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Loss of Functional Processing Gain with Few Chips Per Symbol

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Abstract

Ever since direct-sequence spread-spectrum systems have been built, it has been common practice to estimate error-rate performance using standard (thermal-noise) formulas. Military DSSS systems employed very large numbers of chips per symbol (N_c), and in this regime the use of such formulas is reasonable. Commercial DSSS systems emphasise high data rates, making attractive the use small N_c. For Nc<10 there can be considerable error in using the standard formulas. A recent FCC rule change appears to indicate that equipment need only demonstrate a processing gain greater than 10 as computed using a formula from spread-spectrum practice. However, for small N_c this formula greatly overestimates the processing gain. This means that system calculations will be wrong if based upon the assumption that the computed processing gain is the functional processing gain for interference suppression.

The premise of Part 15.247 is that unregulated users can effectively and fairly share the spectrum because of the incorporation of processing gain into the link designs. In a typical deployment the transmitted power needs to be high enough to close the link at some separation between co-channel users. Because equipment employing a small N_c will not actually have processing gain, or at least not as much as computed, the effect of co-channel users will be apparent at considerably longer separations than would have been the case if the functional processing gain were the computed value. This, in turn, will cause links to be operated at a higher transmitter power than would have been required with the corresponding functional processing gain, and this only makes the mutual interference worse.

The present note illuminates the difference between computed and functional processing gain by computing the error rate for several example transmission formats. In an extreme example, it is shown that a simple (non-spread-spectrum) DPSK transmission can pass the processing gain test with a computed processing gain of 10 dB. However, this would only be useful for passing such a test; there would clearly be no functional processing gain using such a modulation.

Loss of Functional Processing Gain with Few Chips Per Symbol

Introduction

Ever since direct-sequence spread-spectrum systems have been built, it has been common practice to estimate error-rate performance using standard (thermal-noise) Pe vs. E/N_0 formulas. While it is well-known that the distribution of de-spread interference in a DSSS system is not identical to the Gaussian distribution ascribed to thermal noise, traditional (military) DSSS systems employed very large numbers of chips per symbol (N_c), and in this regime the difference in distributions is of little consequence. Only since DSSS systems entered the commercial world has the emphasis on higher data rates motivated the use extremely small spreading factors.

A recent FCC rule change appears to indicate that equipment need only demonstrate a processing gain greater than 10 as computed using a familiar formula (see below) from spread-spectrum practice. However, for small N_c this formula greatly overestimates the processing gain. This means that system calculations will be wrong if based upon the assumption that the computed processing gain is the functional processing gain for interference suppression.

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The present note illuminates the difference between computed and functional processing gain by computing the error rate for several example transmission formats. In the extreme example, it is shown that a simple DPSK (non-spread-spectrum) transmission can readily pass the processing gain test with a computed processing gain of 10 dB. However, this would only be useful for passing such a test; there would clearly be no functional processing gain using such a modulation.

Background

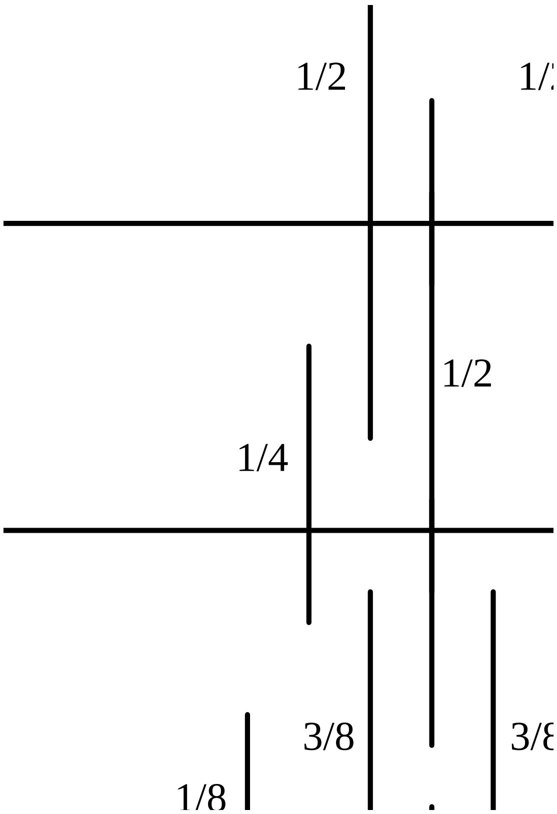


Figure 1 - Processor output distributions for constant and Gaussian interference (baseband case).

