Submission to
IEEE P802.11
Wireless LANs

Title: **Comparing Vector Modulations Using Similarity Transformations**

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Abstract

This submission examines various proposed modulations using linear algebraic concepts. This is possible because the proposed modulations can be viewed as vector transmissions. Vector transmissions have been embraced as a way to meet the FCC's Part 15.247 requirement for the 2.4 GHz PHY. The analysis examines the various proposals from the viewpoint of transmission in an N-dimensional space, where N is equal to the number of chips per symbol. The mathematical properties reveal several candidate modulations which have not been proposed earlier, but have equivalent mathematical performance. It is shown how data rates up to 22 Mbps with 11 Mchips/sec can be sent using an extended Barker set compatible with the current processing gain definition. Finally it is shown that the processing-gain rule-of-thumb "more than 10 chips per symbol" appears to break down under certain circumstances by revealing a paradox.

1. INTRODUCTION

This submission attempts to place the proposed 2.4 GHz PHY high-rate modulations into a common framework. New insights are derived relating the performance of canonical classes of vector modulations.

2. VECTOR MODULATIONS

Due to the FCC's Part 15.247 requirement for 10 dB processing gain imposed upon the ISM band, the proposed 2.4 GHz high-rate modulations all use a spreading scheme. Consequently a transmit symbol is generated as a vector of chips. The chips are transmitted in a time-serial fashion.

A conventional unspread signal is shown in Fig. 2.1. The symbol can use any form a digital modulation. This submission will focus on QPSK symbols, with BPSK used as a fallback rate. QAM can also be used. In all these cases the baseband signal is a complex scalar with amplitude and phase.

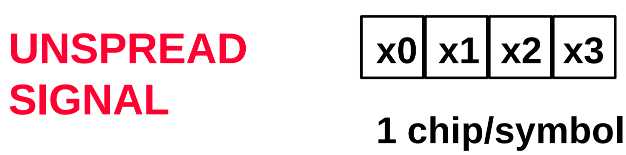


Figure 2.1 Conventional signal viewed as complex scalar elements.

To meet the processing gain requirement, most systems use 10 chips or more per symbol. The logic stated is 10 chips has 10 dB processing gain according to the common equations presented in textbooks on direct sequence spread spectrum (DSSS). The process gain alludes to the robustness to a narrowband jammer. This submission will address the processing gain issue in Section 9.

The conventional signal shown in Fig. 2.1 is shown spread in Fig. 2.2. The individual digital modulation symbol scalars are chopped-up using an extra sequence called the chipping sequence. N chips are used per symbol time. The chipping sequence conventionally does not carry any information data by itself. The chipping sequence is modulated (weighted or multiplied) by the original scalar symbols. Each set of N-chips can be viewed as a vector.

In military DS systems the chips are usually driven by a pseudo-random generator. In certain wireless systems each user is given a unique chipping sequence called a signature sequence. For 1 and 2 Mbps 802.11 DS, the chipping sequence is an 11 bit Barker word. For 1 Mbps the Barker word is modulated with a BPSK scalar. For 2 Mbps the Barker word is modulated with a QPSK scalar. The chipping sequence is fixed and identical for all users. The Barker chipping sequence is modulated by the data modulation: BPSK or QPSK.

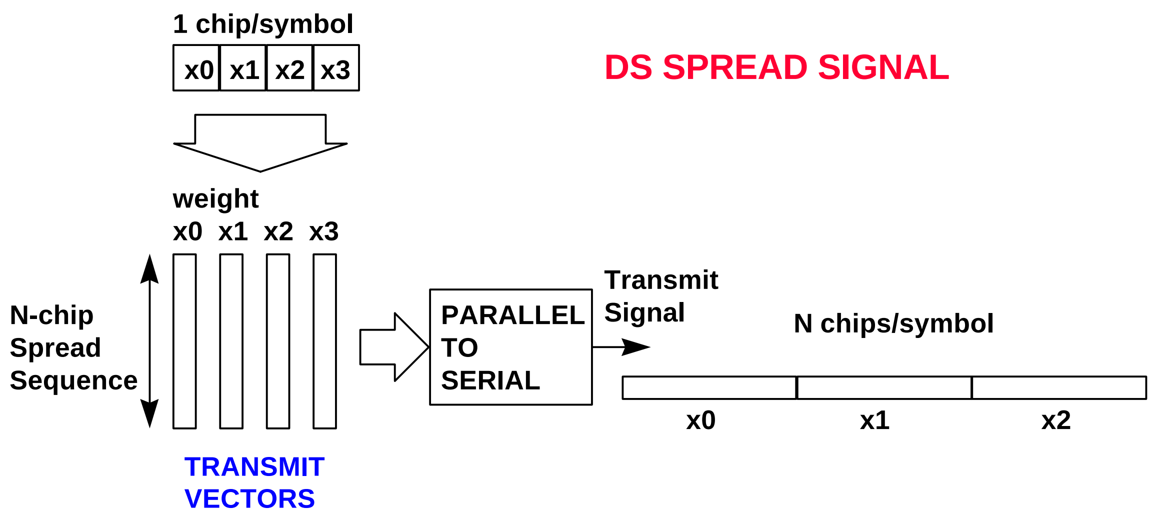


Figure 2.2 Scalar signal expanded (spread) to a vector signal to achieve processing gain.

To carry a higher data rates, the BPSK and QPSK could be extended to a multilevel QAM signal or high-order PSK. The 11 bit Barker words would be modulated by the higher-order modulation. The proposals have shied away from this concept since constant envelops with simple symbol level slicing in the demodulator is preferred for implementation and SNR reasons.

As an alternative scheme, in an effort to carry a high data rate, most of the proposals have decided either to carry information directly on the chipping sequences or to transmit (stack) several orthogonal sequences simultaneously.

CARRY HIGH-ORDER INFORMATION ON CHIP SEQUENCE

To carry information on the chipping sequence several schemes have been floated. Walsh modulation chip sequences are proposed by HARRIS and MICRILOR. HARRIS is placing Walsh sequences on the I or Q channel. MICRILOR is using MSK. LUCENT is proposing using PPM'd Barker chips. These can all be viewed as M-ary orthogonal modulations (MOK). The MOK signals are biphase modulated by all. HARRIS and LUCENT plan to send the respective BMOK chip set in a quadrature fashion to double the data rate. This is BMOK. There is also quadrature MOK or quadrature BMOK. Both MOK and BMOK are commonly described in textbooks. In all cases, the fundament sequence is MOK, so this submission will describe MOK as a canonical vector modulation. The extensions to BMOK and quadrature BMOK are easy.

Although not proposed, it will be shown later that FSK is also a MOK which can be used to form a set of chipping sequences.

M-ary orthogonal modulations viewed as vector modulations is shown in Fig. 2.3. A set of orthogonal chipping sequences are stored as candidates for transmission each vector time (WALSH set, PPM set or FSK set). To push the data rate, one particular orthogonal chip sequence is selected using information bits. In the example, 2 bits of information select one of four orthogonal basis functions. To carry more information, the individual chipping sequence is modulated with a basic digital modulation. The composite modulated chips are transmitted by a parallel-to-serial operation.

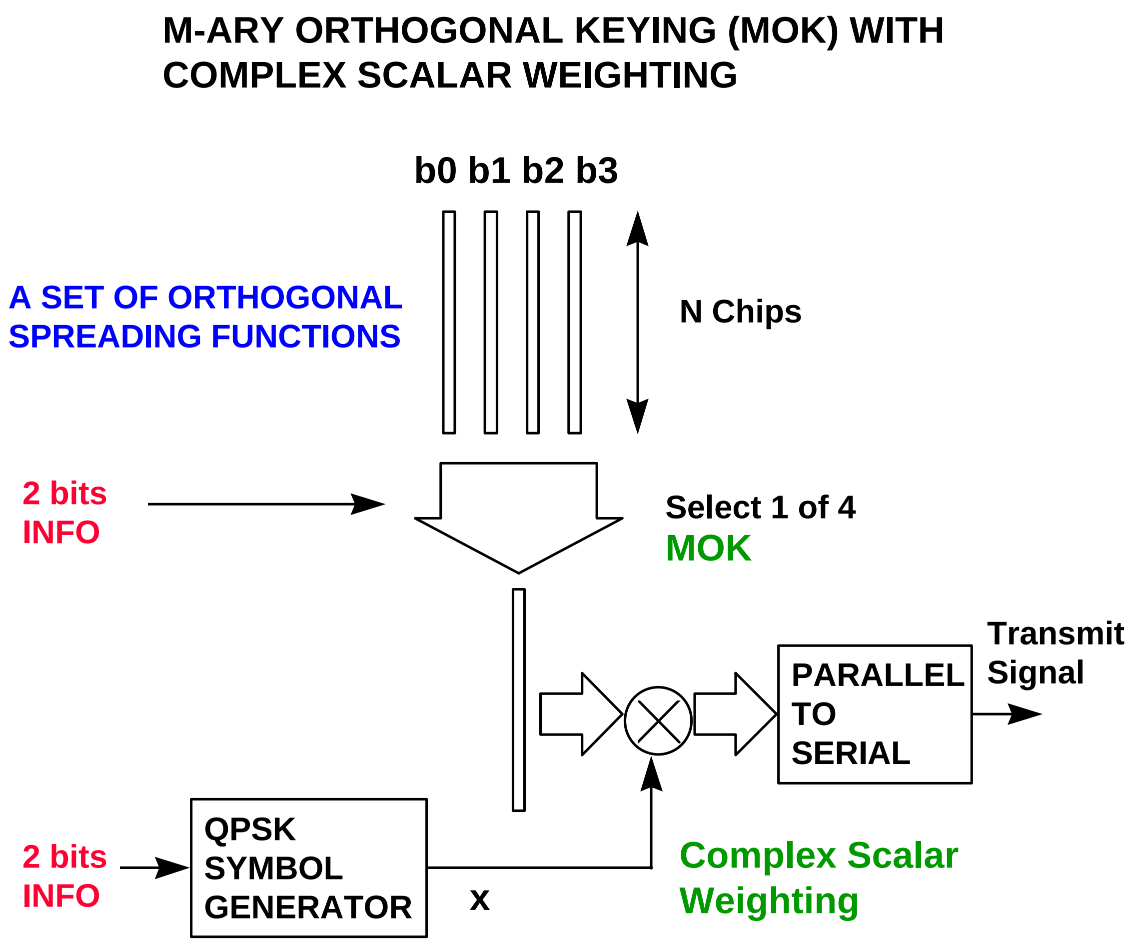


Figure 2.3 An example M-ary orthogonal vector modulator. One orthogonal function is sent at a time.

