

# Correlated Lognormal Shadowing Model

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Purpose:

[To recommend a correlated lognormal shadowing model for .16j network simulation](#)

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# Introduction

- In a network of base stations the lognormal shadowing from two different base sites at a given subscriber location will have some level of correlation. In order to correctly model the benefits of relaying this correlation needs to be modelled.
- In addition, the shadowing from a given base site at two different subscriber locations will be correlated if they are within the spatial decorrelation distance of the shadowing. Therefore relays need to be beyond the spatial decorrelation distance to have a beneficial effect for a subscriber, and the spatial correlation of the shadowing also needs to be modelled
- This contribution presents a relatively simple method for modelling real world shadowing correlation effects

# Lognormal Correlation

- C/I distribution in cellular networks is dependent on lognormal shadowing
  - Correlation between base stations
    - For a MS at a given location how similar is the signal from two different base stations?
  - Decorrelation distance between shadowing
    - For a given MS, how far does it have to move for the shadowing to change?
- Realistic models are required to include these correlations

# Considerations for real world effects

- In practice, the spatial decorrelation distance is not equal in all directions
  - MSs moving along a radial line from the BS have longer decorrelation distances than mobiles moving along a circumferential path
- More realism can be added by defining wavenumbers in polar coordinates  $(r, \phi)$  instead of Cartesian coordinates  $(x, y)$ 
  - The shadowing is highly likely to be lowest (left tail of the Gaussian distribution) when the mobile is moving along a radial direction from the base station (street aligned with the BS), as in many cases it will have a good LOS back to the base site.
- This needs to be accounted for in the model. The standard deviation will also be reduced in this case.
  - The lognormal standard deviation will have some dependence on the subscriber range, and will tend to decrease as range decreases.
  - Some account of this is required in the model to prevent very large values of the standard deviation occurring at very short ranges in the network simulation.

# Modified MOSAIC

$$L(\text{dB}) = \begin{cases} -1.5\sigma \cos(|\phi - \phi_s|) + \sqrt{\frac{2\sigma}{3N}} \sum_{n=1}^N \cos(k_n^r r + \psi_n^r) & \text{for } |\phi - \phi_s| < 5^\circ \\ \sqrt{\frac{4\sigma^2}{N}} \sum_{n=1}^N \cos(k_n^r r + \psi_n^r) \cos(k_n^\phi \phi + \psi_n^\phi) & \text{for } |\phi - \phi_s| \geq 5^\circ \end{cases}$$

where,

$\sigma$  = standard deviation of the lognormal shadowing

$\phi$  = mobile bearing from the basestation

$\phi_s$  = Street orientation at mobile location

$r$  = range of mobile from basestation

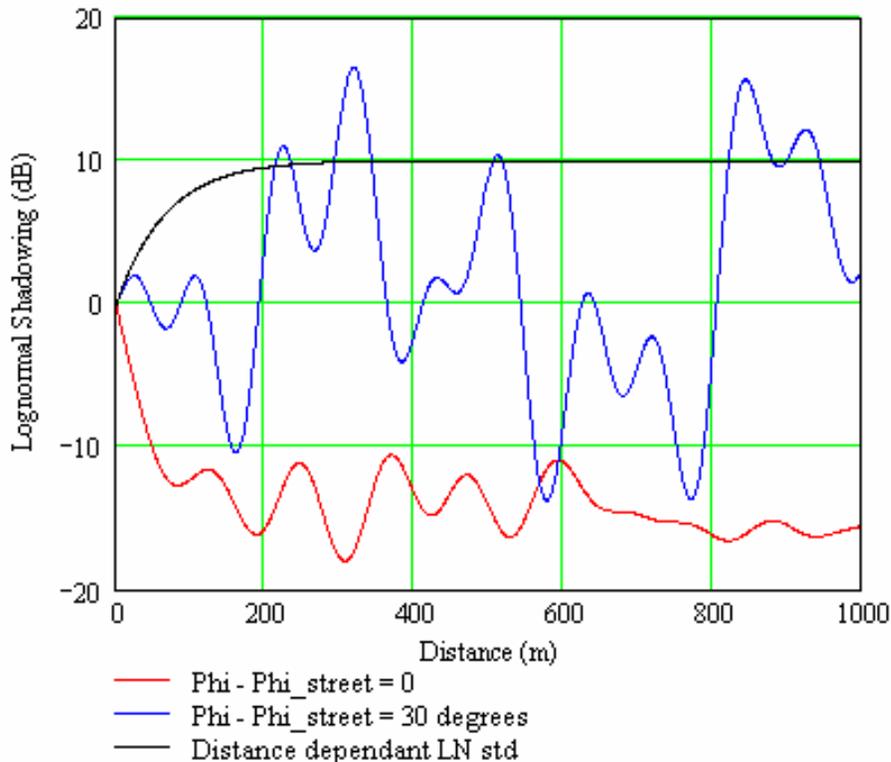
$k_n^r$  =  $n^{\text{th}}$  wavenumber in the radial direction

