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Re:	IEEE 802.16m-08/024 - Call for Comments and Contributions on Project 802.16m System Description Document (SDD). Topic: UL MIMO
Abstract	This contribution proposes a closed-loop approach for uplink multi-user MIMO transmissions within 802.16m system implementations.
Purpose	To review and adopt the proposed text in the next revision of the SDD.
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Closed-loop Multi-user MIMO for Uplink Transmissions

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1 Introduction

Multiple-input, multiple-output (MIMO) communication systems employ $N_T \geq 1$ transmit antennas and $N_R \geq 1$ receive antennas. An N_R-by-N_T MIMO channel may be decomposed into $N_S \leq \min(N_T, N_R)$ independent spatial subchannels when the MIMO channel matrix is a full-rank matrix. Each spatial subchannel may support an independent spatial data stream. Possible antenna configurations for 802.16m systems are $(N_T, N_R) = (2, 2)$, $(4, 2)$ and $(4, 4)$. Hence, uplink single user MIMO transmissions of two or four independent spatial streams are possible using horizontal (Matrix B) space-time coding. Closed-loop MU-MIMO precoding techniques may be used to further increase the spectral efficiency of uplink transmissions. Using closed-loop MU-MIMO identical subframe time-frequency resources (Physical Resource Units) may be concurrently transmitted by two or more MSs to a BS. This contribution describes a closed-loop multi-user MIMO technique for uplink 802.16m transmissions.

2 Received Base Station Signal Model

Figure 1 shows a conceptual block diagram of an multi-MS MIMO system for uplink transmissions. To describe the signals and the proposed technique we use Figure 1 and the following notation:

- N_R denotes the number of BS receive antennas.
- $N_{T,m}$ denotes the number of transmit antennas for the mth MS. $N_{T,m}$ may equal 1, 2 or 4. The number of transmit antennas used at each MS can be different
- M denotes the number of active MSs serviced by the BS.
- $N_{S,m} \leq N_{T,m}$ denotes the number of independent spatial streams transmitted by the mth MS.
- $\mathbf{W}_m \in \mathbb{C}^{N_{T,m} \times N_{S,m}}$ denotes the linear precoding matrix for the mth MS.
- $\mathbf{s}_m \in \mathbb{C}^{N_{S,m} \times 1}$ denotes the data symbol vector for the mth MS. Vector $\mathbf{s}_m$ supports $N_{S,m}$ independent spatial streams. Each MS's data symbol vector can support space-time coded data or spatially multiplexed data.
- $\mathbf{H}_m \in \mathbb{C}^{N_R \times N_{T,m}}$ denotes the channel matrix for the mth MS. The (i, j)th element of $\mathbf{H}_m$ represents the channel gain and phase associated with the signal path from MS transmit antenna j to BS receive antenna i. The channel matrices are assumed fixed during the transmission duration but may change independently from one uplink subframe to the next uplink subframe.
- $\mathbf{v}_m \in \mathbb{C}^{N_R \times 1}$ denotes the BS's interference-plus-noise vector.

The uplink signal transmitted by the mth MS in Figure 1 is defined as

$$\mathbf{x}_m = \mathbf{W}_m \mathbf{s}_m \in \mathbb{C}^{N_{T,m} \times 1} \tag{1}$$

We assume that the data symbol vector $\mathbf{s}_m$ has an arbitrary probability distribution, normalized unit energy, and the following mean and covariance

$$
\begin{aligned}
E[\mathbf{s}_m] &= \mathbf{0} && (2)\\
\mathbf{R}_{\mathbf{s}_m} &= E[\mathbf{s}_m\mathbf{s}_m^H] = \mathbf{I}_{N_{T,m}} \in \mathbb{R}^{N_{T,m}\times N_{T,m}} && (3)
\end{aligned}
$$

Matrix $\mathbf{I}_{N_{T,m}N_T}$ denotes an identity matrix. Hence, the covariance matrix of $\mathbf{x}_m$ is

$$
\mathbf{R}_{\mathbf{x}_m} = E\left[\mathbf{x}_m\mathbf{x}_m^H\right] = \mathbf{W}_m\mathbf{R}_{\mathbf{s}_m}\mathbf{W}_m^H = \mathbf{W}_m\mathbf{W}_m^H \in \mathbb{C}^{N_{T,m}\times N_{T,m}} \tag{4}
$$

The total average transmit power is

$$
P_T = \mathrm{Tr}\left\{\mathbf{R}_{\mathbf{x}_m}\right\} = \mathrm{Tr}\left\{\mathbf{W}_m\mathbf{W}_m^H\right\} \tag{5}
$$

where Tr denotes the trace of the matrix.