The MOK chipping sequence can be viewed as a vector in N dimensional space, where N chips are used per vector as shown in Fig. 2.4. Possibly only K < N orthogonal vectors are in the candidate set. It is not necessary for K to equal N. All of the vectors are orthogonal. The transmitted vector randomly appears are each location dependent upon transmit data. The receiver does not know which will arrive, so it must look (correlate) for all of them.

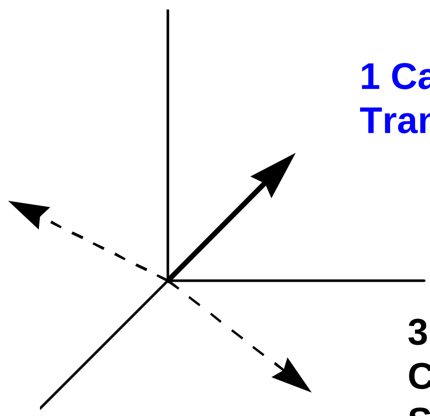


Figure 2.4 All candidate MOK vectors are orthogonal. One is transmitted each vector interval. The particular one is randomly selected by information bits.

STACKING MULTIPLE ORTHOGONAL SEQUENCES

As a second idea for pushing high data rates while maintaining multiple chips per symbol, some proposals are recommended sending more than one orthogonal function at a time. GOLDEN BRIDGE is accomplishing this by using a set of augmented Barker words [4]. KDD is accomplishing this by using multiple carrier offsets for the 11 bit Barker [3]. The carrier offset frequency is equal to the Barker word-rate in MHz to produce decorrelation over the Barker correlation time in the demodulator. A common trait is an increase peak-to-average ratio in the transmit waveform through the sum of multiple vectors.

Although not one of the 2.4 GHz proposals, OFDM also falls in this category. OFDM uses orthogonal complex sinusoidals.

This stacked-MOK canonical form is illustrated in Fig. 2.5. Here a set a orthogonal functions are sent simultaneous. To push even higher data rates even more orthogonal functions can be used. GOLDEN BRIDGE is proposing using a variable number for 1 to 11 orthogonal functions simultaneously, giving a fine resolution on the selectable data rates. Later it will be shown how GOLDEN BRIDGE's orthogonal-set can be extended to 12 through mathematical insights provided by this submission.

In Fig. 2.5 each orthogonal basis is modulated with data in the same fashion as for the 1 and 2 Mbps 802.11 DS specification. Each orthogonal basis is then summed to form the transmit composite.

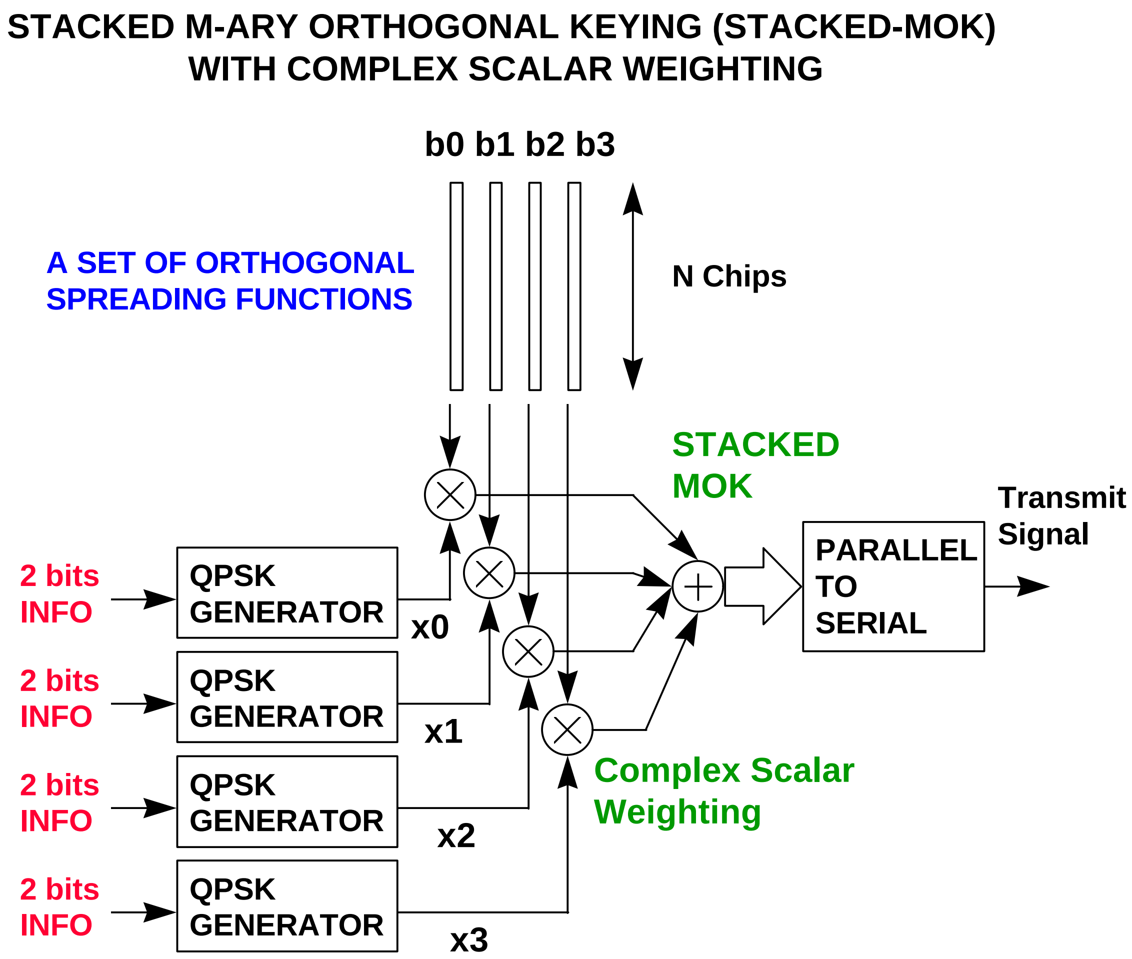


Figure 2.5 An example stacked-MOK vector modulator. A single transmit vector is formed by adding (stacking) several vectors. Two or more orthogonal functions are added to form the composite.

The stacked-MOK chipping sequence can be viewed as a sum of K vectors in N dimensional space, where N chips are used per vector as shown in Fig. 2.6. Possibly there are only $K < N$ orthogonal vectors are in the candidate set. It is not necessary for K to equal N. All of the vectors are orthogonal. The transmitted vector always appears at each basis location independent of the transmit data. The receiver must look at (correlate for) all basis components to recover the transmitted data.

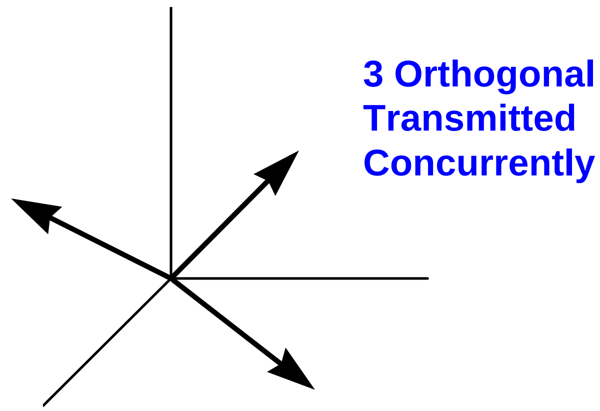


Figure 2.6 All orthogonal basis vectors are sent each vector interval. The signal does not jump around in N-space.

3. VECTOR MODULATORS VIEWED AS LINEAR TRANSFORMATIONS

This section attempts to put the vector modulations of the previous section into a common mathematical setting. Since vectors are part of the theory of linear algebra, the signals will be viewed as linear transformations.

Figure 3.1 shows how the spreading functions for the examples of Fig. 2.3 and Fig. 2.5 can be loaded as columns into a basis matrix **B**. The matrix need not be square. The number of chips may exceed the number of orthogonal vectors. For example, LUCENT uses 11 chips but only 8 positions (11 x 8 matrix dimension). GOLDEN BRIDGE uses 12 chips but a variable number of columns: 1 to 11. KDD uses 5 frequency channels with a frequency-rotated 11 bit Barker in each (11 x 5 matrix).

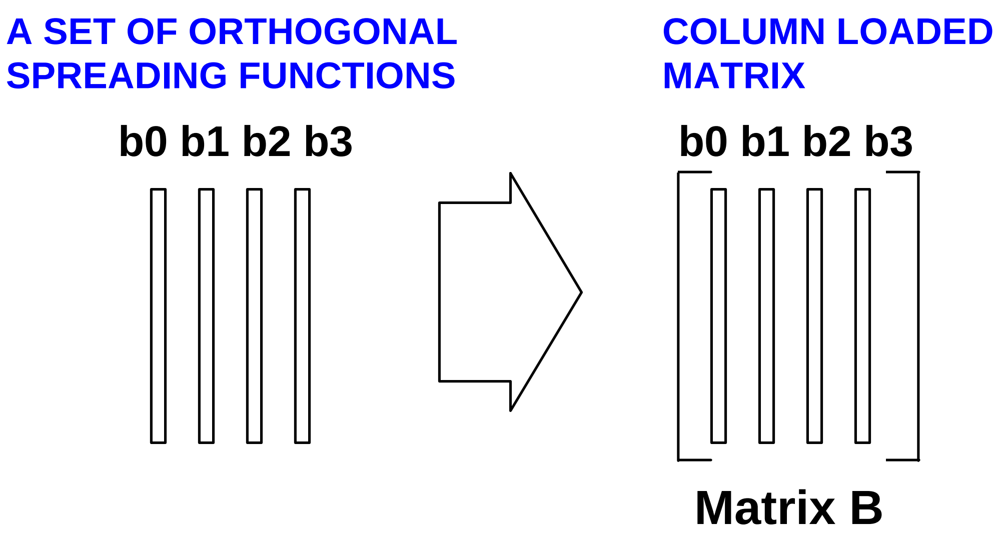


Figure 3.1 For notational and mathematical convenience the spreading functions can be loaded into a matrix. Each column in the matrix is one of the spreading functions. The matrix is frequently tall (more rows than columns), corresponding to more chips/function than number of functions.

It was shown in Fig. 2.5 that stacked MOK summed together multiple orthogonal functions. Each orthogonal function is scalar weighted by a modulation (BPSK or QPSK typically). Fig. 3.2 shows how this operation can be formed using the matrix described in Fig. 3.1. The QPSK-weighting complex-scalars are x0, x1, x2 and x3. These respective complex scalars multiply respective column vectors **b0**, **b1**, **b2** and **b3**. A vector sum forms the composite. This is mathematically equivalent to loading the x0, x1, x2 and x3 complex QPSK scalars in a vector **x**. The product **B x** is now equal to the transmit signal. With stacked-MOK, none of the elements of **x** are zero.

