Project	**IEEE 802.16 Broadband Wireless Access Working Group <http://ieee802.org/16>**
Title	**Improved Feedback for MIMO Precoding**
Date Submitted	**2004-11-12**
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Re:	
Abstract	Improved Feedback for 802.16e MIMO Precoding
Purpose	Adoption of proposed changes into P802.16e ~~Crossed-out indicates deleted text,~~ underlined blue indicates new text change to the Standard
Notice	
Release	
Patent Policy and Procedures	

Improved Feedback for 802.16e MIMO Precoding

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Abstract

Subsection 8.4.5.4.10.6 in 16e D5 standard [1] depicts a MIMO feedback format for downlink MIMO transmit beamforming. The format employs a 32-point constellation (indexed by 5 bits) to quantize each complex element of a channel matrix. The total feedback overhead is $5\times m\times n\times N_f$ bits, where m, n, and N_f are the numbers of receive antennas, transmit antennas, and subcarriers respectively. The overhead is large and may not be supported by CQICH mechanism in 16e since there are only 64 CQICH channels available.

In this contribution, a compact feedback scheme is proposed. The feedback scheme can reduce the overhead in the draft standard by a factor of more than 3 at the cost of additional computations. The feedback overhead is 1-4 CQICH channels per 36 subcarriers for up to 8x4 MIMO configuration. The overhead reduction is achieved by four means. First, the receiver feeds back transmit beamforming vectors instead of the channel matrix. This reduces the overhead by a factor of more than 1.6 on average. Secondly, the elements of each beamforming vector are jointly quantized by vector quantization using small vector codebooks. The vector quantization reduces the overhead by a factor of two compared to the scalar quantization in current draft. Thirdly, the frequency coherence is exploited. Frequency domain downsampling and interpolation are employed. The beamforming matrix feedback is conducted for every 36 subcarriers. Finally, the scheme feeds back the beamforming vectors only for the active spatial channels. This provides a significant overhead reduction in the case of spatial channel puncture, where the spatial channel corresponding to the weakest eigenmode is usually punctured.

1 Signal Model and Current Scheme

The system model is illustrated by an example in Figure 1, where the base station (BS) has 4 antennas and the subscriber station (SU) has 3. The channel matrix, **H**, between the two devices consists of the complex channel gains between each transmit and receive antenna pair. The feedback scheme in subsection 8.4.5.4.10.6 in 16e D5 standard [1] quantizes each element of **H** using a 32-point constellation shown in Figure 2. Each element takes 5 bits to index and each matrix consumes a feedback overhead of $5\times m\times n$ bits, where m and n are the numbers of receive and transmit antennas respectively. After receiving the indexes, the transmitter reconstructs a quantized channel matrix and computes a beamforming matrix based on the reconstructed channel matrix.

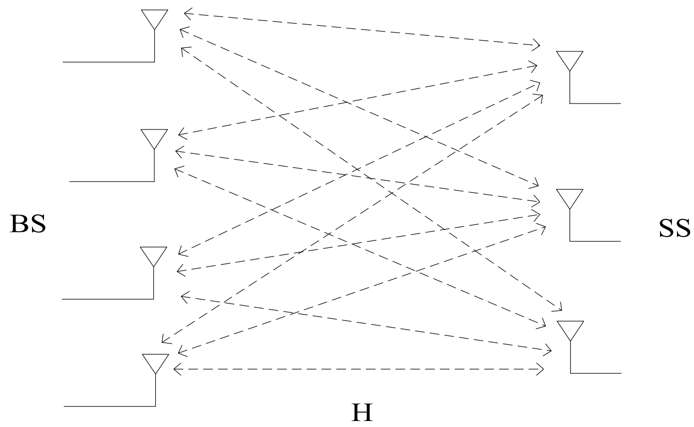

Figure 1. MIMO system model with 4 and 3 antennas at BS and SS respectively.

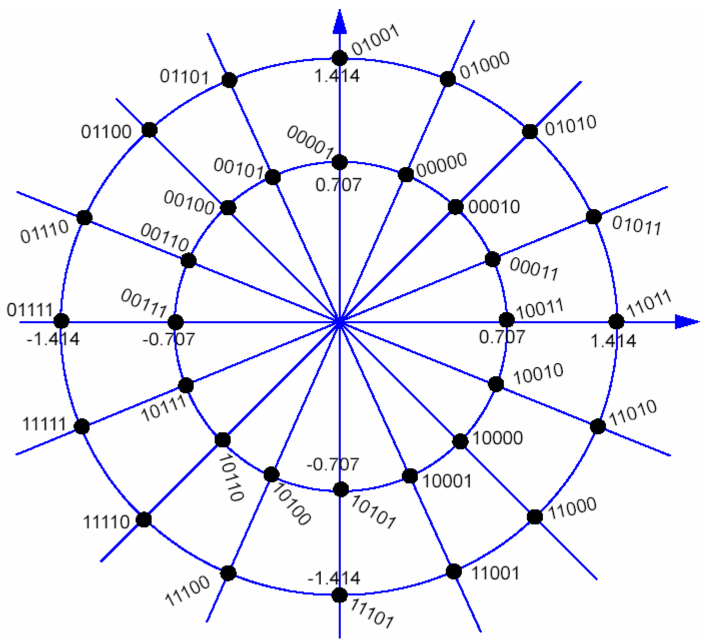

Figure 2 Mapping of complex channel gain in current draft.

2 Compact Feedback Scheme

A compact feedback scheme is proposed to reduce the overhead by a factor from 3.5 to 10 at the cost of additional computations. The overhead reduction is achieved by three means. First, the receiver feeds back transmit beamforming vectors instead of the channel matrix. This reduces the overhead by a factor of more than 1.6 on average. Second, the elements of each beamforming vector are jointly quantized by vector quantization using three small codebooks of sizes 16, 32 and 64 respectively. The vector quantization reduces the overhead by a factor of two compared to the scalar quantization in current draft. Finally, the scheme feeds back the beamforming vectors only for the active spatial channels. This provides a significant overhead reduction in the case of spatial channel puncture, where the spatial channel corresponding to the weakest eigenmode is usually punctured.