De-spreading of a DSSS signal imposes a corresponding spreading upon interference; the result is that the interference sample contained in the correlation output (demodulation) statistic has a noise-like distribution which appears Gaussian. However, for simple CW interference the post-processing distribution is actually Binomial.[1] The key difference between the Binomial distribution and the Gaussian distribution used for approximation purposes is that the Binomial distribution does not possess the characteristic "tails" of the Gaussian, as shown in Figure 1 for a baseband example. As a result, for any DSSS system experiencing signal plus CW interference alone, there will be a threshold in S/I ratio at which the probability of error will drop abruptly to zero. This is in contrast to the case of Gaussian noise, for which the probability of error decreases exponentially, but not abruptly, as a function of S/N.

The key fact is that for very few chips per symbol, this abrupt drop to zero probability of error actually happens at a much lower S/I than would be predicted by using the thermal-noise formula. Appendix A1 demonstrates this effect for the example case of baseband signalling. This is very important because FCC rules allow manufacturers to compute the processing gain using the formula

$$Gp = (S / N)_0 + Mj + Lsys$$

[1] The distribution is actually binomial only if one averages over all possible codes, and in the baseband case. For a fixed spreading code the output will still drop abruptly to zero at some S/I value, even though not a binomial distribution. For bandpass signals it is necessary to average over the unknown relative phase of the interference, which also causes departure from binomial but still drops abruptly.

where

Gp is the processing gain of the system,
$(S/N)_0$ is the SNR required for the chosen BER,
Mj is the J/S ratio (i.e., the threshold I/S ratio), and
Lsys is the system implementation loss.

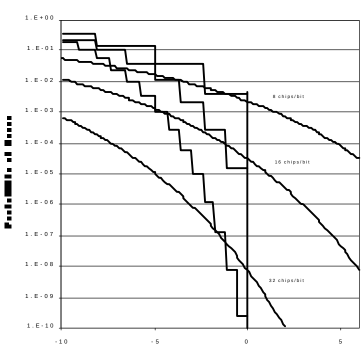
Unfortunately, processing gain determined by this formula cannot be claimed as functional processing gain for determining system performance. In appendices A2 and A3 are two examples of transmission having computed processing gain more than 10 dB. For all examples the probability of error drops abruptly to zero at an input signal-to-noise ratio of 0 dB. If we assume that the system losses are 0 dB, then the processing gain will be computed to be

$$G_P = (S/N)_0 + M_J + L_{SYS} = (S/N)_0$$

Thus, the higher the required SNR, the higher the computed processing gain. Link designs for wireless LANs must exhibit useful probability of successful transfer of an entire data packet (or frame) containing 10^3 to 10^4 bits, or more; thus, the required probability of bit (or symbol) error is usually 10^{-5} to 10^{-6}, or lower. Thus, it is possible to obtain a computed processing gain greater than 10 by insisting upon a low error rate, which demands high $(S/N)_0$.

Discussion

The processing gain equation, which is the basis of the processing gain computation, appears in textbooks on spread-spectrum communications; however, it was never intended that this equation be used for small numbers of chips per bit. The essential flaw in the computation is the estimation of the required SNR based upon



Gaussian statistics. When wireless LAN equipment is actually deployed, the background interference will rarely be of constant amplitude. When only a few independent sources of interference are present, the Gaussian-like tails of the interference distribution will cause higher error rates than would be predicted using the computed processing gain. The functional processing gain will be substantially lower.

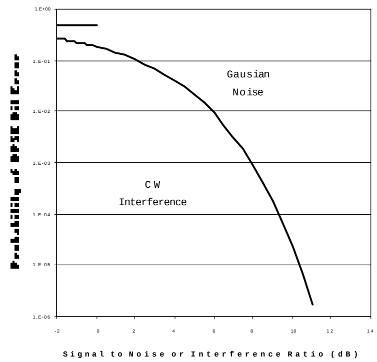


Figure 2 - Figures from Appendices.

Figure 2 reproduces figures from the three appendices. The top graph shows the probability of bit error for a baseband, binary example using 8, 16, and 32 chips per bit; the staircase functions are for constant-amplitude interference, while the smooth functions are for Gaussian noise. The middle graph shows the probability of symbol error for a 4-ary Orthogonal transmission using 4 chips per symbol, for CW interference and for Gaussian noise. The bottom graph shows the probability of bit error for simple DPSK (one chip per bit) for CW interference and for Gaussian noise.

The baseband example is presented only to show the binomial-distribution behaviour. The later two examples are true bandpass transmissions, and hence could actually be implemented. However, few would advocate the extreme of using DPSK transmission under Part 15.247 as spread-spectrum signaling. In all three cases, there is an abrupt drop to zero probability of error at an input S/I of 0 dB for CW interference. Since probability of error curves based upon thermal noise demand considerably higher S/I, the computed processing gain will be substantial for all three cases; how substantial depends upon how low a probability of error one argues for.

