

Project	IEEE 802.16 Broadband Wireless Access Working Group < http://ieee802.org/16 >
Title	Draft text for LDPC coding scheme for OFDMA
Date Submitted	2004-04-26
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Re:	802.16REVd Sponsor Ballot Recirculation comment
Abstract	This contribution describes the LDPC coding scheme
Purpose	Proposal to include the LDPC FEC scheme as an option for OFDMA
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Following is text intended for 802.16RevD. The equations, figures, tables and appendix will need to be numbered appropriately for the full document.

8.4.9.2.4 Low Density Parity Check Code (optional)

8.4.9.2.4.1 Code Description

The fundamental LDPC code is a systematic linear block code with $(n,k) = (2000, 1600)$, eighteen edges per check node, and Rate = 4/5. There is only one code definition for the system and lower code rates may be achieved by shortening the basic code. Varying data field lengths are accommodated by shortening the basic code, concatenating codewords, and a combination of shortening and concatenation.

8.4.9.2.4.2 LDPC encoding

In a general analysis an (n,k) LDPC code has k information bits and n coded bits with code rate $r = k/n$. The parity-check matrix H is of dimension $(n-k) \times n$, and it defines a set of equations

$$\text{Equation 1} \quad v \cdot H^t = 0$$

for all codewords v .

An example parity-check matrix is shown below for an LDPC code (9,3) as well as the expanded parity-check equations

$$H = \begin{bmatrix} 1 & & 1 & & 1 & & & & \\ & 1 & & 1 & & 1 & & & \\ & & 1 & & 1 & & 1 & & \\ 1 & & & 1 & & & & 1 & \\ & 1 & & & 1 & 1 & & & \\ & & 1 & 1 & & & 1 & & \end{bmatrix} \iff \begin{cases} v_1 + v_4 + v_7 = 0 \\ v_2 + v_5 + v_8 = 0 \\ v_3 + v_6 + v_9 = 0 \\ v_1 + v_5 + v_9 = 0 \\ v_2 + v_6 + v_7 = 0 \\ v_3 + v_4 + v_8 = 0 \end{cases}$$

LDPC codes are encoded via a generator matrix G . For a given information vector u , the corresponding codeword v is encoded via

$$\text{Equation 2} \quad v = u \cdot G$$

From (1), we have $u \cdot G \cdot H^t = 0$. Since u is an arbitrary vector, we have $u \cdot G \cdot H^t = 0$

$$G \cdot H^t = 0$$

For a given parity-check matrix H which defines the code, there are 2^k different G that satisfy (2) provided that H 's rank is $n-k$. One of these generator matrices has the format

Equation 3

$$G = [I_{k \times k} \mid P_{k \times (n-k)}]$$

and is denoted a systematic encoder since the first k code bits are identical with the information bits.

Denote $H=[H_1|H_2]$, where H_1 and H_2 have dimensions $(n-k)*k$ and $(n-k)*(n-k)$, respectively. Then according to (2), (assume H_2 is non-singular), we have

Equation 4

$$I \cdot H_1^t + P \cdot H_2^t = 0 \Rightarrow P = H_1^t H_2^{-t}$$

and the codeword is in the format of

Equation 5

$$\bar{v} = \bar{u} \cdot G = [\bar{u} \mid \bar{u}P] = [\bar{u} \mid \bar{u}H_1^t H_2^{-t}]$$

For some LDPC codes, high encoding complexity arises from the high density of H_2^{-t} in (5). However, for this code H_2 is simply $f(D) = 1 + D$, and hence H_2^{-t} can be implemented by a differential encoder. One possible encoding process is then

Equation 6

$$v = [u \mid uH_1^t H_2^{-t}] = \left[u \mid uH_1^t \frac{1}{1 \oplus D} \right]$$

and is illustrated in Figure 1.

