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Re:	Response to call for contributions on Evaluation Methodology and Key Criteria for P802.16m – Advanced Air Interface
Abstract	
Purpose	For consideration of 802.16 TGm Evaluation Methodology and Key Criteria drafting group
Notice	
Release	
Patent Policy and Procedures	

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1.0 Introduction

3GPP SCM model [1] has been used in system simulation. It models the physical propagation environment using paths and sub-paths with randomly specified angles, delays, phases, and mean powers. The mathematical MIMO channel for simulation is then derived after defining the antenna configuration and array orientation at both MS and BS. Time-variation is realized after defining MS travel direction and speed. Other ray-based channel models include SCME (extension of SCM to >5MHz) and WINNER [4]. The ray-based geometrical models are powerful since they can reproduce truthfully the randomness as observed in measurement campaigns. But there are a few undesirable characteristics for simulation purpose:

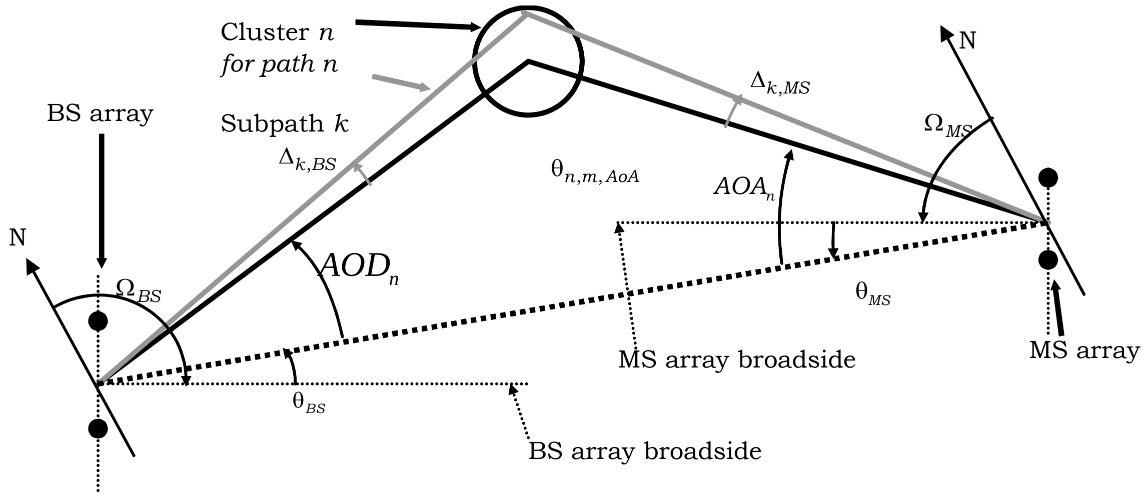
- It may be computationally more expensive to generate the actual channel from the ray-based geometrical models in system simulation, as compared to the conventional tap-delayed-line (TDL) model with fixed power delay profile and a Jakes Doppler profile. This is due to the needs of summing all the rays with different delay, amplitude, and phase to generate each channel tap.
- SCM was defined for system level simulation more than for link level simulation. In a system level simulation, people may want to use link-level performance results that are obtained under a non-SCM link model. For example, simple spatial extension of the GSM TU channels and ITU Ped-A/B and Veh-A/B channels are still widely used or in legacy results.
- SCM defines some second order statistics as random variable (e.g., angular spread, delay spread, etc.), which may require longer simulation to get convergence (i.e., run time concern)

The proposal is a simpler system-level model that is modified from SCM but directly generates a mathematical MIMO channel for each point-to-point link. The link-level model can be built upon any pre-selected conventional SISO TDL model. A per-tap spatial correlation is then derived in closed-form based on predefined geometrical parameter (e.g., a fixed angular spread, antenna orientation, etc.) and the antenna configuration (spacing, cross-polarization, etc.). The sub-path concepts defined in SCM may be used, but just as one of the options for deriving the closed-form spatial correlation matrix, rather than for deriving the MIMO channel coefficients. The methodology used to extend a conventional TDL model to MIMO link-level model is the same as used to derive the MIMO link model for WiMAX Forum's RCT MIMO test [2][3].

