Project	**IEEE 802.16 Broadband Wireless Access Working Group <http://ieee802.org/16>**	
Title	**Closed-Loop Precoding for STC**	
Date Submitted	**2004-11-12~~04~~**	
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Re:	Contribution on comments to IEEE P802.16e/D5	
Abstract	Closed loop STC precoding to improve performance with limited feedback	
Purpose	Discussion and Adoption	
Notice		
Release		
Patent Policy and Procedures		

Closed-loop Precoding for STC

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1. Introduction

The current IEEE 802.16e-D5 amendment suggests the application of complex weights to all four antennas for the transmission from 4-antenna BS in the AMC permutation zone. This corresponds to a precoding matrix

$$W = \begin{bmatrix} w_0 & 0 & 0 & 0 \\ 0 & w_1 & 0 & 0 \\ 0 & 0 & w_2 & 0 \\ 0 & 0 & 0 & w_3 \end{bmatrix}$$

However, this mode of operation requires an unnecessary large amount of feedback from the MSS to the BS to inform the BS about the settings of the~~se~~ four complex valued weights in W.

In this contribution, first we analytically show how exactly the same received signal-to-noise ratio can be achieved with a significant reduction in the amount of the required feedback information and delay. Then we show how even further reduction of the feedback information and delay (which improves performances at non-zero Doppler frequencies) can be achieved by an appropriate quantization of the feedback information. This contribution also contains the text proposal.

2. The 4-antenna STC with rate 1

In the current specification, section 8.4.8.3.5 describes the STC scheme with the AMC permutation zone using the following matrix:

$$A = \begin{bmatrix} s_1 w_0 & -s_2^* w_0 & 0 & 0 \\ s_2 w_1 & s_1^* w_1 & 0 & 0 \\ 0 & 0 & s_3 w_2 & -s_4^* w_2 \\ 0 & 0 & s_4 w_3 & s_3^* w_3 \end{bmatrix} \qquad (1)$$

The matrix (1) defines the transmission format with the row index indicating the antenna number and column index indicating the subchannel symbol time (2 symbols per entry), the entrees defines the transmission from a subchannel used for this transmission configuration.

It is well known [1], that the Alamouti space time code using a Maximum Likelihood receiver can be described as an equivalent SISO channel. With this notation, the Alamouti space time code has an SNR after space time decoding that is dependent on the channel coefficients in the following way

$$SNR_{A1} \propto \sum_{r=1}^{N_r} |w_0|^2 |h_{r1}|^2 + |w_1|^2 |h_{r2}|^2 \qquad (2)$$

$$SNR_{A2} \propto \sum_{r=1}^{N_r} |w_2|^2 |h_{r3}|^2 + |w_3|^2 |h_{r4}|^2 \qquad (3)$$

where h_{rt}, t=1,2 are the channel coefficients from transmitter antenna t to receiver antenna r for subchannel 1 and h_{rt}, t=3,4 are the channel coefficients from transmitter antenna t to receiver antenna r for subchannel 2 . It can thus be seen in (2),(3) that the phase information of the weights w_0, w_1, w_2, w_3 are not present in the expressions for the SNR and therefore, receive SNR cannot be improved by any adjustment of these phases. Hence, the feedback of phase information in this mode is redundant and consumes unnecessary feedback bandwidth.

Due to the constant power constraint $|w_0|^2 + |w_1|^2 + |w_2|^2 + |w_3|^2 = K$, and with the weight amplitudes quantized to two discrete levels as in IEEE 802.16e-D5, only three feedback bits are required to uniquely set all the four weights in (1). This is a considerable reduction in required amount of feedback data.

3. The 4-antenna STC with rate 2

Section 8.4.8.3.5 describes the STC scheme with the AMC permutation zone with rate 2 using the following matrix:

$$B = \begin{bmatrix} s_1 w_0 & -s_2^* w_0 & s_5 w_0 & -s_7^* w_0 \\ s_2 w_1 & s_1^* w_1 & s_6 w_1 & -s_8^* w_1 \\ s_3 w_2 & -s_4^* w_2 & s_7 w_2 & s_5^* w_2 \\ s_4 w_3 & s_3^* w_3 & s_8 w_3 & s_6^* w_3 \end{bmatrix} \tag{4}$$

With this mode, there will be mutual interference between the space time encoded data streams transmitted from the two pairs of antennas, and at least two antennas are required at the MSS to separate the signals. However, as will be shown below, due to the special structure of the Alamouti space time code using a minimum mean squared error receiver, the SNR for the two substreams does not depend on the absolute values of the phases of the four antenna weights w_0, w_1, w_2 and w_3, but rather on a linear combination of these phases.