We assume that the cyclic prefix is greater in length than the channel delay spread and that the maximum Doppler frequency is much smaller than the OFDM symbol subcarrier spacing. We can therefore ignore any inter-subcarrier interference caused by Doppler frequency spreading. Under these assumptions the received BS signal for the mth MS may be written as

$$
\mathbf{y}_m = \mathbf{H}_m\mathbf{x}_m + \sum_{\substack{j=1\\ j\neq m}}^{M} \mathbf{H}_j\mathbf{x}_j + \mathbf{n}_m \in \mathbb{C}^{N_R\times 1} \tag{6}
$$

where $\mathbf{n}_m \in \mathbb{C}^{N_R\times 1}$ denotes a noise vector of arbitrary probability distribution with the following mean and covariance

$$
\begin{aligned}
E[\mathbf{n}_m] &= \mathbf{0} && (7)\\
\mathbf{R}_{\mathbf{n}_m} &= E[\mathbf{n}_m\mathbf{n}_m^H] = \sigma^2_{\mathbf{n}_m}\mathbf{I}_{N_R} \in \mathbb{C}^{N_R\times N_R} && (8)
\end{aligned}
$$

The BS's interference-plus-noise vector is defined as

$$
\mathbf{v}_m = \sum_{\substack{j=1\\ j\neq m}}^{M} \mathbf{H}_j\mathbf{x}_j + \mathbf{n}_m \tag{9}
$$

Let the effective channel matrix be defined as

$$
\mathbf{F}_m = \mathbf{H}_m\mathbf{W}_m \in \mathbb{C}^{N_R\times N_{S,m}} \tag{10}
$$

then the received BS signal from the mth MS may be written more compactly as

$$
\mathbf{y}_m = \mathbf{F}_m\mathbf{s}_m + \mathbf{v}_m \tag{11}
$$

Mean and covariances associated with the received BS signal are defined as

$$
\begin{aligned}
E\left[\mathbf{y}_m\right] &= \mathbf{0} \in \mathbb{C}^{N_R\times 1} && (12)\\
E\left[\mathbf{v}_m\right] &= \mathbf{0} \in \mathbb{C}^{N_R\times 1} && (13)\\
\mathbf{R}_{\mathbf{y}_m} &= E\left[\mathbf{y}_m\mathbf{y}_m^H\right] = \mathbf{F}_m\mathbf{F}_m^H + \mathbf{R}_{\mathbf{v}_m} \in \mathbb{C}^{N_R\times N_R} && (14)\\
\mathbf{R}_{\mathbf{v}_m} &= E\left[\mathbf{v}_m\mathbf{v}_m^H\right] = \sum_{\substack{j=1\\ j\neq m}}^{M} \mathbf{F}_j\mathbf{F}_j^H + \sigma^2_{\mathbf{n}_m}\mathbf{I}_{N_R} \in \mathbb{C}^{N_R\times N_R} && (15)\\
\mathbf{R}_{\mathbf{v}_m\mathbf{s}_m} &= E\left[\mathbf{v}_m\mathbf{s}_m^H\right] = \mathbf{0} \in \mathbb{C}^{N_R\times N_{S,m}} && (16)\\
\mathbf{R}_{\mathbf{y}_m\mathbf{s}_m} &= E\left[\mathbf{y}_m\mathbf{s}_m^H\right] = \mathbf{F}_m \in \mathbb{C}^{N_R\times N_{S,m}} && (17)
\end{aligned}
$$