2.0 Generation of Correlation Matrices

Step 1: *Determine various distance and orientation parameters.*

The placement of the MS with respect to each BS is to be determined according to the cell layout. From this placement, the distance between the MS and the BS (*d*) and the LOS directions with respect to the BS and MS (θ_{BS} and θ_{MS}, respectively) can be determined. Note that θ_{BS} and θ_{MS} are defined relative to the broadside directions. The MS antenna array orientations (Ω_{MS}), are i.i.d., drawn from a uniform 0 to 360 degree distribution. (see figure below)



Step 2: *Calculate the bulk path loss associated with the BS to MS distance.*

A few pathloss models may be considered depending on operating frequency, such as Hata suburban, Erceg, etc. For example in SCM, the macrocell pathloss is based on the modified COST231 Hata urban propagation model:

$$PL[dB] = \left(44.9 - 6.55\log_{10} h_{bs}\right)\log_{10}(\frac{d}{1000}) + 45.5 +$$
$$\left(35.46 - 1.1h_{ms}\right)\log_{10}(f_c) - 13.82\log_{10}(h_{bs}) + 0.7h_{ms} + C$$

where h_{bs} is the BS antenna height in meters, h_{ms} the MS antenna height in meters, f_c the carrier frequency in MHz, d is the distance between the BS and MS in meters, and C is a constant factor (C = 0dB for suburban macro and C = 3dB for urban macro). Setting these parameters to h_{bs} = 32m, h_{ms} = 1.5m, and f_c =1900MHz, the pathlosses for suburban and urban macro environments become, respectively, $PL = 31.5 + 35\log_{10}(d)$ and $PL = 34.5 + 35\log_{10}(d)$. The distance d is required to be at least 35m.

Step 3: *Determine the Shadowing Factor (SF).*

It is randomly generated from a log-normal distribution with a pre-specified standard deviation value. If desired, *SF* can be generated together with the BS angular spread (*AS*$_{BS}$) as described in appendix-1, after enforcing a certain intra-site correlation between SF and AS, and an inter-site correlation of SFs.

Step 4: *Choose a well-defined power-delay-profile (PDP) (e.g., GSM TU/HT, ITU Ped-A/B, Veh-A/B) with a specified number of taps (N), their average powers (Pn,, n=1…N) and delays (τ_n ,n=1…N).*

It is also possible to use a user-defined PDP. For system with a bandwidth of more than 5MHz, a PDP with a larger number of taps may be considered (e.g., SCME is a simple extension of SCM to handle this case).

Average powers are normalized so that the total average power for all paths is equal to one:

$$P_n^{'} = \frac{P_n}{\sum_{n=1}^{N} P_n}.$$

Step 5: *Determine, for each path, the mean "AOD_n" (Angle-of-Departure at BS) and the mean "AOA_n" (Angle-of Arrival at MS).*

For the *N* departure paths, their mean AOD_n are determined as

$$AOD_1 = \theta_{BS}, \quad AOD_n = \theta_{BS} + \delta_n \quad n = 2...N$$

where first path (path with τ_1 =0) is assigned with a mean $AOD_1 = \theta_{BS}$ (i.e., LOS direction) and the rest of paths deviate from the LOS direction by some random values δ_n, where δ_n are ordered in increasing absolute value so that $|\delta_2| < |\delta_3| < ... < |\delta_N|$ and they are re-ordered from the random realizations of

$$\delta_n^{'} \sim \eta(0, \sigma_{AoD}^2) \quad , \quad n = 2, \ldots, N,$$

where $\sigma_{AoD} = r_{AS} AS_{BS}$ with AS_{BS} being the angular spread of the composite signal with all *N* paths (as opposed to the per-path $AS_{BS, Path}$) and the value r_{AS} is given in Table 1 (The table could be expanded to accommodate various PDP. Other values "r_{AS} " can be defined for different PDPs).

Table 1: Value of r_{AS}

PDP	TU	PB	VA
r_{AS}	1.51	1.78	1.94

The quantity r_{AS} describes the distribution of powers in angle, i.e. the spread of angles to the power weighted angle spread. A value of r_{AS} greater than "1" indicates that there is more power being concentrated in paths that have a smaller AoD. r_{AS} is pre-calculated based on the per-path angular spread $AS_{BS, Path}$ and the PDP pre-defined in Step 4. When r_{AS} and $AS_{BS, Path}$ are chosen properly, the resulting AS of the composite signal should satisfy the mean value of AS_{BS} for the pre-specified PDP. The per-path AS can be chosen as (the choice of "2/5" for per-path AS is to simply try to make the overall AS as close to the desired value as possible)

$$AS_{BS,Path} = \frac{2}{5} AS_{BS}$$

where in the simplest model we can define a fixed value for AS_{BS} (e.g., 2-degree, 5-degree, 15-degree), but it can also be generated randomly together with the Shadowing Factor (SF) as described in Appendix-1.

