Differential Feedback Scheme for Closed-Loop Beamforming

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Re:

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Base Contribution:

None

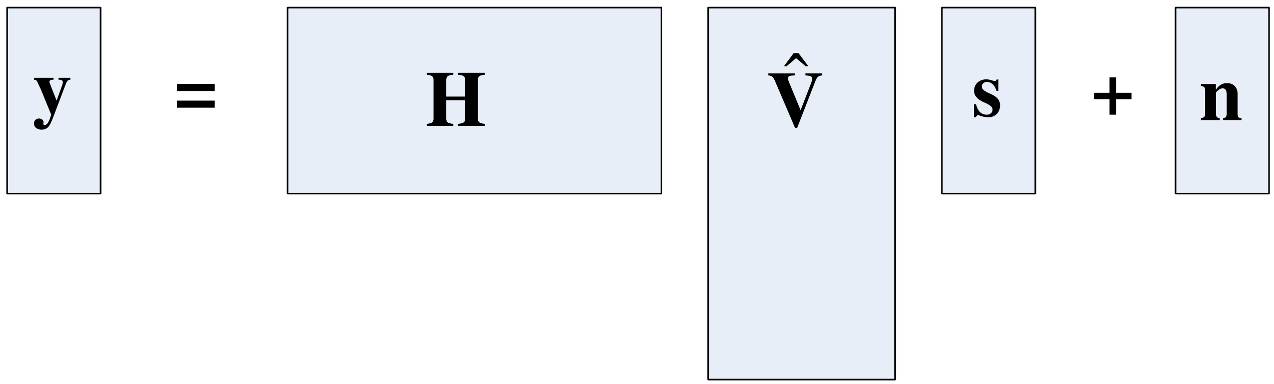
Purpose:

Discussion and adoption by TGm AWD

Outline

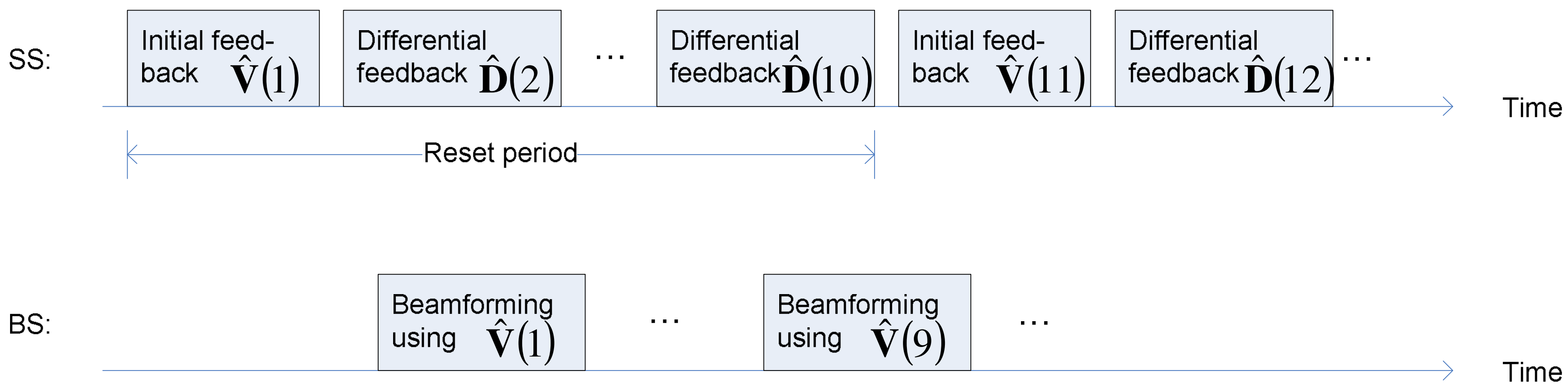
- Introduction
- Differential feedback
  - **Scheme I: C80216m-09_0528r4, Qinghua Li, et al., Intel.**
  - **Scheme II: S80216m-09_0790r1, Bruno Clerckx, et al., Samsung.**
- Comparison of throughput, reliability, overhead, and complexity
- Conclusions
- Proposed text