STACKED M-ARY ORTHOGONAL KEYING

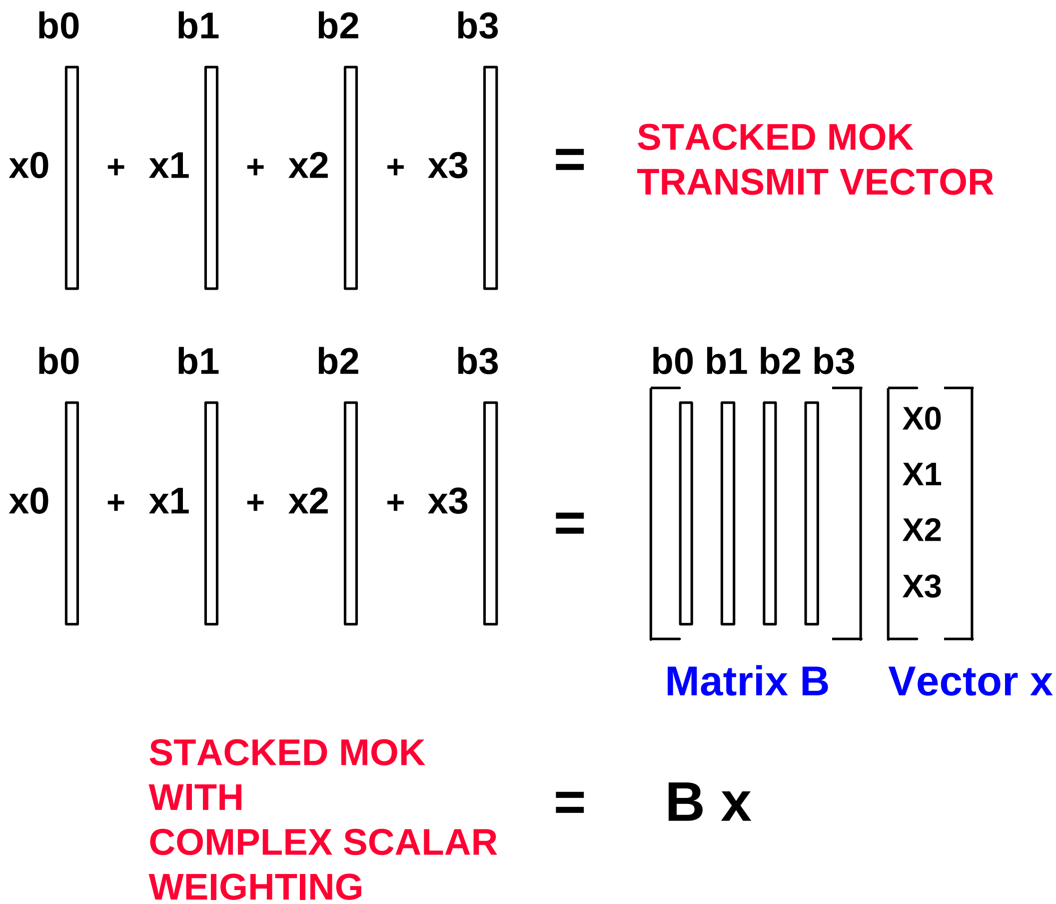


Figure 3.2 Stacked-MOK viewed as a matrix multiply.

The same product **B x** description can be used to formulate MOK. The only difference is all the elements of **x** are zero except for one. Only one orthogonal basis is transmitted at a time. Consequently, only one nonzero element of **x** is used to enable and weight the particular basis.

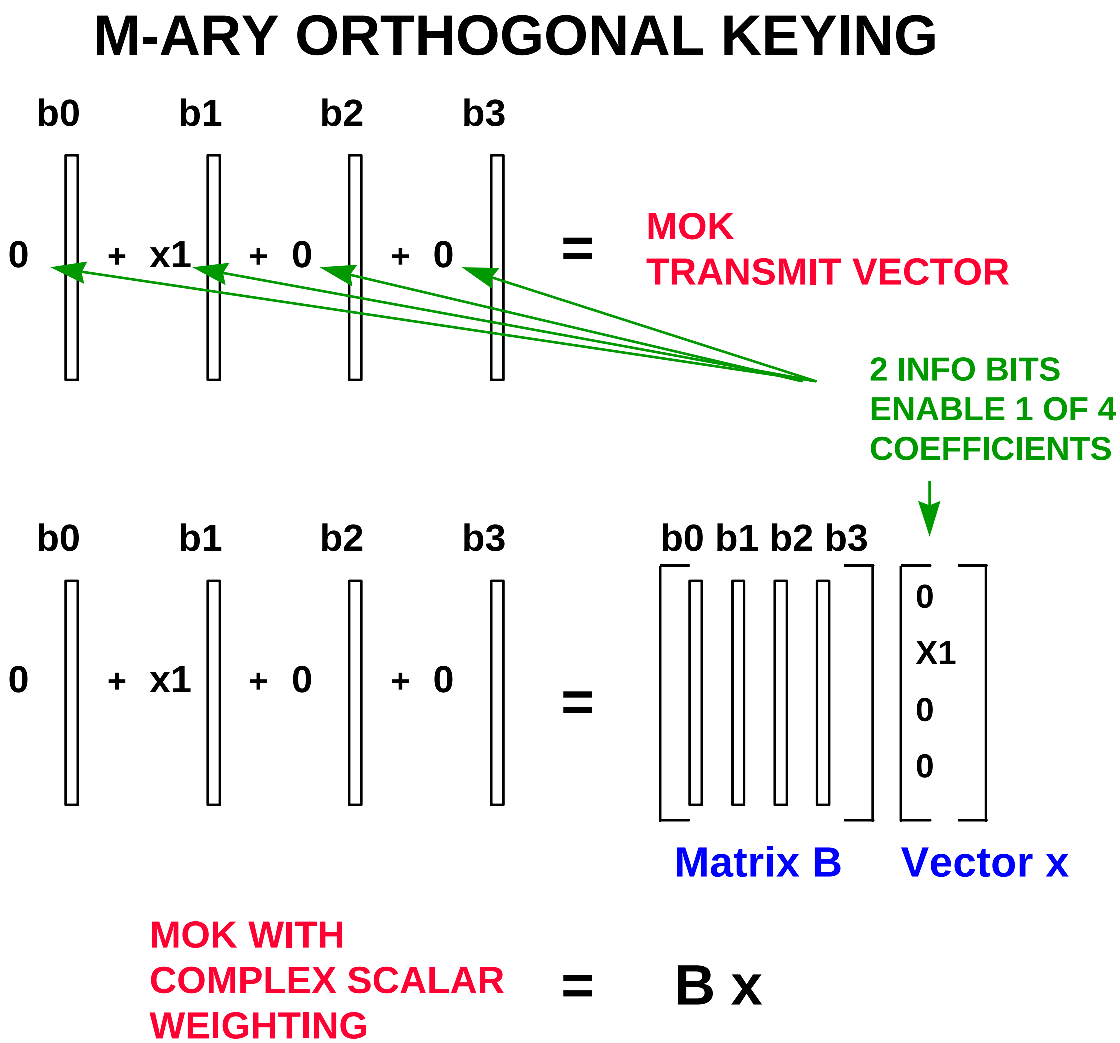


Figure 3.3 MOK viewed as a matrix multiply.

The transmit operation for both MOK and stacked MOK can be viewed as shown in Fig. 3.4. The information bits load the modulation vector **x**. The basis matrix **B** then performs a linear transformation to create the transmit vector. The parallel-to-serial operation converts the vector signal into practical transmit samples. This places vector modulations into a unified mathematical context.

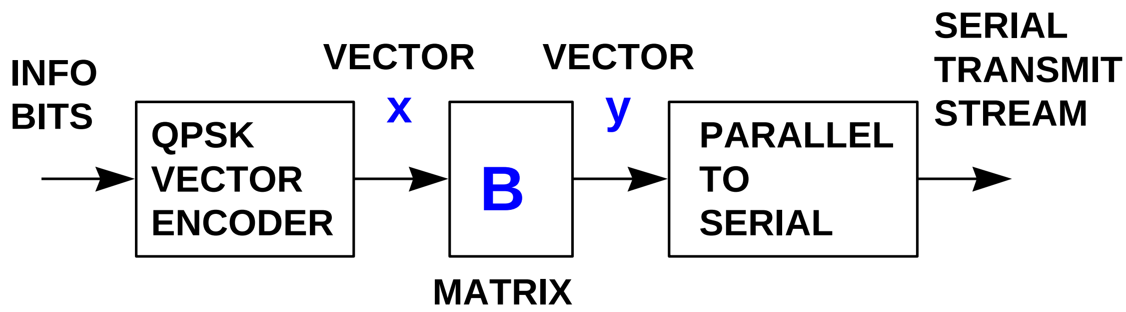


Figure 3.4 Generalized vector modulator.

The transmit basis vectors for MOK are shown in Fig. 3.5. Only one is sent at a time.

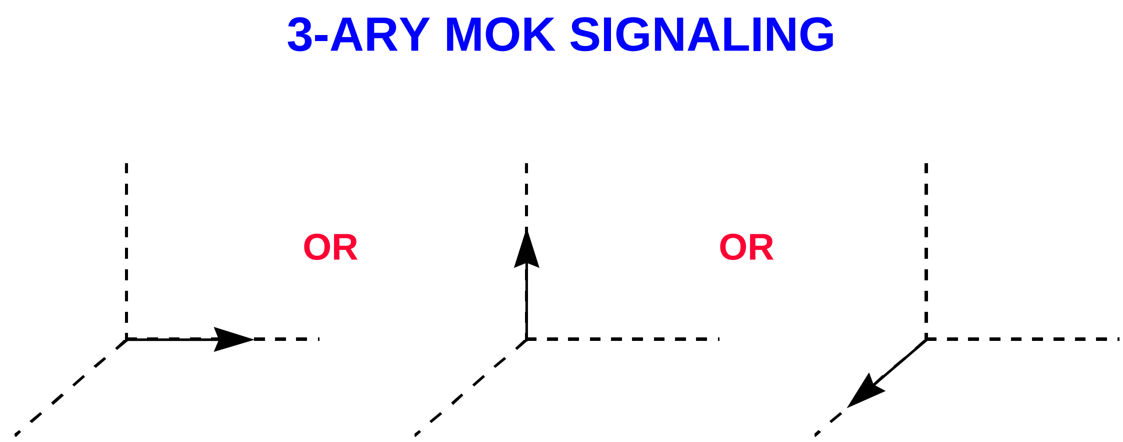


Figure 3.5 MOK transmits only one basis vector at a time.