Hence, the exactly the same SNR can be obtained if the phases of arbitrary three antenna weights are set to zero, and if the phase of the remaining forth antenna weight is set to an appropriate value. This means that only the phase information of *one* of the four antenna weights needs to be feed back to the receiver, which will lead to a significant reduction in the feedback bandwidth and delay.

Assume that the weights applied to the four transmitted signals are composed of the magnitude and phase as

$$w_i = a_i e^{j\alpha_i} \quad i = 1,..,4 . \tag{5}$$

Using a linear MMSE receiver to detect the two data streams on each subchannel (the symbols s_1 to s_4 in subchannel 1 and symbols s_5 to s_8 in subchannel 2), the SNR for the two data streams can be written as proportional to (the analysis is assuming subchannel 1, an equivalent analysis can be made for subchannel 2)

$$SNR_{B1} \propto K_1 \left(a_2^2 |h_{12}|^2 + a_1^2 |h_{21}|^2 + a_1^2 |h_{11}|^2 + a_2^2 |h_{22}|^2 \right) - 2\varepsilon_1 \operatorname{Re}\{\Lambda\} \tag{6}$$

$$SNR_{B2} \propto K_2 \left(a_3^2 |h_{13}|^2 + a_3^2 |h_{23}|^2 + a_4^2 |h_{14}|^2 + a_4^2 |h_{24}|^2 \right) - 2\varepsilon_2 \operatorname{Re}\{\Lambda\} \tag{7}$$

where

$$\begin{aligned} \Lambda = {} & a_1 a_2 a_3 a_4 \left(e^{j\Delta} \left(h_{12} h_{21} h_{14}^* h_{23}^* - h_{12} h_{21} h_{13}^* h_{24}^* \right) + e^{-j\Delta} \left(h_{11}^* h_{22}^* h_{13} h_{24} - h_{11}^* h_{22}^* h_{14} h_{23} \right) \right) + \\ & + a_1 a_3 h_{11}^* h_{21} h_{13} h_{23}^* + a_1 a_4 h_{11}^* h_{21} h_{14} h_{24}^* + a_2 a_4 h_{12} h_{22}^* h_{14}^* h_{24} + a_2 a_3 h_{12} h_{22}^* h_{13}^* h_{23} \end{aligned} \tag{8}$$

and

$$\varepsilon_1 = \frac{1}{\sigma^2 \left(a_3^2 |h_{13}|^2 + a_4^2 |h_{14}|^2 + a_3^2 |h_{23}|^2 + a_4^2 |h_{24}|^2 + \sigma^2 \right)}, \tag{9}$$

$$\varepsilon_2 = \frac{1}{\sigma^2 \left(a_1^2 |h_{11}|^2 + a_2^2 |h_{12}|^2 + a_1^2 |h_{21}|^2 + a_2^2 |h_{22}|^2 + \sigma^2 \right)}, \tag{10}$$

$$K_1 = \frac{1}{\sigma^2} + \varepsilon_1 \sigma^2 , \tag{11}$$

$$K_2 = \frac{1}{\sigma^2} + \varepsilon_2 \sigma^2 , \tag{12}$$

$$\Delta = \alpha_1 + \alpha_2 - \alpha_3 - \alpha_4 . \tag{13}$$

Several things can now be noted from the expressions above:

- Both SNRs given by (6) and (7) can be maximized *concurrently* by finding the optimum angle Δ.
- By adjusting the weight amplitudes, a_1, a_2, a_3, a_4, the SNR's in (6),(7) will change. However, due to the cross-interference terms, it is not in general possible to achieve the highest achievable SNR for both streams simultaneously for one unique set of weights a_1, a_2, a_3, a_4. If more power is put on one of the substreams, the other one will generally suffer a lower SNR.
- The value Δ depends only on the linear combination of the phases of weights according to (13). We can thus without loss of generality set the combining coefficients for α_2, α_3 and α_4 to be equal to zero and adjust the SNR for both substreams with $\Delta = \alpha_1$ only. This gives a significant reduction in the amount of feedback since only the phase weight of a single antenna needs to be feed back to the transmitter.
- Due to the constant power constraint, $a_1^2 + a_2^2 + a_3^2 + a_4^2 = K$ must hold, so it is only necessary to feed back information about three of the weight amplitudes and then the fourth follows directly from this power constraint. This leads to a further reduction in the amount of required feedback.