Signal model — matrix dimensions

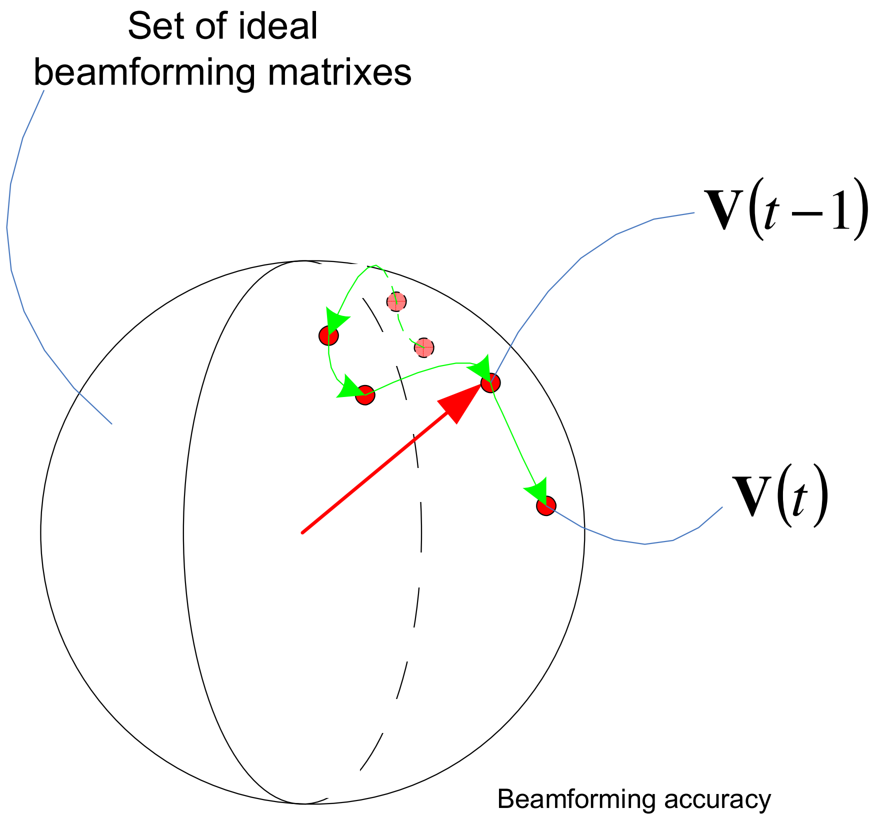


- $\mathbf{H}$ is channel matrix of dimension $N_r \times N_t$.
- $\hat{\mathbf{V}}$ is beamforming matrix of dimension $N_t \times N_s$.
- $\mathbf{s}$ is transmitted signal vector of dimension $N_s \times 1$.

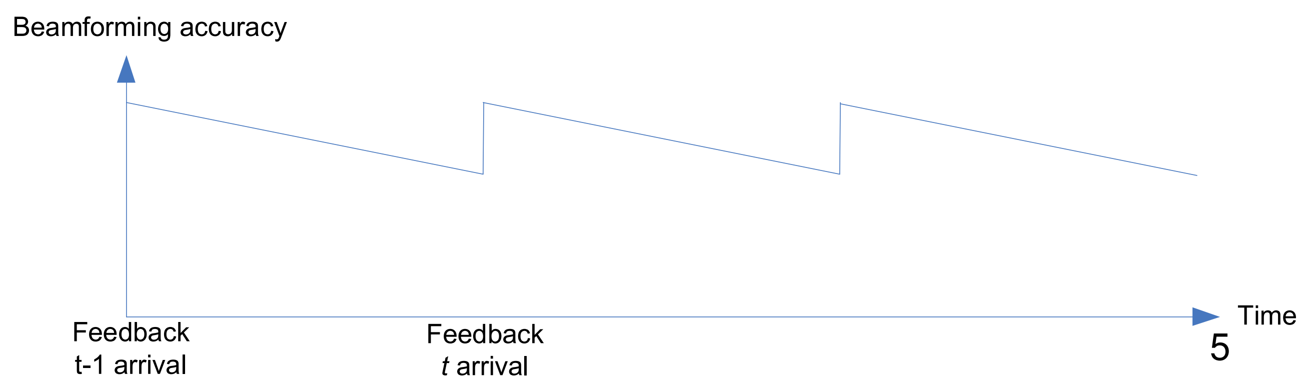
One-shot reset and differential feedbacks



There is always correlation between adjacent precoders that can be utilized.