For the *N* arrival paths, their mean AOA_n are determined as

$$AOA_1 = \theta_{MS}, \quad AOA_n = \theta_{MS} + \Phi_n \quad n = 2...N$$

where first path (path with τ_1 =0) is assigned with a mean $AOA_1 = \theta_{MS}$ (i.e., LOS direction) and the rest of paths deviate from the LOS direction by some random values Φ_n. Φ_n (in degrees) are random realizations of the following Gaussian variables

$$\Phi_n \sim \eta(0, \sigma_{n,AoA}^2), \quad n = 2, \ldots, N,$$

where $\sigma_{n,AoA} = 104.12\left(1\text{-}\exp\left(-0.2175\left|10\log_{10}(P_n^{'})\right|\right)\right)$ and $P_n^{'}$ is the mean power of the *n*th path specified in Step 4. Note that a uniform power angular profile over [0,360] degree will result in an angular spread of 104.12 degree (i.e., the $\sigma_{n,AoA}$ value at $P_n^{'}$=1). The per-path AOAs are taken from Gaussian distributions with different variances that are specified according to the average path power of each path, and there is no need to order the values. The per-path angular spread $AS_{MS,\ Path}$ is set as 35 degrees in SCM.

To compute the final AS_{BS} and AS_{MS} of the composite signal, refer to Appendix-2. If the value is not desirable, for example, for link level simulation purpose, a value as close to the specified value AS_{BS} (or AS_{MS}) as possible may be desired, this step can be repeated until a satisfactory value is obtained.

Step 6: *Calculate the per-path spatial correlation matrix based on per-path $AS_{BS,\ Path}$, $AS_{MS,\ Path}$, AOD_n, AOA_n, and BS/MS antenna configurations (broadside direction, number and spacing of antennas, polarization., etc.)*

Once the per-path AS, mean AOA, and mean AOD are defined, the theoretical spatial correlation at both BS and MS can be derived, assuming Laplacian power angular distribution. In particular, the antenna spatial correlations between the p-th and q-th antenna at the BS and MS, respectively, are

$$r_{n,BS}(p,q) = \int_{-\infty}^{\infty} p(\alpha)\exp\left\{ j\frac{2\pi d_{BS}}{\lambda}(p-q)\sin(AOD_n+\alpha)\right\} d\alpha$$

$$r_{n,MS}(p,q) = \int_{-\infty}^{\infty} p(\beta)\exp\left\{ j\frac{2\pi d_{MS}}{\lambda}(p-q)\sin(AOA_n+\beta)\right\} d\beta$$

where d_{BS} (d_{MS}) is the antenna spacing at BS (MS) and λ is the wavelength. α is the angular offset around the mean AOD at BS, and β is the angular offset around the mean AOA at MS. The pdf of angular offsets is

$$p(\alpha) = \frac{1}{\sqrt{2}AS_{BS,Path}}\exp\left\{ -\frac{\sqrt{2}|\alpha|}{AS_{BS,Path}}\right\}$$

$$p(\beta) = \frac{1}{\sqrt{2}AS_{MS,Path}}\exp\left\{ -\frac{\sqrt{2}|\beta|}{AS_{MS,Path}}\right\}$$

The above integration can be computed with two approaches (other alternatives may also exist). See appendix-3 for details. In summary, the first approach is to approximate the Laplacian PDF with 20 subpaths, after which the integration is reduced to a summation. The second approach is to pre-compute the integration using a numerical method. Since the integration depends on mean AOA and AOD, it is possible to quantize them and then pre-compute the integration for each quantized AOA and AOD values.

Denoting the spatial correlation matrix at BS and MS as $\mathbf{R}_{BS,n}$ and $\mathbf{R}_{MS,n}$, the per-path spatial correlation is determined as

$$\mathbf{R}_n = \mathbf{R}_{BS,n} \otimes \mathbf{R}_{MS,n} \text{ (Kronecker product)}$$

In the case that the antenna elements are cross-polarization antennas, the per-path channel correlation is determined as

$$\mathbf{R}_n = \mathbf{R}_{BS,n} \otimes \mathbf{\Gamma} \otimes \mathbf{R}_{MS,n}$$

where $\mathbf{\Gamma}$ is a cross-polarization matrix defined in Appendix-4.