The current proposal for the closed-loop STC precoding is to use the weights from the latest 802.16e/D5 specification. According to that solution, 5 bits feedback information is required for each antenna, 1 bit for the amplitude information, and 4 bits for the phase information. Hence, a total of 20 feedback bits are needed to update all antenna weights.

Based on the findings shown above, exactly the same SNR can be achieved by transmitting 7 feedback bits, where 4 bits are for the phase information (required for just 1 weight), and 3 bits are required for the amplitude information. So the amount of feedback bits to update all antenna weights is reduced by almost a factor of 3.

To further reduce, i.e. minimize the amount of feedback, with almost negligible loss in the received SNR (as it is shown by simulations), we propose to quantize the phase information to 4 values. In that way, 2 bits for the phase information are required, and with the 3 bits for the amplitude information, we come up to the total of 5 required feedback bits for each update of the precoding weights for all 4 antennas. The amount of feedback is thus minimized to be contained in a single feedback payload entity of 5 bits. We shall refer to this solution as "minimum feedback" solution.

To compare the performances of the current precoding solution with the other solutions with reduced feedback discussed above, for several discussed feedback configuration we have performed Monte-Carlo evaluations of the raw bit error rate as function of the received SNR. We have assumed BPSK modulation in quasi-static i.i.d Rayleigh fading channel with two receive antennas. The results can be seen in Figure 1~~Figure 1~~~~Figure 1~~.

The adaptation of the weight amplitudes in the SNR expressions (6),(7) can lead to conflicting adaptation criterions due to the fact that an increase of one of the substream's SNR, eventually gives a decrease of the other substream's SNR. Therefore, these two substream SNR's are aimed to be as equal as possible and concurrently also as high as possible. Therefore, the antenna weights are selected to maximize the smallest of the SNR of the two substreams. The feedback delay is assumed to be zero for all evaluated solutions.

As it can be seen in Figure 1~~Figure 1~~~~Figure 1~~, the minimum feedback solution results in negligible raw bit error rate degradation compared to the current STC precoding solution. On the other side, as the current solution requires 4 times more feedback data than the minimized feedback solution, its performances in terms of coded bit error rate and block error rate will be significantly worse than for the minimum feedback solution.