• Adjacent beamforming matrixes are not independent. Otherwise, the beamforming gain vanishes before the next feedback arrives.



Differential codebook — polar cap

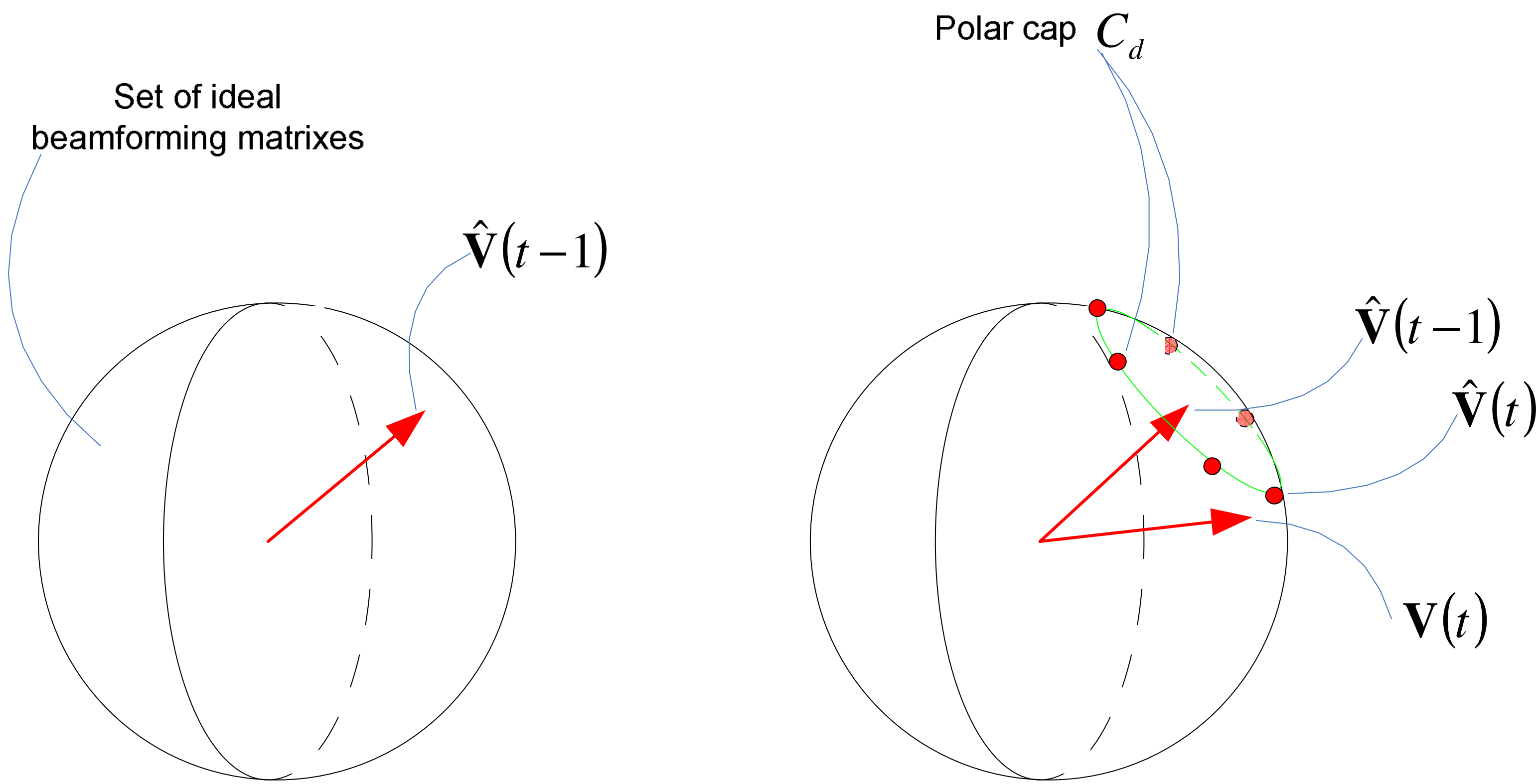


Illustration of Scheme I

- Differentiation at SS: $\mathbf{D}=\mathbf{Q}^{H}\left(t-1\right)\mathbf{V}\left(t\right)$

Beamforming matrix needs to be fed back.