Step 8: *Determine the antenna gains of the BS and MS paths as a function of their respective AoDs and AoAs. Calculate the per-path average power with BS/MS antenna gain as*

$$P_n^" = P_n^" * G_{BS}(AOD_n) * G_{MS}(AOA_n)$$

Step 9: *Generate time-variant MIMO channels with above-defined per-tap spatial correlations.*
For each tap, generate M_{BS}xM_{MS} *i.i.d.* channels first assuming Jakes Doppler spectrum $\mathbf{H}_{iid}$ (each tap is a M_{BS}xM_{MS} matrix).

Compute the correlated channel at each tap as

$$\mathbf{H}_n = unvec\left\{R_n^{1/2} vec(H_{iid})\right\}$$

where *vec(H)* denotes the column-wise stacking of matrix H and *unvec* is the reverse operation. $Rn^{1/2}$ denotes the square-root of matrix R.

Step 10: *If a non-zero K-factor wants to be enforced (i.e., K≠0), adjust the LOS path power*
This step is not needed if a K-factor wants to be enforced (see Appendix-5)

Step 11: *Introduce receive antenna gain imbalance or coupling, if need.*
See Appendix-6.

3.0 Reference

[1] "Spatial Channel Model (SCM) Text Description" v7.0, Spatial Channel Model Ad-Hoc Group (Combined ad-hoc from 3GPP & 3GPP2), August 19, 2003

[2] "MIMO Channel Model for MTG RCT", Working Document Draft, http://www.wimaxforum.org/apps/org/workgroup/twg/download.php/12670/MIMO%20channel%20model%20for%20MTG%20RCT_271206.doc

[3] "MIMO Extension of ITU Models for RCT", Presentation to TWG/RCT, http://www.wimaxforum.org/apps/org/workgroup/twg/document.php?document_id=11517

[4] "IST-2003-507581 WINNER: Final Report on Link Level and System Level Channel Models, D5.4 v. 1.4", WINNER project report

Appendix-1: Correlation of Angular Spread and Shadowing Factor

Assuming correlation between AS and SF (both are log-normal distributed), randomly determine the AS and SF for the *d*th base station (*d* = 1 … *D*) as in the following with respect to a given mobile user:

$$\sigma_{AS,d} = 10 \wedge \left(\varepsilon_{AS}\beta_d + \mu_{AS}\right)$$

$$\sigma_{SF,d} = 10 \wedge \left(\sigma_{SH}\gamma_d / 10\right)$$

where $\mu_{AS} = E(\log_{10}(\sigma_{AS}))$ is the logarithmic mean of the distribution of AS (pre-defined), and $\varepsilon_{AS} = \sqrt{E\left[\log_{10}(\sigma^2_{AS,n})\right] - \mu^2_{AS}}$ is the logarithmic standard deviation of the distribution of AS (pre-defined). σ_{SH} is the shadow fading standard deviation given in dB (pre-defined). β_n and γ_n are generated together by:

$$\begin{bmatrix}\beta_n\\ \gamma_n\end{bmatrix} = \begin{bmatrix}c_{11} & c_{12}\\ c_{21} & c_{22}\end{bmatrix}\begin{bmatrix}w_{n1}\\ w_{n2}\end{bmatrix} + \begin{bmatrix}0 & 0\\ 0 & \sqrt{\zeta}\end{bmatrix}\begin{bmatrix}\xi_1\\ \xi_2\end{bmatrix}$$

where the two correlated Gaussian random variables are in turn respectively generated from independent Gaussian random variables w_{n1} and w_{n2}, as well as two global (applicable to all bases) independent Gaussian random variables ξ_1, ξ_2. The matrix **C** with elements c_{ij} multiplying the w's is given by

$$\mathbf{C} = (\mathbf{A} - \mathbf{B})^{1/2} = \left(\begin{bmatrix}1 & \rho_{\gamma\beta}\\ \rho_{\gamma\beta} & 1\end{bmatrix} - \begin{bmatrix}0 & 0\\ 0 & \zeta\end{bmatrix}\right)^{1/2}$$

where the superscript "½" denotes the matrix square root. The intra-site correlation between SF and AS is $\rho_{\gamma\beta} = -0.6$ and inter-site SF correlation is $\zeta = 0.5$ in SCM.