The transmit basis vector for stacked MOK are shown in Fig. 3.6. All are sent simultaneously. Note that Fig. 3.5 and Fig. 3.6 are very similar. Consequently, the demodulators will be very similar.

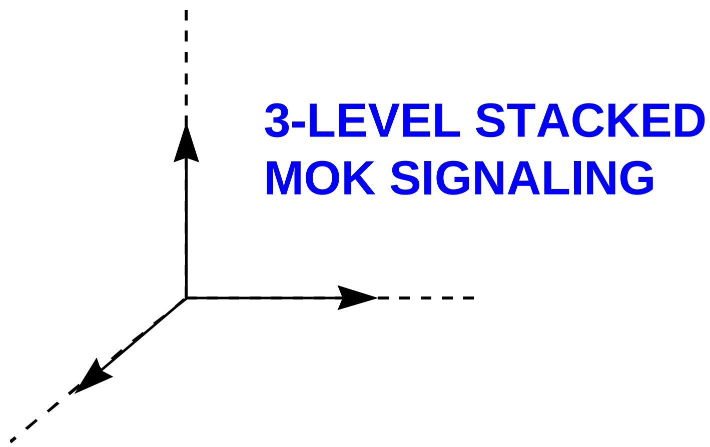


Figure 3.6 Stacked-MOK transmits only all basis vectors at a time.

4. VECTOR DEMODULATION IN AWGN CHANNEL

This section examines the demodulation architectures for vector modulations and places the operations in a linear algebraic context. The additive white Gaussian noise (AWGN) channel will be the focus.

The basic receiver structure is shown in Fig. 4.1. This is the common matched filter or correlation receiver which is described in communications textbooks. This architecture is used by both MOK and stacked MOK. The MOK receiver must decide which one is the active basis and then strip the modulation information off. The stacked-MOK receiver knows that all basis functions are active and strips the modulation information off each.

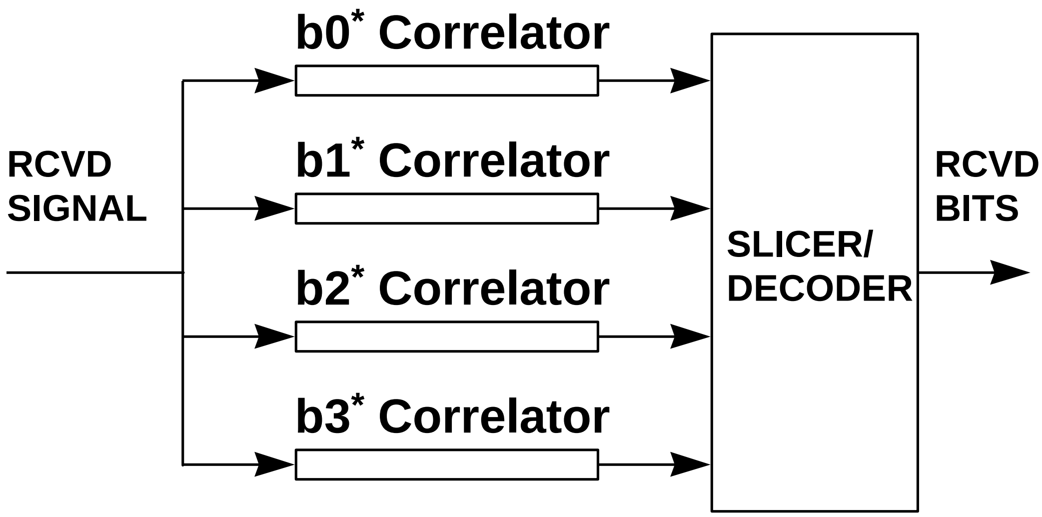


Figure 4.1 The generic demodulation structure for both MOK and stacked MOK.

As with the transmitter, the receiver's correlation vectors can be loaded into a matrix to make mathematical manipulation easy as shown in Fig. 4.2.

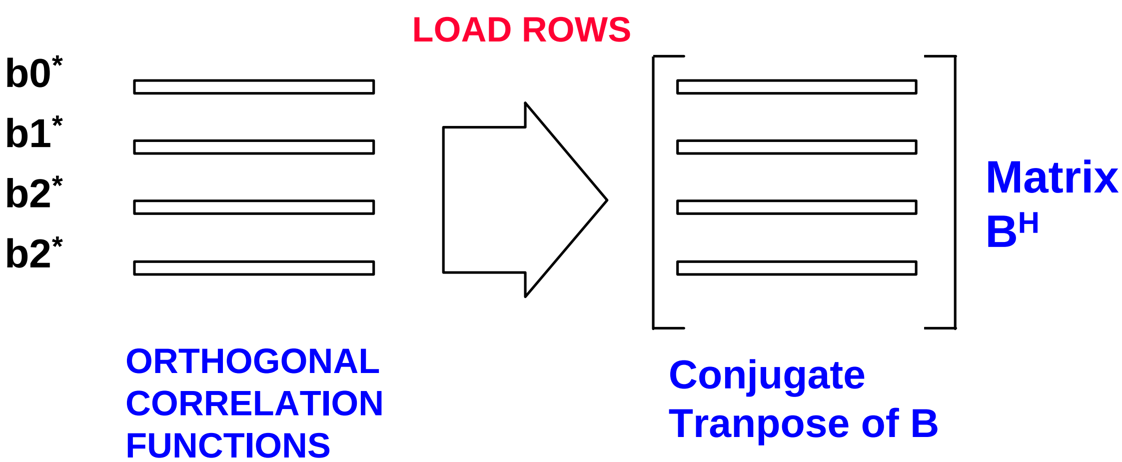


Figure 4.2 Loading the correlation filter chip sequences into the rows of a matrix.

To see how the matrix in Fig. 4.2 relates to the earlier shown transmit frame, let's progress through a set of steps. The natural inverse receive structure for the architectures shown in Fig. 3.4 is shown in Fig. 4.3. If the matrix is not square, the singular value decomposition is equivalently used to define an inverse for the active subspace.

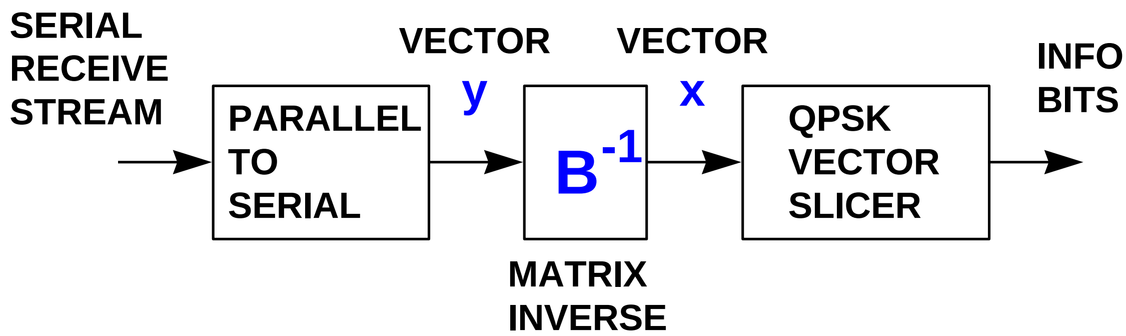


Figure 4.3 A matrix inverse is the natural mathematical operation for reversing the operation of Fig. 3.4.

For orthonormal spreading sequences, the mathematical identities shown in Fig. 4.4 hold. If the sequence is merely orthogonal, it can be made orthonormal via a normalization. All the basis vectors are made unit length in a Euclidean sense. Again, the singular value decomposition holds for a non-square matrix. Only the active subspace is important. For example GOLDEN BRIDGE works in a 12 dimensional space, but maybe only 4 augmented Barkers are used. Here, the active subspace is only of dimension 4.

ORTHONORMAL SPREADING FUNCTIONS

$$B^H B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

IDENTITY MATRIX

$$B^{-1} = B^H$$

INVERSE EQUAL TO CONJUGATE TRANSPOSE

Figure 4.4 The inverse matrix is equal to the correlation matrix for an orthonormal chipping sequences.

Now Fig. 4.3 is redrawn as Fig. 4.4. This is the mathematically derived equivalent to Fig. 4.1, which is described in communications textbooks.

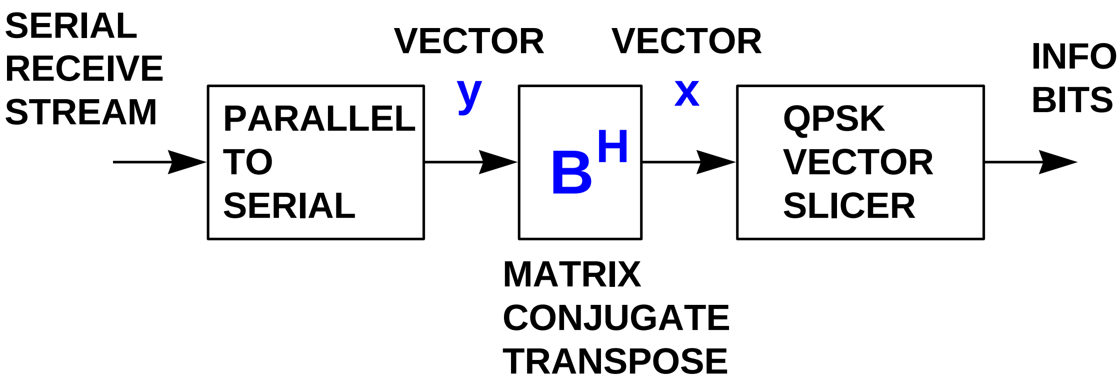


Figure 4.4 The mathematically derived equivalent to Fig. 4.1.

From linear algebra it is known that $\mathbf{B}^{\mathbf{H}}$ is a projection operator. It projects the noisy received signal onto the known transmit subspaces in a minimum-norm sense. In practical terms a projection is made onto to each orthonormal basis vector.

The projection operation can be viewed in Fig. 4.4 and Fig. 4.5. As long as noise does not perturb a transmit basis to a point where it is closer to another basis vector, an error will not be made. The decision regions are the hard slicing regions which determine whether or not a error is made. Crossing a boundary with a noise perturbation causes an error.