- Quantization at SS: $\hat{\mathbf{D}}=\arg\max_{\mathbf{D}_i\in C_d}\left\|\mathbf{D}^{H}\mathbf{D}_i\right\|_F$

Should be replaced by maximizing channel capacity in practice.

- Beamforming matrix reconstruction at BS:

$$\hat{\mathbf{V}}\left(t\right)=\mathbf{Q}\left(t-1\right)\hat{\mathbf{D}}$$

- Beamforming at BS: $\mathbf{y}=\mathbf{H}\,\hat{\mathbf{V}}\left(t\right)\mathbf{s}+\mathbf{n}$

Sanity check: $\hat{\mathbf{V}}\left(t\right)=\mathbf{Q}\left(t-1\right)\hat{\mathbf{D}}\approx\underbrace{\mathbf{Q}\left(t-1\right)\mathbf{Q}^{H}\left(t-1\right)}_{\mathbf{I}}\mathbf{V}\left(t\right)=\mathbf{V}\left(t\right)$

Actual implementation

- Actual quantization at SS:

$$\hat{\mathbf{D}} = \arg\max_{\mathbf{D}_i \in C_d} \det\left(\mathbf{I} + \frac{\gamma}{N_s} \mathbf{D}_i^H \mathbf{Q}(t-1)^H \mathbf{H}^H \mathbf{H} \mathbf{Q}(t-1) \mathbf{D}_i \right)$$

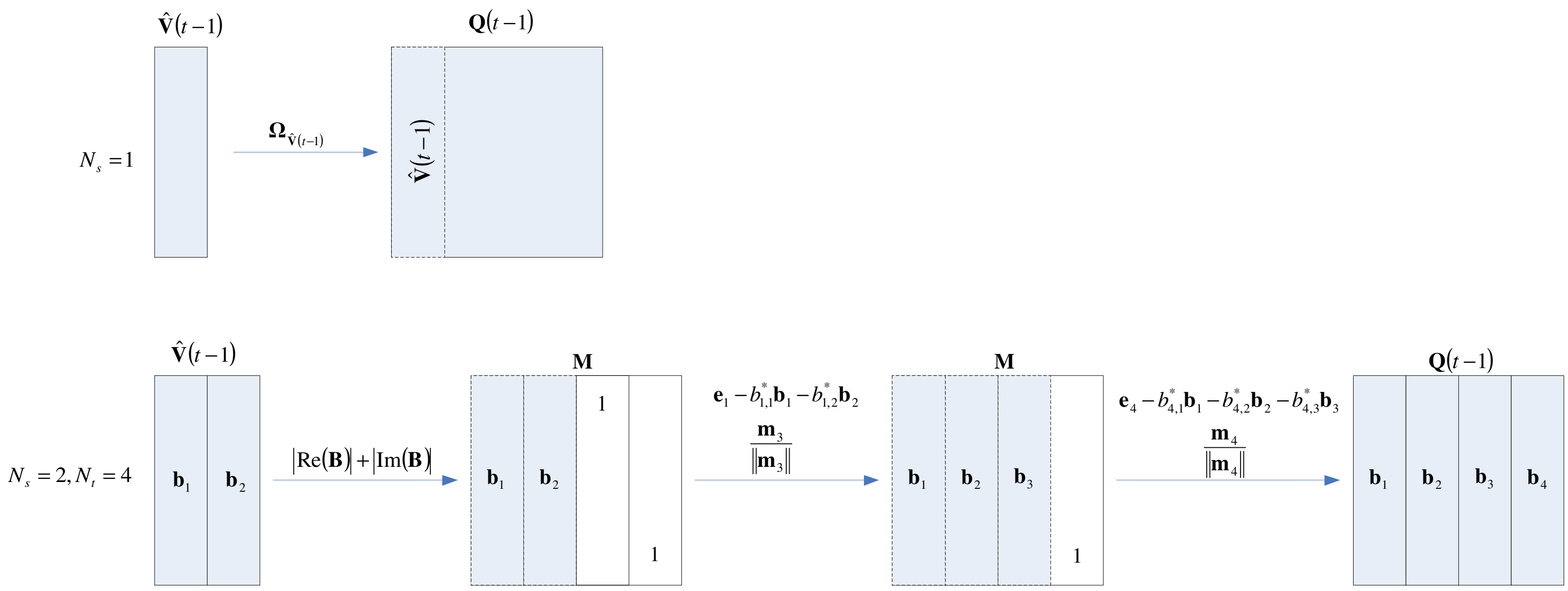
- Beamforming matrix reconstruction at BS:

$$\hat{\mathbf{V}}(t) = \mathbf{Q}(t-1)\hat{\mathbf{D}}$$

- Beamforming at BS: $\mathbf{y} = \mathbf{H}\hat{\mathbf{V}}(t)\mathbf{s} + \mathbf{n}$

Note that quantization criterion here is better than that in C80216m-09_0058r4.

Computation of Q(t-1)



- Low computational complexity
  - Householder matrix for rank 1
  - Gram-Schmidt for rank 2
- Add no hardware
  - Reuse hardware of the mandatory, transformed codebook

Scheme II

- Actual quantization at SS:

$$\hat{\mathbf{D}} = \arg\max_{\mathbf{D}_i \in C_d} \det\left(\mathbf{I} + \frac{\gamma}{N_s} \hat{\mathbf{V}}^H (t-1) \mathbf{D}_i^H \mathbf{H}^H \mathbf{H} \mathbf{D}_i \hat{\mathbf{V}}(t-1) \right)$$

- Beamforming matrix reconstruction at BS:

$$\hat{\mathbf{V}}(t) = \hat{\mathbf{D}} \hat{\mathbf{V}}(t-1)$$

•Beamforming at BS: $\mathbf{y} = \mathbf{H} \hat{\mathbf{V}}(t) \mathbf{s} + \mathbf{n}$

Note that quantization criterion here is better than that in S80216m-09_0790r1. Maximizing the inner product is much worse than maximizing channel capacity.

Updates of Scheme II

- Identity matrix is included into the codebook lately.
- Recommend using ρ 0.9 for overall channels.

Scheme I tracks e_1 perturbation of small dimensions while Scheme II tracks $[e_1\ e_2\ e_3\ e_4]$ perturbation of large dimensions.

- **Scheme I's codewords quantize perturbations of e_1 with 6 degrees of freedom (DOF), while Scheme II's codewords quantize perturbations of whole identity matrix on Stiefel manifold with 12 DOF.**

Scheme I

$$\begin{bmatrix} \mathbf{1} \\ 0 \\ 0 \\ 0 \end{bmatrix} \longrightarrow \mathbf{D}_i = \begin{bmatrix} \mathbf{0.9} \\ 0.05 \\ 0.01 \\ 0.00 \end{bmatrix}$$

