38 20

Pattern 43:

2 66 51 36 21 6 70 55 40 25 10 74 59 44 29 14 78 63 48 33 18 3 67 52 37 22 7 71 56 41 26 11 75 60 45 30 15 79 64 49

34 19 4 68 53 38 23 8 72 57 42 27 12 76 61 46 31 16 80 65 50 35 20 5 69 54 39 24 9 73 58 43 28 13 77 62 47 32 17

Pattern 44:

2 69 57 45 33 21 9 76 64 52 40 28 16 4 71 59 47 35 23 11 78 66 54 42 30 18 6 73 61 49 37 25 13 80 68 56 44 32 20 8

75 63 51 39 27 15 3 70 58 46 34 22 10 77 65 53 41 29 17 5 72 60 48 36 24 12 79 67 55 43 31 19 7 74 62 50 38 26 14

Pattern 45:

2 72 63 54 45 36 27 18 9 79 70 61 52 43 34 25 16 7 77 68 59 50 41 32 23 14 5 75 66 57 48 39 30 21 12 3 73 64 55 46

37 28 19 10 80 71 62 53 44 35 26 17 8 78 69 60 51 42 33 24 15 6 76 67 58 49 40 31 22 13 4 74 65 56 47 38 29 20 11

Second set of 22 patterns

```
Pattern 47:
2 10 18 26 34 42 50 58 66 74 3 11 19 27 35 43 51 59 67 75 4 12 20 28 36 44 52 60 68 76 5 13 21 29 37 45 53 61 69 77
6 14 22 30 38 46 54 62 70 78 7 15 23 31 39 47 55 63 71 79 8 16 24 32 40 48 56 64 72 80 9 17 25 33 41 49 57 65 73
Pattern 48:
2 13 24 35 46 57 68 79 11 22 33 44 55 66 77 9 20 31 42 53 64 75 7 18 29 40 51 62 73 5 16 27 38 49 60 71 3 14 25 36
47 58 69 80 12 23 34 45 56 67 78 10 21 32 43 54 65 76 8 19 30 41 52 63 74 6 17 28 39 50 61 72 4 15 26 37 48 59 70
Pattern 49:
2 16 30 44 58 72 7 21 35 49 63 77 12 26 40 54 68 3 17 31 45 59 73 8 22 36 50 64 78 13 27 41 55 69 4 18 32 46 60 74
9 23 37 51 65 79 14 28 42 56 70 5 19 33 47 61 75 10 24 38 52 66 80 15 29 43 57 71 6 20 34 48 62 76 11 25 39 53 67
Pattern 50:
2 19 36 53 70 8 25 42 59 76 14 31 48 65 3 20 37 54 71 9 26 43 60 77 15 32 49 66 4 21 38 55 72 10 27 44 61 78 16 33
50 67 5 22 39 56 73 11 28 45 62 79 17 34 51 68 6 23 40 57 74 12 29 46 63 80 18 35 52 69 7 24 41 58 75 13 30 47 64
Pattern 51:
2 22 42 62 3 23 43 63 4 24 44 64 5 25 45 65 6 26 46 66 7 27 47 67 8 28 48 68 9 29 49 69 10 30 50 70 11 31 51 71
12 32 52 72 13 33 53 73 14 34 54 74 15 35 55 75 16 36 56 76 17 37 57 77 18 38 58 78 19 39 59 79 20 40 60 80 21 41 61
Pattern 52:
2 25 48 71 15 38 61 5 28 51 74 18 41 64 8 31 54 77 21 44 67 11 34 57 80 24 47 70 14 37 60 4 27 50 73 17 40 63 7 30
53 76 20 43 66 10 33 56 79 23 46 69 13 36 59 3 26 49 72 16 39 62 6 29 52 75 19 42 65 9 32 55 78 22 45 68 12 35 58
Pattern 53:
2 28 54 80 27 53 79 26 52 78 25 51 77 24 50 76 23 49 75 22 48 74 21 47 73 20 46 72 19 45 71 18 44 70 17 43 69 16 42 68
15 41 67 14 40 66 13 39 65 12 38 64 11 37 63 10 36 62 9 35 61 8 34 60 7 33 59 6 32 58 5 31 57 4 30 56 3 29 55
Pattern 54:
2 31 60 10 39 68 18 47 76 26 55 5 34 63 13 42 71 21 50 79 29 58 8 37 66 16 45 74 24 53 3 32 61 11 40 69 19 48 77 27
56 6 35 64 14 43 72 22 51 80 30 59 9 38 67 17 46 75 25 54 4 33 62 12 41 70 20 49 78 28 57 7 36 65 15 44 73 23 52
Pattern 55:
2 34 66 19 51 4 36 68 21 53 6 38 70 23 55 8 40 72 25 57 10 42 74 27 59 12 44 76 29 61 14 46 78 31 63 16 48 80 33 65
18 50 3 35 67 20 52 5 37 69 22 54 7 39 71 24 56 9 41 73 26 58 11 43 75 28 60 13 45 77 30 62 15 47 79 32 64 17 49
Pattern 56:
2 37 72 28 63 19 54 10 45 80 36 71 27 62 18 53 9 44 79 35 70 26 61 17 52 8 43 78 34 69 25 60 16 51 7 42 77 33 68 24