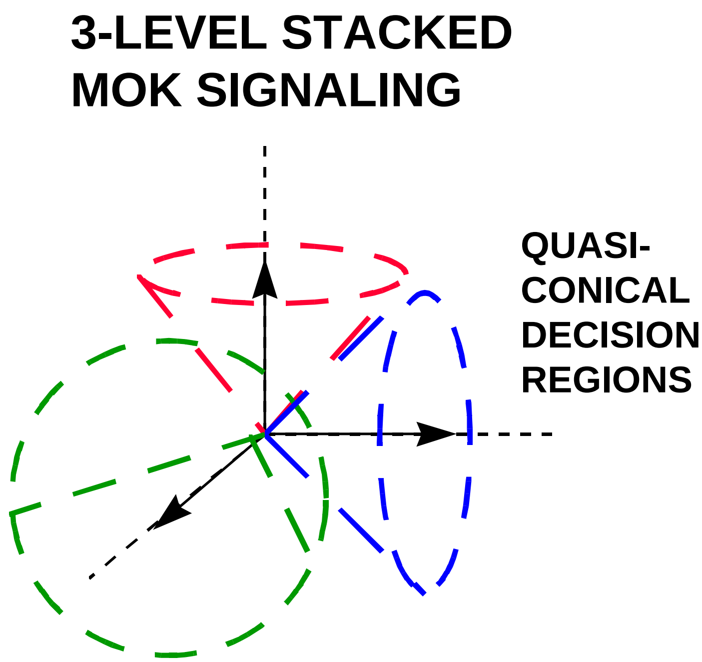


Figure 4.5 Decision regions for stacked MOK.

Note for MOK and the equivalent stacked-MOK, the decision regions are the same. In high SNR conditions the probability of a symbol error for stacked-MOK is just K times the symbol error rate for MOK. This is because the stacked MOK makes a symbol-error if an error is made in the first basis, or the second basis vector, or the third basis, … to the Kth basis.

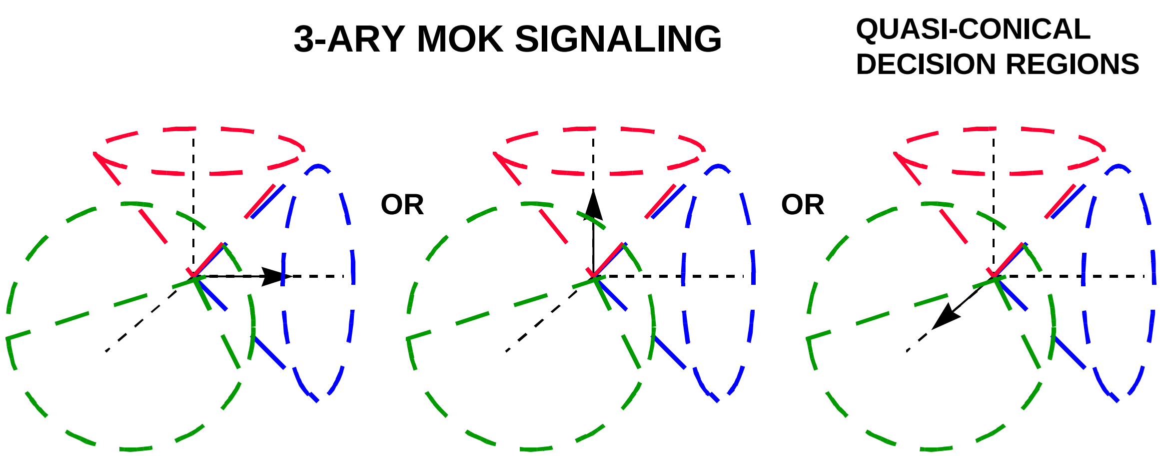


Figure 4.6 Decision regions for MOK.

5. DISTANCE PROPERTIES

This section examines the distance properties associated with the vector modulations. This information will be used later to compare the distance properties of different vector modulation techniques. The distances between constellation points defines the error rate. For an equal amount of noise, comparing two modulations, the modulation with the greater distance between constellation points has the lower error rate.

Fig. 5.1 shows that the transmit vector can be described in terms of orthogonal components. Each orthogonal component is a QPSK modulation scalar-weighted vector. The modulation scalars are x0, x1, x2 and x3. The orthogonal basis vectors are **b0**, **b1**, **b2** and **b3**.

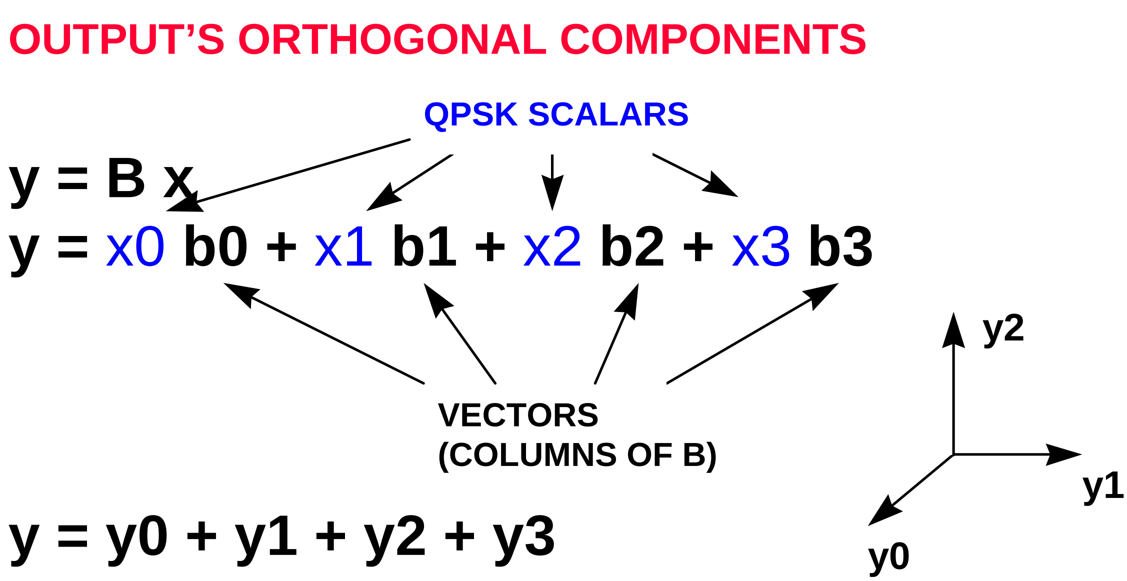


Figure 5.1 The transmit vector decomposed into orthogonal components.

The squared Euclidean distance can be computed as shown in Fig. 5.2. Since each basis vector is orthonormal, they are decoupled and of unit length. The result is very simple. The transmit vector's magnitude is equal to the modulation vector's magnitude.

EUCLIDEAN DISTANCE SQUARED

$$|y|^2 = |x0|^2\,|b0|^2 + |x1|^2\,|b1|^2 + |x2|^2\,|b2|^2 + |x3|^2\,|b3|^2$$

$$|y|^2 = |x0|^2 + |x1|^2 + |x2|^2 + |x3|^2 = |x|^2$$

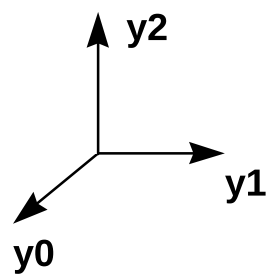
OUTPUT ENERGY = INPUT QPSK VECTOR ENERGY

Figure 5.2 The magnitude-squared of the transmit vector is simply equal to the magnitude squared of the transmit modulation vector **x** for a orthonormal basis matrix **B**.

The distance between constellation points can also be easily computed as shown in Fig. 5.3. The squared distance between constellation points is simply equal to the sum of the squares of the individual modulation components. This is a higher-order Pythagorean theorem. This distance defines error rate robustness.

DISTANCE BETWEEN ORTHOGONAL TRANSMISSION POINTS

$$|y1 - y0|^2 = (y1 - y0)^H (y1 - y0)$$
$$|y1 - y0|^2 = |y1|^2 + |y0|^2$$
$$|y1 - y0|^2 = |x1|^2 + |x0|^2$$



DEFINES ERROR-RATE PERFORMANCE

Figure 5.3 The distance between constellation points establishes the error rate performance.

6. SIMILAR MOK MODULATIONS

This section will examine MOK modulations which are equal to within an orthogonal similarity transformation. This will highlight some interesting properties. In particular, it will be shown that two modulations which are orthogonally-similar have the same error rate performance (distance properties).

First Walsh signaling will be examined. WALSH signaling is proposed by both HARRIS and MICRILOR. Walsh is a popular orthogonal modulation because the basis vectors are constant envelop and can be efficiently detected with the Fast Walsh Transform.

The 4-ary Walsh signal set is shown in Fig. 6.1. By performing an eigen-decomposition on the Walsh basis matrix **B**, a similarity transformation is defined which maps the Walsh basis matrix to a diagonal matrix **D**. Linear algebra textbooks describe a similarity transformation as a change of basis for a linear transform (vector modulator). Since the eigenvectors are orthonormal, the new basis vectors in **D** are orthonormal as were the original Walsh vectors.

4-ary WALSH

B =

0.5000	0.5000	0.5000	0.5000
0.5000	-0.5000	0.5000	-0.5000
0.5000	0.5000	-0.5000	-0.5000
0.5000	-0.5000	-0.5000	0.5000

EIGEN DECOMPOSITION

[Q,D] = eig(B)

EIGENVECTORS

Q =

0.8599	-0.1031	0.0000	-0.5000
0.3352	0.3710	-0.7071	0.5000
0.3352	0.3710	0.7071	0.5000
0.1894	-0.8451	0	0.5000

EIGENVALUES

D =

1.0000	0	0	0
0	1.0000	0	0
0	0	-1.0000	0
0	0	0	-1.0000

ORTHOGONAL SIMILARITY TRANSFORMATION

$$Q^H \; Q = I$$

$$Q^H * B * Q = D$$

ORTHOGONAL SIMILARITY TRANSFORMATION

$$Q * D * Q^H = B$$

Figure 6.1 Performing an eigen-decomposition of 4-ary Walsh. The eigen-decomposition defines a orthogonal similarity transformation.

The properties of an orthogonal similarity transformation are shown in Fig. 6.2. In short, an orthogonal similarity transformation creates a new modulation with the same linear and Euclidean properties as the original. Consequently, the transmit vector energy remains unchanged. Also, the distance properties are retained so the error rate performance is the same. Textbooks on linear algebra emphasize these traits for orthogonal similarity transformations.

ORTHOGONAL SIMILARITY TRANSFORMATION

- **Same Linear Transformation Except a Change of Basis**
- **Maps One Orthogonal Basis into Another Orthogonal Basis**
- **All Angles Maintained**
- **All Distances Maintained**

Figure 6.2 The properties of an orthogonal similarity transformation as described in any good textbook on linear algebra.

