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Re:	IEEE 802.16m-07/048 “Call for contributions on Draft 802.16m Evaluation Methodology Document”
Abstract	This contribution proposes a post processing SINR calculation method for MIMO receivers with HARQ.
Purpose	For discussion and approval of the proposed text
Notice	
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Post-Processing SINR Calculation for MIMO H-ARQ

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Introduction

For MIMO systems with H-ARQ, there are many different methods of combining the received signals. One method is doing bit-level combining, which is a straightforward extension of the SISO H-ARQ case. This bit-level combining is applicable both for Chase Combining (CC) H-ARQ and Incremental Redundancy (IR) H-ARQ. Another method of combining is doing symbol-level combining before MIMO equalization or MIMO ML decoding. This symbol-level combining method is known to perform better than the bit-level combining in case of CC H-ARQ.

The current draft of the evaluation methodology document [1] contains the PHY abstraction methodology for H-ARQ, which is applicable to bit-level combining. However, it does not deal with the case of symbol-level combining for MIMO CC H-ARQ. This contribution describes how the PHY abstraction can be done for symbol-level combining for MIMO CC H-ARQ.

Combining for CC H-ARQ with MIMO Equalizers

For H-ARQ with MIMO equalization, there are three different methods to combine the information from each transmission depending on the location of combining in the receiver. The first method is bit-level combining. The combining for bit-level combining happens at the output of the bit-metric calculator. The second method is symbol-level combining after MIMO equalization. This combining happens at the output of the MIMO equalizer. Finally, the third method is symbol-level combining before MIMO equalization. This symbol-level combining happens before the MIMO equalizer.

In the following section, we describe the symbol-level combining before MIMO equalization.

Symbol-Level Combining before MIMO Equalization

The received signal model at the i-th H-ARQ transmission for MIMO is

$$\underline{Y}_i = \underline{H}_i \underline{X}_i + \underline{Z}_i ,$$

where

$\underline{Y}_i$ is the received signal for the i-th transmission

$\underline{H}_i$ is the MIMO channel for the i-th transmission,

$\underline{X}_i$ is the transmit signal for the i-th transmission,

$\underline{Z}_i$ is the noise plus interference vector for the i-th transmission.

In case of Chase-Combining, the same signal is repeatedly transmitted, i.e.,

$$\underline{X}_i = \underline{X}\ .$$

In this case, the symbol-level combining before MIMO equalization can be done by concatenation approach. The concatenated received signal model after the m-th H-ARQ transmission is

$$\underline{Y}_{c,m} = \underline{H}_{c.m}\underline{X} + \underline{Z}_{c,m}\ ,$$

where

$\underline{Y}_{c,m} = \begin{bmatrix} \underline{Y}_1 \\ \underline{Y}_2 \\ \vdots \\ \underline{Y}_m \end{bmatrix}$ is the concatenated received signal after the m-th transmission,

$\underline{H}_{c,m} = \begin{bmatrix} \underline{H}_1 \\ \underline{H}_2 \\ \vdots \\ \underline{H}_m \end{bmatrix}$ is the concatenated channel matrix after the m-th transmission,

$\underline{Z}_{c,m} = \begin{bmatrix} \underline{Z}_1 \\ \underline{Z}_2 \\ \vdots \\ \underline{Z}_m \end{bmatrix}$ is the noise vector after the m-th transmission.

The equalization can be performed based on this concatenated received signal model. The MIMO equalization is followed by the bit-metric or log-likelihood ratio (LLR) calculation, which is used as an input to a decoder. Thus, the post-processing SINR can be calculated after doing the above concatenation operation before MIMO equalization. This way of calculating the post-processing SINR can be found in [2].

Although the above method can calculate the post-processing SINR accurately, it has the shortcoming that the channel matrices and the received signals for each transmission. Thus, we describe an alternative way of calculating the post-processing SINR.

For symbol-level combining before MIMO equalization with H-ARQ, the post-processing SINR can be obtained by computing the post-processing SINR of the combined receive signal.

As in Section 4.5.4, for the i-th transmission, the received signal at the n-th subcarrier is

$$\underline{Y}_i^{(0)}(n) = \sqrt{P_{tx,i}^{(0)} P_{loss,i}^{(0)}}\,\underline{H}_i^{(0)}(n)\underline{X}_i^{(0)}(n) + \sum_{j=1}^{N_I}\sqrt{P_{tx,i}^{(j)} P_{loss,i}^{(j)}}\,\underline{H}_i^{(j)}(n)\underline{X}_i^{(j)}(n) + \underline{U}_i^{(0)}(n),$$

where the subscript *i* represent the transmission index. For every transmission, the same signal is transmitted for the intended user, i.e., $\underline{X}_i^{(0)}(n) = \underline{X}^{(0)}(n)$ for every i.

