Project	**IEEE 802.16 Broadband Wireless Access Working Group <http://ieee802.org/16>**
Title	**Correlation Models for Shadow Fading Simulation**
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Re:	IEEE 802.16m-07/005r2, “Call for Contributions on Evaluation Methodology and Key Criteria for P802.16m – Advanced Air Interface”
Abstract	This contribution introduces the shadow fading correlation models and their simulation methodologies for system level simulation.
Purpose	Propose the correlation models for shadow fading simulation into the IEEE 802.16m evaluation methodology
Notice	
Release	
Patent Policy and Procedures	

Correlation Models for Shadow Fading Simulation

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1 Introduction

In system level simulation, the shadow fading is usually modeled by the log-normal random variable. When dynamic simulation is considered for simulating the channel variation due to user mobility, the shadow fading effect shall be updated from time to time. Due to the nature of the propagation environment, the shadow fading for the same radio link at the nearby locations will not change very much. Therefore, certain correlation shall be applied when simulating the shadow fading effect, and this kind of correlation is called auto-correlation for shadow fading.

In addition, multiple radio links may be simulated for the same user in multi-cell environment. For the radio links which has similar propagation path, the shadow fading experienced by each of them will not differ too much. In order to proper model this effect, the cross-correlation model shall be considered when simulating the shadow fading effect for multiple radio links. In the following text proposal, both the auto-correlation and cross-correlation model for shadow fading will be introduced. Moreover, the corresponding simulation methodology will also be performed.

2 Text Proposal

---Start of the Text---

Add the following text to the evaluation methodology document

Correlation Models for Shadow Fading

By given mean (usually equal to 0dB) and standard deviation, the level of shadow fading (in dB) is usually simulated by dropping a Normal distributed random variable. This refers to typical log-normal shadow fading model. However, the correlation models shall be considered to capture the characteristics of the propagation environment when performing the system level simulation. Two types of correlation models for shadow fading are introduced as following.

Auto-correlation Model for Shadow Fading

The auto-correlation of shadow fading indicates the correlation among the shadowing effects among the same radio link in different locations, which is illustrated in Figure 1.

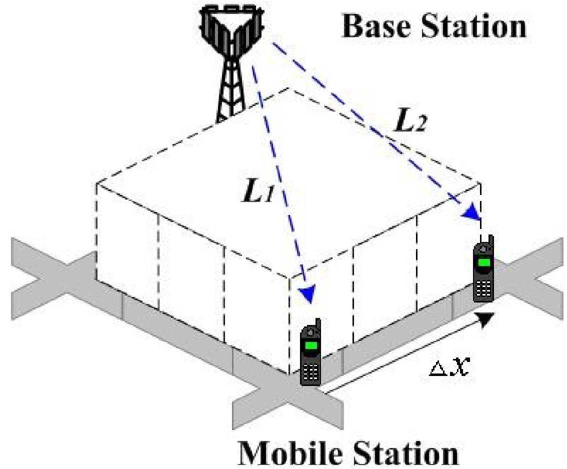

Fig.1 Auto-correlation of shadow fading

A popular model proposed by Gudmundson [1] is well understood in following form [2]:

$$\rho(\Delta x) = e^{-\frac{|\Delta x|}{d_{cor}}\ln 2} \qquad (13)$$

where ρ is the correlation coefficient and Δx is the distance between adjacent observation locations. d_{cor} is the de-correlation distance, which was suggested as 20m in vehicular test environment in [2].

The way to apply this model in system level simulation is briefly summarized as follows:

- Consider the log-normal shadow fading model with zero mean and variance σ^2 in dB. If L_1 is the log-normal component at position P_1 and L_2 is the one for P_2, which is Δx away from P_1. Then L_2 will be normally distributed in dB with mean $\rho(\Delta x)\cdot L_1$ and variance ($1-[\rho(\Delta x)]^2$) $\cdot\ \sigma^2$.

This result can be derived through the analysis in Appendix A.

Cross-correlation Model for Shadow Fading

Instead of the aforementioned auto-correlation model, the cross-correlation of shadow fading indicates the correlation among the shadow effect of different radio links at the same time. In general, longer common propagation path will induce higher correlation. For example, the cross-correlation among the shadow fading of the radio links in Figure 2(a) should be lower than the one in Figure 2(b).

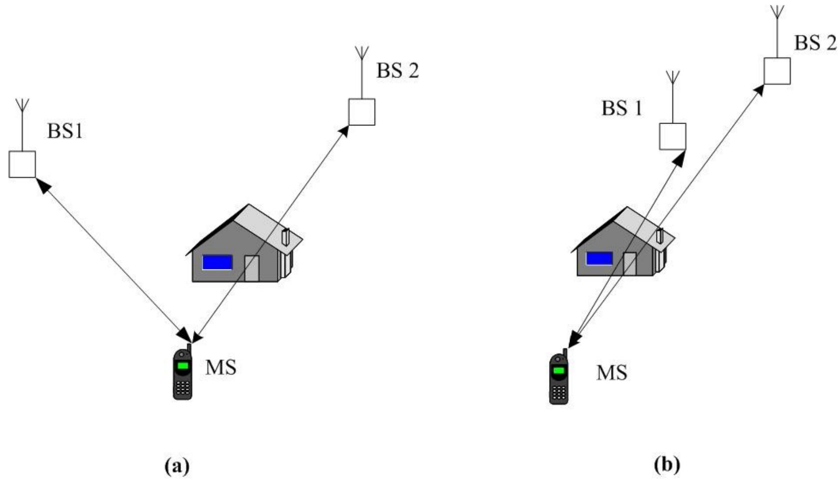

Fig.2 Cross-correlation of shadow fading

A simple cross-correlation model proposed in [3] can be considered for this:

$$r(q) = \begin{cases} 0.8 - \dfrac{|q|}{150} & if\ |q| \le 60^o \\ 0.4 & if\ |q| > 60^o \end{cases} \quad (14)$$

where r is the correlation coefficient and q is the angle of arrival difference.