The eigen-decomposition creates real eigenvectors and real eigenvalues because the Walsh basis matrix **B** is real symmetric. Notice **B** is no longer symmetric if the columns of **B** are reordered. The modulation characteristics do not change with this reordering, but the eigenvectors and eigenvalues can become complex. Nevertheless, the eigenvectors remain orthonormal.

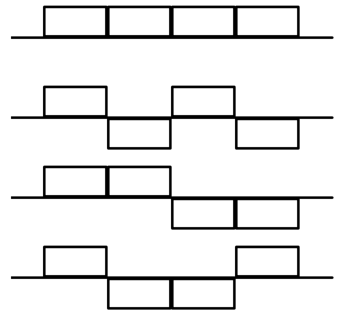
The eigen-decomposition shown in Fig. 6.1 shows that 4-ary Walsh is related to 4-ary PPM as shown in Fig. 6.3. It is obvious the PPM basis in the matrix **D** is orthonormal as is the 4-ary Walsh basis **B**.

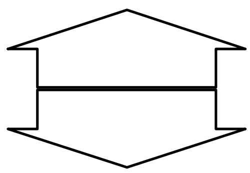
EQUIVALENT MOK MODULATIONS

4-ary WALSH

B =

0.5000	**0.5000**	**0.5000**	**0.5000**
0.5000	**-0.5000**	**0.5000**	**-0.5000**
0.5000	**0.5000**	**-0.5000**	**-0.5000**
0.5000	**-0.5000**	**-0.5000**	**0.5000**





ORTHOGONAL
SIMILARITY
TRANSFORMATION

PPM

D =

1.0000	**0**	**0**	**0**
0	**1.0000**	**0**	**0**
0	**0**	**-1.0000**	**0**
0	**0**	**0**	**-1.0000**

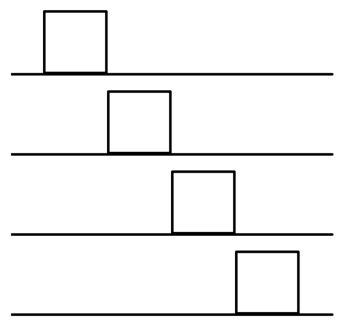


Figure 6.3 PPM and Walsh signaling is equivalent to within an orthogonal similarity transformation.

In more detail, Fig. 6.4 shows the two different vector modulations as shown in Fig. 6.3. The Walsh MOK modulation uses **B** as a basis vector for transmission. The PPM MOK modulation uses **D** as a basis vector. Both can be stimulated by the same QPSK modulation vector **x**. Notice than the energy in the transmit vectors (magnitude squared) are equal. Also, notice that the distance properties are equal. Consequently the error rate performance is identical.

$$\mathbf{y = B\,x} \quad \text{Original Modulation}$$
$$\mathbf{v = D\,x} \quad \text{Similar Modulation}$$

LENGTHS MAINTAINED

$$|\,y\,|^2 = |x0|^2 + |x1|^2 + |x2|^2 + |x3|^2 = |\,x\,|^2$$
$$|\,v\,|^2 = |x0|^2 + |x1|^2 + |x2|^2 + |x3|^2 = |\,x\,|^2$$

DISTANCES MAINTAINED

ERROR-RATE PERFORMANCE MAINTAINED

$$|\,y1 - y0\,|^2 = |y1|^2 + |y0|^2 = |x1|^2 + |x0|^2$$
$$|\,v1 - v0\,|^2 = |v1|^2 + |v0|^2 = |x1|^2 + |x0|^2$$

Figure 6.4 Transmit vector lengths and constellation point distances are maintained by the orthogonal similarity transformation.

Identical error-rate performance is comforting because textbooks provide the error rate curves for a MOK modulation. The textbook does not describe the particular modulation (Walsh, PPM, FSK) since they are all equivalent.

Although different MOK modulations have the same AWGN error performance and transmit vector energy, they can different dramatically in implementation and signal characteristics. The Walsh modulation has a constant envelop, so the peak-to-average is one. The PPM signal has a large peak-to-average ratio.

A simple contrast between Walsh and PPM is shown in Fig. 6.5.

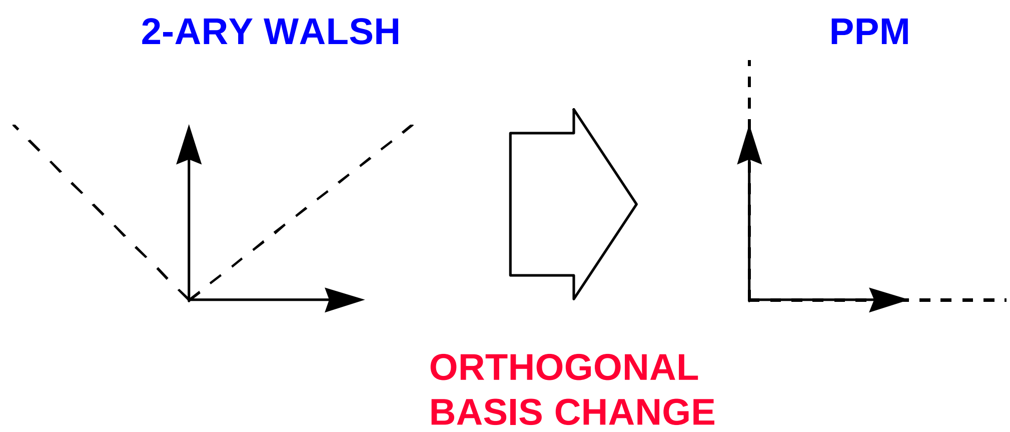


Figure 6.5 A comparison between 2-ary Walsh and 2-ary PPM which is easily visualized. The equivalence is obvious.

Fig. 6.6 shows that orthogonal FSK is also equivalent to PPM (and Walsh). This has to be true because it is a MOK waveform also. However, it is nice that linear algebra identifies the equivalence for us.

The eigen-decomposition of the DFT matrix (basis matrix) is not symmetric so the eigenvectors and eigenvalues become complex. The eigenvector matrix becomes unitary matrix. This does not fundamentally change the results because we are working with complex baseband equivalent signals anyhow. QPSK symbols are complex scalars loaded in the complex symbol vector **x**. Transmissions are accomplished with the matrix vector multiply **DFT x**.

4-ary FSK

DFT =

1.0000	1.0000	1.0000	1.0000
1.0000	- 1.0000i	-1.0000	1.0000i
1.0000	-1.0000	1.0000	-1.0000
1.0000	1.0000i	-1.0000	- 1.0000i

ORTHOGONAL SIMILARITY TRANSFORMATION

EIGEN DECOMPOSITION

[Q,D] = eig(DFT)

$$Q^H \; Q = I$$

$$Q^H * DFT * Q = D$$

EIGENVECTORS

Q =

0.5000	0.8494+ 0.0000i	0.0000+ 0.0000i	0.0526+ 0.1297i
-0.5000	0.3306+ 0.0638i	-0.6974+ 0.1169i	0.4127+ 0.1218i
-0.5000	0.1883- 0.1277i	0.0000- 0.0000i	-0.7728- 0.1139i
-0.5000	0.3306+ 0.0638i	0.6974- 0.1169i	0.4127+ 0.1218i

ORTHOGONAL SIMILARITY TRANSFORMATION

EIGENVALUES

D =

-2.0000	0	0	0
0	2.0000	0	0
0	0	2.0000i	0
0	0	0	2.0000

$$Q * D * Q^H = DFT$$

Figure 6.7 Performing an eigen-decomposition of 4-ary Walsh. The eigen-decomposition defines a orthogonal similarity transformation.

No one has proposed using FSK as a 2.4 GHz PHY high-rate option, but linear algebra has identified it as an equivalent option. Maybe the primary reason it has not been proposed is the DFT required in the receiver. But this cannot be too big a hurdle if the OFDM camp's arguments for the 5 GHz PHY are sound. On the other hand, the FCC may reject it because the spectral density is too discrete.

While being mathematically equivalent in AWGN, the FSK waveform is again quite a different signal than the Walsh or PPM waveform. While it is a MOK, the basic signaling elements are complex sinusoids at baseband.

Pure PPM which shifts an impulse function could be used also, but no one has proposed that. LUCENT is using a shifted Barker.

7. SIMILAR STACKED-MOK MODULATIONS

This section compares stacked MOK modulations which are related by orthogonal similarity transformations. Recall that the only difference between MOK and stacked MOK is the formulation of the modulation vector **x**. For MOK, only one element of **x** is nonzero. For stacked MOK more than one element of **x** is nonzero.

The obvious proposed stacked MOK modulations are by GOLDEN BRIDGE with stacked augmented Barker words and by KDD [3] which is uses orthogonal carrier offsets of the 11 bit Barker. Less obvious is LUCENT's stacking of orthogonal data on the I and Q channels, and HARRIS's stacking of orthogonal data on the I and Q channels. The I and Q channels are orthogonal to one another, so they are stacked (sent simultaneously).

While they have not been proposed, other schemes are possible: carrier-offset Walsh, stacked Walsh, OFDM, carrier-offset PPM, etc. Any permutation of the basic schemes is possible.

The first modulation to be examined is GOLDEN BRIDGE's stacked augmented Barker words. The 1 and 2 Mbps 802.11 DS 11 bit Barker word is augmented to create a larger set. The augmentation algorithm takes the basic Barker word and produces 11 cyclic-shifted copies. To make these copies orthogonal, an additional positive bit is appended to make a total of 12 bits. The augmented Barkers **augBarkerBits** are shown in Fig. 7.1.

Let us compute the maximum supportable data rate. If the chip rate is 11 Mchips/sec and there are 12 chips per Barker symbol and if QPSK (2 bits) is loaded on each augmented Barker, the data rate is (11/12) times 22 Mbits/sec, or 20.167 Mbps.

Since the eleven 12 bit augmented Barkers are orthogonal, they span 11 dimensions in a 12 dimensional space. Since a 12 dimensional space has 12 orthogonal basis vectors, an additional orthogonal vector must exist. To find the missing basis vector, MATLAB was used to compute the null-space of the **augBarkerBits** matrix. The null space is orthogonal to the space spanned by the **augBarkerBits**'s column vectors. The vector spanning the null space was appended as an extra column to form a full rank matrix as shown in Fig. 7.1. Further analysis will use the full 12 orthogonal vectors.