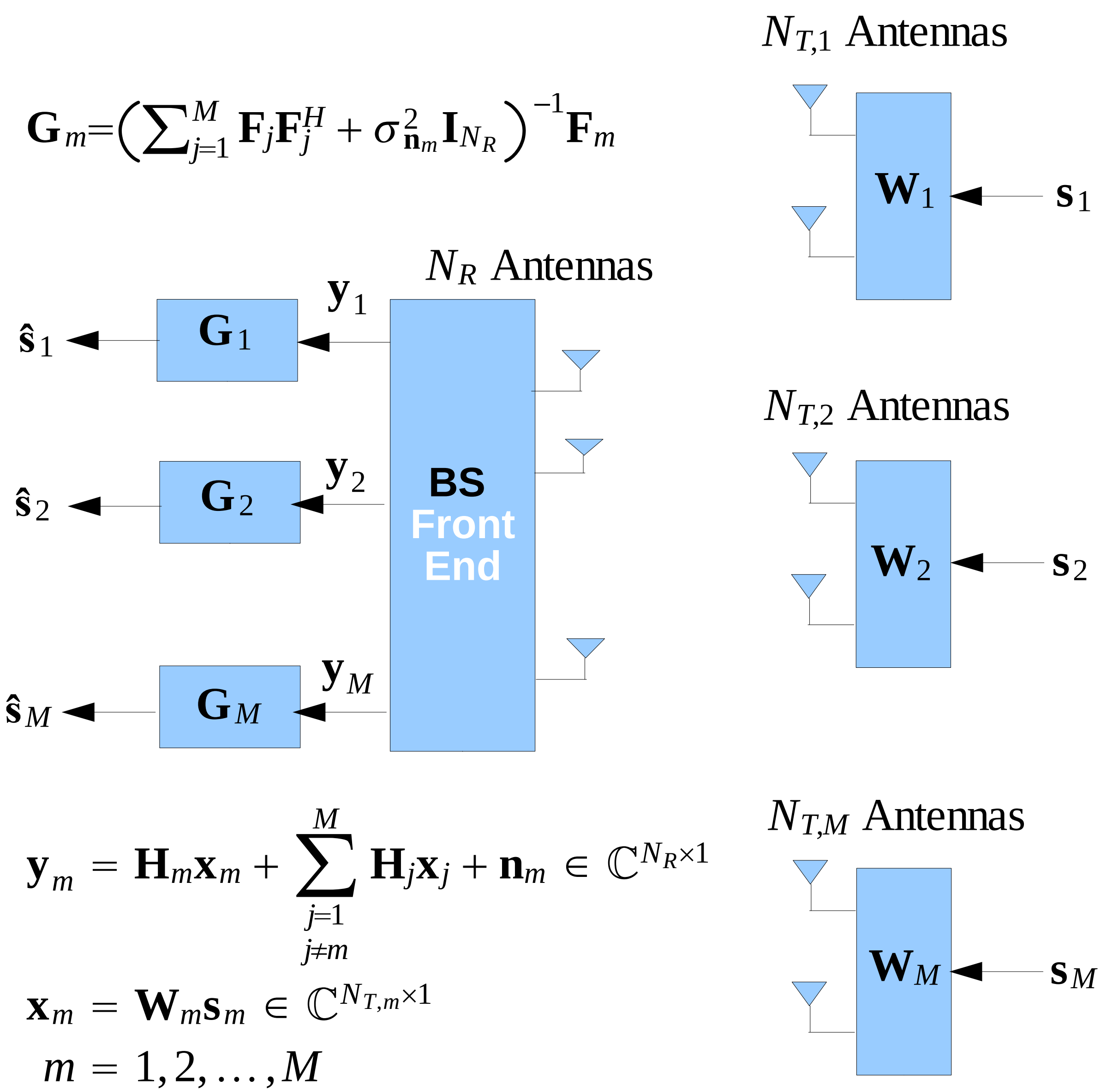

Figure 1: Conceptual block diagram of the proposed technique for uplink MU-MIMO.

3 Joint Precoder-Decoder Design Problem

In a TDD mode of operation channel reciprocity allows MU-MIMO transmissions to be implemented in three ways: (1) using only MS precoding or "pre-equalization" matrices; (2) using only BS decoding or equalization matrices; and (3) using both a precoding matrix at the MS and a decoding matrix at the BS. Precoding and decoding of UL spatial streams improves the system performance compared to only spatial precoding or spatial decoding (equalization). A linear estimate of the transmitted vector $\mathbf{s}_m$ may be written as

$$\hat{\mathbf{s}}_m = \mathbf{G}_m^H \mathbf{y}_m \in \mathbb{C}^{N_{S,m} \times 1} \tag{18}$$

where $\mathbf{G}_m^H \in \mathbb{C}^{N_R \times N_{S,m}}$. The estimation error covariance matrix as a function of $\mathbf{W}_m$ and $\mathbf{G}_m$ is defined as

