Project	**IEEE 802.16 Broadband Wireless Access Working Group <http://ieee802.org/16>**
Title	**Ranging Code Performance for Multi-element Arrays**
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Re:	Call for Contributions on P802.16a Mergers and Consolidation Document Number: IEEE 802.16a-02/22, April 15, 2002. URL: < http://ieee802.org/16/docs/02/80216-02_22.pdf >
Abstract	Demonstrates that the performance of the OFDMA ranging codes are more than adequate for multiple antennae.
Purpose	This document is submitted as a technical reference to support the inclusion of allocation method 2 for the ranging codes used in systems incorporating multi-element arrays.
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1. Introduction

This document discusses the performance of the ranging code detector at the basestation for a system incorporating multiple antenna adaptive arrays.

Some concerns were raised as part of IEEE 802.16 TGa discussions at Session 18 (based on [1]) that the OFDMA ranging code specification is inadequate for AAS operation. Analysis and simulations presented here indicate that the ranging codes are adequate when using the OFDMA2 PHY signal structure (IEEE P802.16a/D3). One of the key concerns is that the detection of the ranging codes will suffer in the presence of multipath, causing the frequency response of each subcarrier bin to be independent. For the OFDMA2 PHY structure, the narrow band assumption causes the channel response to be dominated by the phase ramp associated with the bulk propagation delay and is quite insensitive to most multipath. This improves detection performance at the theoretical expense of reducing delay estimation accuracy. However numerical experiments show that even with subchannel bandwiths as narrow as 50 kHz, delay estimation error for the adaptive array detector can easily be reduced below 200 ns, which is more than adequate for proper operation of the system.

The other issue that arises for systems with multi-element arrays is the nature of the detector itself. The maximum likelihood function for estimating the transmitted signal delay offset is not given by the traditional matched filter. The data must first be whitened before the matched filter can be applied. The resulting detector has the ability to pierce interference, placing soft nulls in the directions of the ranging signals transmitted by other users. A description of this detector and the results of a few numerical simulations follow.

2. Signal Model and Ranging Code

This analysis deals with the detection statistic derived from the Maximum Likelihood (ML) function for estimating signal parameters for a single emitter received by an adaptive antenna array. In this context, the ML detection statistic is used to determine whether or not a signal was received from a set of possible allowable but known signal waveforms and to determine the correct delay offset for the unknown signal.

The detector is appropriate for processing a ranging signal transmitted by a user to the basestation. We neglect temporarily the issue of determining the required transmit power, which is traditionally part of the ranging problem, and instead focus on user detection and transmit synchronization.

Assume the signals are received over a wireless narrow-band channel and consist of a grouping of several subcarriers, indexed by n. The detection statistic itself is derived from the following signal model,

$$\mathbf{x}(n) = \mathbf{a}s(n) + \mathbf{i}(n), \tag{2.1}$$

where $\mathbf{x}(n)$ is an $M \times 1$ complex received data vector at frequency sample n , $\mathbf{a}$ is an $M \times 1$ received spatial signature or aperture vector and $\mathbf{i}(n)$ is an interference vector due to environmental noise and other signals in the environment. We assume that $s(n)$ is known up to a possible unknown delay or advance offset, so that,

$$s(n) \equiv e^{2\pi j \Delta f n \tau} d(n), \tag{2.2}$$

where Δf is the frequency spacing between carriers in a ranging sub-channel, τ is an unknown transmit delay offset, and where $d(n)$ is a known complex symbol/codeword. (For the ranging codes $d(n)$ is specified to be ± 1, depending on the pseudo-random code chosen by the remote user.)