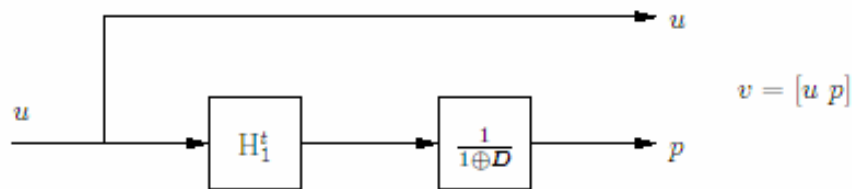


Figure 1. Block Diagram of one possible encoder architecture for specified LDPC code.

For the described code H_1 is low-density, with a uniform column weight of four. A detailed description of the H matrix of the code is contained in the LDPC Code Definition Appendix. The code is fully described by the definition of the H matrix as indicated in the appendix.

8.4.9.2.4.3 Rate Adjustment

The code is rate flexible in that it can be easily shortened to reduce the overall rate, R , of the code. By “shortening” of the codewords we mean reducing the length of the systematic portion of the codeword by stuffing zeroes in the unused portion, encoding, and transmitting the systematic information and parity bits, but not the stuffed zeroes. Table 1 shows the parameters for shortening the code to the desired code rates.

R Code Rate	N_{cw} (bits)	N_i (bits)	N_i (bytes)
$R = 4/5$	2000	1600	200
$R = 3/4$	1600	1200	150
$R = 2/3$	1200	800	100
$R = 1/2$	800	400	50
$R = 1/3$	600	200	25

Table 1. Parameters for code rate adjustment using code shortening. For each rate shown, N_{cw} is the length of the codeword, and N_i is the length of the information field. The $R = 1/3$ code is not used as a basic code rate but represents the lowest code rate realized in a codeword using the algorithm in the Packet Encoding section.

8.4.9.2.4.4 Packet Encoding

Since transported data packets can be any size from typically about 40 bytes up to 12000 bytes and larger, the system must be able to encode variable length packets in a consistent manner. This consistency is required to ensure that the receiver always knows how to reconstruct the information field from the encoded transmitted data.

Each packet is encoded as an entity. In other words, the data boundary of a packet is respected by the encoder. Headers are encoded using the Viterbi decoder at $R = 1/2$ and BPSK modulation.

The length of the packet that is to be encoded is all that is needed to encode or decode the packet using the following rules:

1. If $\text{Length} \leq N_i$ bits, then shorten the code as necessary to the size of the packet.
2. If $\text{Length} > N_i$ bits and $\leq 2 N_i$ bits, divide the data field in half and encode with two LDPC codewords that have been shortened equally. If the total number of bytes is odd, the even-length portion of the field goes in the first codeword (corresponding to the first (beginning) portion of the data).
3. If $\text{Length} > 2 N_i$ bits, then compute $N_r = \text{modulo}(\text{Length}, N_i)$ (in bits). If N_r is less than or equal to $N_i/2$ bits, encode the first $\text{Length} - (N_r + N_i)$ bits using full-length, unshortened codewords, and encode the remaining $N_r + N_i$ bits using rule #2. If N_r is greater than $N_i/2$ bits, encode the first $\text{Length} - N_r$ bits using full-length, unshortened codewords and encode the remaining N_r bits as in rule #1.

The intent of the above rule set is to provide a means for data transmission without the need for additional information beyond the packet field length. This scheme does so with a simple rule set that reduces the rate of the last codewords in order to reduce the number of iterations (and therefore the latency) that must be performed on the last portion of the data. By “shortening” of the codewords we mean reducing the length of the systematic portion of the codeword by stuffing zeroes in the unused portion, encoding, and transmitting the

systematic information and parity bits, but not the stuffed zeroes. The length and position of the shortened codewords and erased bits are deterministic when the above rules are followed.

