Submission
IEEE P802.11
Wireless LANs

Title: **General Partial-Response Pulse Shape Recommendation**

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Summary

This paper is offered to the 5 GHz PHY subgroup of IEEE 802.11 in two parts: 1) "Introduction" and 2) "Mathematical Derivation and Example Result for a General Partial Response Pulse Shape."

The introduction by C. Rypinski describes the relevance and application of this work to the selection of the 802.11 PHY, and briefly reviews some background for theory and application. This minimized spectrum pulse shape is directly applicable to coherent and non-coherent detection and to any amplitude, phase or time coding of information. Once the shape is established at baseband, all remaining steps must be linear. The theory and mathematics are believed to have been in the public domain for decades. Some implementations might have proprietary features.

The "Mathematical Derivation…" is the development of a method for calculating a desirable pulse shape, and it applies the result to a sample pulse shape providing a power density spectrum which would be suitable to 802.11 reducing the amount of bandwidth lost to guard bands at the edge of the allocation and fir getting a larger proportion of a channel defined at the 27 down points for data.

Part I Introduction—C. A. Rypinski

A recommended definition of pulse shape is given, which if applied to any or all of the modulations proposed to IEEE 802.11 will result in systems meeting the required out-of-band emissions requirements with reduced analog filtering.

The method of computation used to obtain the recommended pulse shape has a long history.[1] In the last few years, the implementations of designs, using this mathematical approach to digital pulse shaping, have become increasingly frequent. The hardware implementation feasibility and economy was shown in a presentation to IEEE 802.9 in 1990.[2] My Company no longer thinks it appropriate to hold the detail of the calculation as a trade secret. The method has appeared in contributions to 802.14 and is believed to be used in the NEC 802.11 PHY proposal (98/34, 98/35) and possibly others. Some implementation details have appeared in patents.[3]

The initial pulse shape picked in 1989 for IEEE 802.9 was the 2/3rds compressed duobinary ***(2T/3)^sin(3pt/2T) / p t((2T/3)-t)*** rather than the raised cosine now commonly used. There is probably little difference in the end result after filtering. The choices for window size and sample frequency are also important. The values used in the example are reasonable, but not necessarily the best choices.

There may be Committee members who do not realize the power and importance of this technique, or think that it is so expensive to implement that it need not be considered. The cost of ignoring this possibility is high. In the alternative.the cost of analog filters and additional signal processing that are required will be much more. The loss of eye opening from intersymbol intererence is also significant showing as decreased tolerance to noise and interference. As compared with GMSK the potential bandwidth required for a given data rate will be much smaller. The realized improvement can be greater than the benefits of FEC, diversity or low-degree spreading in systems with bit-at-a-time out-of-band emissions filtering.

Implementation in hardware could require a shift register of the required speed for one cell per coefficient (32 in the example) and a resistor value representing the magnitude of that coefficient. It can also be implemented in DSP with a D/A converter and coefficient ROM, however this might use more power.

This pulse envelope shape is applicable to almost all modulations. If the first pulse has a defined and suitable spectrum, others may follow that are altered in polarity or amplitude. Assuming linearity, the spectrum is the same as the elementary pulse.

The selection of this pulse shape does not contain the issue of coherent vs. non-coherent modulation nor does it favor any of the proposed modulations.

It does require linear tranfer of the baseband envelope shape to rf. It does not require linear rf amplifiers as does ODFM or non-offset QPSK. The designs employed by my Company do not use linear amplification even though the detail of the envelope shape is preserved. In this case, the potential backoff loss of 7-11 dB of maximum power output from a given transistor is not present.

Attention is directed to Figure 18 in Part II. This is the video modulating wave form for the selected bit sequence. It shows amplitude values slightly exceeding 1 and below 0. These must not be clipped. Figure 20 shows the power density spectrum. The first null for sin(x)/x would be at 1x10^-7; and in this case, the edge of the roll-off is about 70% of this frequency. In addition, the side lobes do not exist as normally seen. The high level of carrier is the result of using simple AM modulation. The same envelope for QPSK would not show the carrier or near carrier frequency components with white bit patterns. The implications on filtering requirements and adjacent channel spacing should be obvious.

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[1] "Handbook of Digital Signal Processing Engineering Applications," Edited by D. F. Elliott, Academic Press Inc., San Diego, 1987, ISBN 0-12-237075-9

[2] "Report on NRZST Line signal Experimental Results," C. A. Rypinski, IEEE 802.9-89/123, and "theory and Practic of NRZST Modulation with Line Compensation at the Transmitter, C. A. Rypinski, IEEE 802.9-90/7

[3] "RF Modem with Improved Binary Transversal Filter," G. L. Somer, U. S. Patent 4,773,082

Part II Mathematical Derivation and Sample Result for a General Partial Response Pulse Shape—J. J. Tavormina

The following derivation and simulation is constructed using Matchcad version 6.0. Mathcad is available from MathSoft, Inc., 101 Main Street Cambridge, Massachusets 02142.