The danger at hand is that some might predict system performance for a LAN deployment based upon having the <u>computed</u> processing gain as <u>functional</u> processing gain for rejection of interference.

If a system has a specified <u>functional</u> processing gain, then the error-rate performance is roughly independent of whether the interference is from a single source or multiple sources. However, if link performance is estimated based upon the <u>computed</u> processing gain against a single CW interferer, then when multiple interferers are present at the same average interfering power the link will degrade tremendously. This effect is the loss of <u>functional</u> processing gain.

A1: Baseband Example

This example is intended to highlight the mathematical anomaly encountered when using small N_c. It does not apply to any specific implementation in detail; rather it will illuminate the statistical behaviour in general. We consider baseband, bi-phase signalling in the presence of a constant-level interferer. This is a very simple case to analyze, which is the motivation for its selection. (It corresponds to RF transmission using coherent PSK with the unlikely condition that the CW interferer is somehow phase-locked to the signal carrier.

We consider a baseband PSK-spread signalling waveform

$$x(t) = d\sum_{n=0}^{N_c-1} C_n p(t - nT_c)$$

Where

- d={±1} is the data;
- C_n={±1} is the spreading code;
- p(t) is the chip pulse waveform =1 for 0<t<T_c, 0 elsewhere;
- T_c is the chip duration.

Constant-level interference of amplitude A is added to x(t), and the results are correlated with C_n at the receiver to yield the demodulation statistic

$$R = dN_c + A\sum_{n=0}^{N_c-1} C_n$$

Rather than select a particular code, we average over all possible spreading codes. The values ±1 each have probability ½, so that the distribution of the interference component of the correlator output has a binomial distribution.

$$P\left(A\sum_{n=0}^{N_c-1} C_n = a(-N_c + 2k) \right) = \begin{pmatrix} N_c \\ k \end{pmatrix} 2^{-M}$$

The summation over the ±1 code values can be positive or negative. Thus, independent of the signs of d and A, the probability of error is the cumulative sum of the probabilities for which

$$\left|A\right|\sum_m C_m \geq N_c\left|d\right| = N_c$$

We define k_0 to be the largest integer for which
The input signal to
interference ratio is γ=A^{-2}.

$$-M + 2k_0 < \frac{N_c}{\left|A\right|} = N_c\sqrt{g}$$

Then the probability of error is

$$P(g) = 1 - \sum_0^{k_0} \begin{pmatrix} M \\ k \end{pmatrix} 2^{-M}$$

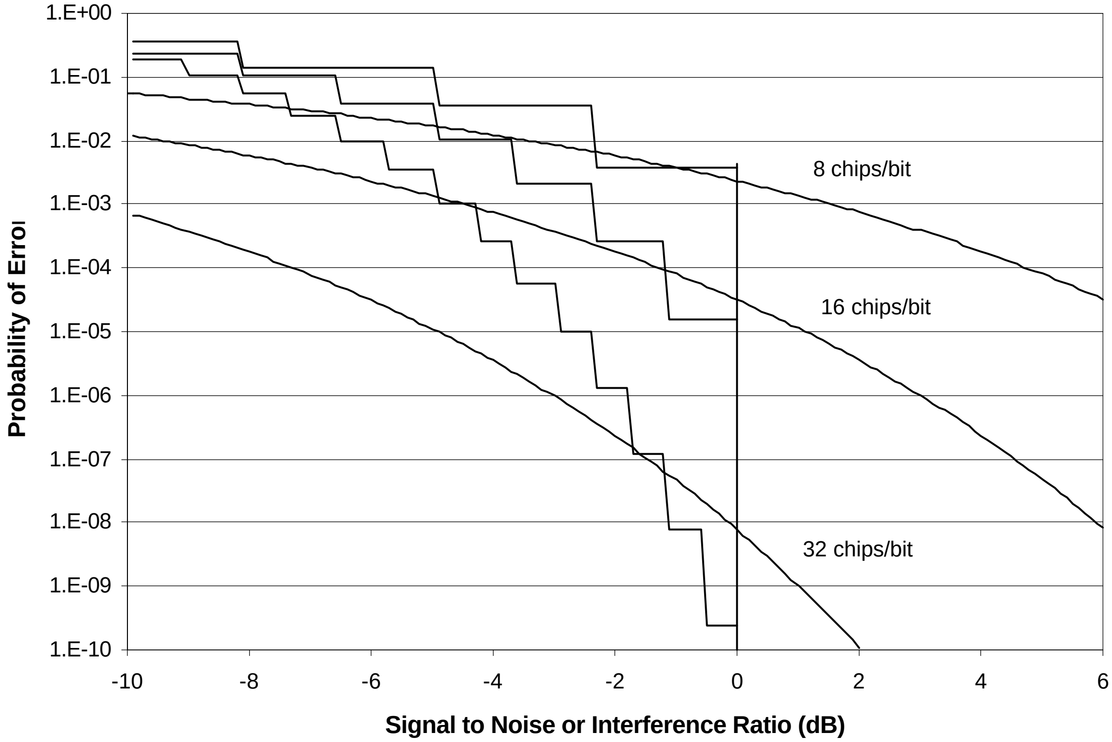


Figure 3 - Probability of error vs. input S/I for baseband signaling in Constant and Gaussian Interference.