2.1 Compute Beamforming Vector — SVD Algorithm

The optimal transmit beamforming scheme is the SVD algorithm [5], where the transmit beamforming matrix is obtained through singular value decomposition of the channel matrix **H**.

$$\mathbf{H}_{m\times n} = \mathbf{U}_{m\times m}\mathbf{D}_{m\times n}\mathbf{V}'_{n\times n} \quad \textbf{(1)}$$

$$\mathbf{x}_{n\times 1} = \mathbf{V}_{n\times n}\,\mathbf{d}_{n\times 1} \quad \textbf{(2)}$$

where **d** is the n-vector of data symbols containing k non-zero elements, where k is the number of spatial channels (see next paragraph); **x** is the beamformed, transmitted signal vector on n transmit antennas; **H** is the channel matrix; **H**'s singular value decomposition is **H** = **UDV**'; **U** and **V** are unitary; **D** is a diagonal matrix with **H**'s eigenvalues; **V** is n by n. Expression (2) is the beamforming step at the transmitter after the beamforming matrix **V** is feedback.

The SS(subscriber station) estimates the channel **H** from pilots and/or midamble. The SS computes and feedbacks the beamforming matrix **V** in **(1)**. If SS knows beforehand that the BS only employs k spatial streams, the SS may only feedbacks the first k columns of the **V** matrix, which corresponds to the k strongest eigenmodes of **H**. This offers another overhead reduction, which can't be provided by that in current draft. The degree of freedom of **H** is $2n^2$ while the degree of freedom of **V** is $n^2 - n$ for m=n. Since only V is useful for transmit beamforming and **V** contains less information than **H**, feeding back **V** is more efficient than feeding **H**.

2.2 Quantize Beamforming Matrix — Column by Column VQ

We illustrate the quantization of the beamforming matrix **V** by the example in Figure 1, where there are 4 transmit antennas and 3 receive antennas. The generalization of the example to any other antenna configuration is straightforward. The receiver received pilots and computed the beamforming matrix, **V** in (1). Next, the receiver only needs to quantize the first 3 columns of **V** since the channel supports at most three modes. If the receiver knows the transmitter only employs two spatial channels, the receiver only quantizes the first two columns of **V** in the scheme depicted next.

The **V** matrix is quantized column by column and recursively as illustrated in Figure 3. After the quantization of one column, we reduce the size of the problem by one on both row and column dimensions. Denote the beamforming matrix as

$$\mathbf{V} = \begin{bmatrix} v_{11} & v_{12} & v_{13} \\ v_{21} & v_{22} & v_{23} \\ v_{31} & v_{32} & v_{33} \\ v_{41} & v_{42} & v_{43} \end{bmatrix}. \qquad \textbf{(3)}$$

We quantize the first column of **V** denoted as $\mathbf{v}_1$ as follows.

$$\hat{\mathbf{v}}_1 = \arg\max_{\mathbf{u}\in C_1} \left\| \mathbf{u}^H \mathbf{v}_1 \right\| \qquad \textbf{(4)}$$

where C_1 is a codebook containing unit 4-vectors for quantization shown in Appendix A.3. $\hat{\mathbf{v}}_1$ has the maximum inner product among all unit vectors in the codebook. The codebook is constructed such that the codeword vectors distribute on the n-dimension complex unit sphere as uniformly as possible. Additionally, the first element of each codeword is set to be real for the next step.

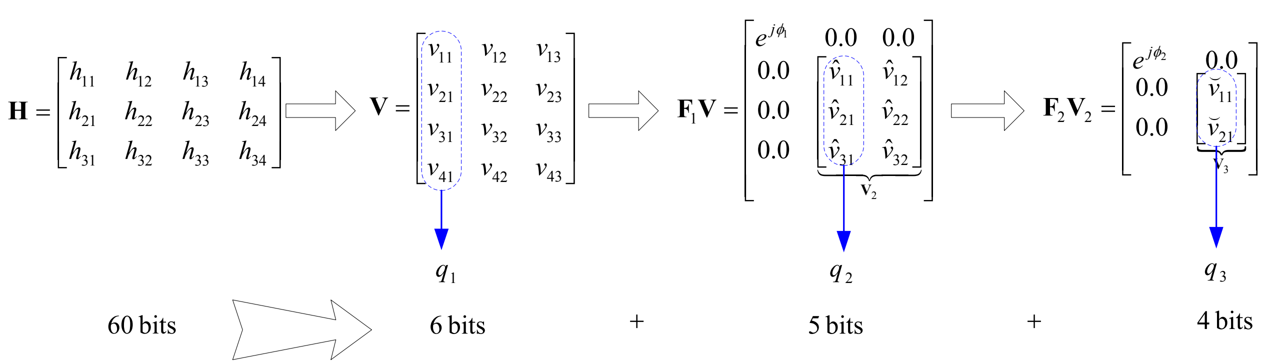

Figure 3 Illustration of the iterative quantization of beamforming matrix.