For convenience, we define the noise plus interference term as

$$\underline{T}_i^{(0)}(n) = \sum_{j=1}^{N_I} \sqrt{P_{tx,i}^{(j)} P_{loss,i}^{(j)}}\, \underline{H}_i^{(j)}(n) \underline{X}_i^{(j)}(n) + \underline{U}_i^{(0)}(n) .$$

The covariance matrix of $\underline{T}_i^{(0)}(n)$ is defined as $\tilde{\sigma}_i^2(n)$.

The pre-equalization combining can be optimally done by weighting the received signal with the conjugate transpose of the intended channel matrix when the noise plus interference is white. If the noise plus interference is not white, whitening of the noise plus interference should come first. Thus, the combining of the received signal should be done as follows after the i-th transmission:

$$\underline{V}_i^{(0)}(n) = \sqrt{P_{tx,i}^{(0)} P_{loss,i}^{(0)}}\, \underline{H}_i^{(0)*}(n) \underline{\tilde{\sigma}}_i^{-2}(n)(n) \underline{Y}_i^{(0)}(n) + \underline{V}_{i-1}^{(0)}(n) ,$$

with $\underline{V}_0^{(0)}(n) = 0$.

The combined squared channel matrix is

$$\underline{S}_i^{(0)}(n) = P_{tx,i}^{(0)} P_{loss,i}^{(0)} \underline{H}_i^{(0)*}(n) \underline{\tilde{\sigma}}_i^{-2}(n) \underline{H}_i^{(0)}(n) + \underline{S}_{i-1}^{(0)}(n) .$$

with $\underline{S}_0^{(0)}(n) = 0$.

The estimate of the transmit signal with a linear MMSE receiver is

$$\begin{aligned}
\hat{\underline{X}}_i^{(0)}(n) &= \left(\underline{S}_i^{(0)}(n) + \underline{I}\right)^{-1} \underline{V}_i^{(0)}(n) \\
&= \left(\underline{S}_i^{(0)}(n) + \underline{I}\right)^{-1} \sum_{m=1}^{i} \sqrt{P_{tx,m}^{(0)} P_{loss,m}^{(0)}}\, \underline{H}_m^{(0)*}(n) \underline{\tilde{\sigma}}_m^{-2}(n) \underline{Y}_m^{(0)}(n) \\
&= \left(\underline{S}_i^{(0)}(n) + \underline{I}\right)^{-1} \sum_{m=1}^{i} \sqrt{P_{tx,m}^{(0)} P_{loss,m}^{(0)}}\, \underline{H}_m^{(0)*}(n) \underline{\tilde{\sigma}}_m^{-2}(n) \left[\sqrt{P_{tx,m}^{(0)} P_{loss,m}^{(0)}}\, \underline{H}_m^{(0)}(n) \underline{X}^{(0)}(n) + \underline{T}_m^{(0)}(n)\right] \\
&= \left(\underline{S}_i^{(0)}(n) + \underline{I}\right)^{-1} \underline{S}_i^{(0)}(n) \underline{X}^{(0)}(n) + \left(\underline{S}_i^{(0)}(n) + \underline{I}\right)^{-1} \sum_{m=1}^{i} \sqrt{P_{tx,m}^{(0)} P_{loss,m}^{(0)}}\, \underline{H}_m^{(0)*}(n) \underline{\tilde{\sigma}}_m^{-2}(n) \underline{T}_m^{(0)}(n)
\end{aligned}$$

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The first term in the last time of the above expression is the sum of the desired signal component $D_i(n) = diag\left[\left(\underline{S}_i^{(0)}(n) + \underline{I}\right)^{-1} \underline{S}_i^{(0)}(n)\right]$ and the self interference between MIMO streams $I_{self,i}(n) = \left(\underline{S}_i^{(0)}(n) + \underline{I}\right)^{-1} \underline{S}_i^{(0)}(n) - D_i(n)$. The second term represents the aggregated inter-cell interference plus noise.