$k_n^\phi$  =  $n^{\text{th}}$  wavenumber in the  $\phi$  direction

$\psi_n^r, \psi_n^\phi$  = Random phase terms

**$\sigma$  is reduced by one third  
in this (radial, LOS) case**

# Variation of shadowing with range, r



The plot shows the variation of the shadowing for an MS moving along a street aligned with the mobile bearing, and for an MS moving along a street whose orientation is at 30° to the mobile bearing.

For the case where the street is aligned with the mobile bearing the shadowing is clearly in the lower portion of the shadowing distribution, and it has a much reduced standard deviation.

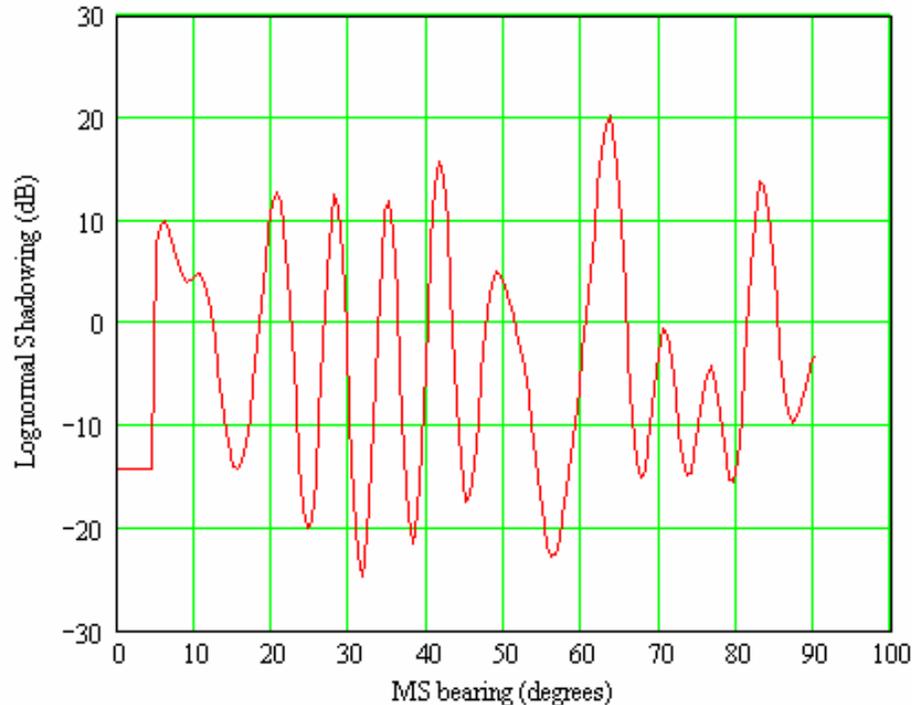
The plot also shows the distance dependant lognormal  $\sigma$ .