A Householder reflection matrix [6] is constructed as follows

$$\mathbf{F}_1 = \mathbf{I} - \frac{2}{\left\|\mathbf{w}_1\right\|^2} \mathbf{w}_1 \mathbf{w}_1^H , \qquad \textbf{(5)}$$

where $\mathbf{w}_1 = \hat{\mathbf{v}}_1 - \mathbf{e}_1 = \begin{bmatrix} \hat{v}_{11} - 1 \\ \hat{v}_{21} \\ \hat{v}_{31} \\ \hat{v}_{41} \end{bmatrix}$. If $\hat{\mathbf{v}}_1 = \mathbf{v}_1$, Householder reflection converts the first column and row of $\mathbf{V}$ into $[e^{j\phi_1} \quad 0 \quad 0 \quad 0]^T$ and $e^{j\phi_1}[1 \quad 0 \quad 0]$ as shown in (6), where ϕ_1 is the phase of v_{11}. Since usually $\hat{\mathbf{v}}_1 \approx \mathbf{v}_1$, there will be nonzero residuals in the off diagonal entries of the first column and row.

$$\mathbf{F}_1\mathbf{V} = \begin{bmatrix} e^{j\phi_1} & 0.0 \quad 0.0 \\ 0.0 & \\ 0.0 & \underbrace{\begin{bmatrix} \hat{v}_{11} & \hat{v}_{12} \\ \hat{v}_{21} & \hat{v}_{22} \\ \hat{v}_{31} & \hat{v}_{32} \end{bmatrix}}_{\mathbf{V}_2} \\ 0.0 & \end{bmatrix}, \tag{6}$$

where two properties are employed to get the result, i.e. $\hat{v}_{11}$ is real and the unitary property of $\mathbf{V}$. Since both $\mathbf{F}_1$ and $\mathbf{V}$ are unitary, $\mathbf{V}_2$ is unitary. From (6), we see that the size of $\mathbf{V}_2$ is 3 by 2 and it is reduced from that of $\mathbf{V}_1$ by one on both row and column dimensions. Recursively, we repeat steps in (4), (5), and (6) on $\mathbf{V}_2$ as follows. First, we quantize the first column of $\mathbf{V}_2$ denoted as $\mathbf{v}_2$, using another codebook of unit 3-vectors shown in Appendix A.2, whose first element of each codeword is real. Then, we construct a Householder reflection matrix and multiply it with $\mathbf{V}_2$ as follows. These steps are illustrated in Figure 3.

$$\mathbf{F}_2\mathbf{V}_2 = \begin{bmatrix} e^{j\phi_2} & 0.0 \\ 0.0 & \\ 0.0 & \underbrace{\begin{bmatrix} \breve{v}_{11} \\ \breve{v}_{21} \end{bmatrix}}_{\mathbf{V}_3} \end{bmatrix} \tag{7}$$

Finally, we quantize the vector $\mathbf{v}_3$ using a codebook of unit 2-vectors. The quantization indexes of $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ are feedback to the BS, i.e. the transmitter, for beamforming. It is worth noting that the phases ϕ_i do not need to be sent back. For high speed and low complexity, the codebooks are optimized so that their sizes are no greater than 64. Since the codebook size is small, the Householder matrix for each codeword can be stored beforehand to reduce computational complexity.

The pseudo code of quantization algorithm for a general beamforming matrix is listed as follows.

1. Compute singular value decomposition of the downlink channel matrix $\mathbf{H}$ with size *m* by *n* as **(1)**, and obtain the first *k* columns of the beamforming matrix $\mathbf{V}$, where *k* is the number of active spatial channels.
2. Let $\widetilde{\mathbf{V}} = \mathbf{V}_{:,1:k}$, which is a temporary matrix and is formed by the first *k* columns of $\mathbf{V}$.
3. For $i = 1 : \min(k,n-1)$
   3.1. Let $\mathbf{v}_i = \widetilde{\mathbf{V}}_{:,1}$, which is the first column of $\widetilde{\mathbf{V}}$.
   3.2. Quantize $\mathbf{v}_i$ by finding $\hat{\mathbf{v}}_i = \arg\max_{\mathbf{u}\in C_i} \left\| \mathbf{u}^H \mathbf{v}_i \right\|$, where C_i is a codebook of unit n-i+1vectors.
   3.3. Record the index of $\hat{\mathbf{v}}_i$ in the codebook for feedback.
   3.4. Construct a Householder reflection matrix as $\mathbf{F}_i = \mathbf{I} - \frac{2}{\left\|\mathbf{w}_i\right\|^2}\mathbf{w}_i\mathbf{w}_i^H$, where $\mathbf{w}_i = \hat{\mathbf{v}}_i - \mathbf{e}_1$ and e_1 is the unit vector with all zero elements except the first equal to one.
   3.5. Conduct Householder reflection on $\widetilde{\mathbf{V}}$ as $\widehat{\mathbf{V}} = \mathbf{F}_i\widetilde{\mathbf{V}}$. To reduce complexity, one only needs to compute columns and rows of $\widehat{\mathbf{V}}$ other than the first one.
   3.6. Update $\widetilde{\mathbf{V}} = \widehat{\mathbf{V}}_{2:n-i+1,2:k}$.
4. End

Since there is strong coherence between adjacent subcarriers, frequency domain downsampling can be employed. In Band AMC mode, the pilot insertion rate is 1/9 but the feedback rate can be 1/36. The matrix feedback can be feedback for every 36 subcarriers. This significantly reduces the overhead. Interpolation is required to recover the beamforming matrixes on skipped subcarriers.

2.3 Reconstruction of Beamforming Matrix

At the transmitter side, the reconstruction of the beamforming matrix $\mathbf{V}$ is as follows and shown in Figure 4.