For all packets the codeword bits can be indexed using the corresponding column indices of the H matrix. Using this convention the systematic codeword bits comprise the leftmost bits starting at bit location zero, and fill the codeword to bit 1599. The remaining four hundred bits of the codeword, from indices 1600 to 1999 are the parity bits. The codeword systematic bits are filled in an order consistent with the indices, so that the first bits of the packet fill the codeword from the lowest indices linearly to the highest indices. The codeword is then transmitted in a linear fashion starting from the lowest indices so that the systematic bits are transmitted first, followed by the parity bits. For shortened codewords the zeros are padded in the low order bits, so that the final codeword starting at the lowest indices contains first zero-padded bits and then the systematic data bits followed by the parity bits.

Appendix (normative) LDPC Code Definition

A full definition of an LDPC code can be accomplished through identification of the locations of the “edges” between the variable nodes (codeword bits) and check nodes (parity relationships). Figure 2 shows a Tanner graph of an example LDPC code, depicting the arrangement of the check nodes, variable nodes, and the “edges” connecting them.

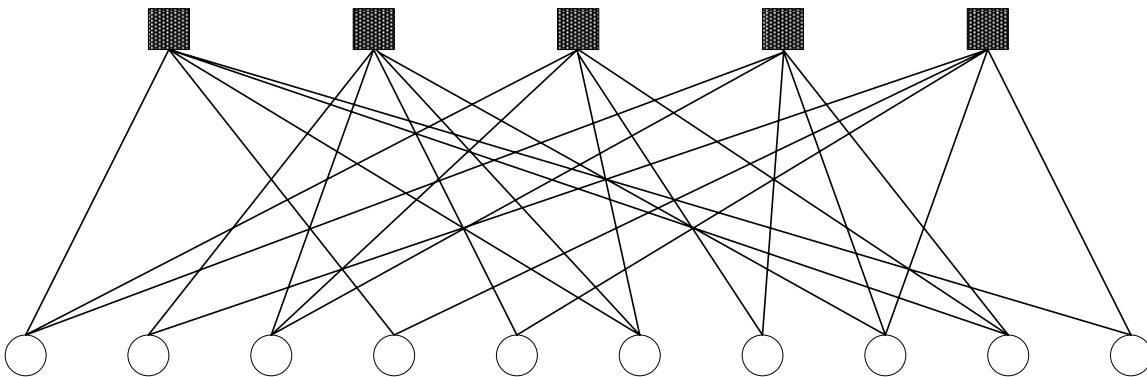


Figure 2. This is an example Tanner graph of an LDPC code, showing the check nodes, variable nodes, and edges. The codeword is made up of the bits represented by the variable nodes. In this case the codeword has ten bits.

Each check node represents a parity relationship between the codeword bits represented by the variable nodes connected to it by the edges. The number of edges connected to a check node is the “degree” of the check node, and the number of edges connected to a variable node is the “degree” of the variable node. For the specified code all check nodes are of degree eighteen, all variable nodes related to the systematic information bits are of degree four, and all variable nodes corresponding to parity bits are of degree two except for the last, which is of degree one.

28 121 170 277
61 273 351 386
71 76 232 328
62 109 190 201
111 162 190 227
189 272 288 302
14 49 147 334
33 53 213 238
53 219 368 379
126 149 188 339
108 118 182 393
0 37 160 295
158 200 335 356
11 20 229 397
77 86 212 250
79 193 262 336
43 104 125 376
55 114 134 293
240 283 299 333
0 24 57 100
46 84 322 341
5 43 45 221
29 217 274 301
81 93 116 278
93 174 213 231
64 201 251 385
76 134 278 370
71 93 182 398
38 174 250 377
19 116 357 372
81 91 164 307
180 186 241 251
239 254 331 342
107 149 250 295
73 221 295 362
75 97 242 279
32 197 244 313
245 248 276 296
59 230 322 347
17 246 291 364
125 157 227 390
122 205 279 348
61 298 340 380
12 31 256 328
119 163 178 217
61 129 185 200
34 38 104 295
119 289 349 377
50 314 322 367
28 48 248 382
32 41 128 201
91 115 220 368
45 151 196 265
152 190 198 317
157 212 242 275
2 40 249 283
195 280 299 345
142 151 220 395
70 121 252 382
52 244 279 297
22 131 256 349
47 52 339 346