The desired signal component power for the *k-th* MIMO stream is $diag\left[\sigma_0^2 D_i(n) D_i^*(n)\right]_{kk}$, whereas the power of the self interference between MIMO streams is $diag\left[\sigma_0^2 I_{self,i}(n) I_{self,i}^*(n)\right]_{kk}$. The power of the inter-cell interference plus noise can be calculated as follows:

$$E\left[\left[\left(\underline{S}_i^{(0)}(n)+I\right)^{-1}\sum_{m=1}^{i}\sqrt{P_{tx,m}^{(0)}P_{loss,m}^{(0)}}\,\underline{H}_m^{(0)*}(n)\underline{\tilde{\sigma}}_m^{-2}(n)\underline{T}_m^{(0)}(n)\right]\left[\left(\underline{S}_i^{(0)}(n)+I\right)^{-1}\sum_{m=1}^{i}\sqrt{P_{tx,m}^{(0)}P_{loss,m}^{(0)}}\,\underline{H}_m^{(0)*}(n)\underline{\tilde{\sigma}}_m^{-2}(n)\underline{T}_m^{(0)}(n)\right]^*\right]$$

$$=E\left[\left(\underline{S}_i^{(0)}(n)+I\right)^{-1}\sum_{m=1}^{i}P_{tx,m}^{(0)}P_{loss,m}^{(0)}\underline{H}_m^{(0)*}(n)\underline{\tilde{\sigma}}_m^{-2}(n)\underline{T}_m^{(0)}(n)\underline{T}_m^{(0)*}(n)\underline{\tilde{\sigma}}_m^{-2}(n)\underline{H}_m^{(0)}(n)\left(\underline{S}_i^{(0)}(n)+I\right)^{-1}\right]$$

$$=\left(\underline{S}_i^{(0)}(n)+I\right)^{-1}\sum_{m=1}^{i}P_{tx,m}^{(0)}P_{loss,m}^{(0)}\underline{H}_m^{(0)*}(n)\underline{\tilde{\sigma}}_m^{-2}(n)\underline{H}_m^{(0)}(n)\left(\underline{S}_i^{(0)}(n)+I\right)^{-1}$$

$$=\left(\underline{S}_i^{(0)}(n)+I\right)^{-1}\underline{S}_i^{(0)}(n)\left(\underline{S}_i^{(0)}(n)+I\right)^{-1}$$

Finally, the post processing SINR of the desired MS for the *n-th* subcarrier and the *k-th* MIMO stream is thus given as:

$$SINR_{k,i}^{(0)}(n)=\frac{diag\left[\sigma_0^2D_i(n)D_i^*(n)\right]_{kk}}{diag\left[\sigma_0^2I_{self,i}(n)I_{self,i}^*(n)+\left(\underline{S}_i^{(0)}(n)+I\right)^{-1}\underline{S}_i^{(0)}(n)\left(\underline{S}_i^{(0)}(n)+I\right)^{-1}\right]_{kk}}$$

Note that the post-processing SINR can be represented as a function of $\underline{S}_i^{(0)}(n)$ only since $D_i(n)$ and $I_{self,i}(n)$ are a function of $\underline{S}_i^{(0)}(n)$. For the post-processing SINR calculation, the calculation of the combined received signal $\underline{V}_i^{(0)}(n)$ does not need to be computed.

Comparison of Various Combining Methods

As was stated earlier, there are three different combining methods for MIMO CC H-ARQ depending on at which stage in the receiver the combining is done: bit-level combining, symbol-level combining after equalization, and symbol-level combining before equalization.

As the combining happens at the later stage, there is greater amount of information loss in general. The information loss at the stage of the bit-metric calculator may be minimal, and there is little performance difference between the bit-level combining and the symbol-level combining after equalization. So, both the bit-level combining and the symbol-level combining performance can be predicted easily by calculating the SINR at the output of MIMO equalizer for each transmission and summing the SINRs over all the past and current transmissions.

However, the information loss at the stage of the MIMO equalizer can be quite significant, and the performance of the symbol-level combining before equalization can be much better than the performance of the symbol-level combining after equalization. For example, consider a MIMO system with two transmit antennas, two receive antennas, and two spatial streams. For the first transmission, let's assume that the channel matrix happens to be $\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$. Because the rank of the channel matrix is only one, two streams are not likely to be passed reliably. So, the receiver asks for retransmission. And for the second transmission, let's assume that the channel matrix happens to be $\begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$. Again the rank of the channel matrix is only one, and the MIMO equalizer will not handle this second transmission well for the case of symbol-level combining after equalization. However, for the case of symbol-level combining before equalization, the concatenated channel matrix has a rank of two; two column vectors of the concatenated channel matrix are orthogonal. Thus, the symbol-level combining before equalization can handle the second transmission well.

The above example illustrates the importance of the symbol-level combining before equalization in case of MIMO systems. Roughly speaking, for MIMO systems, decoding error happens for the three reasons:

1. Deep fading in channel gain
2. Large noise
3. Self-interference among multiple streams

The symbol-level combining before equalization handles the self-interference among multiple streams better than the symbol-level combining after equalization.