$$\hat{\mathbf{V}} = \mathbf{F}_1 \begin{bmatrix} 1 & 0 \quad 0 \\ 0 & \\ 0 & \mathbf{F}_2 \begin{bmatrix} 1 & 0 \\ 0 & \\ 0 & \hat{\mathbf{V}}_3 \end{bmatrix} \\ 0 & \end{bmatrix}$$

q_1 q_2 q_3

Figure 4 Illustration of the reconstruction of beamforming matrix at BS

It starts from the lowest dimension and recursively constructs the whole matrix. In each step, a Householder matrix is computed from a reconstructed unit vector. The Householder matrix can be computed and stored beforehand for small codebooks. Even in the case that there is no quantization error, the reconstructed matrix could be different from the original $\mathbf{V}$ by a global phase on each column and this is fine with closed loop MIMO. First, two vectors, $\mathbf{v}_3$ and $\mathbf{v}_2$, are reconstructed using the feedback quantization indexes and the corresponding 2-vector and 3-vector codebooks. Second, we compute a Householder matrix using the reconstructed $\hat{\mathbf{v}}_2$ as

$$\mathbf{F}_2 = \mathbf{I} - \frac{2}{\|\mathbf{w}\|^2}\mathbf{w}\mathbf{w}^H, \tag{8}$$

where $\mathbf{w} = \hat{\mathbf{v}}_2 - \mathbf{e}_1$ and $\hat{\mathbf{v}}_2$ is the reconstructed 3-vector; $\mathbf{F}_2$ can be stored beforehand to reduce computation. Third, $\mathbf{V}_2$ can be reconstructed as

$$\hat{\mathbf{V}}_2 = \mathbf{F}_2 \begin{bmatrix} 1 & 0 \\ 0 & \\ 0 & \hat{\mathbf{v}}_3 \end{bmatrix} \tag{9}$$

Fourth, we reconstruct the first column of $\mathbf{V}$ using the quantization index and compute a Householder matrix as

$$\mathbf{F}_1 = \mathbf{I} - \frac{2}{\|\mathbf{w}\|^2}\mathbf{w}\mathbf{w}^H, \tag{10}$$

where $\mathbf{w} = \hat{\mathbf{v}}_1 - \mathbf{e}_1$ and $\hat{\mathbf{v}}_1$ is the reconstructed first column of $\mathbf{V}$. Finally, the beamforming matrix $\mathbf{V}$ is given by

$$\hat{\mathbf{V}} = \mathbf{F}_1 \begin{bmatrix} 1 & 0 \quad 0 \\ 0 & \\ 0 & \hat{\mathbf{V}}_2 \\ 0 & \end{bmatrix}. \tag{11}$$

Since the codebook size is less than 64, which is small, the Householder matrix for each codebook entry can be stored beforehand to speedup the reconstruction. Interpolation is conducted on the fed back matrixes to recover the beamforming matrixes on the skipped subcarriers.

2.4 Overhead Computation

Table 1 The feedback overheads of proposed scheme for seven antenna and data stream configurations are listed. The notation $n \times m$ means *n* transmit antennas and *m* receive antennas. The numbers are the feedback index bits for every 36 subcarriers.

	2x2, 1 stream	4x2, 1 stream	4x2, 2 streams	4x4, 2 streams	4x4, 4 streams	8x4, 2 streams	8x4, 4 streams
Bits per 36 subcarriers	4 bits	6 bits	10 bits	12 bits	12 bits	15 bits	24 bits
CQICHs per feedback (6 bits per CQICH)	1	1	2	2	2	3	4

Table 2 The feedback overheads of current 802.16e scheme for seven antenna and data stream configurations are listed. The notation $n \times m$ means *n* transmit antennas and *m* receive antennas. The numbers are the feedback index bits for every 36 subcarriers.

	2x2, 1 stream	4x2, 1 stream	4x2, 2 streams	4x4, 2 streams	4x4, 4 streams	8x4, 2 streams	8x4, 4 streams
Bits per 36 subcarriers	20 bits	40 bits	40 bits	80 bits	80 bits	160 bits	160 bits
CQICHs per feedback (6 bits per CQICH)	4	7	7	14	14	27	27

The comparison between Table 1 and Table 2 demonstrates that the proposed scheme reduced the feedback overhead by a factor 4-10.

4 Simulations

We evaluate the throughput performance of the proposed scheme by simulations. Two antenna configurations for closed-loop MIMO, i.e. 4x2 and 8x4 with up to 4 spatial streams, are simulated. All seven modulation coding schemes (MCSs) of 802.16e are simulated for all available number streams. We employed 10 MHz bandwidth with band AMC structure. MMSE receiver is assumed at the receiver. ITU-R channel model, pedestrian B [3] with antenna correlations 0.2 and 0.5 are employed for 4 transmit antennas and 8 transmit antennas. Packet error rates (PERs) are generated. The packet size is 64 bytes, which is the length of TCP/IP acknowledgement. In the legend, ‘closed-loop’ is the proposed scheme and ‘open loop’ is spatial multiplexing scheme in 802.16e without space-time coding. The throughput hull curves are computed using PERs for all available MCSs for each antenna configurations, whose results are shown in Figure 5 and Figure 6. The proposed feedback scheme achieve 4-7 dB gain over the open loop as the cost of 1-4 CHQICH feedback channels per AMC band.

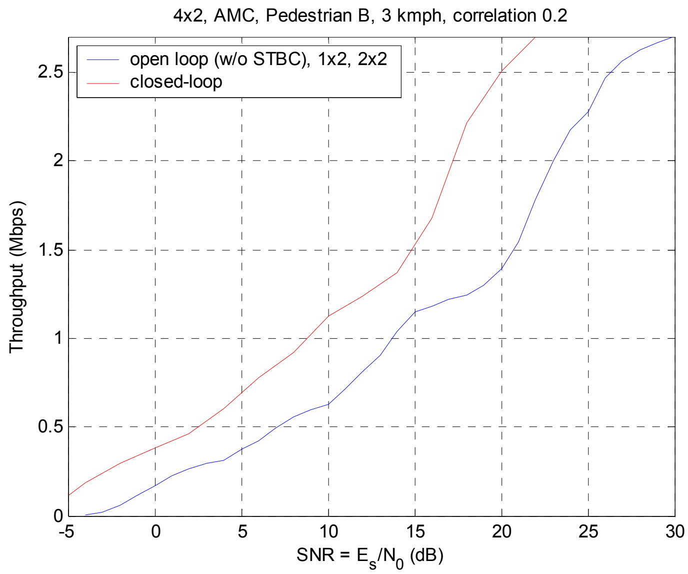

Figure 5 Throughput of closed-loop and open loop MIMO with up to 2 streams and up to 4x2 antenna configuration.