50 288 342 388
26 87 247 283
67 127 132 136
146 264 321 323
210 275 319 346
57 160 252 261
26 54 170 197
120 218 229 341
44 53 124 323
0 113 315 358
110 144 246 298
89 91 99 346
21 32 216 393
37 170 209 342
49 58 357 399
18 23 31 373
159 172 195 366
213 335 337 378
1 103 159 277
96 159 209 387
102 165 234 378
173 245 356 376
57 230 240 314
1 89 153 166
25 32 264 342
265 276 321 324
57 211 274 360
12 291 311 348
34 220 258 282
52 58 109 379
116 248 337 369
87 146 183 278
42 96 318 361
32 176 312 361
69 258 310 389
1 84 182 300
45 124 161 396
15 76 99 101
62 248 354 375
78 258 262 311
181 265 364 368
60 168 227 254
162 231 270 377
14 102 139 158
28 79 155 318
28 40 63 236
163 181 258 279
158 176 273 334
80 236 256 380
74 156 214 358
176 229 251 283
19 104 114 162
141 284 291 358
77 123 157 361
141 154 215 338
55 294 296 298
80 109 272 364
43 206 287 363
81 175 206 261
31 94 275 317
10 123 141 279
44 64 157 270

160 243 290 373
39 217 262 324
19 185 312 389
211 271 277 291
19 148 155 324
24 94 124 314
3 85 193 349
68 175 202 253
139 160 337 377
21 224 249 398
113 122 206 327
7 10 156 245
140 182 192 235
161 291 324 387
31 232 237 350
30 184 235 387
136 226 269 327
4 93 136 167
47 148 309 348
73 225 252 290
44 213 361 386
79 319 361 381
74 251 339 356
100 105 246 293
68 101 191 285
32 103 323 355
122 188 228 305
6 77 291 397
70 76 259 276
72 270 335 348
93 147 255 312
92 112 259 388
9 18 61 308
3 137 139 257
165 217 345 354
78 134 263 280
186 213 227 303
68 194 294 346
35 225 284 312
117 188 340 346
258 299 306 331
83 194 207 349
43 141 175 329
0 68 170 262
25 36 153 309
57 62 273 323
7 19 75 264
21 254 259 366
8 97 156 172
9 185 313 330
55 219 253 393
86 120 185 233
41 136 191 242
194 265 303 393
256 285 310 399
103 247 275 378
115 218 225 285
98 196 217 328
177 267 306 350
82 299 320 395
139 251 364 381
42 118 178 194

73 100 198 286
68 249 292 376
13 216 221 256
127 138 177 398
20 69 239 264
3 126 132 163
66 88 169 271
88 197 201 387
1 51 135 149
257 294 331 356
204 260 288 294
45 144 185 383
173 310 329 362
15 165 305 348
27 66 85 182
47 235 238 246
230 276 293 367
118 150 267 324
68 82 309 398
72 154 226 231
76 135 151 384
39 48 80 309
0 178 305 353
88 136 196 321
37 95 222 300
23 343 358 369
195 252 303 349
9 81 102 317
20 219 285 316
219 281 304 354
33 121 319 351
21 157 191 260
0 88 303 307
13 23 62 268
13 173 279 320
117 189 253 392
32 40 57 350
57 123 148 368
18 96 164 326
84 103 107 359
92 338 350 355
16 70 242 338
20 74 141 179
159 246 248 365
207 292 387 399
38 148 303 347
68 113 296 389
12 257 286 325
50 287 294 327
149 259 356 367
3 12 178 309
63 92 166 368
97 190 199 363
13 86 92 308
132 141 221 322
213 257 348 396
91 147 294 325
14 27 48 222
11 81 110 360
10 50 357 393
35 89 248 252
6 55 319 345