In order to apply this effect in system level simulation, following procedure can be considered:

1. Consider the shadow fading effect of the radio links of one mobile station and N base stations at specific time instance, drop N independent Normal distributed random variables $C = \left[X_1, X_2, X_3, \cdots, X_N\right]$.

2. Obtain the matrix G, which is

$$G = \begin{bmatrix} 1 & r_{12} & \cdots & r_{1N} \\ r_{21} & 1 & \cdots & r_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ r_{N1} & r_{N2} & \cdots & 1 \end{bmatrix} \quad (15)$$

where $r_{12} = r_{21}$ is the correlation coefficient of the shadow fading component among the radio link to base station #1 and #2.

Since G is a symmetric and positive definite matrix, it can be decomposed into a lower and upper triangular matrix by Cholesky decomposition technique. Therefore, $G = C^T C$, where C is an upper triangular matrix.

3. $Y = XC = \left[Y_1, Y_2, Y_3, \cdots, Y_N\right]$ will be the cross-correlated log-normal shadow fading components for each radio link.

--End of the Text---

References

[1] Gudmundson, M., “Correlation Model for Shadow Fading in Mobile Radio Systems,” *Electronics Letters*, pp.2145-2146, vol. 27, No 23, Nov. 1991.

[2] ETSI TR 101.112 v3.2.0, “Selection procedures for the choice of radio transmission technologies of the UMTS”, Apr. 1998.

[3] T. Klingenbrunn and P. Mogensen, “Modeling Cross-Correlated Shadowing in Network Simulations,” *IEEE Vehicular Technology Conference*, pp.1407-1411, Vol.3, Sep. 1999.

Appendix A

- Consider *X* and *Y* are both Normal distributed random variables. Given $Y = y$ and the correlation coefficient as ρ, the probability distribution function $f_X(x\,|\,y)$ can be derived as:

Consider μ_1, μ_2 and σ_1^2, σ_2^2 are the mean and variance of X and Y

$$f_X(x\,|\,y) = \frac{f_{X,Y}(x,y)}{f_Y(y)}$$

$$= \frac{\exp\left\{\frac{-1}{2(1-\rho_{X,Y}^2)}\left[\left(\frac{x-\mu_1}{\sigma_1}\right)^2 - 2\rho_{X,Y}\left(\frac{x-\mu_1}{\sigma_1}\right)\left(\frac{y-\mu_2}{\sigma_2}\right) + \left(\frac{y-\mu_2}{\sigma_2}\right)^2\right]\right\}}{2\pi\sigma_1\sigma_2\sqrt{1-\rho_{X,Y}^2}}\left[\frac{\exp\left\{-\frac{(y-\mu_2)^2}{2\sigma_2^2}\right\}}{\sqrt{2\pi\sigma_2^2}}\right]^{-1}$$

$$= \frac{1}{\sqrt{2\pi\sigma_1^2(1-\rho_{X,Y}^2)}}\exp\left\{\frac{-1}{2(1-\rho_{X,Y}^2)}\left[\left(\frac{x-\mu_1}{\sigma_1}\right)^2 - 2\rho_{X,Y}\left(\frac{x-\mu_1}{\sigma_1}\right)\left(\frac{y-\mu_2}{\sigma_2}\right) + \left(\frac{y-\mu_2}{\sigma_2}\right)^2 - (1-\rho_{X,Y}^2)\left(\frac{y-\mu_2}{\sigma_2}\right)^2\right]\right\}$$

$$= \frac{1}{\sqrt{2\pi\sigma_1^2(1-\rho_{X,Y}^2)}}\exp\left\{\frac{-1}{2(1-\rho_{X,Y}^2)}\left[\frac{x-\mu_1}{\sigma_1} - \rho_{X,Y}\frac{y-\mu_2}{\sigma_2}\right]^2\right\}$$

$$= \frac{1}{\sqrt{2\pi}\sqrt{\sigma_1^2(1-\rho_{X,Y}^2)}}\exp\left\{-\frac{\left[x-\mu_1-\rho_{X,Y}\frac{\sigma_1}{\sigma_2}(y-\mu_2)\right]^2}{2\sigma_1^2(1-\rho_{X,Y}^2)}\right\}$$

$\therefore$ the mean of $f_X(x\,|\,y)$ is $\mu_1 + \rho_{X,Y}\frac{\sigma_1}{\sigma_2}(y-\mu_2)$ and variance is $\sigma_1^2(1-\rho_{X,Y}^2)$

If *X* and *Y* are both with zero mean and variance σ^2, and the correlation coefficient between *X* and *Y* are equal to ρ. Then, the mean of $f_X(x\,|\,y)$ will be ρy and its variance will be $\sigma^2(1-\rho^2)$.