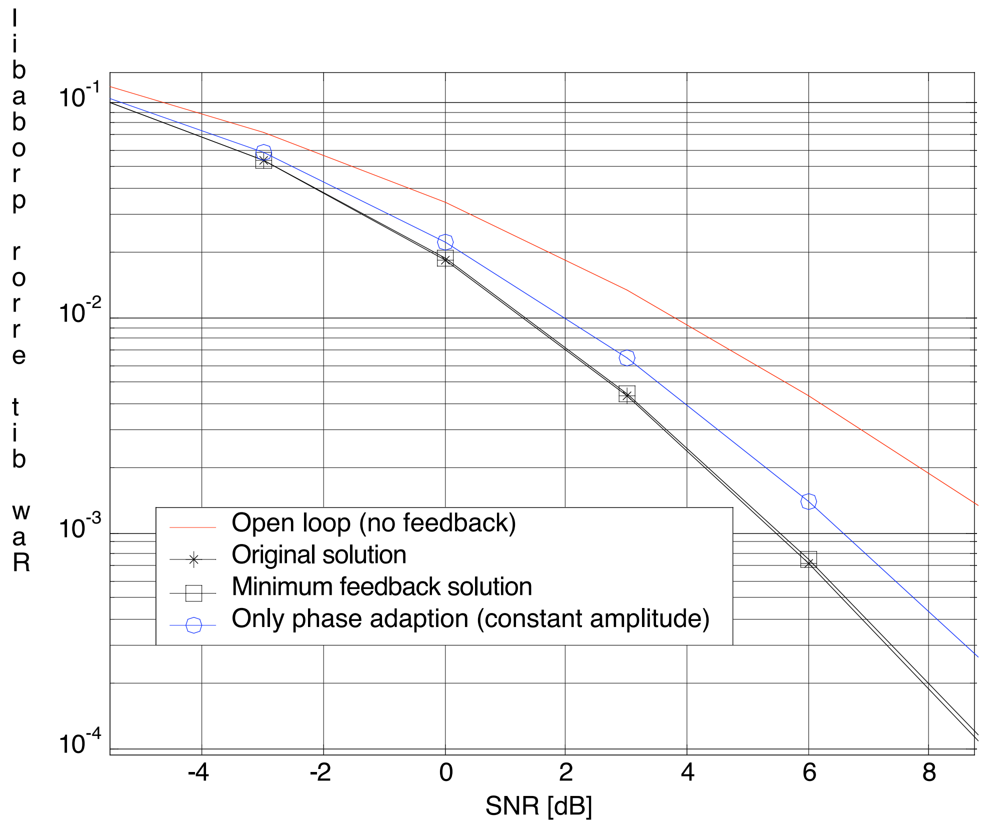

Figure 1 Performances of closed loop precoded STC with several different feedback schemes on quasi-static flat Rayleigh fading channel.

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4. Precoding for STC with 2,3 or 4 BS antennas

The number of outputs of the STC matrices, is denoted in Beceems contribution [4] as the rank of the precoder matrix, and in Samsungs notation as Mt [2],[3]. Furthermore, Nt is the number of BS transmit antennas. For the STC case, which is considered here, we shall assume square precoding matrices[1], hence Mt=Nt. Then, for 2,3, or 4 BS antennas, we can define the precoding matrix W_{PC} as follows, where the amplitudes a_0, a_1, a_2, a_3 and the phase α_0 are real valued variables.

Table 1 Precoding matrices for different rank Mt

Precoding matrix W_{PC}				
Spatial rate 1 (Matrix A)			Spatial rate 2 (Matrix B)	
Rank Mt=2	Rank Mt=3	Rank Mt=4	Rank Mt=3	Rank Mt=4

[1] It is however possible to combine this method with an antenna selection scheme to allow schemes with Nt>Mt.

$\begin{pmatrix} a_0 & 0 \\ 0 & a_1 \end{pmatrix}$	$\begin{pmatrix} a_0 & 0 & 0 \\ 0 & a_1 & 0 \\ 0 & 0 & a_2 \end{pmatrix}$	$\begin{pmatrix} a_0 & 0 & 0 & 0 \\ 0 & a_1 & 0 & 0 \\ 0 & 0 & a_2 & 0 \\ 0 & 0 & 0 & a_3 \end{pmatrix}$	$\begin{pmatrix} a_0 & 0 & 0 \\ 0 & a_1 & 0 \\ 0 & 0 & a_2 \end{pmatrix}$	$\begin{pmatrix} a_0 e^{j\alpha_0} & 0 & 0 & 0 \\ 0 & a_1 & 0 & 0 \\ 0 & 0 & a_2 & 0 \\ 0 & 0 & 0 & a_3 \end{pmatrix}$

As is seen in **Table 1**~~Table 11~~, if the spatial rate is 1, all weights are real valued since as was shown in section 2, there is no spatial cross-interference between the Alamouti encoded streams and therefore, phase weights does not give any additional SNR gain. If the spatial rate is 2 and rank Mt=4, we have the double Alamouti encoded substreams as discussed in section 3 and thus three real valued weights and one complex weight is used.

When the rank is Mt=3, it can be shown, using the calculations above with $h_{14}=h_{24}=0$ that the phase of the weights has no influence on SNR so the weights in this case are also real valued.

The precoding matrix W_{PC} from **Table 1**~~Table 11~~ above can, if desired, be combined with Samsung's antenna grouping scheme [2],[3]

$$W = W_{AG} W_{PC}$$

where W_{AG} is the antenna grouping matrix. The required number of feedback bits for STC precoding, antenna grouping (from(2],[3]) and if these methods are compared in **Table 2**~~Table 22~~. Since antenna grouping and STC precoding with spatial rate 2 and rank Mt=4 requires 8 feedback bits we propose not to use the antenna grouping in this mode. Thereby the required number of feedback bits will for all cases be limited to 5 bits.