The mathematical analysis is simplified if we consider processing a block of data of N frequency samples at a given time and stack our received vectors in a matrix. We therefore define the conjugate received data and signal matrices,

$$\mathbf{X} \triangleq \begin{bmatrix} \mathbf{x}(1)^H \\ \mathbf{x}(2)^H \\ \vdots \\ \mathbf{x}(N)^H \end{bmatrix}, \tag{2.3}$$

$$\mathbf{s} \triangleq [s(1), s(2), \cdots, s(N)]^H \tag{2.4}$$

$$\mathbf{\Upsilon} \triangleq [\mathbf{i}(1), \mathbf{i}(2), \cdots, \mathbf{i}(N)]^H. \tag{2.5}$$

This permits (2.1) to be written as,

$$\mathbf{X} = \mathbf{s}\mathbf{a}^H + \mathbf{\Upsilon} \tag{2.6}$$

If we assume that $\mathbf{i}(n)$ is a complex, circularly symmetric, Gaussian random vector, with unknown interference covariance matrix $\mathbf{R_{ii}}$, and that the aperture vector $\mathbf{a}$ is unknown and deterministic, we can write the log-likelihood function for this signal model.

$$\rho_{ML}(\mathbf{R_{ii}}, \mathbf{a}) = -NM\ln(\pi) - \ln|\mathbf{R_{ii}}| - tr\left\{\mathbf{R_{ii}}^{-1}(\mathbf{X} - \mathbf{s}\mathbf{a}^H)^H(\mathbf{X} - \mathbf{s}\mathbf{a}^H)\right\}. \tag{2.7}$$

We optimize the likelihood function over unknown $\mathbf{a}$ and $\mathbf{R_{ii}}$, to arrive at the detection statistic [2],

$$\rho_d \equiv \frac{\mathbf{s}^H P(\mathbf{X})\mathbf{s}}{\mathbf{s}^H\mathbf{s}}, \tag{2.8}$$

where $P(\mathbf{X}) \equiv \mathbf{X}(\mathbf{X}^H\mathbf{X})^{-1}\mathbf{X}^H$. This statistic can be computed efficiently by first obtaining the QR-decomposition of the matrix $\mathbf{X}$, wherein

$$\mathbf{Q}_x\mathbf{R}_x = \mathbf{X}, \tag{2.9}$$

and $\mathbf{Q}_x$ is an $N \times M$ orthonormal matrix and $\mathbf{R}_x$ is an $M \times M$ upper triangular matrix. The detection statistic can then be written as,

$$\rho_d(\tau) \equiv \frac{\|\mathbf{Q}_x^H\mathbf{s}(\tau)\|^2}{\|\mathbf{s}(\tau)\|^2}, \tag{2.10}$$

where the dependency on the unknown transmit delay offset τ is shown. The denominator of (2.10) is normally a constant for the ranging application.

The detection statistic in (2.10) can be shown to be equal to one minus the normalized mean square error at the output of a linear beamformer whose weights are set to the optimal Wiener spatial filter [2]. The detection statistic can also be analyzed under the assumption that the aperture vectors behave like Gaussian noise. It can be shown under these conditions that the OFF detection statistic is beta distributed, where the OFF detection statistic is $\rho_{off} \equiv \rho_d$ in (2.10) when the basestation assumes a code sequence $d(n)$ that has not been transmitted .

In fact ρ_{off} is approximately the ratio of two χ^2 random variables. The numerator has $2M$ degrees of freedom and the denominator is the sum of one with $2M$

degrees of freedom and one with $2(N-M)$ degrees of freedom. Symbolically we can write this as,

$$\rho_{off} = \frac{\chi_1^2(2M)}{\chi_1^2(2M) + \chi_2^2(2(N-M))}. \tag{2.11}$$

The first random variable in the denominator is the same as the random variable in the numerator. This is just the definition of the beta distribution with parameters $2M$ and $2(N-M)$. Thus ρ_{off} is beta distributed and has PDF,

$$p(t) = \frac{t^{M-1}(1-t)^{N-M-1}}{B(M, N-M)}, \tag{2.12}$$

where $B(a,b)$ is the beta function,

$$B(a,b) = \int_0^1 t^a (1-t)^b dt. \tag{2.13}$$

In the case where the base station attempts to detect a code that has been transmitted the ON detection statistic can be shown to be approximately equal to,

$$\rho_{on} \approx \frac{\gamma}{\gamma+1}, \tag{2.14}$$

where γ is the maximum obtainable signal to interference noise ratio (SINR) at the output of the beamformer [2]. When the number of signals is smaller than the number of antennae, the ON detection statistic does not vary by much as the later CDF plots show. A full analysis of the CDF of the ON detection statistic requires the use of the inverse complex non-central Wishart distribution and is beyond the scope of this report.