The maximum values of the wavenumbers can be adjusted to give the desired decorrelation distance. The precise values used in the equation for  $\sigma$  can also be adjusted as desired.

$$\max(k_n^r) = \frac{2\pi}{100} \quad \max(k_n^\phi) = 20\pi$$

$$\sigma = 10 \left( 1 - e^{-\frac{3r}{200}} \right)$$

# Variation of shadowing with mobile bearing



The model is constructed so that the shadowing is low (signal level high) when the street is aligned with the mobile bearing ( $\phi=0^\circ$  for the above case). At  $5^\circ$  there is a sharp discontinuity as the mobile moves out of the aligned street. This is consistent with effects observed in real environments.

# Summary

- A model has been presented which allows the lognormal shadowing between BSs in a network to be correlated, for the BS-RS or BS-MS links.
- The model includes a simple model for including spatial correlation of the shadowing, which has an  $(r, \phi)$  dependence, and which ensures that the shadowing is low when the street orientation is aligned with the mobile bearing.
- A distance dependant lognormal  $\sigma$  is proposed to ensure that large shadowing variations do not occur at very short ranges.
- Recommend to adopt the model for network simulation of .16j relaying system

# Appendix

# MOSAIC

## Model for ShAdowing Including Correlation

MOSAIC generates spatially correlated fading using a sum of  $N$  products of cosines, whose periodicity is dependent on the  $x$  and  $y$  spatial coordinates. The  $N$  wavenumbers ( $k$ ) are random numbers, uniformly distributed within a range from 0 to  $k_{max}$ . The value of  $k_{max}$  is dependent on the decorrelation distance required.

$$L(dB) = \sum_{n=1}^N \left( a \cdot \cos(k_{n-1} \cdot x + \phi_n) \cdot \cos(k_{n-2} \cdot y + \psi_n) \right)$$

**Shadowing model for a single base**

$$a = \sqrt{\frac{4 \cdot \sigma^2}{N}} \quad \text{where } \sigma \text{ is the lognormal standard deviation in dB.}$$

$\phi_n$  and  $\psi_n$  are random phase terms uniformly distributed between 0 -  $2\pi$ .

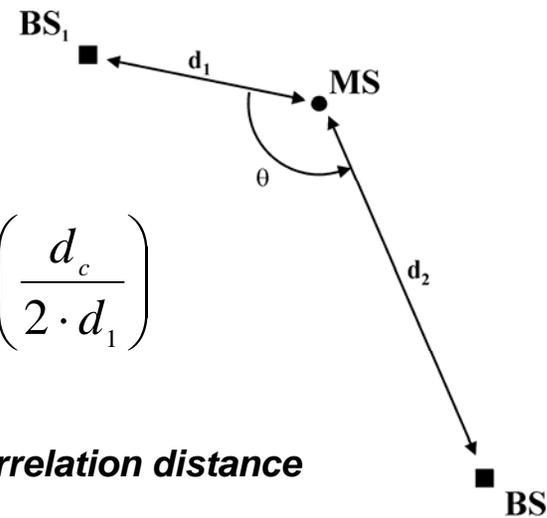
# Correlation between Basestations

## Modified Saunder's correlation model

$$p = \begin{cases} \sqrt{\frac{d_1}{d_2}} & \text{for } 0^\circ \leq \theta \leq \theta_T \text{ and } d_1 \geq \frac{d_c}{2} \\ \left(\frac{\theta_T}{\theta}\right)^\gamma \cdot \sqrt{\frac{d_1}{d_2}} & \text{for } \theta_T \leq \theta \leq \pi \text{ and } d_1 \geq \frac{d_c}{2} \\ \sqrt{\frac{d_c}{2 \cdot d_2}} & \text{for } d_1 < \frac{d_c}{2} \end{cases}$$