Table 2 The required number of feedback bits to select *W* from the codebook .

Number of feedback bits					
	Spatial rate 1 (Matrix A)			Spatial rate 2 (Matrix B)	
Matrix	**Rank Mt=2**	**Rank Mt=3**	**Rank Mt=4**	**Rank Mt=3**	**Rank Mt=4**
W_{PC}	1	3	3	3	5
W_{AG}	0	2	2	2	3
Total	1	5	5	5	8

Example

Assume that Nt=3 antennas is used with a spatial rate 2 STC matrix B of rank Mt=3. Then the precoding matrix is selected from **Table 1**~~Table 11~~ as

$$W = W_{AG}\begin{pmatrix} a_0 & 0 & 0 \\ 0 & a_1 & 0 \\ 0 & 0 & a_2 \end{pmatrix}$$

where W_{AG} is selected as one of the following matrices

$$W_{AG} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \text{or} \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

A total of 3+2 = 5 bits is in this case required to determine the final precoding matrix W.

5. Proposed text changes

~~*[Add the following text into section 8.4.8.3.5]*~~

~~When the AMC permutation zone is chosen, BS may further enhance the system performance by multiplying complex weights to antennas as follows;~~

$$A = \begin{bmatrix} s_1 w_0 & -s_2^* w_0 & 0 & 0 \\ s_2 w_1 & s_1^* w_1 & 0 & 0 \\ 0 & 0 & s_3 w_2 & -s_4^* w_2 \\ 0 & 0 & s_4 w_3 & s_3^* w_3 \end{bmatrix}$$

$$B = \begin{bmatrix} s_1 w_0 & -s_2^* w_0 & s_5 w_0 & -s_7^* w_0 \\ s_2 w_1 & s_1^* w_1 & s_6 w_1 & -s_8^* w_1 \\ s_3 w_2 & -s_4^* w_2 & s_7 w_2 & s_5^* w_2 \\ s_4 w_3 & s_3^* w_3 & s_8 w_3 & s_6^* w_3 \end{bmatrix}$$

$$C = \begin{bmatrix} s_1 w_0 \\ s_2 w_1 \\ s_3 w_2 \\ s_4 w_3 \end{bmatrix}$$

~~The weights w_0, w_1, w_2 and w_3 are defined by the 5 bits in the payload $b_4b_3b_2b_1b_0$ as follows. The bits b_0, b_1 and b_2 determines the absolute value of the weight for antenna 0,1 and 2.~~

~~b_0~~	~~b_1~~	~~b_2~~	~~w_0~~	~~w_1~~	~~w_2~~
~~0~~	~~0~~	~~1~~	~~$\sqrt{1/2}$~~	~~$\sqrt{1/2}$~~	~~$\sqrt{2}$~~
~~0~~	~~1~~	~~0~~	~~$\sqrt{1/2}$~~	~~$\sqrt{2}$~~	~~$\sqrt{1/2}$~~
~~0~~	~~1~~	~~1~~	~~$\sqrt{1/2}$~~	~~$\sqrt{2}$~~	~~$\sqrt{2}$~~
~~1~~	~~0~~	~~0~~	~~$\sqrt{2}$~~	~~$\sqrt{1/2}$~~	~~$\sqrt{1/2}$~~
~~1~~	~~0~~	~~1~~	~~$\sqrt{2}$~~	~~$\sqrt{1/2}$~~	~~$\sqrt{2}$~~
~~1~~	~~1~~	~~0~~	~~$\sqrt{2}$~~	~~$\sqrt{2}$~~	~~$\sqrt{1/2}$~~

~~All others~~	~~unused~~

~~The weight w_3 is real and determined from $\sum_{i=0}^{3}|w_i|^2 = 5$. The bits b_3 and b_4 determines the phase of w_0 as $\arg(w_0) = \alpha_0$ in matrix B as follows~~

~~b_3~~	~~b_4~~	~~α_0~~
~~0~~	~~0~~	~~$\pi/2$~~
~~0~~	~~1~~	~~π~~
~~1~~	~~0~~	~~0~~
~~1~~	~~1~~	~~$-\pi/2$~~

~~In matrix A, b_3 and b_4 are unused and set to $b_3=0$ and $b_4=0$.~~

The text proposal considers the STC precoding and assumes that 5 feedback bits is used. Hence the codebook contains 32 entries. The proposal can be modified to 6 feedback bits if this becomes of interest.