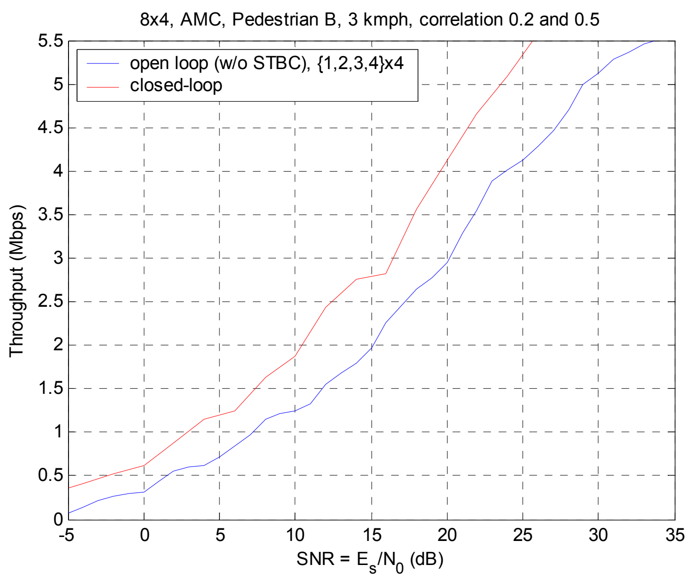

Figure 6 Throughput of closed-loop and open loop MIMO with up to 4 streams and up to 8x4 antenna configuration.

5 Conclusion

We proposed an efficient feedback scheme for the 802.11e closed-loop MIMO. Compared to that in the existing draft, the new scheme has an overhead at least four times lower. This proposal does require additional SVD computation and codebook searching on the subscriber side, which requires only about 11k gates.

6 Specific Text Changes

Replace section 8.4.5.3.17.2 in page 128 of [1] as follows

8.4.5.3.17.2 Fast MIMO feedback

When the FAST_FEEDBACK subheader Feedback Type field is '01' or '10', the SS shall report the MIMO beam forming vector index so that beamforming vectors can be generated accordingly at BS using the calculation in **8.4.5.3.17.2.1**.

Add section 8.4.5.3.17.2.1 on page 128 of [1] as follows

8.4.5.3.17.2.1 Beam forming vector calculation

The receiver of the beam forming vector index shall reconstruct the beam forming vector according to the following algorithm.

1 Receive indices $n_i, i = 1:k$.

2 If the spatial stream is less than m, the number of transmitter antennas:

  2.1 $\widehat{V}_k = \hat{v}_{n_k}$, the n_kth element of the codebook of (m-k+1) dimension unit vector.

  2.2 For i=k-1:-1:1

    2.2.1 $\hat{v}_i = \text{the } \mathrm{n_i} \text{th element of the codebook of } (\mathrm{m - i + 1}) \text{ dimension unit vector}$

    2.2.2 $F_i = I - \frac{2}{\|w\|^2} ww^H, w = \hat{v}_i - e_1$

    2.2.3 $V_i = F_i \begin{bmatrix} 1 & 0 \\ 0 & \hat{V}_{i+1} \end{bmatrix}$

  2.3 End

  2.4 $V = V_1$

3 Else

  3.1 $\widehat{V}_m = [1]$

  3.2 For i=m-1:-1:1

    3.2.1 $\hat{v}_i = \text{the } \mathrm{n_i} \text{th element of the codebook of } (\mathrm{m - i + 1}) \text{ dimension unit vector}$

    3.2.2 $F_i = I - \frac{2}{\|w\|^2} ww^H, w = \hat{v}_i - e_1$

    3.2.3 $V_i = F_i \begin{bmatrix} 1 & 0 \\ 0 & \hat{V}_{i+1} \end{bmatrix}$

  3.3 End

  3.4 $V = V_1$

4 End

Add Annex G at the end of [1]

Annex G, codebooks

These optimized codebooks are chosen to minimize the mean quantization errors. These codebooks are equivalent to any of its unitary transformed versions. In choosing the normalization, we made the following choices: The first entry is a unit vector in $\hat{e}_1$ direction and the first element of all codebook entries is real. This choice minimizes the number of memory elements. Each n-vector m-entry codebook needs $2mn - m - 2n$ real memory spaces.

A.1. Codebook for unit 2-vector with 16 entries

1.0000	0.8997	0.8997	0.8970
0	0.0150 - 0.4362i	0.3612 + 0.2452i	-0.4388 - 0.0533i
0.8969	0.8463	0.7259	0.7250
-0.2129 + 0.3875i	0.4748 - 0.2417i	-0.4396 - 0.5290i	0.1703 + 0.6674i
0.6409	0.6409	0.6102	0.6099
0.3045 - 0.7046i	0.7491 + 0.1679i	-0.7922 - 0.0056i	-0.4612 + 0.6444i
0.3730	0.3722	0.3236	0.2278
-0.3442 - 0.8616i	0.4959 + 0.7845i	0.8426 - 0.4304i	-0.8683 + 0.4406i