The purpos of the derivation and simulation is to demonstrate the performance on an Amplitude Modulator that employs raised cosine waveform shaping, 6-bit digital quantization of the analog waveform, and digitization at a sampling rate of 4 per symbol period. The parameter values used in the model result in a shaping filter with 32 taps (8 symbols) in length. The data sequence used to demonstrate performance of the modulator is 35 bits in length, and is formed from a random bit pattern.

For the selected parameter values, the performance of the modulator is shown to produce a signal bandwidth at the 26 dB down points that is equal to the reciprocal of the symbol period. At 1.4 times the signal bandwidth, out-of-band emissions are shown to be −50 dB with respect to the AM carrier and −40 dB with repect to the shoulder of the modulated signal. No modulation sidelobes are visible.

For the selected parameter values, out-of-band emissions are limited by the resolution of the analog-to-digital conversion. The simulation indicates that greater suppression of out-of-band emissions may be achieved through use of a higher performance digitizer.

Shaped AM modulation is employed as an example use. The symbol period is given by:

$T := 100 \cdot 10^{-9} \cdot sec$

$\frac{1}{T} = 10 \cdot MHz$

The amplitude s(t) of the unshaped baseband signal is characterized by sharp transitions at symbol (bit) boundaries. Consider a single "on" bit at t=0:

$s(t) := if(t \ge 0, if(t < T, 1, 0), 0)$

$t := -2 \cdot T, -1.9 \cdot T .. 5 \cdot T$

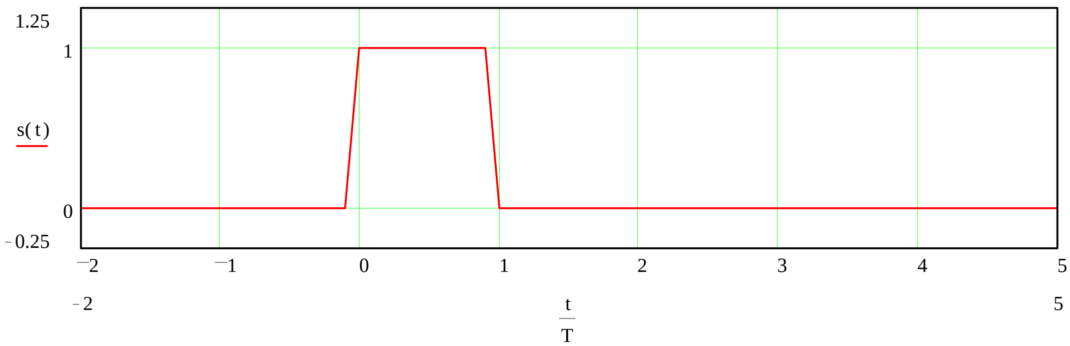

Figure 1

The digitized representation of the baseband signal contains D samples per symbol period:

$D := 4$

$N := 256$

$k := 0, 1 .. N - 1$

$s_k := s\left(\frac{k}{D} \cdot T\right)$

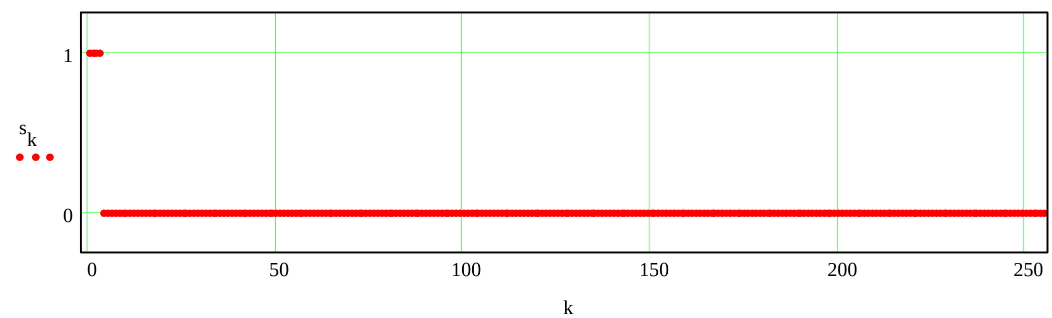

Figure 2

The Fourier components of the digitized signal are given by:

$n := 0, 1 .. \frac{N}{2}$

$S := \frac{N}{D} \cdot FFT(s)$

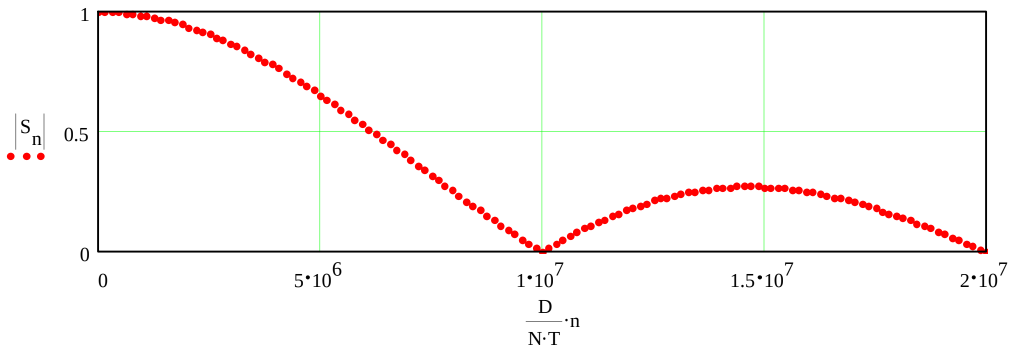

Figure 3

Waveform shaping is used to control the spectral content of the transmitted signal. Industry practice is to apply a square-root raised-cosine pulse shaping filter g(t), with a spectral roll-off factor α. Typical values for α range from 0.2 for 16QAM modulation to 0.5 for QPSK modulation (See DAVIC 1.0 Specification Part 08 Physical Layer Interface for transmission over copper pairs and coax at bandwidths up to 40 MHz). The impulse response g(t) given by:

$\alpha := .36$

$$g(t) := \frac{\sin\left[\frac{\pi\cdot t}{T}\cdot(1-\alpha)\right] + \frac{4\cdot\alpha\cdot t}{T}\cdot\cos\left[\frac{\pi\cdot t}{T}\cdot(1+\alpha)\right]}{\frac{\pi\cdot t}{T}\cdot\left[1-\left(\frac{4\cdot\alpha\cdot t}{T}\right)^2\right]}$$

The singular point at g(0) evaluates to:

$$g_0 := (1-\alpha) + 4\cdot\frac{\alpha}{\pi}$$

$$g_0 = 1.098$$

A graphical representation is shown below:

$t := -5\cdot T, -4.99\cdot T .. 5\cdot T$

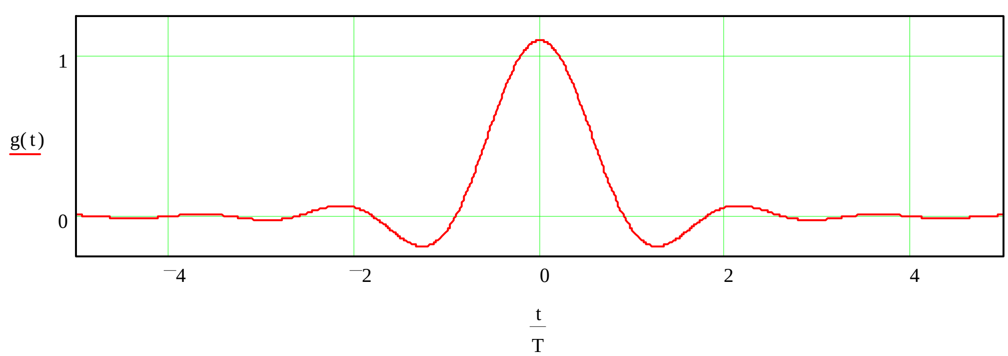

Figure 4

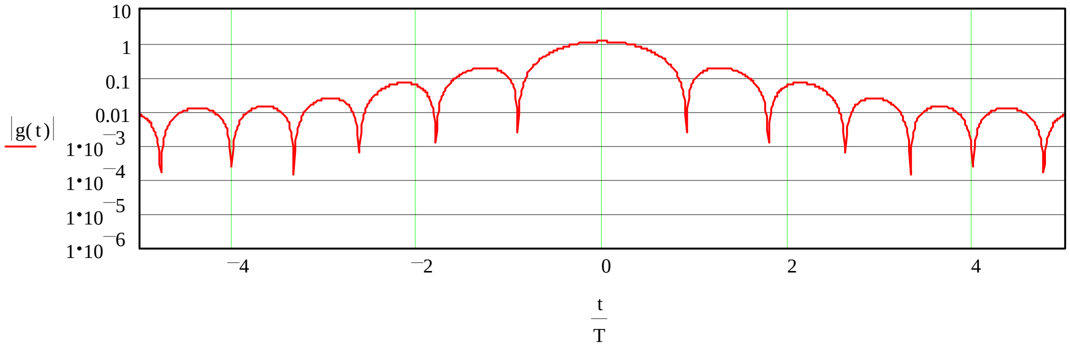

Figure 5

In practice g(t) is implemented as a weighting function in a Finite Impulse Response filter which by necessity is truncated in time. Residual power spectral density is minimized when the points of truncation coincide with zero crossings; the spectral roll-off factor may be chosen to produce zero crossings at these points. For truncation points $\pm t_L$:

$$t_L := 4 \cdot T$$
$$t_R := 0.99 \cdot t_L$$
$$\frac{\mathrm{root}\left(g\left(t_R\right), t_R\right)}{T} = 4.004$$

The Fourier components of g(t) truncated at $\pm t_L$ is calculated below. Because g(t) has a singularity at t=0 the Fourier integral is split into two parts:

$$G(\omega) := \frac{1}{T} \cdot \int_{-t_L}^{-0.0001 \cdot T} g(t) \cdot \exp(-j \cdot \omega \cdot t)\, dt + \frac{1}{T} \cdot \int_{0.0001 \cdot T}^{t_L} g(t) \cdot \exp(-j \cdot \omega \cdot t)\, dt$$

$$\omega := -1.5 \cdot \left(\frac{2 \cdot \pi}{T}\right), -1.48 \cdot \left(\frac{2 \cdot \pi}{T}\right) .. 1.5 \cdot \left(\frac{2 \cdot \pi}{T}\right)$$