59 15 50 6 41 76 32 67 23 58 14 49 5 40 75 31 66 22 57 13 48 4 39 74 30 65 21 56 12 47 3 38 73 29 64 20 55 11 46
Pattern 57:
2 40 78 37 75 34 72 31 69 28 66 25 63 22 60 19 57 16 54 13 51 10 48 7 45 4 42 80 39 77 36 74 33 71 30 68 27 65 24 62
21 59 18 56 15 53 12 50 9 47 6 44 3 41 79 38 76 35 73 32 70 29 67 26 64 23 61 20 58 17 55 14 52 11 49 8 46 5 43
Pattern 58:
2 43 5 46 8 49 11 52 14 55 17 58 20 61 23 64 26 67 29 70 32 73 35 76 38 79 41 3 44 6 47 9 50 12 53 15 56 18 59 21
62 24 65 27 68 30 71 33 74 36 77 39 80 42 4 45 7 48 10 51 13 54 16 57 19 60 22 63 25 66 28 69 31 72 34 75 37 78 40
Pattern 59:
2 46 11 55 20 64 29 73 38 3 47 12 56 21 65 30 74 39 4 48 13 57 22 66 31 75 40 5 49 14 58 23 67 32 76 41 6 50 15 59
24 68 33 77 42 7 51 16 60 25 69 34 78 43 8 52 17 61 26 70 35 79 44 9 53 18 62 27 71 36 80 45 10 54 19 63 28 72 37
Pattern 60:
2 49 17 64 32 79 47 15 62 30 77 45 13 60 28 75 43 11 58 26 73 41 9 56 24 71 39 7 54 22 69 37 5 52 20 67 35 3 50 18
65 33 80 48 16 63 31 78 46 14 61 29 76 44 12 59 27 74 42 10 57 25 72 40 8 55 23 70 38 6 53 21 68 36 4 51 19 66 34
Pattern 61:
2 52 23 73 44 15 65 36 7 57 28 78 49 20 70 41 12 62 33 4 54 25 75 46 17 67 38 9 59 30 80 51 22 72 43 14 64 35 6 56
27 77 48 19 69 40 11 61 32 3 53 24 74 45 16 66 37 8 58 29 79 50 21 71 42 13 63 34 5 55 26 76 47 18 68 39 10 60 31
Pattern 62:
2 55 29 3 56 30 4 57 31 5 58 32 6 59 33 7 60 34 8 61 35 9 62 36 10 63 37 11 64 38 12 65 39 13 66 40 14 67 41 15
68 42 16 69 43 17 70 44 18 71 45 19 72 46 20 73 47 21 74 48 22 75 49 23 76 50 24 77 51 25 78 52 26 79 53 27 80 54 28
Pattern 63:
2 58 35 12 68 45 22 78 55 32 9 65 42 19 75 52 29 6 62 39 16 72 49 26 3 59 36 13 69 46 23 79 56 33 10 66 43 20 76 53
30 7 63 40 17 73 50 27 4 60 37 14 70 47 24 80 57 34 11 67 44 21 77 54 31 8 64 41 18 74 51 28 5 61 38 15 71 48 25
Pattern 64:
2 61 41 21 80 60 40 20 79 59 39 19 78 58 38 18 77 57 37 17 76 56 36 16 75 55 35 15 74 54 34 14 73 53 33 13 72 52 32 12
71 51 31 11 70 50 30 10 69 49 29 9 68 48 28 8 67 47 27 7 66 46 26 6 65 45 25 5 64 44 24 4 63 43 23 3 62 42 22
Pattern 65:
2 64 47 30 13 75 58 41 24 7 69 52 35 18 80 63 46 29 12 74 57 40 23 6 68 51 34 17 79 62 45 28 11 73 56 39 22 5 67 50
33 16 78 61 44 27 10 72 55 38 21 4 66 49 32 15 77 60 43 26 9 71 54 37 20 3 65 48 31 14 76 59 42 25 8 70 53 36 19
Pattern 66:
2 67 53 39 25 11 76 62 48 34 20 6 71 57 43 29 15 80 66 52 38 24 10 75 61 47 33 19 5 70 56 42 28 14 79 65 51 37 23 9
74 60 46 32 18 4 69 55 41 27 13 78 64 50 36 22 8 73 59 45 31 17 3 68 54 40 26 12 77 63 49 35 21 7 72 58 44 30 16
Pattern 67:
2 70 59 48 37 26 15 4 72 61 50 39 28 17 6 74 63 52 41 30 19 8 76 65 54 43 32 21 10 78 67 56 45 34 23 12 80 69 58 47
36 25 14 3 71 60 49 38 27 16 5 73 62 51 40 29 18 7 75 64 53 42 31 20 9 77 66 55 44 33 22 11 79 68 57 46 35 24 13
Pattern 68:
2 73 65 57 49 41 33 25 17 9 80 72 64 56 48 40 32 24 16 8 79 71 63 55 47 39 31 23 15 7 78 70 62 54 46 38 30 22 14 6
77 69 61 53 45 37 29 21 13 5 76 68 60 52 44 36 28 20 12 4 75 67 59 51 43 35 27 19 11 3 74 66 58 50 42 34 26 18 10
```