A.2. Codebook for unit 3-vector with 32 entries

1.0000	0.7526	0.7509	0.7481
0	-0.3439 - 0.0598i	0.3036 - 0.1884i	-0.0646 - 0.4021i
0	-0.4612 + 0.3148i	0.1404 - 0.5374i	0.5170 - 0.0847i
0.7452	0.7449	0.7439	0.7438
0.2966 + 0.2876i	0.1001 + 0.2808i	0.6040 - 0.2058i	-0.5992 - 0.1147i
-0.3700 + 0.3703i	0.5965 + 0.0199i	0.1521 + 0.1279i	0.2120 + 0.1724i
0.7436	0.7434	0.7425	0.7412
-0.2467 + 0.5858i	0.4184 + 0.4540i	0.0402 + 0.1029i	0.0482 - 0.3614i
-0.0021 + 0.2075i	-0.0535 - 0.2516i	-0.5397 - 0.3810i	0.0199 + 0.5633i
0.7395	0.7170	0.6983	0.4699
-0.2918 + 0.2879i	-0.4693 - 0.2755i	0.0587 - 0.6672i	0.6648 - 0.2402i
0.2295 - 0.4821i	-0.1499 - 0.4091i	-0.2478 - 0.0486i	-0.5151 + 0.1191i
0.3996	0.3786	0.3600	0.3570
-0.1100 + 0.4286i	-0.4105 + 0.4145i	-0.4324 - 0.1688i	0.4915 - 0.2007i
0.1781 + 0.7828i	-0.7176 + 0.0373i	0.7806 + 0.2137i	0.3794 + 0.6684i
0.3527	0.3502	0.3464	0.3366
-0.1710 - 0.1652i	-0.1031 - 0.4821i	0.3551 + 0.2984i	0.2923 - 0.6986i
0.3188 - 0.8470i	-0.6503 - 0.4598i	-0.0099 - 0.8153i	0.3858 - 0.4055i
0.3362	0.3358	0.3305	0.3255
-0.8816 - 0.0760i	0.1212 - 0.0659i	-0.2162 - 0.8560i	0.5691 + 0.7060i
-0.2927 + 0.1350i	-0.7672 + 0.5288i	0.2964 + 0.1529i	-0.1068 + 0.2455i
0.3192	0.3191	0.3172	0.2793
-0.4631 - 0.4748i	0.7029 + 0.3684i	-0.4168 + 0.7629i	-0.0442 + 0.6588i
-0.2546 + 0.6272i	0.4362 - 0.2794i	0.3153 - 0.2104i	-0.5048 - 0.4808i

A.3. Codebook for unit 4-vector with 64 entries

1.0000	0.6899	0.6892	0.6884
0	0.2646 - 0.6236i	-0.4949 + 0.4988i	-0.3373 + 0.0843i

0	-0.1134 + 0.0228i	-0.1389 - 0.0687i	0.0189 - 0.3053i
0	-0.0291 + 0.2257i	0.0640 - 0.0561i	-0.5428 - 0.1305i
0.6882	0.6873	0.6867	0.6867
0.4005 - 0.1592i	0.0675 + 0.2053i	-0.0025 + 0.2047i	0.1403 - 0.3819i
0.0492 - 0.1322i	-0.3177 - 0.4477i	0.4546 + 0.2540i	0.0575 + 0.5078i
0.5584 + 0.0944i	0.2144 - 0.3654i	-0.4570 - 0.0803i	-0.0735 - 0.3103i
0.6865	0.6835	0.6834	0.6815
-0.1019 - 0.1807i	0.4833 + 0.2398i	0.0489 + 0.4950i	-0.0967 + 0.0900i
0.1758 - 0.2421i	-0.0778 - 0.2194i	0.3846 + 0.2144i	-0.3679 + 0.4953i
0.1078 + 0.6202i	-0.4325 - 0.0217i	0.2971 + 0.0584i	0.2643 + 0.2599i
0.6812	0.6811	0.6801	0.6798
0.2106 - 0.0503i	-0.0850 - 0.0071i	0.4167 + 0.4068i	-0.3638 - 0.2822i
-0.0361 + 0.4444i	0.4651 + 0.0155i	-0.2684 + 0.2810i	-0.4686 - 0.2498i
-0.3578 + 0.4028i	0.1476 - 0.5390i	0.1064 - 0.1897i	0.2001 + 0.0626i
0.6779	0.6776	0.6691	0.6674
-0.4620 - 0.2641i	-0.3111 + 0.0354i	0.1055 + 0.4814i	-0.1586 - 0.4905i
0.2995 + 0.2884i	-0.2317 + 0.1869i	-0.1206 - 0.0501i	0.3874 - 0.3680i
0.2473 + 0.1525i	-0.0726 - 0.5907i	-0.0295 + 0.5398i	-0.0082 - 0.0569i
0.6219	0.6158	0.6110	0.6067
0.0306 - 0.2794i	0.5008 - 0.3037i	0.1066 + 0.2804i	-0.5547 + 0.0351i
-0.5549 - 0.1114i	0.1027 - 0.1870i	0.4186 - 0.5915i	-0.0738 + 0.4088i
-0.4610 - 0.0382i	-0.0817 - 0.4749i	-0.0353 + 0.1017i	-0.3616 + 0.1404i
0.6024	0.5944	0.5670	0.4713
-0.2557 + 0.1666i	0.3618 - 0.0342i	0.6426 - 0.0416i	-0.2584 - 0.5426i
-0.0702 + 0.0171i	-0.5930 - 0.2736i	0.4669 + 0.1481i	0.1850 + 0.0064i
0.7304 - 0.0725i	0.1523 + 0.2549i	-0.0506 + 0.1462i	-0.5943 + 0.1709i
0.4671	0.4434	0.4130	0.4033
0.1507 - 0.3379i	-0.3875 + 0.2337i	-0.0986 - 0.4272i	0.1335 - 0.1322i
0.0319 - 0.6058i	-0.2220 - 0.6510i	-0.1590 + 0.4303i	0.6346 + 0.3346i
-0.4595 + 0.2564i	-0.0071 + 0.3543i	0.6257 - 0.1879i	0.4870 + 0.2240i
0.3917	0.3819	0.3741	0.3623
-0.6602 - 0.5622i	-0.3886 + 0.4925i	0.1750 - 0.5460i	0.3505 + 0.3552i
-0.0387 - 0.0060i	0.3083 - 0.3061i	-0.5397 - 0.0018i	0.2157 + 0.2191i
0.0738 - 0.2961i	0.3959 + 0.3392i	0.1165 - 0.4759i	-0.2216 - 0.6900i
0.3581	0.3581	0.3571	0.3413
-0.2724 + 0.5525i	-0.3470 + 0.6183i	-0.5480 + 0.2149i	0.0131 + 0.6704i
-0.1459 + 0.6570i	0.4409 + 0.0466i	0.3061 - 0.5573i	-0.1876 + 0.1707i
-0.0374 - 0.1947i	-0.1185 - 0.3980i	0.0936 - 0.3360i	-0.6079 + 0.0024i
0.3392	0.3385	0.3379	0.3343
0.0093 + 0.3250i	-0.2840 + 0.1067i	0.1396 + 0.3295i	-0.0767 - 0.3157i
-0.8233 + 0.2046i	-0.0565 + 0.3029i	0.5730 + 0.0330i	0.7591 + 0.2427i
-0.2318 - 0.0761i	0.0812 + 0.8317i	0.0396 + 0.6533i	-0.2271 + 0.3099i