Appendix-2: Calculation of Circular Angular Spread

The following derivation of the angular spread assumes that each multipath further contains M sub-paths. The creation of subpath is to approximate the power angular distribution (Laplacian for example) with a finite number of points. With the introduction of subpath, the AS of the overall signal can be more easily computed as, for a signal with N multi-paths,

$$\sigma_{AS} = \sqrt{\frac{\sum_{n=1}^{N}\sum_{m=1}^{M}(\theta_{n,m,\mu})^2 \cdot P_{n,m}}{\sum_{n=1}^{N}\sum_{m=1}^{M}P_{n,m}}}$$

where $P_{n,m}$ is the power for the mth subpath of the nth path, $\theta_{n,m,\mu}$ is defined as

$$\theta_{n,m,\mu} = \begin{cases} 2\pi + (\theta_{n,m} - \mu_\theta) & \text{if } (\theta_{n,m} - \mu_\theta) < -\pi \\ (\theta_{n,m} - \mu_\theta) & \text{if } |\theta_{n,m} - \mu_\theta| \le \pi \\ 2\pi - (\theta_{n,m} - \mu_\theta) & \text{if } (\theta_{n,m} - \mu_\theta) > \pi \end{cases},$$

μ_θ is defined as

$$\mu_\theta = \frac{\sum_{n=1}^{N}\sum_{m=1}^{M}\theta_{n,m} \cdot P_{n,m}}{\sum_{n=1}^{N}\sum_{m=1}^{M}P_{n,m}}$$

and $\theta_{n,m}$ is the AoA (or AoD) of the mth subpath of the nth path. Note that we have dropped the AoA (AoD) subscript for convenience.

We note that the angle spread should be independent of a linear shift in the AoAs. In other words, by replacing $\theta_{n,m}$ with $\theta_{n,m} + \Delta$, the angle spread $\sigma_{AS}(\Delta)$ which is now a function of Δ should actually be constant no matter what Δ is. However, due to the ambiguity of the modulo 2π operation, this may not be the case. Therefore the angle spread should be the minimum of $\sigma_{AS}(\Delta)$ over all Δ:

$$\sigma_{AS} = \min_{\Delta} \sigma_{AS}(\Delta) = \sqrt{\frac{\sum_{n=1}^{N}\sum_{m=1}^{M}\left(\theta_{n,m,\mu}(\Delta)\right)^2 \cdot P_{n,m}}{\sum_{n=1}^{N}\sum_{m=1}^{M} P_{n,m}}}$$

where $\theta_{n,m,\mu}(\Delta)$ is defined as

$$\theta_{n,m,\mu}(\Delta) = \begin{cases} 2\pi + \left(\theta_{n,m}(\Delta) - \mu_\theta(\Delta)\right) & \text{if } \left(\theta_{n,m}(\Delta) - \mu_\theta(\Delta)\right) < -\pi \\ \left(\theta_{n,m}(\Delta) - \mu_\theta(\Delta)\right) & \text{if } \left|\theta_{n,m}(\Delta) - \mu_\theta(\Delta)\right| \le \pi \\ 2\pi - \left(\theta_{n,m}(\Delta) - \mu_\theta(\Delta)\right) & \text{if } \left(\theta_{n,m}(\Delta) - \mu_\theta(\Delta)\right) > \pi \end{cases},$$

$\mu_\theta(\Delta)$ is defined as

$$\mu_\theta(\Delta) = \frac{\sum_{n=1}^{N}\sum_{m=1}^{M}\theta_{n,m}(\Delta) \cdot P_{n,m}}{\sum_{n=1}^{N}\sum_{m=1}^{M} P_{n,m}}$$

and $\theta_{n,m}(\Delta) = \theta_{n,m} + \Delta$.

Note that when the angular spread is small, the angular spread computed conventionally and the circular angular spread often give the same value.

Appendix-3: Spatial Correlation Calculation

In order to compute the integration defined in step 6, two methods can be considered here:

Method-1: Using 20 subpaths to approximate the Laplacian PDF

For each path, generate 20 subpaths with some angular offsets from the per-path AOD_n and AOA_n. The angular offsets of the k-th (k=1..20) subpath are determined by (the offsets are the same for all paths)

$$\psi_{k,BS} = \Delta_k AS_{BS,Path}$$

$$\psi_{k,MS} = \Delta_k AS_{MS,Path}$$

where the values of Δ_k are given in **Error! Reference source not found.**.