3. Numerical Experiments

We illustrate the performance of the ranging code detector using a few simple numerical experiments. First we show the detector output as a function of the transmit delay offset. Our initial set of experiments assume processing is performed using a subset of 16 of the 53 subcarriers proposed for a ranging channel. The 16 subcarriers are spaced at a 3.125 kHz carrier spacing (roughly corresponding to the carrier spacing for a 6 - 7 MHz channel bandwidth) over 4 OFDM symbols and a basestation with 8 antennae. The key point here is that the component of the ranging code actually used only occupies 50 kHz and is not affected thereby by multipath. The multiple antennae mitigate against flat fades, which do not appear to be a factor based on the observed CDF plots. It is assumed that the transmit gains are set so that the signal is received at the basestation 15 dB above the noise floor. Signals from other remotes are assumed to all be received at the same 15 dB signal to white noise power ratio (SWNR). For these experiments we simulate 5 users simultaneously attempting to range over the same set of subcarriers. The true delay offset in Figure 2 is shown by the dotted vertical line at 0 μs. The measured delay error for this experiment is 300 ns. The channel model used here is a modification of the Stanford University Interim channel model (SUI) proposed in [3] and further described in [4].

For the SUI 3 and SUI 4 Omni channels we performed a Monte Carlo experiment containing a thousand trials. The results of these experiments are tabulated in Table 1.

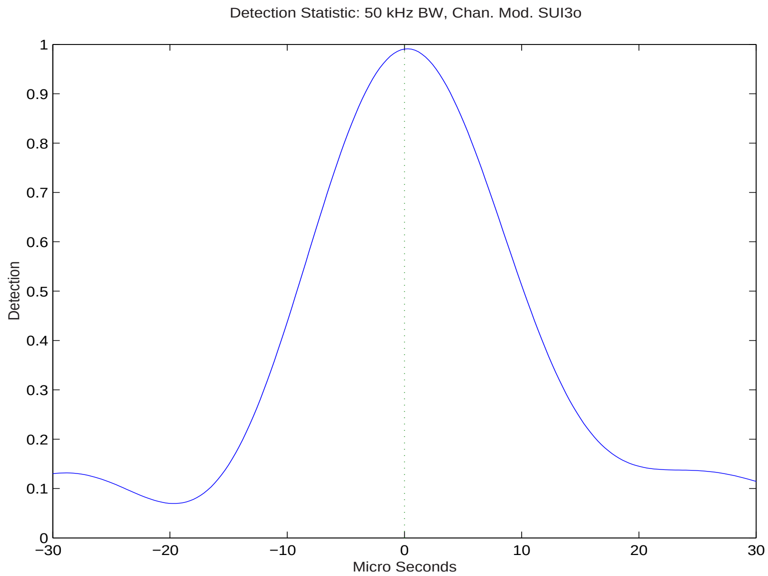

FIGURE 1. Detection Statistic vs Delay Offset for the SUI 3 Omni Channel Model

	rms(τ) μsec.	**mean Detect**	**std Detect**	**Prob. Detect. Error**
SUI 3 Omni ON	0.2574	0.9914	0.0049	0
SUI 3 Omni OFF	-	0.1826	0.0372	0
SUI 4 Omni ON	1.0026	0.9829	0.0098	0
SUI 4 Omni OFF	-	0.1779	0.0396	0

TABLE 1. Detection Performance

An ON detection error is logged if the delay error associated with the peak is larger than the largest specified multipath delay. An OFF detection error, however, is logged if the maximum peak exceeds a threshold of 0.8. After a thousand trials no detection errors were logged for either experiment.

Plots of the CDF of the ON and OFF detection statistics for both the SUI 3 and SUI 4 omni channels follow in Figures 2 and 3.