0.3173	0.3109	0.2932	0.2850
0.7447 + 0.5251i	-0.2910 - 0.3256i	0.6426 - 0.2371i	0.7010 - 0.2362i
0.0619 - 0.1883i	0.0600 - 0.5515i	-0.5571 + 0.2499i	-0.0449 + 0.4844i
0.1607 + 0.0627i	0.6321 - 0.0733i	-0.2523 - 0.0921i	0.3288 + 0.1636i
0.2803	0.2718	0.2692	0.2611
0.2262 - 0.4122i	-0.1135 - 0.3920i	0.2484 + 0.3635i	-0.6202 + 0.0302i
0.0557 - 0.7946i	0.0387 - 0.2933i	0.3151 - 0.5331i	0.5699 + 0.0380i
0.1077 - 0.2328i	0.1071 - 0.8128i	0.1524 - 0.5718i	-0.4642 - 0.0676i
0.2601	0.2550	0.2543	0.2491
0.5093 - 0.4079i	0.1973 - 0.0627i	0.3491 - 0.0428i	0.4927 + 0.2139i
-0.0508 - 0.5008i	-0.3691 + 0.2462i	0.5519 + 0.5917i	-0.2198 + 0.1684i
0.2102 + 0.4571i	-0.6112 - 0.5672i	0.1156 - 0.3788i	0.7212 - 0.2293i
0.2468	0.2440	0.2299	0.2133
-0.0489 + 0.0375i	-0.6799 - 0.4190i	0.0532 + 0.1712i	-0.6352 + 0.3807i
-0.7189 + 0.1380i	-0.3260 + 0.1995i	0.1764 - 0.2053i	-0.4685 + 0.0174i
0.5304 - 0.3436i	0.0631 + 0.3906i	-0.7566 + 0.5189i	-0.2440 + 0.3560i
0.1948	0.1916	0.1558	0.0304
-0.3185 - 0.1529i	0.1084 + 0.1450i	0.1261 - 0.5681i	-0.5753 - 0.6342i
-0.0069 + 0.9135i	-0.6424 - 0.2670i	-0.0431 + 0.2171i	0.2372 - 0.3286i
0.0505 - 0.0090i	-0.4735 + 0.4716i	0.0910 + 0.7615i	0.0895 + 0.3060i

A.3. Codebook for unit 6-vector with 64 entries

1	0.508	0.508	0.508
0	-0.287-0.206i	0.626+0.269i	-0.461+0.433i
0	0.0437 +0.22i	-0.308+0.107i	0.395-0.216i
0	0.0494-0.205i	0.00293-0.144i	0.164+0.158i
0	-0.433-0.432i	0.0421+0.0512i	-0.0553+0.131i
0	0.0466-0.382i	0.00256-0.382i	0.11-0.235i
0.508	0.508	0.508	0.508
0.0211+0.244i	0.103 -0.4i	-0.0712-0.698i	-0.675+0.0277i
0.162+0.0101i	0.105-0.182i	0.0738-0.324i	-0.0851+0.0829i
-0.226+0.0821i	-0.446-0.307i	0.124+0.188i	0.0318+0.0429i
0.0605 +0.75i	-0.019-0.142i	-0.167+0.123i	0.344-0.0472i
-0.0445 +0.17i	-0.457+0.0662i	0.214-0.00355i	-0.197 +0.33i
0.508	0.508	0.508	0.508
0.156-0.0263i	0.141+0.166i	0.571+0.207i	0.377+0.201i
-0.389+0.0578i	-0.345-0.375i	0.48-0.144i	-0.0662+0.324i
0.251+0.435i	0.237+0.225i	0.237 -0.21i	-0.0327-0.288i
0.395+0.0771i	0.0903-0.493i	-0.0543+0.0997i	0.0355-0.349i
-0.273+0.271i	0.272+0.0531i	0.00944+0.0904i	0.37+0.325i
0.508	0.508	0.508	0.508
0.0108-0.267i	-0.306-0.125i	0.252-0.0481i	-0.204-0.0421i