Table 2: Value of Δ_k

Sub-path number *k*	Δ_k
1,2	± 0.0447°
3,4	± 0.1413°
5,6	± 0.2492°
7,8	± 0.3715°
9,10	± 0.5129°
11,12	± 0.6797°
13,14	± 0.8844°
15,16	± 1.1481°

17,18	± 1.5195°
19,20	± 2.1551°

Derive the antenna spatial correlation at the BS and MS between the p-th and q-th antenna as:

$$r_{n,BS}(p,q) = \frac{1}{20}\sum_{k=1}^{20}\exp\left\{ j\frac{2\pi d_{BS}}{\lambda}(p-q)\sin(AOD_n + \psi_{k,BS})\right\}$$

$$r_{n,MS}(p,q) = \frac{1}{20}\sum_{k=1}^{20}\exp\left\{ j\frac{2\pi d_{MS}}{\lambda}(p-q)\sin(AOA_n + \psi_{k,MS})\right\}$$

where d_{BS} (d_{MS}) is the antenna spacing at BS (MS) and λ is the wavelength.

Method-2:Pre-compute the correlation values with quantized AOA, AOD

Pre-calculate the BS spatial correlation matrices for a set of $AOD \in \{-90^o, -80^o, \cdots, 0^o, \cdots, 80^o, 90^o\}$ and the MS spatial correlation matrices for a set of $AOA \in \{-90^o, -80^o, \cdots, 0^o, \cdots, 80^o, 90^o\}$

$$R_{BS}(m,p,q) = \int_{-\infty}^{\infty} p(\alpha)\exp\left\{ j\frac{2\pi d_{BS}}{\lambda}(p-q)\sin(AOD[m]+\alpha)\right\} d\alpha$$

$$R_{MS}(m,p,q) = \int_{-\infty}^{\infty} p(\beta)\exp\left\{ j\frac{2\pi d_{MS}}{\lambda}(p-q)\sin(AOA[m]+\beta)\right\} d\beta$$

where m is the quantization step index, α , β are the angular offset at BS and MS, respectively with Laplacian PDF as defined in step-6.

For each path, determine the index m_{BS} corresponding to AOD_n,

$$m_{BS} = \left\lfloor \frac{AOD_n}{10} \right\rfloor$$

and the index m_{MS} *corresponding to* AOA_n

$$m_{MS} = \left\lfloor \frac{AOA_n}{10} \right\rfloor$$

The spatial correlation matrix for this path is then

$$r_{n,BS}(p,q) = R_{BS}(m_{BS}, p, q)$$

$$r_{n,MS}(p,q) = R_{MS}(m_{MS}, p, q)$$

Appendix-4: Polarized Antenna

Correlation between polarized antennas results from the cross polarization power ratio (XPR). The polarization matrix is given by:

$$\mathbf{S} = \begin{bmatrix} s_{vv} & s_{vh} \\ s_{hv} & s_{hh} \end{bmatrix},$$

where *v* denotes vertical and *h* horizontal polarization, the first index denoting the polarization at BS and the second the polarization at MS. In the ITU scenarios we assume -8 dB per-tap power ratio between vertical-to-horizontal and vertical-to-vertical polarisations (also P_{hv}/P_{hh} = -8dB). This results in the following mean power per polarization components

$$p_{vv} = E\{|s_{vv}|^2\} = 0\text{ dB} = 1$$

$$p_{vh} = E\{|s_{vh}|^2\} = -8\text{ dB} = 0.1585$$

$$p_{hv} = E\{|s_{hv}|^2\} = -8\text{ dB} = 0.1585$$

$$p_{hh} = E\{|s_{hh}|^2\} = 0\text{ dB} = 1$$

If the MS polarizations are assumed to be vertical and horizontal, but the BS polarizations are slant +45° and -45°. The MS and BS polarization matrices $\mathbf{P}_{MS}$ and $\mathbf{P}_{BS}$ respectively are rotation matrices, which map vertical and horizontal polarizations to MS and BS antenna polarizations.

$$\mathbf{P}_{MS} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\mathbf{P}_{BS} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

The total channel is the matrix product of the BS polarization, the channel polarization, and the MS polarization:

$$\mathbf{Q} = \mathbf{P}_{BS}\,\mathbf{S}\,\mathbf{P}_{MS} = \frac{1}{\sqrt{2}}\begin{bmatrix} s_{vv}+s_{hv} & s_{vh}+s_{hh} \\ s_{vv}-s_{hv} & s_{vh}-s_{hh} \end{bmatrix}$$