4. CONCLUSIONS

This report shows that by adopting the ranging subchannel allocation method 2 in Section 6.2.11.2 of the IEEE P802.16a/D3-2002 draft standard and using the narrow band subchannel structure proposed in OFDMA2 PHY in Section 8.3.4.5, we obtain excellent detection statistics, provided that we use an appropriate detection statistic for the number of antennae available at the basestation.

REFERENCES

[1] Jerry Krinock, Manoneet Singh, and Mike Paff. Comments on OFDMA ranging scheme described in IEEE 802.16ab-01/01r1. *IEEE 802.16 Broadband Wireless Access Working Group*, August 2001.

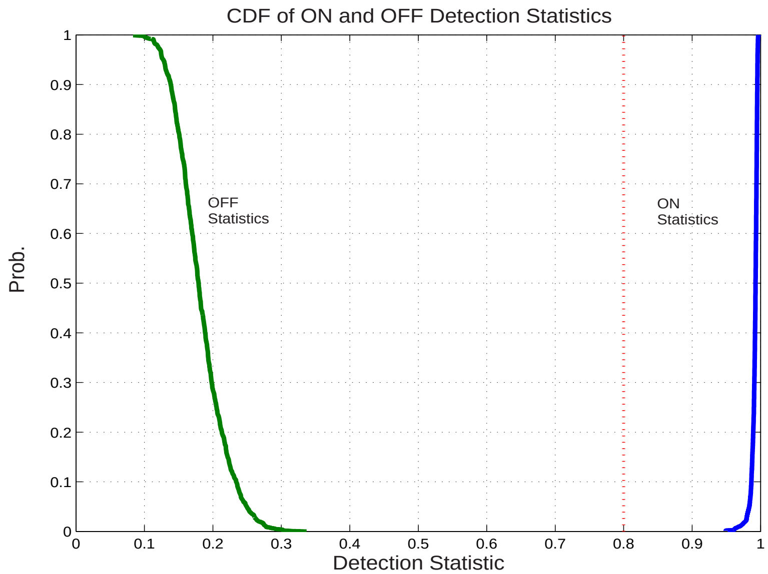

FIGURE 2. Detection Statistic CDF for the SUI 3 Omni Channel Model

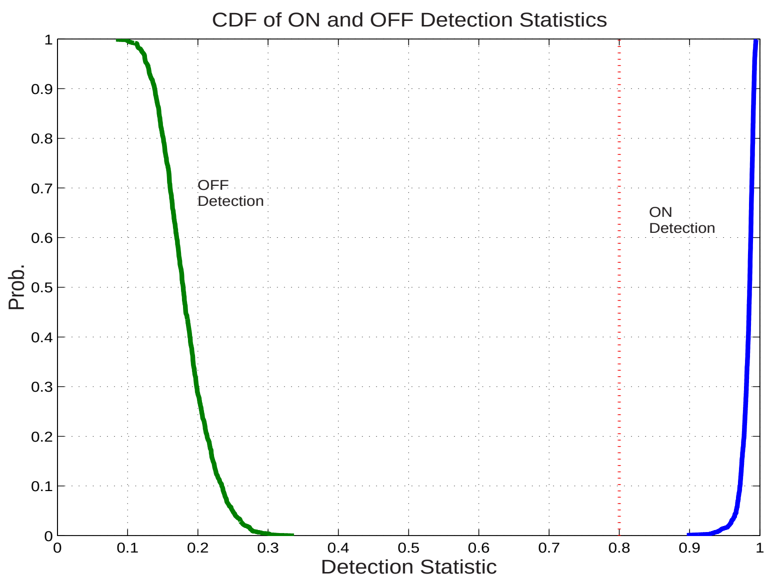

FIGURE 3. Detection Statistic CDF for the SUI 4 Omni Channel Model

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[3] V. E. Erceg, K.V.S. Hari, and M.S. Smith Et. Al. Channel models for fixed wireless applications. *IEEE 802.16 Broadband Wireless Access Working Group*, 2001.

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