$$\begin{aligned} \mathbf{E}_m &= E\left[(\hat{\mathbf{s}}_m - \mathbf{s}_m)(\hat{\mathbf{s}}_m - \mathbf{s}_m)^H \right] \in \mathbb{C}^{N_{S,m} \times N_{S,m}} && (19) \\ &= \mathbf{G}_m^H \mathbf{R}_{\mathbf{y}_m} \mathbf{G}_m + \mathbf{I}_{N_{S,m}} - \mathbf{G}_m^H \mathbf{F}_m - \mathbf{F}_m^H \mathbf{G}_m && (20) \\ &= \mathbf{G}_m^H \left(\mathbf{F}_m \mathbf{F}_m^H + \mathbf{R}_{\mathbf{v}_m} \right) \mathbf{G}_m + \mathbf{I}_{N_{S,m}} - \mathbf{G}_m^H \mathbf{F}_m - \mathbf{F}_m^H \mathbf{G}_m && (21) \\ &= \left(\mathbf{G}_m^H \mathbf{F}_m - \mathbf{I}_{N_{S,m}} \right) \left(\mathbf{F}_m^H \mathbf{G}_m - \mathbf{I}_{N_{S,m}} \right) + \mathbf{G}_m^H \mathbf{R}_{\mathbf{v}_m} \mathbf{G}_m && (22) \end{aligned}$$

As shown in [3,4,5] the joint linear precoder and decoder design problem can be formulated as the following constrained optimization problem

$$\begin{aligned} (\mathbf{W}_m, \mathbf{G}_m) &= \arg \min_{\mathbf{W},\mathbf{G}} \operatorname{Tr}\{\mathbf{E}_m\} && (23) \\ \text{Subject to} &: \operatorname{Tr} \quad \left\{ \mathbf{W}_m \mathbf{W}_m^H \right\} \le P_T && (24) \end{aligned}$$

4 BS Decoder Design

4.1 Wiener Decoder

The linear decoder matrix $\mathbf{G}_m$ can be determined using a fixed precoder matrix $\mathbf{W}_m$. The Wiener decoder of filter is defined as

$$\begin{aligned} \mathbf{G}_m &= \{\mathbf{R}_{\mathbf{y}_m}\}^{-1} \mathbf{R}_{\mathbf{y}_m \mathbf{s}_m} \in \mathbb{C}^{N_R \times N_{S,m}} && (25) \\ &= \left(\mathbf{F}_m \mathbf{F}_m^H + \mathbf{R}_{\mathbf{v}_m} \right)^{-1} \mathbf{F}_m && (26) \\ &= \left(\mathbf{H}_m \mathbf{W}_m \mathbf{W}_m^H \mathbf{H}_m^H + \mathbf{R}_{\mathbf{v}_m} \right)^{-1} (\mathbf{H}_m \mathbf{W}_m) && (27) \end{aligned}$$

Note that the Wiener decoder is independent of the distribution shapes of $\mathbf{y}_m$ and $\mathbf{v}_m$; it is function only of the statistic $\mathbf{R}_{\mathbf{v}_m}$. A Wiener decoder $\mathbf{G}_m$, $m = 1, 2, \ldots, M$, can be used for each of the M active MS's. The UL-MIMO receiver can therefore be viewed as a bank of M Wiener decoders that each estimate an MS's spatial streams. This structure is illustrated in Figure 1.

Using the matrix inversion identity $\mathbf{F}_m^H \left(\mathbf{F}_m \mathbf{F}_m^H + \mathbf{R}_{\mathbf{v}_m} \right)^{-1} = \left(\mathbf{I}_{N_R} + \mathbf{F}_m^H \mathbf{R}_{\mathbf{v}_m}^{-1} \mathbf{F}_m \right)^{-1} \mathbf{F}_m^H \mathbf{R}_{\mathbf{v}_m}^{-1}$ we can rewrite $\mathbf{G}_m$ as

$$\mathbf{G}_m = \mathbf{R}_{\mathbf{v}_m}^{-1} \mathbf{F}_m \left(\mathbf{I}_{N_R} + \mathbf{F}_m^H \mathbf{R}_{\mathbf{v}_m}^{-1} \mathbf{F}_m \right)^{-1} \tag{28}$$