For comparison, the probability of error for Gaussian interference is

Figure 3 plots the probability of error vs. (input) signal-to-interference ratio for a baseband binary DSSS system using 8,16, and 32 chips per bit. (N_c=1 corresponds to using a data-whitening sequence, although there is no spreading.) The staircase behaviour of the summed binomial distribution is finer for larger N_c. The probability of error drops abruptly to zero at S/I=0 dB <u>for all cases</u>. This effect occurs because, in the absence of noise, at S/I > 0dB, the highest value of the interference at the output of the correlator is always smaller than the signal component out of the correlator. The correct data decision will always be made. When the average power of the interference is higher than that of the signal, then depending on how the elements of the interference add up, the result may be larger or smaller than the signal component. The largest amplitude in the binomial distribution occurs when all the interference elements add in the same polarity. This is the least likely case, having probability 2^{-N_c}, but as the signal-to-noise ratio approaches 0 dB this is the last non-zero probability of error.

For probability of error values of interest, e.g. 10^{-4} to 10^{-6}, we see that using N_c=16 and 32 give reasonable correspondence with expectations. However, for N_c=8 the error drops abruptly from .4% to zero. The smooth curves overlaid are the probability of error for Gaussian interference. The accuracy of approximating the actual distribution by using the Gaussian-noise formula is useful for typical operating points for systems using large N_c. However, as the number of chips per bit becomes small the standard formula <u>tremendously overestimates</u> the required S/I. This overestimate, in turn, gives a greatly inflated value for the processing gain when using the formula method.

A2: 4-ary Orthogonal with 4 Chips/Symbol

We consider a 4-ary Orthogonal signalling technique with PSK spreading; for each symbol transmission one of the four Walsh functions is selected (specifying the value of N in the next expression) in order to convey 2 bits of information. The Walsh function is combined chip-by-chip with the PN spreading code, then modulated onto a carrier and transmitted as x(t).

$$x(t) = e^{j\omega_0 t}\sum_{m=0}^{3} C_m W_{Nm}\, p(t - mT_c) \qquad N = 0,1,2\, or\, 3$$

where

ω_0 is the (radian) carrier frequency;

C_m={1,1,1,-1} is the 4-chip PN code pattern; [2]

W_{Nm} is one of the 4-chip Walsh functions

W_{0m}={1,1,1,1} W_{1m}={1,1,-1,-1}

W_{2m}={1,-1,1,-1} W_{3m}={1,-1,-1,1};

p(t) is the PSK chip waveform =1 for $0<t<T_c$, 0 elsewhere;

T_c is the chip duration;

In transmission the signal is imparted a random carrier phase. At the receiver, the aggregate received signal and interference are passed through a chip matched filter[3], downconverted to form a baseband complex(in-phase and quadrature) signal, sampled at the peak (chip) correlation, then applied to a correlation processor for the code correlation.[4] The resulting baseband signal plus interference is

$$r(t) = \sum_{m=0}^{3} C_m W_{Nm} e^{j\phi} + A\frac{\sin(\frac{\Delta\omega T_c}{2})}{\frac{\Delta\omega T_c}{2}} e^{j(\Delta\omega t+\theta)} \qquad N = 0,1,2\, or\, 3$$

where

ϕ is the unknown phase of the signal carrier;

A is the interference amplitude;

Sin($\Delta\omega$Tc/2)/($\Delta\omega$Tc/2) is the shaping of the chip matched filter;

$\Delta\omega$ is the interference offset (radian) frequency; and,

θ is the interference phase.

The correlator channel signal components are

$$S_M = e^{j\phi}\sum_{m=0}^{3} C_m^2 W_{Mm} W_{Nm} = e^{j\phi}\sum_{m=0}^{3} W_{Mm} W_{Nm} \qquad M = 0,1,2,3$$

---

[2] Note that the PN code and Walsh functions are represented algebraically as having ±1 elements; these are, of course, isomorphic to the {0,1} elements that would be used in a discussion of logic implementation circuitry.

[3] It is not required that the received filter actually match the chip waveform exactly; the present formulation is used for simplicity.

[4] Adjustment of the sampler timing at the peak is actually based upon the overall magnitude from the correlation processor during signal acquisition.