Third set of Patterns

Pattern 2:

2 11 20 29 38 47 56 65 74 4 13 22 31 40 49 58 67 76 6 15 24 33 42 51 60 69 78 8 17 26 35 44 53 62 71 80 10 19 28 37

46 55 64 73 3 12 21 30 39 48 57 66 75 5 14 23 32 41 50 59 68 77 7 16 25 34 43 52 61 70 79 9 18 27 36 45 54 63 72

Pattern 3:

2 14 26 38 50 62 74 7 19 31 43 55 67 79 12 24 36 48 60 72 5 17 29 41 53 65 77 10 22 34 46 58 70 3 15 27 39 51 63 75

8 20 32 44 56 68 80 13 25 37 49 61 73 6 18 30 42 54 66 78 11 23 35 47 59 71 4 16 28 40 52 64 76 9 21 33 45 57 69

Pattern 4:

2 17 32 47 62 77 13 28 43 58 73 9 24 39 54 69 5 20 35 50 65 80 16 31 46 61 76 12 27 42 57 72 8 23 38 53 68 4 19 34

49 64 79 15 30 45 60 75 11 26 41 56 71 7 22 37 52 67 3 18 33 48 63 78 14 29 44 59 74 10 25 40 55 70 6 21 36 51 66

Pattern 5:

2 20 38 56 74 13 31 49 67 6 24 42 60 78 17 35 53 71 10 28 46 64 3 21 39 57 75 14 32 50 68 7 25 43 61 79 18 36 54 72

11 29 47 65 4 22 40 58 76 15 33 51 69 8 26 44 62 80 19 37 55 73 12 30 48 66 5 23 41 59 77 16 34 52 70 9 27 45 63

Pattern 6:

2 23 44 65 7 28 49 70 12 33 54 75 17 38 59 80 22 43 64 6 27 48 69 11 32 53 74 16 37 58 79 21 42 63 5 26 47 68 10 31

52 73 15 36 57 78 20 41 62 4 25 46 67 9 30 51 72 14 35 56 77 19 40 61 3 24 45 66 8 29 50 71 13 34 55 76 18 39 60

Pattern 7:

2 26 50 74 19 43 67 12 36 60 5 29 53 77 22 46 70 15 39 63 8 32 56 80 25 49 73 18 42 66 11 35 59 4 28 52 76 21 45 69

14 38 62 7 31 55 79 24 48 72 17 41 65 10 34 58 3 27 51 75 20 44 68 13 37 61 6 30 54 78 23 47 71 16 40 64 9 33 57

Pattern 8:

2 29 56 4 31 58 6 33 60 8 35 62 10 37 64 12 39 66 14 41 68 16 43 70 18 45 72 20 47 74 22 49 76 24 51 78 26 53 80 28

55 3 30 57 5 32 59 7 34 61 9 36 63 11 38 65 13 40 67 15 42 69 17 44 71 19 46 73 21 48 75 23 50 77 25 52 79 27 54

Pattern 9:

2 32 62 13 43 73 24 54 5 35 65 16 46 76 27 57 8 38 68 19 49 79 30 60 11 41 71 22 52 3 33 63 14 44 74 25 55 6 36 66

17 47 77 28 58 9 39 69 20 50 80 31 61 12 42 72 23 53 4 34 64 15 45 75 26 56 7 37 67 18 48 78 29 59 10 40 70 21 51

Pattern 10:

2 35 68 22 55 9 42 75 29 62 16 49 3 36 69 23 56 10 43 76 30 63 17 50 4 37 70 24 57 11 44 77 31 64 18 51 5 38 71 25

58 12 45 78 32 65 19 52 6 39 72 26 59 13 46 79 33 66 20 53 7 40 73 27 60 14 47 80 34 67 21 54 8 41 74 28 61 15 48

Pattern 11:

2 38 74 31 67 24 60 17 53 10 46 3 39 75 32 68 25 61 18 54 11 47 4 40 76 33 69 26 62 19 55 12 48 5 41 77 34 70 27 63

20 56 13 49 6 42 78 35 71 28 64 21 57 14 50 7 43 79 36 72 29 65 22 58 15 51 8 44 80 37 73 30 66 23 59 16 52 9 45

Pattern 12:

2 41 80 40 79 39 78 38 77 37 76 36 75 35 74 34 73 33 72 32 71 31 70 30 69 29 68 28 67 27 66 26 65 25 64 24 63 23 62 22

61 21 60 20 59 19 58 18 57 17 56 16 55 15 54 14 53 13 52 12 51 11 50 10 49 9 48 8 47 7 46 6 45 5 44 4 43 3 42

Pattern 13:

2 44 7 49 12 54 17 59 22 64 27 69 32 74 37 79 42 5 47 10 52 15 57 20 62 25 67 30 72 35 77 40 3 45 8 50 13 55 18 60

23 65 28 70 33 75 38 80 43 6 48 11 53 16 58 21 63 26 68 31 73 36 78 41 4 46 9 51 14 56 19 61 24 66 29 71 34 76 39

Pattern 14:

2 47 13 58 24 69 35 80 46 12 57 23 68 34 79 45 11 56 22 67 33 78 44 10 55 21 66 32 77 43 9 54 20 65 31 76 42 8 53 19