Substituting this form of $\mathbf{G}_m$ into $\mathbf{E}_m$ above we obtain

$$\mathbf{E}_m = \left(\mathbf{I}_{N_T} + \mathbf{F}_m^H \mathbf{R}_{\mathbf{v}_m}^{-1} \mathbf{F}_m\right)^{-1} \tag{29}$$

As shown in the references the mean-squared error (MSE) associated with the ith spatial stream is defined as

$$MSE_{m,i} = [\mathbf{E}_m]_{ii} = \frac{1}{1 + \left(\mathbf{w}_{m,i}^H \mathbf{H}_m^H\right) \mathbf{R}_{\mathbf{v}_{m,i}}^{-1} \left(\mathbf{H}_m \mathbf{w}_{m,i}\right)} \leq 1 \tag{30}$$

where $[\mathbf{E}_m]_{ii}$ denotes the ith diagonal element of $\mathbf{E}_m$ and $\mathbf{w}_{m,i} \in \mathbb{C}^{N_T \times 1}$ the ith column of $\mathbf{W}_m$. Matrix

$$\begin{aligned} \mathbf{R}_{\mathbf{v}_{m,i}} &= E\left[\mathbf{v}_{m.i} \mathbf{v}_{m,i}^H\right] && (31) \\ &= \mathbf{R}_{\mathbf{v}_m} + \mathbf{F}_m \mathbf{F}_m^H - \mathbf{H}_m \mathbf{w}_{m,i} \mathbf{w}_{m,i}^H \mathbf{H}_m^H && (32) \\ &= \mathbf{R}_{\mathbf{v}_m} + \mathbf{H}_m \left(\mathbf{W}_m \mathbf{W}_m^H - \mathbf{w}_{m,i} \mathbf{w}_{m,i}^H\right) \mathbf{H}_m^H \in \mathbb{C}^{N_R \times N_R} && (33) \end{aligned}$$

is the covariance matrix of $\mathbf{v}_{m.i}$ which denotes the interference-plus-noise associated with the ith spatial stream. The post-processing SINR SINR of the ith spatial stream may be written in terms of $MSE_{m,i}$ as

$$SINR_{m,i} = \frac{1 - MSE_{m,i}}{MSE_{m,i}} = \mathbf{w}_{m,i}^H \mathbf{H}_m^H \mathbf{R}_{\mathbf{v}_{m,i}}^{-1} \mathbf{H}_m \mathbf{w}_{m,i} \tag{34}$$

If the interference-plus-noise term $\mathbf{v}_{m.i}$ has a Gaussian distribution the BER for the ith stream may be written in terms of $MSE_{m,i}$ and $SINR_{m,i}$ as

$$\begin{aligned} BER_{m,i} &= \frac{4}{\log_2 M}\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3}{M-1} SINR_{m,i}}\right) && (35) \\ &= \frac{4}{\log_2 M}\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3}{M-1} \frac{1 - MSE_{m,i}}{MSE_{m,i}}}\right) && (36) \end{aligned}$$

where parameter M defines an M-QAM constellation and the Gaussian Q-function is defined as

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-p^2/2} dp \tag{37}$$

If the interference-plus-noise $\mathbf{v}_{m.i}$ is non-Gaussian $BER_{m,i}$ may still serve as an approximation to the true BER.

4.2 Successive Interference Cancellation

It may be possible to upgrade the performance of the above Wiener decoder approach by using a nonlinear Successive Interference Cancellation (SIC) approach. There are some drawbacks with the SIC approach. First, is the problem of error propagation. If one of the spatial streams has been detected incorrectly detected, the remaining spatial streams may also be incorrectly detected. Second, the decoding procedure is serial which increases the processing time. Third, the performance is unequal on various spatial streams (best on first spatial stream, worst on the last).