This is $4e^{j\phi}$ for M=N, and 0 for M≠N. The correlator channel interference components are

$$I_M = A\frac{\sin(\frac{\Delta\omega T_c}{2})}{\frac{\Delta\omega T_c}{2}}e^{j\theta}\sum_{m=0}^{3}C_m W_{Mm}e^{j\Delta\omega m T_c} \qquad M = 0,1,2,3$$

The indicated DFT sum will provide the performance for interference at any specific Δω offset from the signal carrier.

Center-Frequency Interference

First, we shall find the error rate for Δω=0 (hence,|H|=1).

$$I_M = Ae^{j\theta}\sum_{m=0}^{3}C_m W_{Mm} \qquad M = 0,1,2,3$$

The summation yields

$$\begin{aligned}\sum_{m=0}^{3}C_m W_{Mm} &= 2 \qquad M = 0\\ &= 2 \qquad M = 1\\ &= 2 \qquad M = 2\\ &= -2 \quad M = 3\end{aligned}$$

Since the magnitude of the interference output is the same for all channels (in the case of Δω=0), and since both the signal and interference have independent unknown phases, the error is independent of which channel corresponds to the signal. An error will occur whenever

$$| 4e^{j\phi} + 2Ae^{j\theta} | \le 2A$$

since this will cause selection of the wrong channel. We substitute Θ=θ-φ, and solve[5] the inequality in Θ.

$$1 + \frac{1}{4}A^2 + A\cos(\Theta) \le \frac{1}{4}A^2$$

The boundary Θ_c may be written

$$\cos(\Theta_c) \le \frac{-1}{A}$$

The bound cannot be reached by cos(Θ) for A<1,at which point there is no probability of error. For A > 1 we find the boundary phases to be

$$\Theta_B = \pm\cos^{-1}(\frac{-1}{A})$$

The net unknown phase Θ has a uniform distribution over 2π radians; the probability of symbol error for demodulation is the probability that Θ falls into the range for which cos(Θ) exceeds the upper bound, or

$$P_e(A) = \frac{1}{\pi}\left[\pi - \cos^{-1}(\frac{-1}{A})\right] = \frac{1}{\pi}\left[\cos^{-1}(\frac{1}{A})\right]$$

For the case of sinusoidal interference, the input signal-to-interference ratio is just $\gamma = A^{-2}$. Thus, we may write the probability of error as a function of signal-to-interference ratio as

$$P_e(\gamma) = \frac{1}{\pi}\cos^{-1}(\sqrt{\gamma})$$

$$P_e(\gamma) = \frac{N_c - 1}{2} e^{-\frac{N_c \gamma}{2}} = \frac{3}{2} e^{-2\gamma}$$

The probability of symbol error for Gaussian noise (applicable to range calculations) is obtained using the union bound

In Figure 4 We plot $P_e(\gamma)$ vs. γ to compare the signal-to-noise ratio required according to the standard (Gaussian-noise) formula to that actually required for CW interference. At 2×10^{-7} P_e Gaussian noise requires 9 dB more input SNR than does CW interference. We compute the processing gain to be