With this full-rank matrix supports an even higher data rate than proposed by GOLDEN BRIDGE. QPSK (2 bits) can be loaded on each of the 12 basis vectors. This achieves a data rate of 22 Mbps.

augBarkerBits =

1	-1	-1	-1	1	1	1	-1	1	1	-1
-1	1	-1	-1	-1	1	1	1	-1	1	1
1	-1	1	-1	-1	-1	1	1	1	-1	1
1	1	-1	1	-1	-1	-1	1	1	1	-1
-1	1	1	-1	1	-1	-1	-1	1	1	1
1	-1	1	1	-1	1	-1	-1	-1	1	1
1	1	-1	1	1	-1	1	-1	-1	-1	1
1	1	1	-1	1	1	-1	1	-1	-1	-1
-1	1	1	1	-1	1	1	-1	1	-1	-1
-1	-1	1	1	1	-1	1	1	-1	1	-1
-1	-1	-1	1	1	1	-1	1	1	-1	1
1	1	1	1	1	1	1	1	1	1	1

12 x 11

Add Basis Vector Spanning Null Space

allBarkerBits =

1	-1	-1	-1	1	1	1	-1	1	1	-1	1
-1	1	-1	-1	-1	1	1	1	-1	1	1	1
1	-1	1	-1	-1	-1	1	1	1	-1	1	1
1	1	-1	1	-1	-1	-1	1	1	1	-1	1
-1	1	1	-1	1	-1	-1	-1	1	1	1	1
1	-1	1	1	-1	1	-1	-1	-1	1	1	1
1	1	-1	1	1	-1	1	-1	-1	-1	1	1
1	1	1	-1	1	1	-1	1	-1	-1	-1	1
-1	1	1	1	-1	1	1	-1	1	-1	-1	1
-1	-1	1	1	1	-1	1	1	-1	1	-1	1
-1	-1	-1	1	1	1	-1	1	1	-1	1	1
1	1	1	1	1	1	1	1	1	1	1	-1

12 x 12

Figure 7.1 Finding 12 orthogonal basis vectors to span the 12 dimensional span defined by the 12 chip signature sequences.

A quick inspection of the **allBarkerBits** matrix reveals that it is not symmetric. Although not necessary, it is convenient to reorder the columns of the **allBarkerBits** matrix to form a symmetric matrix as shown in Fig. 7.2. The symmetric matrix **symBarkerBits** keeps the eigenvectors and eigenvalues real. This makes for more compact bookkeeping.

symBarkerBits

1	1	1	-1	-1	-1	1	-1	1	1	-1	1
1	-1	-1	-1	1	-1	1	1	-1	1	1	1
1	-1	1	1	1	-1	-1	-1	1	-1	1	1
-1	-1	1	-1	1	1	-1	1	1	1	-1	1
-1	1	1	1	-1	-1	-1	1	-1	1	1	1
-1	-1	-1	1	-1	1	1	-1	1	1	1	1
1	1	-1	-1	-1	1	-1	1	1	-1	1	1
-1	1	-1	1	1	-1	1	1	1	-1	-1	1
1	-1	1	1	-1	1	1	1	-1	-1	-1	1
1	1	-1	1	1	1	-1	-1	-1	1	-1	1
-1	1	1	-1	1	1	1	-1	-1	-1	1	1
1	1	1	1	1	1	1	1	1	1	1	-1

EIGENDECOMPOSITION

[Q,D] = eig(symBarkerBits)

ORTHOGONAL SIMILARITY TRANSFORMATION

$$Q^H \; Q = I$$

$$Q^H * \text{symBarkerBits} * Q = D$$

EIGENVECTORS

Q =

0.5805	-0.7224	-0.2214	0.2169	0.0100	0.0141	0.0012	-0.0135	-0.1677	0.1618	0.5403	0.0889
-0.0819	0.0640	0.4481	-0.7134	0.0404	0.4917	-0.2735	-0.0056	0.1167	-0.0525	-0.0170	-0.0559
-0.1941	-0.1975	-0.1282	-0.2570	-0.4057	-0.1273	0.0688	-0.0808	0.1798	-0.4868	-0.1346	0.2821
0.2582	0.1100	-0.0645	-0.0038	0.0966	-0.1493	0.0421	-0.1082	0.3662	-0.2855	0.1559	-0.3956
0.1713	0.3107	0.3779	0.3496	-0.2215	0.2464	-0.6530	-0.0895	-0.2121	-0.3815	0.1112	-0.1615
0.2926	0.0979	-0.0913	-0.1438	-0.1965	-0.0568	-0.2188	0.5515	-0.0772	-0.1678	0.3853	-0.3681
-0.3311	-0.0393	0.2205	0.3005	-0.5108	0.3796	-0.1681	0.0539	0.4730	-0.0354	-0.4252	0.0858
0.2620	0.3355	0.4324	-0.0196	-0.1924	0.4128	0.3606	-0.5911	-0.2135	-0.1516	0.2675	-0.1910
-0.1854	-0.2253	-0.1896	-0.2921	0.1065	-0.0707	-0.2091	-0.1726	-0.5465	-0.1177	-0.3191	-0.0404
-0.2466	-0.1561	-0.0371	-0.0433	-0.2721	-0.0675	0.3022	0.3059	-0.1507	0.0492	-0.3438	-0.6343
0.2526	0.3569	0.4797	0.0188	0.1024	0.4618	0.3544	0.4354	-0.3170	-0.4950	0.1160	0.2872
-0.3158	-0.0147	0.2747	0.2383	0.5856	0.3438	0.1595	0.0639	0.2228	-0.4397	-0.1365	-0.2470

EIGENVALUES

D = sqrt(12) * IdentityMatrix

Figure 7.2 The eigendecomposition of the symmmetric augmented Barker bits matrix, **symBarkerBits**, forms an orthogonal similarity transformation to an all diagonal vector modulation using matrix **D**. The diagonal matrix **D** has the sqrt(12) because **symBarkerBits** is not orthonormal, only orthogonal.

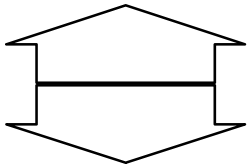
Fig. 7.3 shows the relationship between the use of full set of augmented Barker words and stacked PPM. The **symBarkerBits** basis matrix is related to a PPM diagonal matrix **D** which has the same distance and angle properties as the original **symBarkerbits**. GOLDEN BRIDGE stacks the Barker bits. So equivalently stacked PPM becomes conventional QPSK.

One does not think of QPSK as a vector modulation, but it is mathematically possible to do so. Each chip is orthogonal to the next in a time sense. Note that both square NRZ chips and Raised Cosine chips are time orthogonal.

symBarkerBits distributes the QPSK information in **x** across all 12 chips, while the diagonal matrix **D** places one QPSK complex scalar on each chip. The distance properties remain unchanged, and the performance in either thermal noise or CW jamming is identical for the two. Both the **symBarketBits** and the stacked PPM **D** support 22 Mbps using QPSK scalars.

What happens if fewer than 12 stackings are used? This equivalent to retaining the full rank matrix, but now **x** contains elements of value zero. For example, if only 4 augmented Barker words are used to convey information, reducing the data rate, only 4 complex scalar elements of **x** out of the twelve possible are nonzero.

symBarkerBits



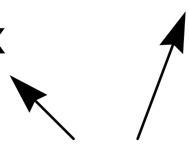
ORTHOGONAL
SIMILARITY
TRANSFORMATION

STACKED PPM = QPSK

$$\begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&0&0&0&0&1 \end{bmatrix} = D$$

SIMILAR MODULATIONS

y = symBarker * x Full Augmented Barker
v = D * x QPSK



QPSK SYMBOL VECTOR

Figure 7.3 If all 12 dimensions are stacked, the augmented Barker technique is mathematically equal to sending straight QPSK, if the modulation vector **x** is filled with QPSK scalars. Both support 22 Mbps under QPSK loading.

This equivalence between the full use of 12 augmented Barkers as stacked orthogonal functons and QPSK raises a paradox as shown in Fig. 7.4 This paradox will be addressed in Section 9.

RULE OF THUMB
10 or more chips gives at least 10 dB PG

BREAKDOWN IN RULE OF THUMB
12 chip augmented Barkers is identical to QPSK. QPSK has no processing gain.

QUESTION
Where is the flaw?

Figure 7.4 Processing gain paradox.

Similarly OFDM is identically equivalent to QPSK if an independent QPSK scalar is used on all OFDM frequency bins as shown in Fig. 7.5. This can be seen by looking at Fig. 6.7. There the DFT matrix was orthogonalized for MOK. Here, with OFDM, the DFT matrix is used in a stacked fashion. More than one element of **x** is non-zero. For the highest data rate all the elements of **x** are non-zero. For FSK only one element of **x** is non-zero at a time.

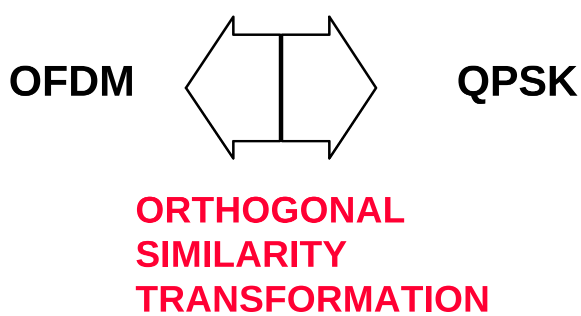


Figure 7.5 OFDM and QPSK are equivalent modulations.

8. COMPARISON SUMMARY

The preceding has placed a large set of modulations into a common mathematical framework. In particular, M-ary orthogonal keying and stacked M-ary orthogonal keying was examined. Even though this mathematical framework exists, this does not mean each proposed modulation for the high-rate 2.4 GHz PHY have identical performance.

Subtle performance differences arise from many sources between the various proposals. If two modulations are identical to within a similarity transformation, then they do have identical distance performance, ignoring implementation effects. However, most of the proposed modulations are not exactly equal to within a similarity transformation. This is primarily due to differences in information loading for each individual proposal. For example, one proposal uses 8 chips, another 11 chips, another 12 chips and another 16 chips. Also, some proposals place information in quadrature on the I and Q channel. Others do not. Proposals also differ in the average information bits/chip loading. Some employ bandwidth expansion by using higher chip rates. Others employ bandwidth expansion through carrier-frequency offset schemes. Some differ in the number of basis vectors versus the number of chips in a basis vector. For example, LUCENT uses only 8 PPM phases but the basis vectors are 11 chip. HARRIS uses 8 Walsh basis vectors with 8 chips.

Other performance sources arise from the peak-to-average ratio requirements in a practical implementation.

How the various modulations handle multipath is an issue also.

9. PROCESSING GAIN THROUGH A DIMENSIONALITY INCREASE

This section attempts to identify the source of processing gain loss experienced by the full-stacked augmented Barker signaling scheme, since its resistance to noise is no better than QPSK. It will be shown that the degrading mechanisms effect all proposed systems. Hard quantified data is not presented, only qualitative principals are discussed.