$$\theta_T = 2 \cdot \sin^{-1} \left( \frac{d_c}{2 \cdot d_1} \right)$$

$d_c = 1/e$  decorrelation distance



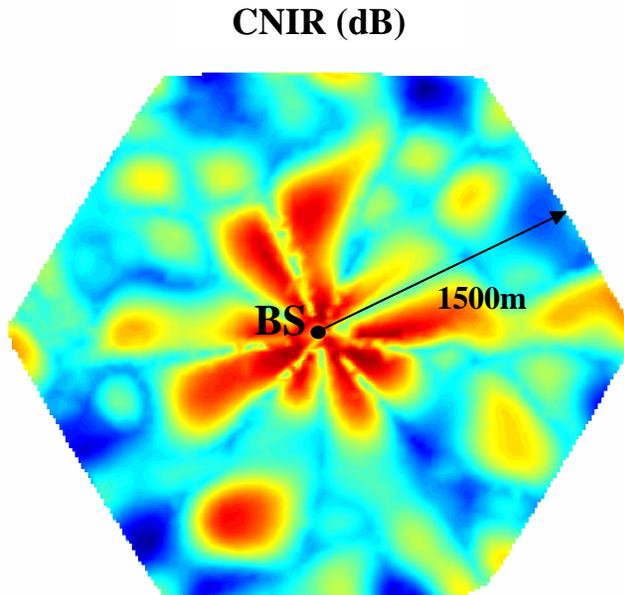
For a given network of MSs, a correlation matrix  $\mathbf{R}_{yy}$  can be calculated using the above model. If independent lognormal samples,  $\mathbf{x}$ , are generated using the model on the previous slide, these can then be correlated using  $\mathbf{R}_{yy}$  to give correlated lognormal samples,  $\mathbf{y}$ .

$$\begin{aligned} \mathbf{y}(t) &= \mathbf{T} \cdot \mathbf{x}(t) \\ \mathbf{R}_{yy} &= E[\mathbf{y}(t)\mathbf{y}(t)^H] = E[\mathbf{T} \cdot \mathbf{x}(t) [\mathbf{T} \cdot \mathbf{x}(t)]^H] \\ &= \mathbf{T} \cdot E[\mathbf{x}(t)\mathbf{x}(t)^H] \cdot \mathbf{T}^H \\ &= \mathbf{T} \mathbf{T}^H = \mathbf{T} \mathbf{T}^H \end{aligned}$$

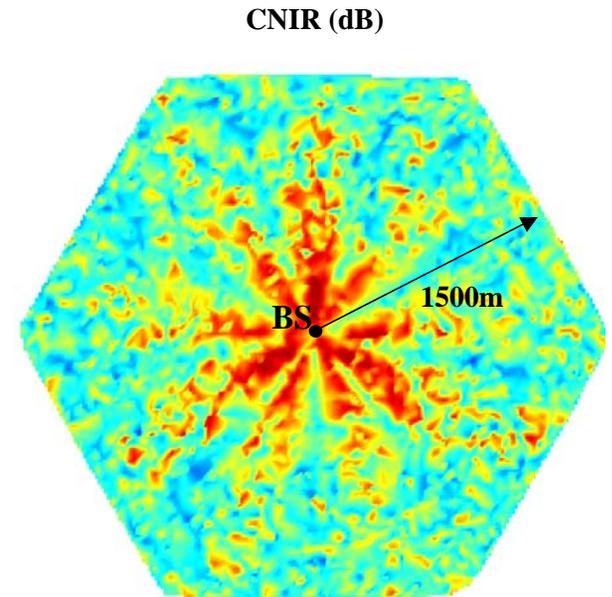
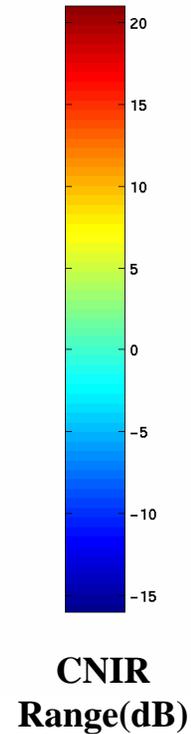
$$\begin{aligned} \mathbf{R}_{yy} &= \mathbf{U} \mathbf{D} \mathbf{U}^H \\ \mathbf{D} &= \mathbf{D}^{1/2} \mathbf{D}^{1/2} \\ \mathbf{R}_{yy} &= (\mathbf{U} \mathbf{D}^{1/2}) (\mathbf{D}^{1/2} \mathbf{U}^H) \\ &= (\mathbf{U} \mathbf{D}^{1/2}) (\mathbf{U} \mathbf{D}^{1/2})^H \\ &= \mathbf{T} \mathbf{T}^H \end{aligned}$$