[Add the following text into section 8.4.8.3.6]

When STC matrix A or B is used, the precoding matrix W is taken from a codebook with 32 entries. The codebook matrices are described as

$$W = W_{AG} W_{PC}$$

where the matrix W_{PC} is defined for 2, 3 and 4 BS transmit antennas as

$$W_{PC} = \begin{pmatrix} a_0 & 0 \\ 0 & a_1 \end{pmatrix} \quad W_{PC} = \begin{pmatrix} a_0 & 0 & 0 \\ 0 & a_1 & 0 \\ 0 & 0 & a_2 \end{pmatrix} \text{ and } W_{PC} = \begin{pmatrix} a_0 e^{j\alpha_0} & 0 & 0 & 0 \\ 0 & a_1 & 0 & 0 \\ 0 & 0 & a_2 & 0 \\ 0 & 0 & 0 & a_3 \end{pmatrix}$$

respectively and W_{AG} is described below in the case of 3 or 4 BS antennas. For 2 BS antennas, antenna grouping is not used and thus $W_{AG} = I$.

The nonzero elements in W_{PC} are defined by the 5 bits in the feedback payload $b_4b_3b_2b_1b_0$. The bits b_0, b_1 and b_2 determines the real coefficients a_0, a_1, a_2 and a_3 as in Table 3~~Table 3~~3

Table 33 Mapping of feedback bits to the amplitude of the elements in the matrix W_{PC}

b_0	b_1	b_2	a_0	a_1	a_2	a_3
0	0	0	Unused			
0	0	1	$\sqrt{1/2}$	$\sqrt{1/2}$	$\sqrt{2}$	$\sqrt{2}$
0	1	0	$\sqrt{1/2}$	$\sqrt{2}$	$\sqrt{1/2}$	$\sqrt{2}$
0	1	1	$\sqrt{1/2}$	$\sqrt{2}$	$\sqrt{2}$	$\sqrt{1/2}$
1	0	0	$\sqrt{2}$	$\sqrt{1/2}$	$\sqrt{1/2}$	$\sqrt{2}$
1	0	1	$\sqrt{2}$	$\sqrt{1/2}$	$\sqrt{2}$	$\sqrt{1/2}$
1	1	0	$\sqrt{2}$	$\sqrt{2}$	$\sqrt{1/2}$	$\sqrt{1/2}$
1	1	1	Unused			

If STC matrix A is used or STC matrix B with 3 BS transmit antennas is used, then the bits b_3 and b_4 determine the antenna grouping matrix W_{AG} as described in Table 4~~Table 4~~4

Table 44 Mapping of feedback bits to select the antenna grouping matrix W_{AG}

b_3	b_4	3 BS Antennas	4 BS Antennas
0	0	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$
0	1	$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$
1	0	$\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix}$
1	1	Unused	Unused

otherwise, if STC matrix B is used with 4 BS transmit antennas, then no antenna grouping is used, hence $W_{AG} = I$ and the bits b_3 and b_4 determines the phase α_0 as in Table 5~~Table 5~~5.

Table 55 Mapping of feedback bits to the phase of element (1,1) in matrix W_{PC}

b_3	b_4	α_0
0	0	$\pi/2$
0	1	π
1	0	0
1	1	$-\pi/2$

6. References

[1] S.Sandhu and A.Paulraj, "Space Time Block Codes: A capacity Perspective", IEEE Communication Letters, vol.4 no.12, pp.384-386, Dec 2000.

[2] Samsung, "Enhancement of Rate 1 STC with Antenna Grouping", C802.16e-04/554

[3] Samsung, "Enhancement of Rate 2 STC with Antenna Grouping", C802.16e-04/555

[4] Beceem, "Closed Loop MIMO Precoding", C802.16e-04/293r2