$$G_P = 9 - 0 + 2 = 11 \quad dB$$

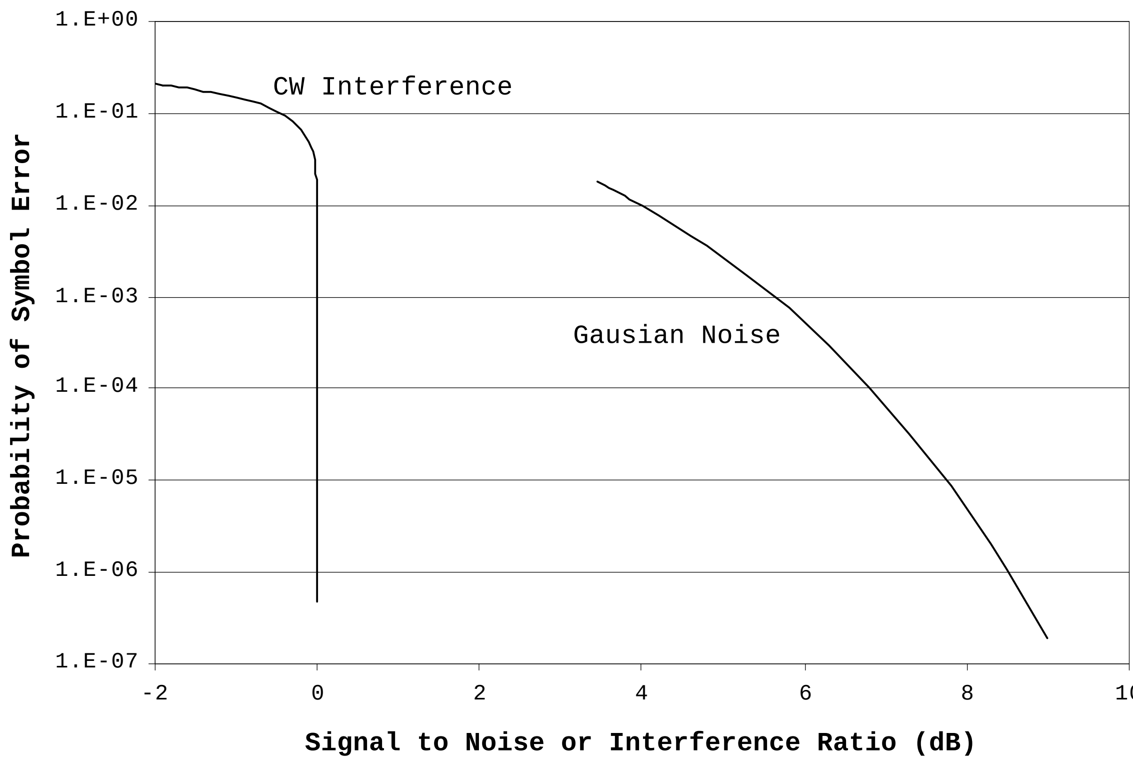


Figure 4 - Comparison of input SNR required for Gaussian vs. center-band CW interference.

[5] If the correct channel corresponds to that for which the interference produces a –2 output, then the phase Q also includes this minus sign; thus, A can be taken as greater than zero.

Off-Center-Frequency Interference

Now, we find the error rate for $\Delta\omega \neq 0$.

$$I_M = A \frac{\sin(\frac{\Delta\omega T_c}{2})}{\frac{\Delta\omega T_c}{2}} e^{j\theta} \sum_{m=0}^{3} C_m W_{Mm} e^{j\Delta\omega m T_c} \qquad M = 0,1,2,3$$

We define four functions $F_M(\Delta\omega T_c)$ which represent the summation in the above expression. We plot these functions in Figure 5.

$$F_M(\Delta\omega T_c) = \left| \sum_{m=0}^{3} C_m W_{Mm} e^{j\Delta\omega m T_c} \right| \qquad M = 0,1,2,3$$

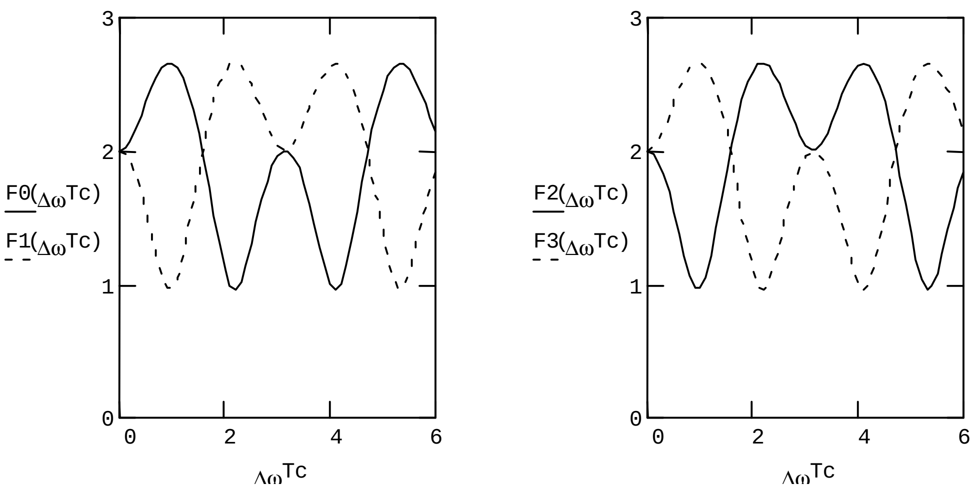


Figure 5 - Correlator interference outputs vs. frequency offset.

We see that two functions initially increase while the other two decrease, then later these functions change role in various ways, always at magnitude 2. At each frequency there is a maximum and minimum value the interference can produce, depending upon which filter is considered. There are two distinct cases to average together in computing the probability of symbol error: either the transmitted symbol waveform corresponds to one of the outputs that will produce an interference maximum, or it will correspond to one which produces an interference minimum. Since we seek the interference amplitude A at which the probability of error drops abruptly to zero,[6]

$$\left| 4e^{j\phi} + F_M(\Delta\omega T_c)_{MAX} A \frac{\sin(\frac{\Delta\omega T_c}{2})}{\frac{\Delta\omega T_c}{2}} e^{j\theta} \right| \leq F_M(\Delta\omega T_c)_{MAX} A \left| H(\Delta\omega) \right|$$

only the first case is important. Proceeding as before, an error will occur for

[6] This determines the J/S value.