# Lognormal Shadowing CINR & spatial correlation

- 19 Node B's
- 3 beams/sector
- $\sigma=10\text{dB}$
- $d_c=280\text{m}$



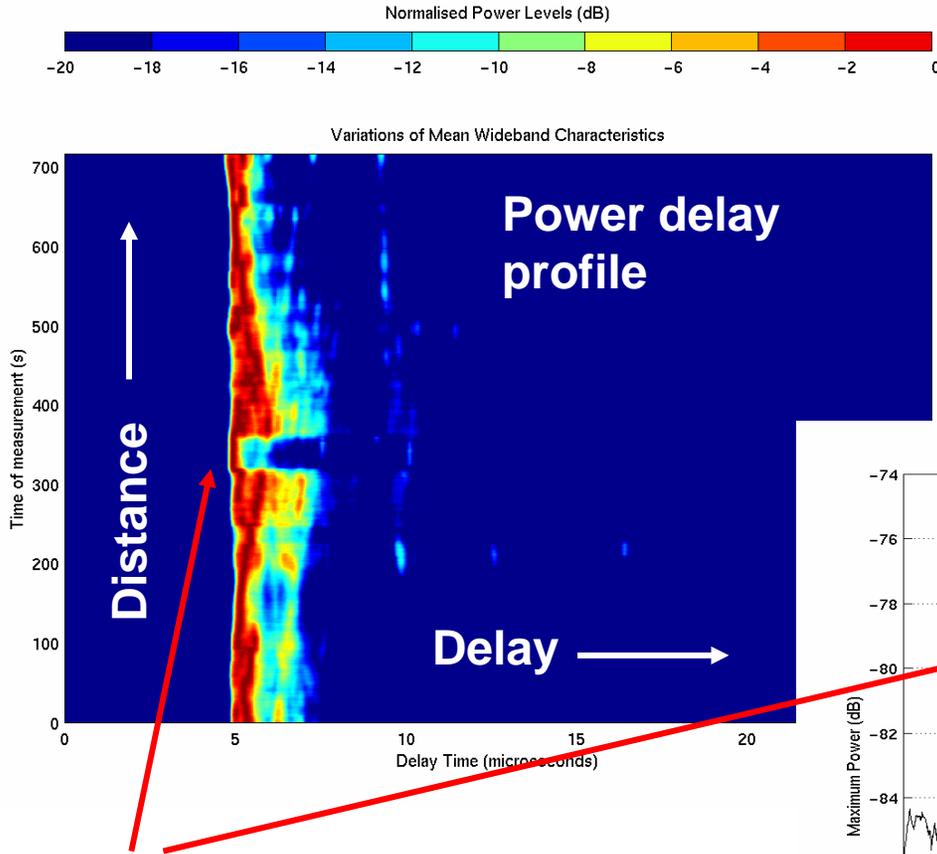
CINR using MOSAIC, including correlation between BS's and spatial correlation.



CINR using ITU model, with no correlation between BS's and no spatial correlation.

**MOSAIC results in a physically representative coverage plot with clearly identifiable black spots**

# Example measurement in Central London



Measurements have been filtered using a moving average window to remove small scale fading. The plot shows the large scale fading. Each 'average' power delay profile in the plot is normalised to its own maximum, so that the plot shows the variation in profile shape with distance. The absolute power variation is shown below :-

Mobile crossing a street junction. Note the increase in absolute received power.