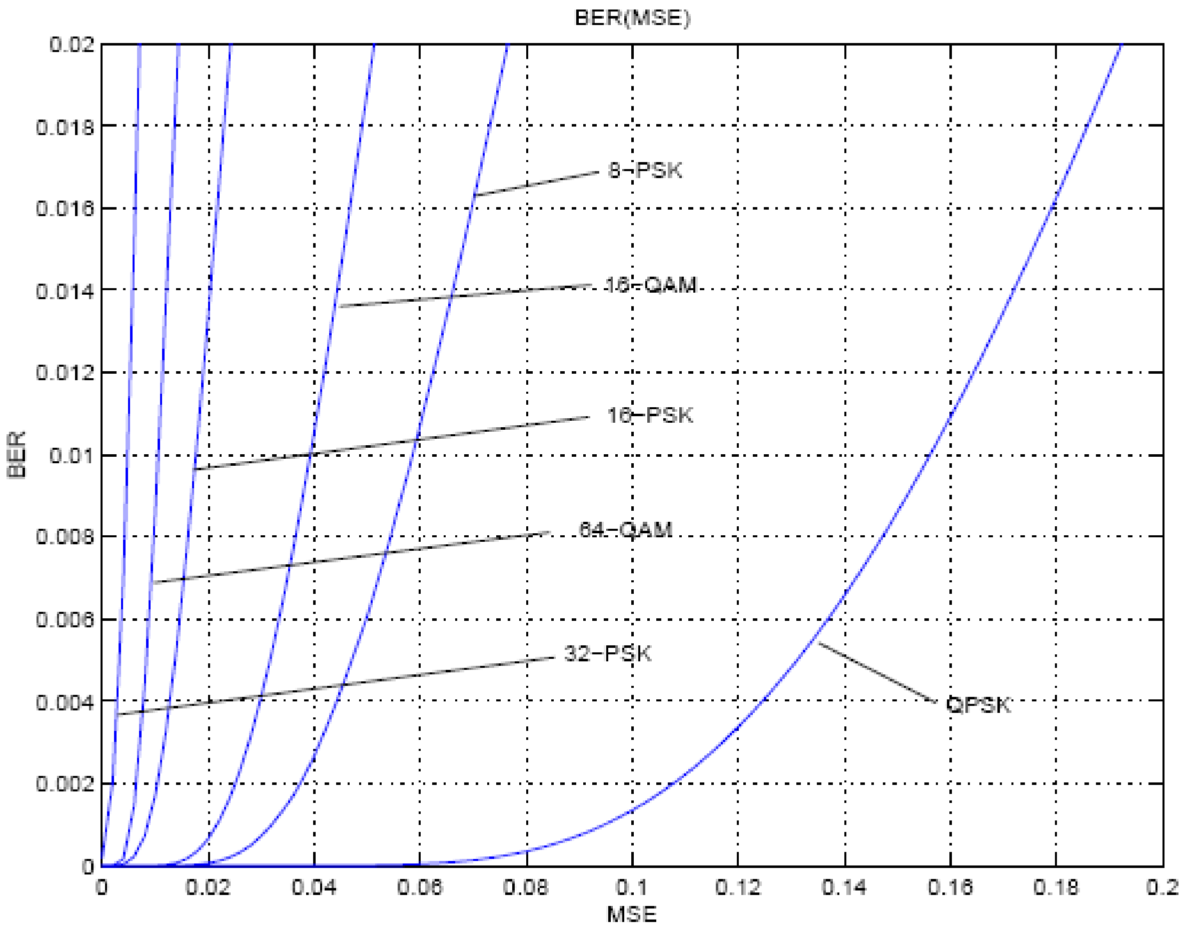

Figure 2: Plots of BER for the ith stream in terms of $MSE_{m,i}$. Since $MSE_{m,i}$ can also be written in terms of SINR the plots can also be used to derive plots for BER versus SINR.

5 MS Precoder Design

Given $\mathbf{G}_m$ the formulation of the constrained optimization problem for the precoder matrix $\mathbf{W}_m$ can be reformulated as

$$\mathbf{W}_m = \arg\min_{\mathbf{W}} \mathrm{Tr}\left\{\mathbf{E}_m\right\} \tag{38}$$

$$\text{Subject to} \; : \mathrm{Tr} \; \left\{\mathbf{W}_m\mathbf{W}_m^H\right\} \leq P_T \tag{39}$$

where

$$\mathbf{E}_m = \left(\mathbf{I}_{N_T} + \mathbf{W}_m^H\mathbf{H}_m^H\mathbf{R}_{\mathbf{v}_m}^{-1}\mathbf{H}_m\mathbf{W}\right)^{-1} \tag{40}$$

The optimum precoding matrix is defined as

$$\mathbf{W}_m = \mathbf{V}_m\mathbf{P}_m \in \mathbb{C}^{N_{T,m}\times N_{S,m}} \tag{41}$$