Solving as before for the condition on the net unknown phase Θ

$$\cos(\Theta) \le \frac{-2}{F_M(\Delta w T_c)_{MAX}\; A \frac{\sin(\frac{\Delta w T_c}{2})}{\frac{\Delta w T_c}{2}}}$$

which can have no solution for

$$A < \frac{2}{F_M(\Delta w T_c)_{MAX}\; \frac{\sin(\frac{\Delta w T_c}{2})}{\frac{\Delta w T_c}{2}}}$$

This value of A corresponds to the threshold where P_e drops abruptly to zero (which is also the J/S threshold); since A^2 is the reciprocal of the SNR this value of A^2 must also be the J/S value appropriate to the computation of the processing gain.

Figure 6 shows the J/S ratio in dB vs. interference offset frequency out to the first null of the PSK spectrum. The center-frequency processing gain computed to be 11 dB (using 2 dB of Lsys); therefore, only frequencies for which J/S<-1 dB would fail the PG test. If we define the passband as frequencies for which ΔωTc<4.5, then we may exclude the range .9< ΔωTc<1.3, for which J/S drops below –1 dB, in order to meet FCC requirements.[7]

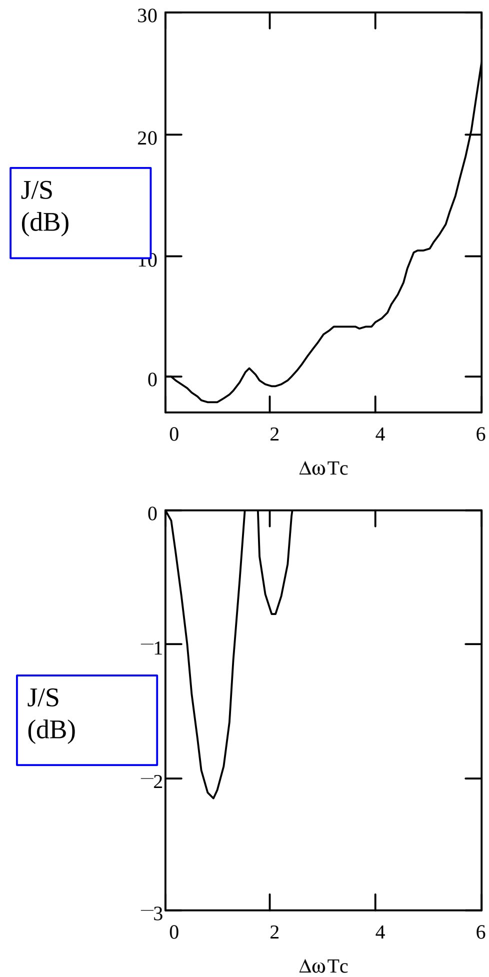


Figure 6 - J/S Ratio vs. Offset Frequency

[7] The processing gain test allows exclusion of the worst 20% of the measured values.

A3: Simple DPSK

We consider a simple DPSK signalling technique

$$x(t) = e^{j\omega_0 t} \sum_m d_m p(t - mT_B)$$

where

ω_0 is the (radian) carrier frequency;
$d_m=\{\pm 1\}$ carries the DPSK information;
p(t) is the DPSK pulse waveform =1 for $0<t<T_B$, 0 elsewhere;
T_B is the bit duration.

In transmission the signal is imparted a random carrier phase. At the receiver, the aggregate received signal and interference are passed through a bit matched filter[8], downconverted to form a baseband complex(in-phase and quadrature) signal, and sampled at the peak correlation.[9] The resulting baseband signal plus interference sequence is

$$r_m = d_m e^{j\phi} + A \frac{\sin(\frac{\Delta\omega T_B}{2})}{\frac{\Delta\omega T_B}{2}} e^{j(\Delta\omega m T_B + \theta)}$$

where

ϕ is the unknown phase of the signal carrier;
A is the interference amplitude;
$\mathrm{Sin}(\Delta\omega T_B/2)/(\Delta\omega T_B/2)$ is the shaping of the bit matched filter;
$\Delta\omega$ is the interference offset (radian) frequency; and,
θ is the interference phase.