As the performance of the symbol-level combining after equalization differs from that of the symbol-level combining before equalization, the post-processing SINR for symbol-level combining after equalization is also different from that the post-processing SINR for symbol-level combining before equalization. The post-processing SINR for symbol-level combining before equalization is the post-processing SINR calculated with the concatenated received signal model. Proposed Text Section shows how the post-processing SINR can be computed.

Conclusion

The symbol-level combining before equalization is one of the promising combining methods for MIMO CC H-ARQ. The PHY abstraction methodology in the current draft of the evaluation methodology document is applicable to the bit-level combining and the symbol-level combining after equalization. However, the PHY abstraction methodology for the symbol-level combining before equalization is not described in the current draft of the evaluation methodology document. So, we propose to include the post-processing SINR calculation method for the symbol-level combining before MIMO equalization.

Proposed Text

4.7.2.1 Symbol-Level Combining before MIMO Equalization with CC H-ARQ

For symbol-level combining before MIMO equalization with H-ARQ, the post-processing SINR can be obtained by computing the post-processing SINR of the combined receive signal.

As in Section 4.5.4, for the i-th transmission, the received signal at the n-th subcarrier is

$$\underline{Y}_i^{(0)}(n) = \sqrt{P_{tx,i}^{(0)} P_{loss,i}^{(0)}}\, \underline{H}_i^{(0)}(n) \underline{X}_i^{(0)}(n) + \sum_{j=1}^{N_I} \sqrt{P_{tx,i}^{(j)} P_{loss,i}^{(j)}}\, \underline{H}_i^{(j)}(n) \underline{X}_i^{(j)}(n) + \underline{U}_i^{(0)}(n),$$

where the subscript *i* represent the transmission index. For every transmission, the same signal is transmitted for the intended user, i.e., $\underline{X}_i^{(0)}(n) = \underline{X}^{(0)}(n)$ for every i.

Define the combined squared channel matrix as

$$\underline{S}_i^{(0)}(n) = P_{tx,i}^{(0)} P_{loss,i}^{(0)} \underline{H}_i^{(0)*}(n) \underline{\tilde{\sigma}}_i^{-2}(n) \underline{H}_i^{(0)}(n) + \underline{S}_{i-1}^{(0)}(n).$$

with $\underline{S}_0^{(0)}(n) = 0$.

In terms of the combined squared channel matrix, the estimate of the transmit signal with a linear MMSE receiver can be represented as

$$.\ \underline{\hat{X}}_i^{(0)}(n) = \left(\underline{S}_i^{(0)}(n) + \underline{I}\right)^{-1} \sum_{m=1}^{i} \sqrt{P_{tx,m}^{(0)} P_{loss,m}^{(0)}}\, \underline{H}_m^{(0)*}(n) \underline{\tilde{\sigma}}_m^{-2}(n) \underline{Y}_m^{(0)}(n)$$

The post-processing SINR can be computed conveniently by defining the following two expressions:
$D_i(n) = diag\left[\left(\underline{S}_i^{(0)}(n) + I\right)^{-1} \underline{S}_i^{(0)}(n)\right]$ which denotes the desired signal component and
$I_{self,i}(n) = \left(\underline{S}_i^{(0)}(n) + I\right)^{-1} \underline{S}_i^{(0)}(n) - D_i(n)$ which is the self interference between MIMO streams.

Then the desired signal component power for the *k-th* MIMO stream is $diag\left[\sigma_0^2 D_i(n) D_i^*(n)\right]_{kk}$, whereas the power of the self interference between MIMO streams is $diag\left[\sigma_0^2 I_{self,i}(n) I_{self,i}^*(n)\right]_{kk}$. It can also be derived that the power of the inter-cell interference plus noise is given as $\left(\underline{S}_i^{(0)}(n) + I\right)^{-1} \underline{S}_i^{(0)}(n) \left(\underline{S}_i^{(0)}(n) + I\right)^{-1}$.

The post processing SINR of the desired MS for the *n-th* subcarrier and the *k-th* MIMO stream is thus given as:

$$SINR_{k,i}^{(0)}(n) = \frac{diag\left[\sigma_0^2 D_i(n) D_i^*(n)\right]_{kk}}{diag\left[\sigma_0^2 I_{self,i}(n) I_{self,i}^*(n) + \left(\underline{S}_i^{(0)}(n) + I\right)^{-1} \underline{S}_i^{(0)}(n) \left(\underline{S}_i^{(0)}(n) + I\right)^{-1}\right]_{kk}}$$

References

[1] IEEE 802.16m-07/037r2, “Draft IEEE 802.16m Evaluation Methodology”, Dec. 14, 2007.

[2] IEEE C802.16m-07/172, “PHY Abstraction for MIMO H-ARQ,” Sep. 7, 2007.