107 116 223 271
168 240 261 384
54 204 295 351
3 51 146 299
74 184 307 361
9 202 272 387
106 198 281 329
36 105 225 236
90 139 183 299
152 160 292 354
11 115 227 236
152 202 211 373
4 173 346 374
132 197 238 279
16 94 150 222
241 344 375 386
31 121 161 231
9 33 197 350
87 197 233 312
100 111 129 368
184 278 289 346
76 177 227 356
11 132 246 314
46 93 103 309
20 33 64 196
111 134 194 204
76 116 140 238
189 298 326 381
235 317 320 333
127 301 348 376
51 286 309 377
17 70 139 187
54 180 184 344
85 311 318 327
263 312 364 369
97 149 198 336
31 141 151 285
72 163 187 311
24 54 249 297
64 143 322 360
53 73 122 256
100 138 214 226
265 348 373 378
42 62 113 174
29 313 349 358
154 179 217 268
164 289 380 392
109 165 236 312
92 141 193 238
190 243 267 275
95 143 203 393
130 213 264 308
102 133 217 226
69 88 116 295
108 217 273 322
26 287 306 343
8 18 136 152
110 240 245 334
225 255 278 310
63 168 170 303
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28 92 98 200

112 201 244 392
134 216 344 383
21 97 115 396
28 69 120 380
34 259 267 314
55 72 87 223
43 180 185 252
23 113 133 277
258 285 347 350
246 253 318 399
12 78 90 369
17 93 96 102
109 162 318 360
22 83 151 290
141 191 240 266
25 90 138 390
81 113 265 382
88 142 210 283
10 40 43 140
2 195 268 328
117 240 257 374
298 332 350 365
60 122 240 313
157 215 274 397
11 41 164 274
67 76 92 104
19 192 305 344
23 35 125 224
152 163 352 385
40 161 165 329
113 215 245 378
80 168 262 382
81 136 165 239
2 42 248 323
111 127 157 330
79 125 239 341
147 172 187 397
230 245 277 352
49 202 350 381
34 56 167 242
36 58 61 83
107 110 133 251
100 245 295 330
16 71 175 397
106 206 229 236
177 308 371 387
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9 141 229 306
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269 296 303 356
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98 281 357 389
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107 203 283 322
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47 82 117 126
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295 318 322 377
78 343 373 377
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160 237 274 285
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0 122 269 346
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187 321 355 378
167 226 281 351
0 200 309 384
36 171 193 328
107 178 228 240
80 146 156 375
75 90 290 312
20 55 131 215
99 127 231 344
156 176 301 313
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75 101 209 349
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63 130 188 352
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75 138 262 293
189 300 366 377
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54 108 270 279
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78 93 95 275
145 169 211 278
29 163 300 320
33 147 219 391
199 214 265 280
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31 34 72 115
246 260 267 286
7 266 309 337
24 69 142 394
98 138 228 351
72 181 336 355
12 47 160 172
84 178 230 343
80 238 321 376
170 213 331 367
12 136 274 326

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85 214 362 395
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103 106 146 344
134 268 295 398
120 220 250 354
115 208 355 398
74 190 343 352
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125 132 210 219
60 67 150 203
18 60 167 328
55 112 179 381
288 317 324 389
43 320 334 382
5 29 145 281
25 124 232 345
11 119 339 359
5 36 231 316
15 138 354 389
25 82 136 180
20 103 167 266
112 292 359 371
184 201 240 328
77 160 307 339
74 147 280 389
127 149 358 387
50 59 117 185
11 189 212 220
123 135 226 372
83 86 149 386
26 95 121 163
30 54 178 315
136 301 341 365
21 59 265 299
111 154 282 297
6 74 290 349
121 142 174 236
108 129 152 261
152 164 205 377
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