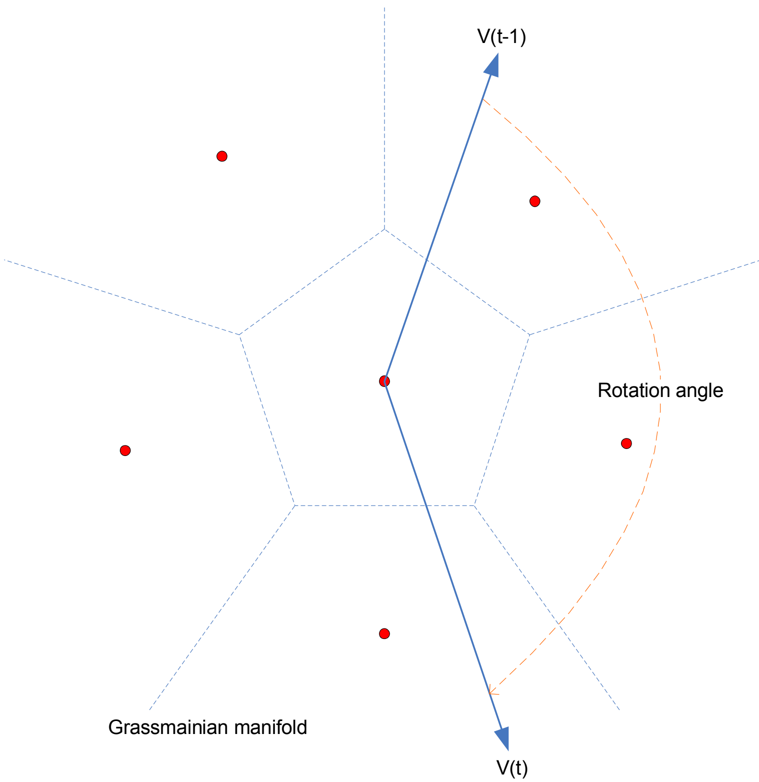
Scheme II

$$\begin{bmatrix} \mathbf{1} & & & \\ & \mathbf{1} & & \\ & & \mathbf{1} & \\ & & & \mathbf{1} \end{bmatrix} \longrightarrow \mathbf{D}_i = \begin{bmatrix} \mathbf{0.9} & 0.02 & 0.00 & 0.00 \\ 0.06 & \mathbf{0.9} & 0.03 & 0.01 \\ 0.01 & 0.05 & \mathbf{0.9} & 0.07 \\ 0.00 & 0.00 & 0.04 & \mathbf{0.9} \end{bmatrix}$$

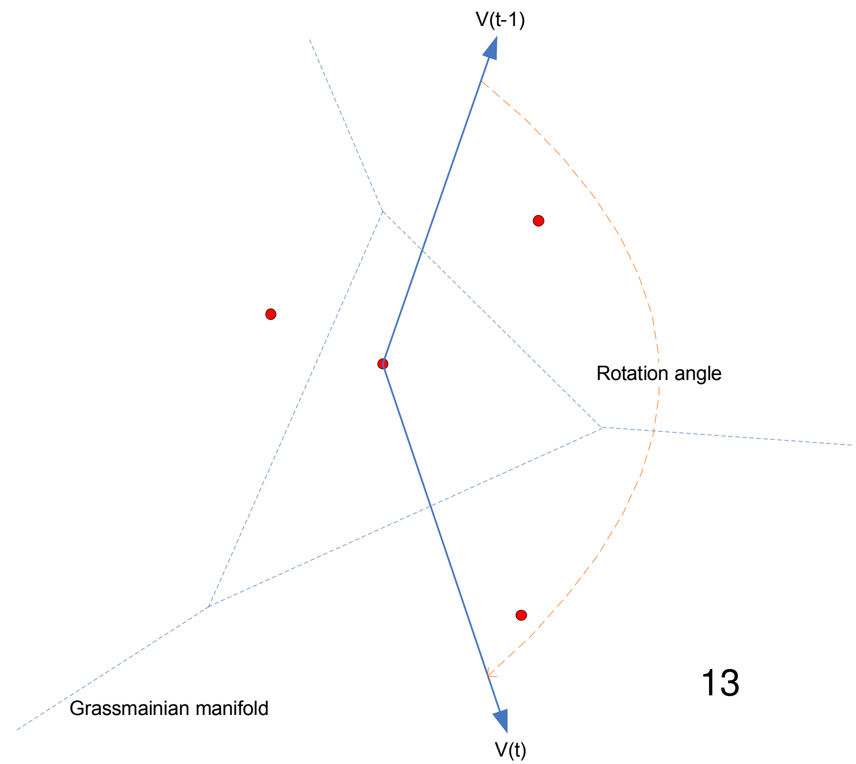
Scheme I codebook matches to precoder delta distribution while Scheme II doesn't.

- Differential matrix is symmetric about center $[e_1]$ or identity matrix.
- Scheme I codebook is symmetric about $[e_1]$, while Scheme II codebook is asymmetric about identity matrix with uneven quantization errors.

Scheme I



Scheme II



Other differences

- Correlation adaptation
  - Scheme I has two codebooks for small and high correlation scenarios, respectively. The two codebooks are pre-defined and stored.
  - Scheme II changes codebook using measured correlation matrix and costly online SVD computation.
- Rank adaptation
  - Scheme I can change precoder rank anytime.
  - Scheme II can not change precoder rank during differential feedbacks and has to wait until next reset.