The DPSK test statistic on the m[th] bit decision is

$$r_m r_{m-1}^* = \left(d_m e^{j\phi} + A \frac{\sin(\frac{\Delta\omega T_B}{2})}{\frac{\Delta\omega T_B}{2}} e^{j(\Delta\omega m T_B + \theta)} \right) \left(d_{m-1} e^{-j\phi} + A \frac{\sin(\frac{\Delta\omega T_B}{2})}{\frac{\Delta\omega T_B}{2}} e^{-j(\Delta\omega (m-1) T_B + \theta)} \right)$$

The sin(x)/x factor multiplying the interference amplitude indicates that the bit matched filter makes center-frequency interference the worst-case situation. Thus, we will evaluate the effect of interference for $\Delta\omega=0$, knowing that the situation will be better across the frequency band.

$$r_m r_{m-1}^* \Big|_{\Delta\omega=0} = \left(d_m e^{j\phi} + A e^{j\theta}\right)\left(d_{m-1} e^{-j\phi} + A e^{-j\theta}\right)$$

The DPSK decision is based upon the real part of the test statistic[10]

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[8] It is not required that the received filter actually match the chip waveform exactly; the present formulation is used for simplicity.
[9] Adjustment of the sampler timing at the peak is actually established during signal acquisition.
[10] This assignment is arbitrary, but commonly the presence of a phase flip is taken as data "one."

$$
\begin{aligned}
data &= 0 \quad \mathrm{Re}\{r_m r^*_{m-1}\} > 0 \\
&= 1 \quad \mathrm{Re}\{r_m r^*_{m-1}\} < 0
\end{aligned}
$$

In the presence of center-frequency interference this test becomes whether

$$\mathrm{Re}\{r_m r^*_{m-1}\big|_{\Delta\omega=0}\} = d_m d_{m-1} + A^2 + A(d_m + d_{m-1})\cos(\phi - \theta)$$

is greater or less than zero. There are two possible errors, mistaking a zero for a one, and mistaking a one for a zero. The corresponding probabilities, defining the relative phase $\Theta \equiv \phi - \theta$,

$$P(1|0) = P(1 + A^2 \pm 2A\cos(\Theta) < 0)$$

$$P(0|1) = P(-1 + A^2 > 0)$$

The $\pm$ in P(1|0) depends upon whether d_m and d_{m-1} are both positive or both negative; however, this is not an issue because Θ must be averaged over range 2π. Thus, in the following we may use the minus sign and take $\Theta=0$ to mean that relative phase at which the minimum of $1+A^2-2A\cos(\Theta)$ occurs.

It is clear that P(0|1) is zero for A<1 and unity for A>1. It is also clear that P(1|0) is zero for A<1. For A>1, P(1|0) is just the fraction of 2π for which the corresponding error inequality holds; however because 1+A2-2A=0 has a double root at A=1, there is actually no range of Θ for which the inequality for P(1|0) holds.[11] Assuming the a priori data probabilities to be ½ each for 1 and 0, the probability of DPSK error for interference becomes

$$
\begin{aligned}
P_{EI}(A) &= 0 \quad A < 0 \\
&= \frac{1}{2} \quad A > 1
\end{aligned}
$$

The signal-to-interference ratio is simply $\gamma = A^{-2}$. Thus, the probability of error becomes

$$
\begin{aligned}
P_{EI}(A) &= 0 \quad \gamma > 0 \\
&= \frac{1}{2} \quad \gamma < 1
\end{aligned}
$$

For comparison, the probability of DPSK error for Gaussian noise (i.e., receiver noise) is

$$P_{EN}(\gamma) = \frac{1}{2} e^{-\gamma}$$

These two probabilities of error are compared in Figure 7.

[11] Equality is satisfied at the point A=1, Θ=0.

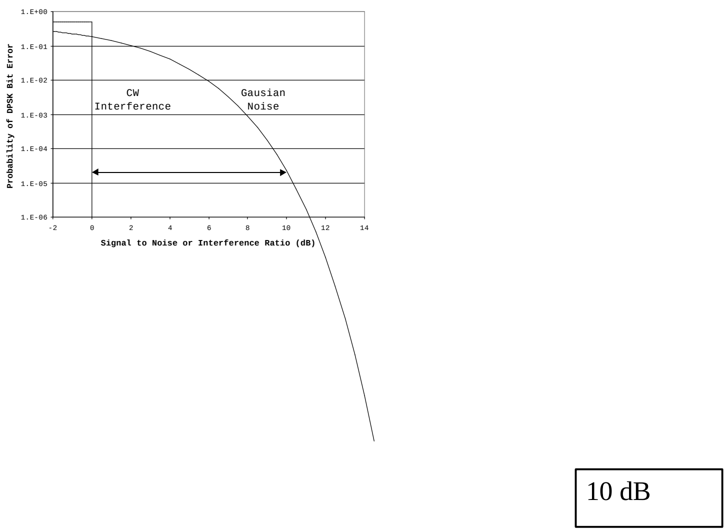

Figure 7 - Comparison of SNR required for Gaussian vs. center-band CW interference for DPSK signaling.

It is clear from this graph that any system design requiring an error probability lower than approximately 10^{-5} will pass the PG test without requiring any Lsys to be claimed because the jamming threshold is 0 dB and the required SNR is greater than 10dB.