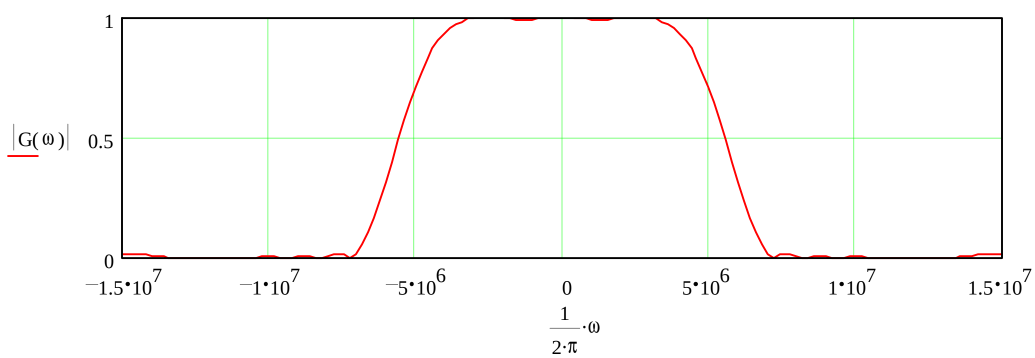

Figure 6

The digitized FIR implementation of the waveform shaping function contains M taps and D samples per symbol period:

$$M := 32$$
$$N := 256$$
$$k := 0, 1 .. N - 1$$
$$g_k := \mathrm{if}\left[k < M, \mathrm{if}\left[\left(k - \frac{M-1}{2}\right) = 0, g_0, g\left[\left(k - \frac{M-1}{2}\right) \cdot \frac{T}{D}\right]\right], 0\right]$$

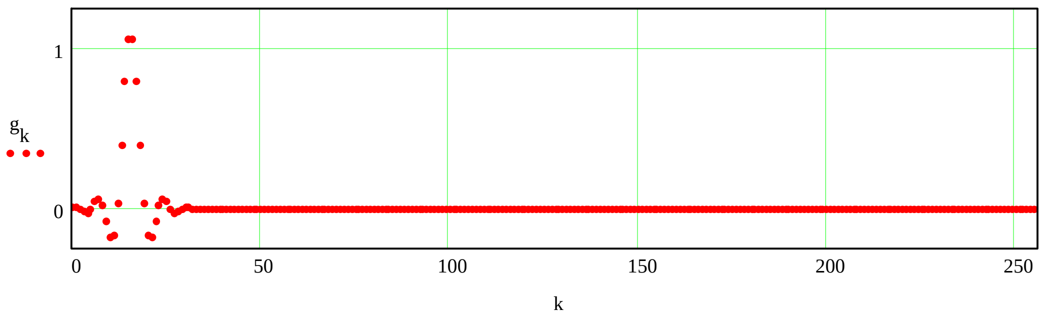

Figure 7

The Fourier components of the digitized shaping function are given by:

$$n := 0, 1 .. \frac{N}{2}$$

$$G := \frac{N}{D} \cdot FFT(g)$$

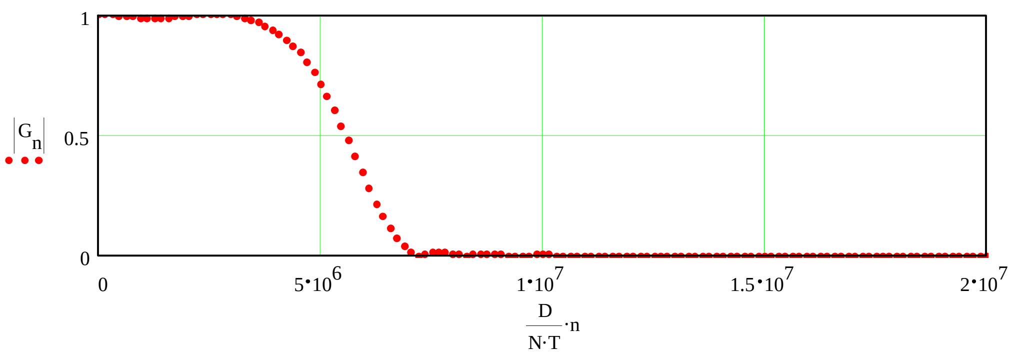

Figure 8

The logarithmic Power Spectral Density of the shaping function is:

$$PSD_n := 10 \cdot \log\left(G_n \cdot \overline{G_n} + 10^{-8}\right)$$

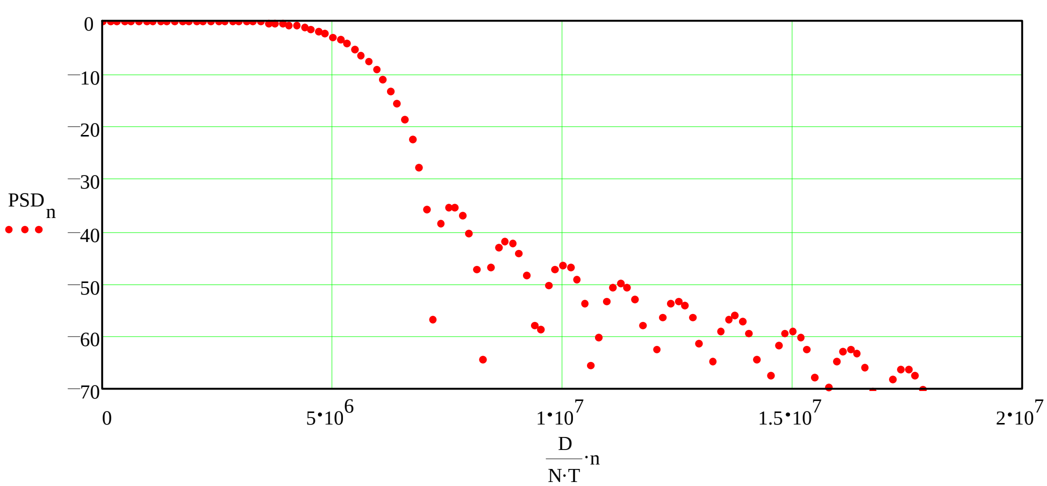

Figure 9

The weighting coefficients g_k are applied to the signal s_k using a tapped delay line:

$$i := 0, 1 .. N - 1$$

$$r_i := \frac{1}{D} \cdot \sum_k s_k \cdot \mathrm{if}\left(k \le i, g_{i-k}, 0\right)$$

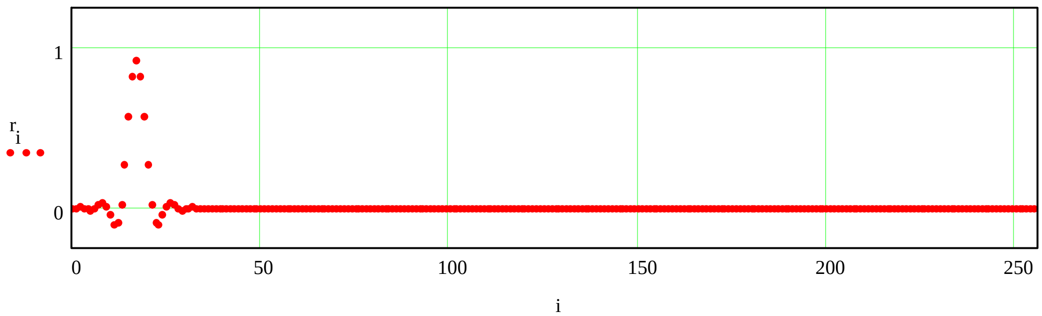

Figure 10

The Power Spectral Density of the shaped digitized signal are shown below:

$$n := 0, 1 .. \frac{N}{2}$$

$$R := \frac{N}{D} \cdot \mathrm{FFT}(r)$$

$$PSD_n := 10 \cdot \log\left(R_n \cdot \overline{R_n} + 10^{-8}\right)$$

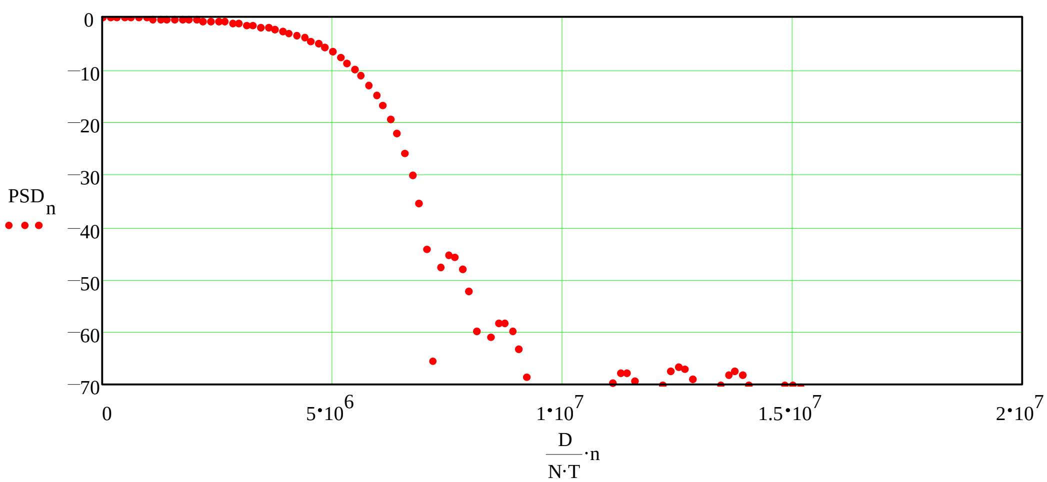

Figure 11

Application of the weighting coefficients g_k to the signal s_k represents convolution in the time domain, which corresponds to multiplication of the spectral components G_n and S_n in the frequency domain. The output spectrum can be computed directly as follows:

$$R_n := G_n \cdot S_n$$

$$PSD_n := 10 \cdot \log\left(R_n \cdot \overline{R_n} + 10^{-8}\right)$$

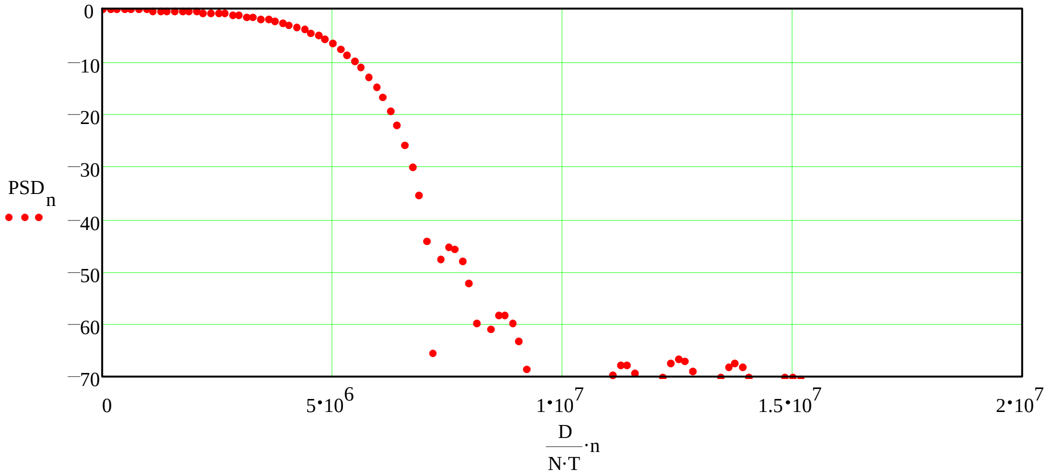

Figure 12

Consider a random 32-bit sequence of digitized data:

$L := 32$

$b := 0, 1 .. L - 1$

$B_b := \mathrm{floor}\left(\mathrm{runif}(L, 0, 2)_b\right)$

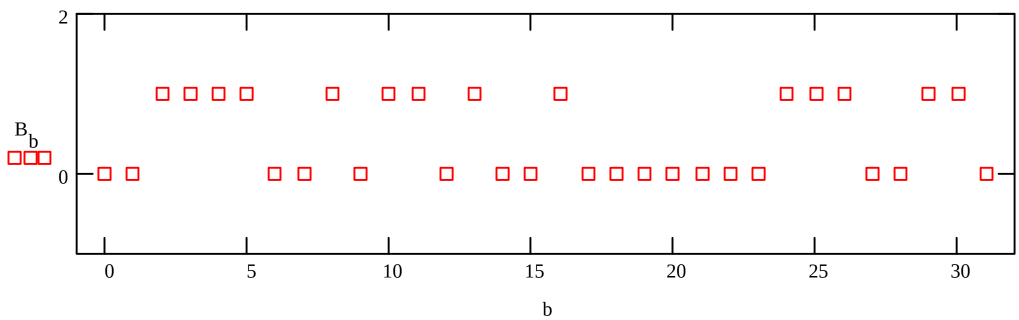

Figure 13

The unshaped amplitude signal that results from the data sequence is characterized by sharp transitions at symbol (bit) boundaries:

$s(t) := \mathrm{if}\left(t \geq 0, \mathrm{if}\left(t < L \cdot T, B_{\mathrm{floor}\left(\frac{t}{T}\right)}, 0\right), 0\right)$

$t := -2 \cdot T, -1.9 \cdot T .. 40 \cdot T$

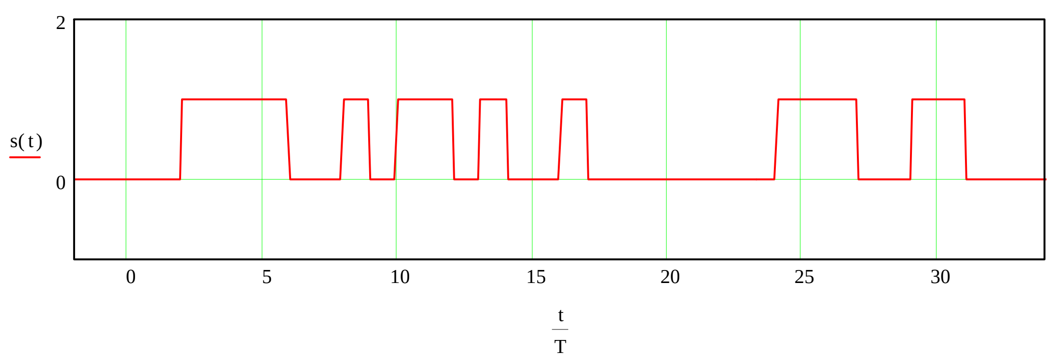

Figure 14

The digitized representation of the signal contains D samples per symbol period:

$N := 256$
$k := 0, 1 .. N - 1$
$s_k := s\left(\frac{k}{D} \cdot T\right)$

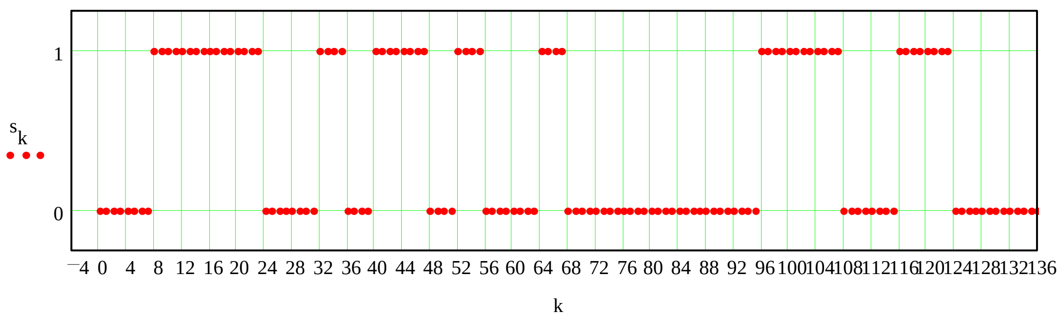

Figure 15

The Power Spectral Density of the unfiltered data sequence is shown below:

$n := 0, 1 .. \frac{N}{2}$

$S := \frac{N}{D \cdot L} \cdot FFT(s)$

$PSD_n := 10 \cdot \log\left(S_n \cdot \overline{S_n} + 10^{-8}\right)$

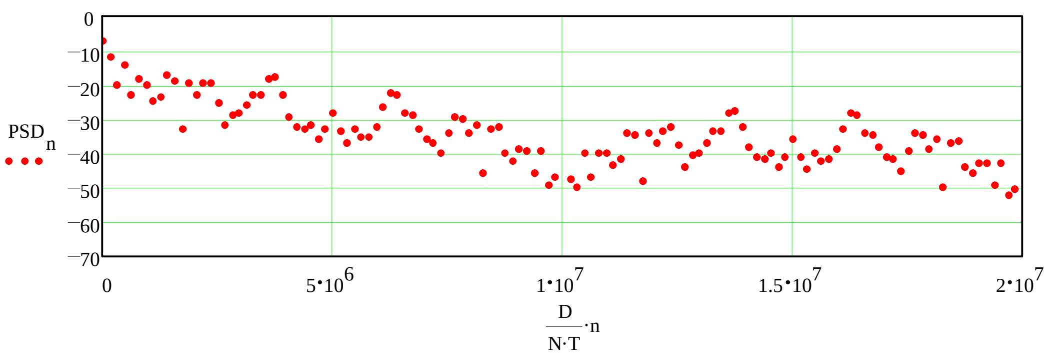

Figure 16

The Power Spectral Density of the filtered data sequence can be computed directly as follows:

$R_n := G_n \cdot S_n$

$PSD_n := 10 \cdot \log\left(R_n \cdot \overline{R_n} + 10^{-8}\right)$

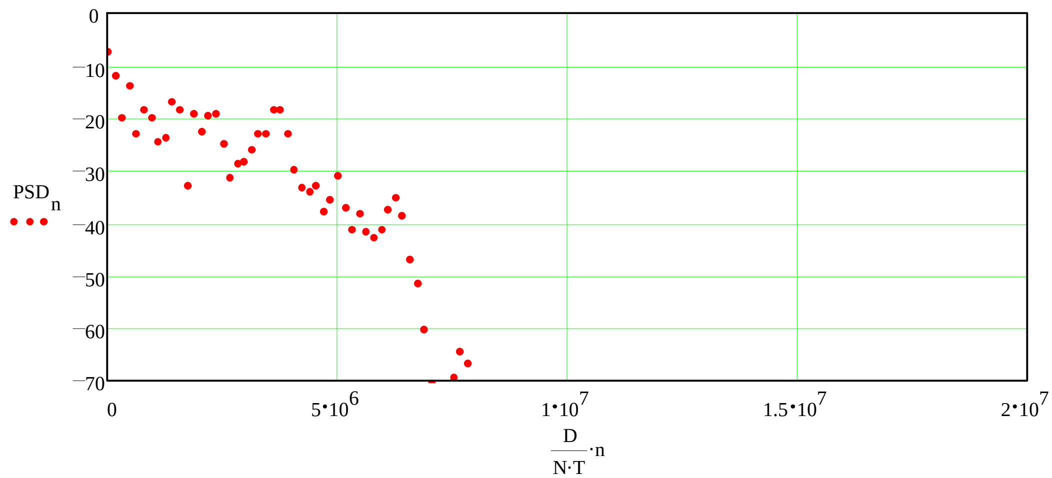

Figure 17

The time domain response of the filtered data sequence is shown below. The response has been adjusted to compensate for the (M+1)/2 sample group delay introduced by the shaping filter:

$$r := \frac{D \cdot L}{N} \cdot \mathrm{IFFT}(R)$$

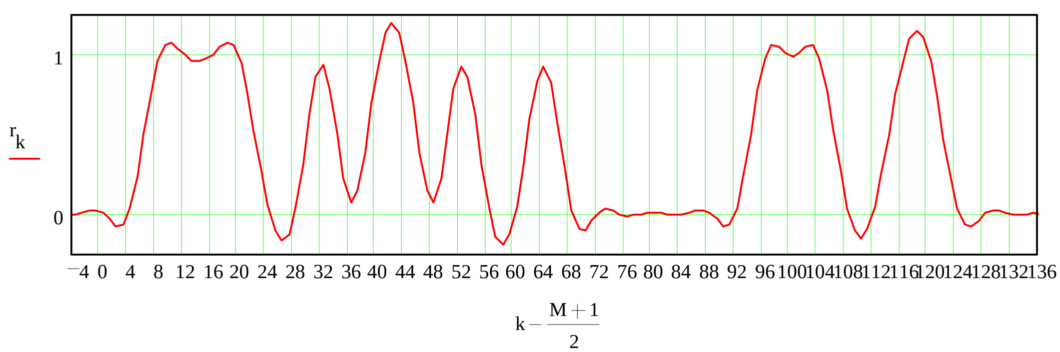

Figure 18

If signal shaping is performed digitally, quantization error will cause the time domain signal at the analog output of the shaping filter to be inexact. An Q-bit digital-to-analog conversion provides 2^Q possible output levels, with the full scale output corresponding to F. Thus, the quantization error results in a systematic roundoff error in the weighting coefficients:

$$\mathrm{quantize}(x, F, Q) := \frac{F}{2^Q} \cdot \mathrm{if}\left(\frac{x}{F} \cdot 2^Q - \mathrm{floor}\left(\frac{x}{F} \cdot 2^Q\right) < 0.5, \mathrm{floor}\left(\frac{x}{F} \cdot 2^Q\right), \mathrm{ceil}\left(\frac{x}{F} \cdot 2^Q\right)\right)$$

$F := 1.2 \cdot g_0$

$F = 1.318$

$Q := 6$

$2^Q = 64$

$$h_k := \mathrm{quantize}\left(r_k, F, Q\right)$$

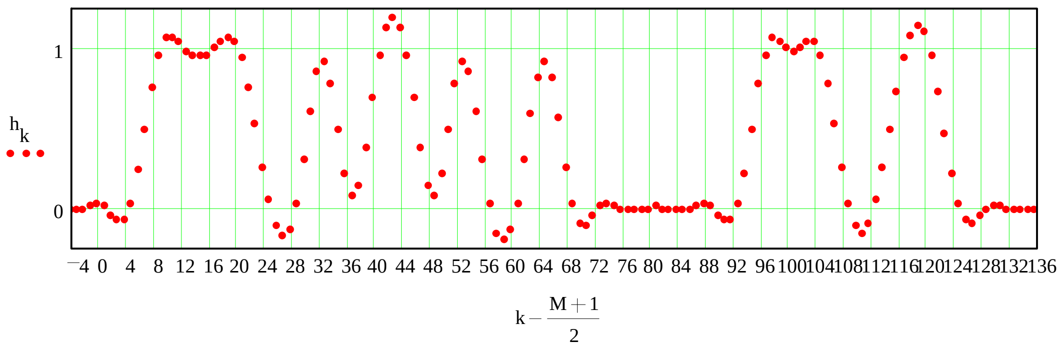

Figure 19

The Power Spectral Density of the quantized shaped signal is:

$$H := \frac{N}{D \cdot L} \cdot FFT(h)$$

$$PSD_n := 10 \cdot \log\left(H_n \cdot \overline{H_n} + 10^{-8}\right)$$

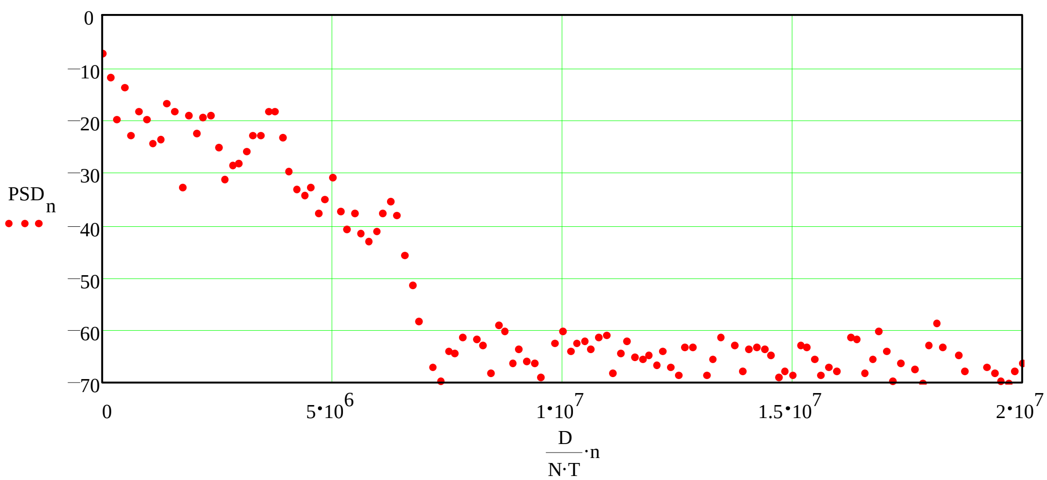

Figure 20