Matrix $\mathbf{V}_m$ is the right singular matrix of the singular value decomposition $\mathbf{H}_m = \mathbf{U}_m\mathbf{\Sigma}_m\mathbf{V}_m^H$ where $\mathbf{U}_m \in \mathbb{C}^{N_R\times N_R}$ and $\mathbf{V}_m \in \mathbb{C}^{N_{T,m}\times N_{T,m}}$ are unitary matrices and $\mathbf{\Sigma}_m \in \mathbb{C}^{N_R\times N_{T,m}}$ a singular value matrix. Matrix $\mathbf{P}_m \in \mathbb{R}^{N_{T,m}\times N_{S,m}}$ denotes the diagonal stream power loading matrix for the mth MS. The ith diagonal element denotes the signal power loaded to the ith spatial stream. Matrix $\mathbf{P}_m$ is of the form

$$\mathbf{P}_m = \begin{bmatrix}\mathbf{D}_m & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix} \in \mathbb{R}^{N_{T,m}\times N_{S,m}} \tag{42}$$

where

$$\mathbf{D}_m = \mathrm{diag}(P_1, P_2, \ldots, P_r) \in \mathbb{R}^{r\times r} \tag{43}$$

and $P_1 \geq P_2 \geq \ldots \geq P_r > 0$ are the stream power loading values with $r = \min(N_{S,m}, N_{T,m})$. Note that matrix $\mathbf{P}_m$ has zero elements except for $\mathbf{D}_m$. Optimal values for $\mathbf{D}_m$ can be found in the literature. However, rather than using optimal power values we propose that available transmit power be equally divided between all transmit antennas and set $\mathbf{D}_m$ equal to an identity matrix. The use of equally divided antenna power simplifies codebook design and the power control problem.

6 Closed-loop Precoding

To obtain $\mathbf{G}_m$ as a function of $\mathbf{W}_m = \mathbf{V}_m\mathbf{P}_m$ we substitute

$$\mathbf{R}_{\mathbf{v}_m} = \sum_{\substack{j=1\\ j\neq m}}^{M} \mathbf{F}_j\mathbf{F}_j^H + \sigma_{\mathbf{n}_m}^2\mathbf{I}_{N_R} \tag{44}$$

into $\mathbf{G}_m$ which gives

$$\mathbf{G}_m = \left(\mathbf{F}_m\mathbf{F}_m^H + \mathbf{R}_{\mathbf{v}_m}\right)^{-1}\mathbf{F}_m \tag{45}$$

$$= \left(\sum_{j=1}^{M}\mathbf{F}_j\mathbf{F}_j^H + \sigma_{\mathbf{n}_m}^2\mathbf{I}_{N_R}\right)^{-1}\mathbf{F}_m \tag{46}$$

where

$$\mathbf{F}_m = \mathbf{H}_m\mathbf{W}_m = \left(\mathbf{U}_m\mathbf{\Sigma}_m\mathbf{V}_m^H\right)(\mathbf{V}_m\mathbf{P}_m) = \mathbf{U}_m\mathbf{\Sigma}_m\mathbf{P}_m \tag{47}$$

In the above equation we see that the above precoding technique requires that the effective channel matrices $\mathbf{F}_m$, $m = 1, 2, \ldots, M$, and $\sigma^2_{\mathbf{n}_m}$ be available at the BS. This can be accomplished using a closed-loop approach where each $\mathbf{F}_m$ is estimated at an MS and then provided to the BS via an uplink codeword. For the closed-loop approach it is simple to quantize MS-estimated effective channels $\mathbf{F}_m$ and transmit codebook indices that encode effective channels back to the BS. A large number M of active MSs results in an unacceptably high computational and feedback load. We therefore propose that the maximum number of active or concurrent MSs is $M = 4$.

It must be emphasized that uplink MS receptions and downlink BS receptions are related as follows

$$\underbrace{\mathbf{H}_m}_{\text{Channel observed at BS for uplink receptions}} = \underbrace{\mathbf{H}_m^T}_{\text{Channel observed at MS for dowmlink receptions}} \tag{48}$$

Hence to estimate $\mathbf{H}_m^T$ the MS requires downlink pilots dedicated to each of the BS's N_R antennas. The same pilot signals can be shared by all active MSs. An alternative is to use uplink pilots dedicated to each of the MS's transmit antennas and let the BS estimate $\mathbf{H}_m$. However, this would require more feedback overhead since there are M active MSs as opposed to one BS.