Conventional direct sequence spread sprectrum (DSSS) is realized by a redundant dimensionality increase as shown in Fig. 9.1. The basic signaling elements are transformed to a higher dimensional space in a redundant fashion.

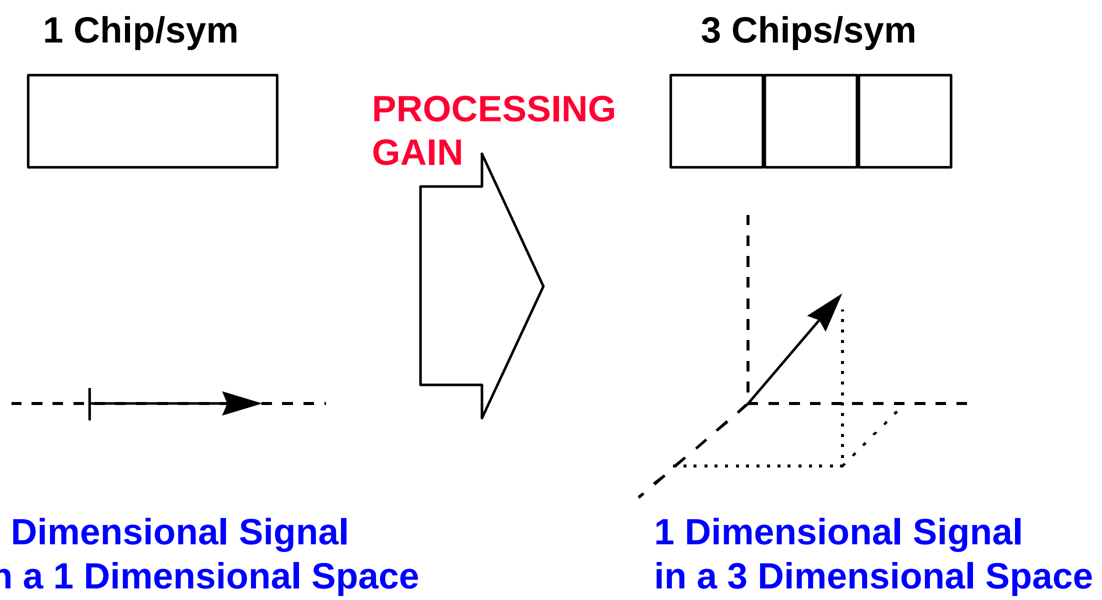


Figure 9.1 A redundant increase in dimensionality from one to three.

Now a jammer must also operate in the higher dimension as shown in Fig. 9.2. Since the jammer does not know in which dimension to place his energy to score a hit, the probability of a jammer hit drops proportional to the number of redundant dimensions. The signal remains a 1-dimensional signal (lies on a line) in N-space. The signaling is mathematically rank one. The signal redundantly lies in N dimensions. If the jammer corrupts N-1 dimensions, but not the Nth dimension, the receiver could recover the data error free if it knew which dimension the jammer failed to corrupt.

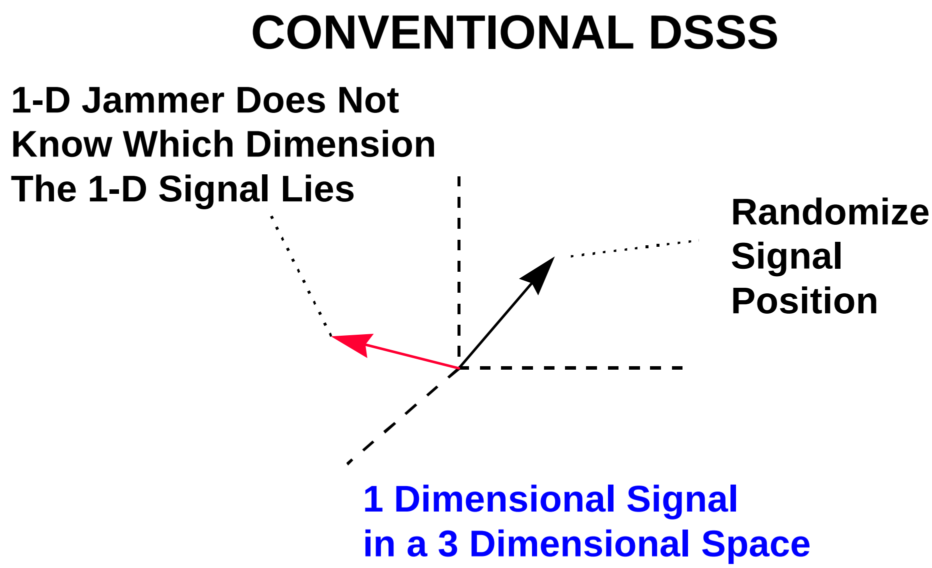


Figure 9.2 The jammer does not know where the 1-dimensional signal lies in the N-dimensional space.

The conventional DS signal can be described as vector modulator since multiple spreading chips are used per symbol. This mathematical description is shown in Fig. 9.3. Note that it is very similar to the mathematical description presented earlier for both MOK and stacked MOK. The primarily difference is information is transmitted redundantly. The **x** vector is loaded with redundant copies of the same information.

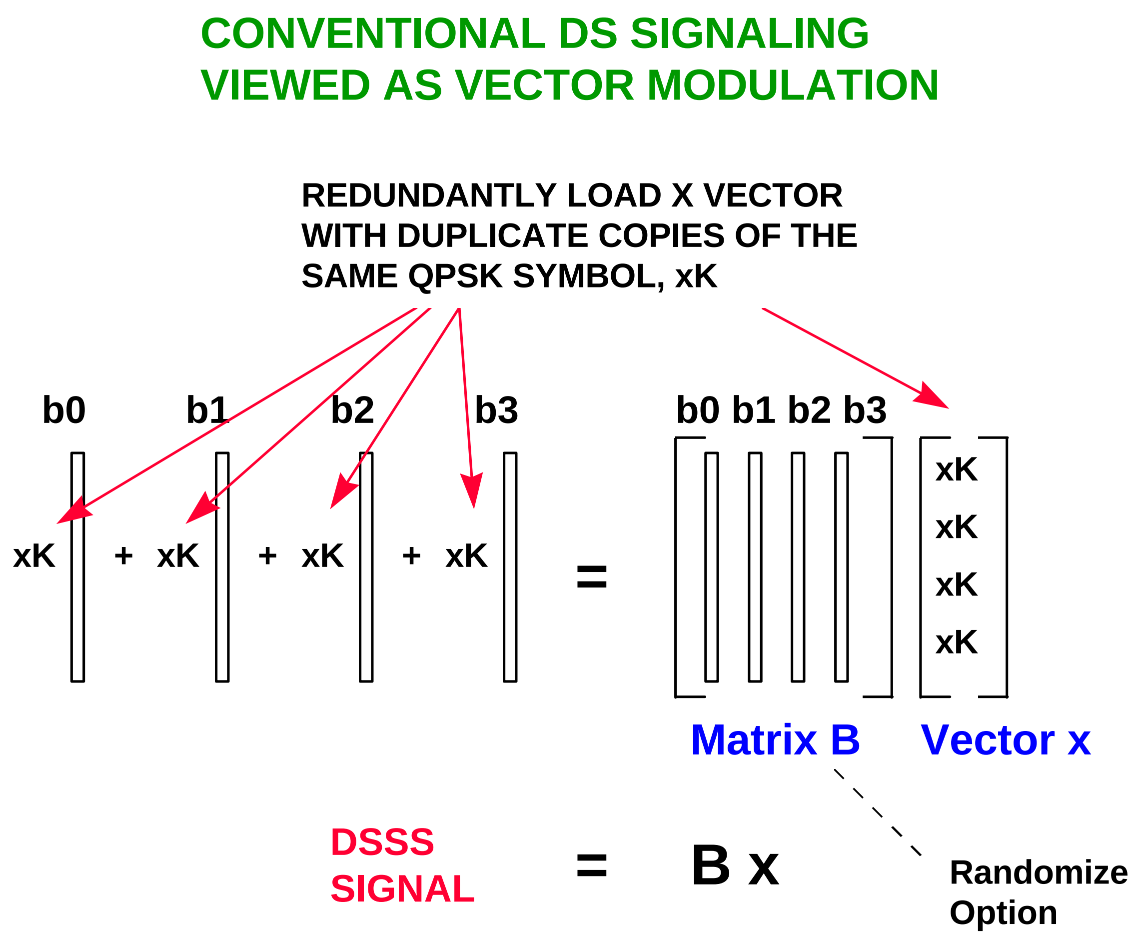
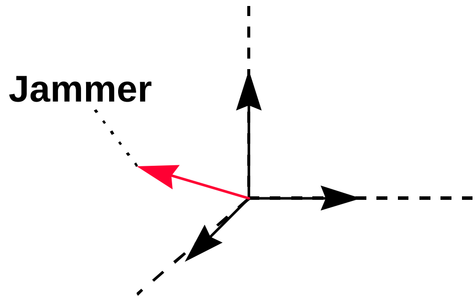


Figure 9.3 Conventional DSSS vector modulator.

Fig. 9.3 differs from our earlier analysis, since earlier data was not explicitly sent redundantly. MOK and stacked-MOK modulations were not sent in a single dimension (on a single line in N-space), but multiple dimensions were used as viewed by the receiver as shown in Fig. 9.4. The signaling space is not rank one from the receiver's viewpoint. Consequently the jammer has much higher probability for corrupting a transmission.

PROBLEM : MOK's and Stacked MOK's Use Multiple Dimensions to Convey Information



PROCESSING GAIN LOSS
Jammer 3 Times More Likely to Score a Hit Compared to Conventional DS

Figure 9.4 MOK and stacked MOK utilize multiple dimensions to carry information each vector time. Conventional DS uses only one dimension.

For example, with a 16-ary Walsh transmission, 16 dimensions are used to convey information. The jammer need not try to find only one dimension but any of the 16 which convey the Walsh information. It is erroneous to think the 16-ary Walsh signal only consumes only 16 dimensions in a signal-space of 2^16 dimensions as some claim.

Note even a OFDM signal meets the conventional definition of DSSS if data is transmitted on all frequency bins.

The conventional processing gain must be measured by the level of redundancy built into the modulation. For example, the amount of redundancy varies for the augmented Barkers. The redundancy decreases as the number of basis vectors stacking increases holding the chip-count constant.

10. CONCLUSION

This submission has viewed various modulations from a linear algebraic viewpoint. Various properties were derived. Various modulation types were derived. It was shown how these relate. Also, it was shown the rule-of-thumb for processing gain breaks down under certain circumstances.

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