64 30 75 41 7 52 18 63 29 74 40 6 51 17 62 28 73 39 5 50 16 61 27 72 38 4 49 15 60 26 71 37 3 48 14 59 25 70 36

Pattern 15:

2 50 19 67 36 5 53 22 70 39 8 56 25 73 42 11 59 28 76 45 14 62 31 79 48 17 65 34 3 51 20 68 37 6 54 23 71 40 9 57

26 74 43 12 60 29 77 46 15 63 32 80 49 18 66 35 4 52 21 69 38 7 55 24 72 41 10 58 27 75 44 13 61 30 78 47 16 64 33

Pattern 16:

2 53 25 76 48 20 71 43 15 66 38 10 61 33 5 56 28 79 51 23 74 46 18 69 41 13 64 36 8 59 31 3 54 26 77 49 21 72 44 16

67 39 11 62 34 6 57 29 80 52 24 75 47 19 70 42 14 65 37 9 60 32 4 55 27 78 50 22 73 45 17 68 40 12 63 35 7 58 30

Pattern 17:

2 56 31 6 60 35 10 64 39 14 68 43 18 72 47 22 76 51 26 80 55 30 5 59 34 9 63 38 13 67 42 17 71 46 21 75 50 25 79 54

29 4 58 33 8 62 37 12 66 41 16 70 45 20 74 49 24 78 53 28 3 57 32 7 61 36 11 65 40 15 69 44 19 73 48 23 77 52 27

Pattern 18:

2 59 37 15 72 50 28 6 63 41 19 76 54 32 10 67 45 23 80 58 36 14 71 49 27 5 62 40 18 75 53 31 9 66 44 22 79 57 35 13

70 48 26 4 61 39 17 74 52 30 8 65 43 21 78 56 34 12 69 47 25 3 60 38 16 73 51 29 7 64 42 20 77 55 33 11 68 46 24

Pattern 19:

2 62 43 24 5 65 46 27 8 68 49 30 11 71 52 33 14 74 55 36 17 77 58 39 20 80 61 42 23 4 64 45 26 7 67 48 29 10 70 51

32 13 73 54 35 16 76 57 38 19 79 60 41 22 3 63 44 25 6 66 47 28 9 69 50 31 12 72 53 34 15 75 56 37 18 78 59 40 21

Pattern 20:

2 65 49 33 17 80 64 48 32 16 79 63 47 31 15 78 62 46 30 14 77 61 45 29 13 76 60 44 28 12 75 59 43 27 11 74 58 42 26 10

73 57 41 25 9 72 56 40 24 8 71 55 39 23 7 70 54 38 22 6 69 53 37 21 5 68 52 36 20 4 67 51 35 19 3 66 50 34 18

Pattern 21:

2 68 55 42 29 16 3 69 56 43 30 17 4 70 57 44 31 18 5 71 58 45 32 19 6 72 59 46 33 20 7 73 60 47 34 21 8 74 61 48

35 22 9 75 62 49 36 23 10 76 63 50 37 24 11 77 64 51 38 25 12 78 65 52 39 26 13 79 66 53 40 27 14 80 67 54 41 28 15

Pattern 22:

2 71 61 51 41 31 21 11 80 70 60 50 40 30 20 10 79 69 59 49 39 29 19 9 78 68 58 48 38 28 18 8 77 67 57 47 37 27 17 7

76 66 56 46 36 26 16 6 75 65 55 45 35 25 15 5 74 64 54 44 34 24 14 4 73 63 53 43 33 23 13 3 72 62 52 42 32 22 12

Pattern 23:

2 74 67 60 53 46 39 32 25 18 11 4 76 69 62 55 48 41 34 27 20 13 6 78 71 64 57 50 43 36 29 22 15 8 80 73 66 59 52 45

38 31 24 17 10 3 75 68 61 54 47 40 33 26 19 12 5 77 70 63 56 49 42 35 28 21 14 7 79 72 65 58 51 44 37 30 23 16 9