We now give an example to describe how memoryless vector quantization (VQ) may be implemented and used for closed-loop precoding. Other applicable quantization approaches may be used, some examples are described in [4]. Memoryless vector quantization is a classification or encoding procedure which concerns the non-linear mapping of MS-estimated matrices $\mathbf{F}_m$ into bit vectors $\mathbf{b}_i$. The bit vectors label entries in a codebook $\mathcal{C}$ known by both the MSs and the BS. The bit vectors are determined by the MSs and are transmitted to a BS. The BS uses the received bit vectors to access a quantized effective channel $\mathbf{F}_m$ from the codebook $\mathcal{C}$.

More specifically, let

$$Q(\mathbf{F}_m) = \arg\min_{\mathbb{F}\in\mathcal{C}} S(\mathbf{F}_m, \mathbb{F}) = \arg\min_{\mathbb{F}\in\mathcal{C}} \left\|\mathbf{F}_m\mathbf{F}_m^H - \mathbb{F}\mathbb{F}^H\right\|_2 \tag{49}$$

denote an operation that maps $\mathbf{F}_m$ to its quantized representation $Q(\mathbf{F}_m)$. Note that for some matrix $\mathbf{A}$ the matrix norm operator $\|\mathbf{A}\|_2$ produces the largest singular value of $\mathbf{A}$. Some other functions that can be used for $S(\mathbf{F}_m, \mathbb{F})$ are defined within [2]. Jacobi rotations can be used to compute the singular value decompositions of the $N_R \times N_{S,m}$ matrix $\mathbf{F}_m$. Antenna values N_R and $N_{S,m}$ are also small so the computation is simple.

Let the effective channel codebook be the K-dimensional set

$$\mathcal{C} = \left\{\mathbb{F}_1 \quad \mathbb{F}_2 \quad \ldots \quad \mathbb{F}_K\right\} \tag{50}$$

and

$$\mathbf{b}_k = f(Q(\mathbf{F}_m)) \tag{51}$$

denote an operation that maps a quantized effective MIMO channel $Q(\mathbf{F}_m)$ to length-$\lceil\log_2 K\rceil$ bit vector $\mathbf{b}_k$ within the set

$$\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_K\} \tag{52}$$

For example, $f(Q(\mathbf{F}_m))$ may be a simple look-up table operation that outputs a bit vector $\mathbf{b}_k$ given $Q(\mathbf{F}_m)$. When the BS receives $\mathbf{b}_k$ from the kth MS the BS uses $\mathbf{b}_k$ to read the corresponding effective MIMO channel stored in the codebook $\mathcal{C}$. An example is given in Table 1, where we assume a codebook with $K = 8$ entries.

Quantized Effective MIMO Channel	**3-bit Channel Index**
$\mathbb{F}_1$	$\mathbf{b}_1 = 000$
$\mathbb{F}_2$	$\mathbf{b}_2 = 001$
$\mathbb{F}_3$	$\mathbf{b}_3 = 010$
$\mathbb{F}_4$	$\mathbf{b}_4 = 011$
$\mathbb{F}_5$	$\mathbf{b}_5 = 100$
$\mathbb{F}_6$	$\mathbf{b}_6 = 101$
$\mathbb{F}_7$	$\mathbf{b}_7 = 110$
$\mathbb{F}_8$	$\mathbf{b}_8 = 111$

Table 1: Example codebook entries and indices for closed-loop precoding

7 References

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8 Proposed Text

11. Physical Layer

11.x UL MIMO

11.x.y. Closed-loop Uplink MU-MIMO

Closed-loop MU-MIMO precoding techniques may be used to further increase the spectral efficiency of uplink transmissions. Using closed-loop MU-MIMO identical subframe time-frequency resources (Physical Resource Units) may be concurrently transmitted by two or more active MSs to a BS. The maximum number of active MSs that may be concurrently transmitting using the same uplink Physical Resource Units is four. MS-estimated effective MIMO channel matrices will be provided to a BS in the form of uplink codewords. The BS will use MS-estimated effective MIMO channel matrices to perform its decoding of concurrent uplink transmissions.