0.277 +0.25i	-0.101+0.132i	-0.102+0.664i	-0.655+0.109i
-0.358-0.0142i	-0.116-0.225i	0.215+0.308i	0.0617+0.0894i
0.551-0.101i	-0.338+0.158i	-0.271+0.0135i	0.0886+0.022i
0.112+0.278i	0.626 +0.1i	0.0953-0.032i	-0.0926-0.479i
0.508	0.508	0.507	0.507
-0.0287+0.091i	0.0225+0.00871i	-0.156-0.224i	0.0115+0.0479i
-0.621-0.115i	0.246-0.322i	0.248+0.293i	-0.136-0.306i
-0.315-0.232i	0.274-8.94e-005i	0.293-0.593i	-0.138-0.623i
-0.0311+0.278i	0.123-0.243i	0.0527-0.0429i	0.0913-0.253i
-0.177+0.268i	-0.473+0.452i	-0.142+0.241i	0.303 -0.24i
0.506	0.497	0.489	0.484
-0.0155+0.504i	0.0983+0.305i	-0.116+0.141i	-0.0332+0.0207i
-0.0751-0.014i	-0.0154-0.288i	-0.14+0.0411i	0.219-0.596i
-0.13-0.00868i	-0.457+0.437i	0.43-0.371i	-0.148+0.119i
0.282-0.471i	-0.0471+0.105i	-0.17+0.498i	-0.321-0.387i
-0.33-0.238i	0.341-0.194i	0.0135-0.328i	-0.132-0.235i
0.472	0.47	0.458	0.457
-0.281+0.284i	-0.218-0.225i	0.43+0.057i	-0.125+0.517i
-0.239-0.322i	-0.385+0.109i	0.271+0.0942i	0.00743-0.106i
0.142+0.284i	0.156+0.149i	-0.209+0.436i	0.121-0.455i
-0.221+0.266i	-0.367 -0.32i	-0.284+0.162i	-0.472-0.0742i
0.222+0.433i	-0.406+0.269i	-0.416-0.0816i	0.0552+0.212i
0.455	0.436	0.421	0.412
-0.0323-0.281i	0.233 -0.18i	0.0598-0.0779i	0.102+0.265i
0.12+0.176i	0.0867-0.266i	0.371-0.0965i	0.0836+0.257i
-0.389+0.191i	-0.257-0.139i	0.456+0.602i	0.497+0.197i
0.218-0.189i	-0.347+0.388i	-0.0357-0.262i	0.453+0.165i
-0.123-0.617i	-0.00554-0.538i	0.0621-0.146i	-0.0204-0.396i
0.396	0.376	0.366	0.354
-0.483+0.188i	0.365-0.0664i	-0.201+0.312i	-0.406-0.233i
-0.17+0.367i	-0.0668-0.00493i	0.177+0.597i	0.345-0.224i
-0.453+0.355i	0.252-0.277i	-0.0765+0.0957i	0.255-0.299i
-0.261+0.0377i	-0.611+0.0812i	0.16-0.303i	0.163-0.0468i
-0.101-0.00763i	-0.442-0.0352i	0.366-0.271i	-0.298-0.463i
0.347	0.343	0.34	0.335
-0.0755-0.333i	-0.182+0.00104i	-0.0534-0.162i	0.323-0.198i
-0.114 +0.22i	-0.137-0.599i	0.563-0.291i	-0.307-0.167i
0.38-0.0431i	0.355-0.0857i	-0.136-0.115i	-0.211+0.163i
0.318+0.565i	0.471+0.306i	0.258+0.297i	0.179+0.0924i
0.319+0.181i	-0.131+0.0768i	0.5-0.129i	0.628+0.341i
0.328	0.313	0.313	0.28
-0.265-0.213i	-0.00596+0.514i	-0.074+0.203i	0.152-0.387i
0.508+0.425i	0.144+0.354i	-0.0741-0.0136i	0.0376+0.372i
-0.23+0.0792i	-0.513-0.211i	-0.272+0.212i	-0.514+0.244i
-0.0932+0.166i	0.0648+0.00837i	-0.554-0.537i	-0.283+0.369i

-0.437 +0.23i	-0.251+0.341i	0.244+0.277i	0.224+0.136i
0.269	0.269	0.255	0.249
0.622-0.167i	0.308-0.574i	0.381-0.283i	-0.0834+0.212i
-0.0929-0.157i	-0.249-0.163i	-0.536+0.0667i	0.609-0.0318i
-0.127-0.0226i	0.274+0.0286i	0.268-0.536i	-0.465-0.242i
0.427-0.418i	0.51+0.0585i	0.045+0.0963i	0.0914+0.218i
-0.161+0.282i	-0.0209-0.274i	0.129+0.179i	-0.254-0.345i
0.226	0.226	0.224	0.196
0.299+0.623i	0.184+0.011i	-0.475-0.213i	0.308+0.104i
-0.279+0.108i	0.558-0.0655i	-0.15 -0.46i	0.12+0.0509i
0.0854 +0.1i	-0.311-0.191i	-0.291-0.139i	0.481+0.302i
-0.217+0.282i	-0.602-0.129i	0.252-0.165i	-0.263-0.0968i
-0.442+0.208i	-0.137+0.263i	0.393+0.308i	0.311+0.584i
0.184	0.181	0.179	0.162
0.445+0.133i	-0.237-0.00752i	-0.439-0.0603i	0.178-0.0598i
0.265-0.108i	-0.00859+0.226i	0.0153-0.00728i	-0.367+0.00717i
0.243+0.035i	0.312+0.303i	0.476 +0.26i	0.183-0.279i
0.308-0.279i	-0.226+0.516i	0.183 -0.24i	0.228+0.025i
0.58-0.314i	-0.563-0.193i	0.602+0.153i	-0.798+0.0602i
0.154	0.149	0.136	0.133
0.521+0.0506i	-0.414+0.393i	-0.0304 -0.19i	-0.29-0.275i
0.503 +0.41i	-0.125-0.0773i	-0.461-0.0394i	0.576-0.275i
-0.163 +0.12i	-0.101-0.499i	-0.466+0.246i	0.253-0.123i
0.325+0.262i	0.266+0.375i	0.643-0.192i	-0.309-0.281i
0.0712-0.246i	0.329+0.226i	-0.00898-0.0429i	0.312+0.255i
0.131	0.116	0.0966	0.09
0.495-0.344i	-0.458+0.0196i	-0.645-0.251i	-0.408+0.00834i
-0.569-0.258i	-0.351 -0.21i	0.219-0.253i	-0.148 +0.5i
-0.354+0.265i	0.344+0.679i	-0.449-0.121i	-0.245+0.0121i
-0.133+0.0387i	-0.016-0.0751i	-0.133+0.401i	0.334+0.579i
-0.0765-0.095i	-0.119-0.0995i	-0.061-0.0391i	-0.132-0.172i

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