The covariance matrix of the channel is

$$\mathbf{\Gamma} = E\{vec(\mathbf{Q})\cdot vec(\mathbf{Q})^H\}$$

$$= E\left\{\frac{1}{2}\begin{bmatrix} (s_{vv}+s_{hv})(s_{vv}+s_{hv})^* & (s_{vv}+s_{hv})(s_{vv}-s_{hv})^* & (s_{vv}+s_{hv})(s_{vh}+s_{hh})^* & (s_{vv}+s_{hv})(s_{vh}-s_{hh})^* \\ (s_{vv}-s_{hv})(s_{vv}+s_{hv})^* & (s_{vv}-s_{hv})(s_{vv}-s_{hv})^* & (s_{vv}-s_{hv})(s_{vh}+s_{hh})^* & (s_{vv}-s_{hv})(s_{vh}-s_{hh})^* \\ (s_{vh}+s_{hh})(s_{vv}+s_{hv})^* & (s_{vh}+s_{hh})(s_{vv}-s_{hv})^* & (s_{vh}+s_{hh})(s_{vh}+s_{hh})^* & (s_{vh}+s_{hh})(s_{vh}-s_{hh})^* \\ (s_{vh}-s_{hh})(s_{vv}+s_{hv})^* & (s_{vh}-s_{hh})(s_{vv}-s_{hv})^* & (s_{vh}-s_{hh})(s_{vh}+s_{hh})^* & (s_{vh}-s_{hh})(s_{vh}-s_{hh})^* \end{bmatrix}\right\}$$

$$= \frac{1}{2}\begin{bmatrix} p_{vv}+p_{hv} & p_{vv}-p_{hv} & 0 & 0 \\ p_{vv}-p_{hv} & p_{vv}+p_{hv} & 0 & 0 \\ 0 & 0 & p_{vh}+p_{hh} & p_{vh}-p_{hh} \\ 0 & 0 & p_{vh}-p_{hh} & p_{vh}+p_{hh} \end{bmatrix}$$

Here the property of uncorrelated fading between different elements in **S** (i.e. $E\{s_{ij}s_{kl}^*\} = 0, \quad i \neq k, j \neq l$) has been used to simplify the expressions. When all of the diagonal elements are equal, the covariance matrix can be further normalised to correlation matrix:

$$\mathbf{R}_{MIMO} = \begin{bmatrix} 1 & \gamma & 0 & 0 \\ \gamma & 1 & 0 & 0 \\ 0 & 0 & 1 & -\gamma \\ 0 & 0 & -\gamma & 1 \end{bmatrix}$$

Value of γ depends only on XPR. With different orientations of MS and BS antenna polarizations, also the covariance matrix structure will be different.

Appendix-5: LOS Option with a K-factor

A single-tap MIMO channel can be added to the TDL channels in this case and then modify the time-domain channels as:

$$\mathbf{H}_n = \begin{cases} \sqrt{\frac{1}{K+1}}\mathbf{H}_n + \sqrt{\frac{K}{K+1}}\mathbf{H}^{LOS} & n = 1 \text{(first tap)} \\ \sqrt{\frac{1}{K+1}}\mathbf{H}_n & n \neq 1 \end{cases}$$

where the K-factor is in decimal and the LOS component is defined as, between p-th BS antenna and q-th MS antenna

$$\mathbf{H}^{LOS}(p,q) = \exp\left(j\frac{2\pi d_{BS}(p-1)}{\lambda}\sin\left(\theta_{BS}\right)\right) \times \exp\left(j\frac{2\pi d_{MS}(q-1)}{\lambda}\sin\left(\theta_{MS}\right)\right)$$

where d_{BS} and d_{MS} are antenna spacing at the BS and MS, respectively, assuming uniform linear array in this case.

Appendix-6: Antenna Gain Imbalance and Coupling

Overall receive correlation matrix is

$$\mathbf{H}_n^{'} = \begin{bmatrix} \sqrt{\frac{1}{c+1}} & \sqrt{\frac{c}{c+1}} \\ \sqrt{\frac{c}{c+1}} & \sqrt{\frac{1}{c+1}} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{a} \end{bmatrix} \mathbf{H}_n$$

Where antenna-1 to antenna-2 coupling coefficient (leakage of ant-1 signal to ant-2) is “c” (linear) and the antenna-1 and antenna gain ratio is “a” (linear).