The complexity of Scheme II 4-bit version is more than triple of Scheme I 3-bit because Scheme II uses 4x4 matrix operation rather than 4x1 or 4x2.

Size of $\mathbf{H}^T$ and V	No. of real multiplications for Scheme I		No. of real multiplications for Scheme II	
4×1	$\mathbf{Q}$(t-1): 58	218	$\mathbf{a}_i$=$\mathbf{HD}_i$: 16×48	960
	$\mathbf{a}$=$\mathbf{HQ}$(t-1): 48			
	$\|\|\mathbf{aD}_i\|\|^2$: 8×14		$\|\|\mathbf{a}_i\mathbf{V}(t-1)\|\|^2$: 16×12	
4×2	$\mathbf{Q}$(t-1): 92	708	$\mathbf{A}_i$=$\mathbf{HD}_i$: 16×96	3536
	$\mathbf{A}=\mathbf{HQ}(t-1)$: 96		$\mathbf{B}_i$=$\mathbf{A}_i\mathbf{V}$(t-1): 16×48	
	$\mathbf{B}_i=\mathbf{AD}_i$: 8×48		$\mathbf{B}_i^H\mathbf{B}_i$: 16×14	
	$\mathbf{B}_i^H\mathbf{B}_i$: 8×14		det( ): 16×3	
	det( ): 8×3			
4×3	$\mathbf{Q}$(t-1): 52	1548	$\mathbf{A}_i$=$\mathbf{HD}_i$: 16×144	5008
	$\mathbf{A}=\mathbf{HQ}(t-1)$: 144		$\mathbf{B}_i$=$\mathbf{A}_i\mathbf{V}$(t-1): 16×108	
	$\mathbf{B}_i=\mathbf{AD}_i$: 8×108		$\mathbf{B}_i^H\mathbf{B}_i$: 16×45	
	$\mathbf{B}_i^H\mathbf{B}_i$: 8×45		det( ): 16×16	
	det( ): 8×16			

Scheme II 4-bit complexity is more than 1.6x of Scheme I 4-bit's because Scheme II uses 4x4 matrix operation rather than 4x1 or 4x2.

Size of $\mathbf{H}^T$ and V	No. of real multiplications for Scheme I		No. of real multiplications for Scheme II	
4×1	Q(t-1): 58	330	$\mathbf{a}_i$=$\mathbf{HD}_i$: 16×48	960
	$\mathbf{a}$=$\mathbf{HQ}$(t-1): 48			
	$\lVert\mathbf{aD}_i\rVert^2$: 16×14		$\lVert\mathbf{a}_i\mathrm{V}(t-1)\rVert^2$: 16×12	
4×2	Q(t-1): 92	1180	$\mathbf{A}_i$=$\mathbf{HD}_i$: 16×96	3536
	$\mathbf{A}=\mathbf{HQ}(t-1)$: 96		$\mathbf{B}_i$=$\mathbf{A}_i$V(t-1): 16×48	
	$\mathbf{B}_i=\mathbf{AD}_i$: 16×48		$\mathbf{B}_i^H\mathbf{B}_i$: 16×14	
	$\mathbf{B}_i^H\mathbf{B}_i$: 16×14		det( ): 16×3	
	det( ): 16×3			
4×3	Q(t-1): 52	2900	$\mathbf{A}_i$=$\mathbf{HD}_i$: 16×144	5008
	$\mathbf{A}=\mathbf{HQ}(t-1)$: 144		$\mathbf{B}_i$=$\mathbf{A}_i$V(t-1): 16×108	
	$\mathbf{B}_i=\mathbf{AD}_i$: 16×108		$\mathbf{B}_i^H\mathbf{B}_i$: 16×45	
	$\mathbf{B}_i^H\mathbf{B}_i$: 16×45		det( ): 16×16	
	det( ): 16×16			

Summary of Differences

	Scheme I	Scheme II
Principle	Track perturbation of $\mathbf{e}_1$ or $[\mathbf{e}_1\ \mathbf{e}_2]$.	Track perturbation of $[\mathbf{e}_1\ \mathbf{e}_2\ \mathbf{e}_3\ \mathbf{e}_4]$.
Codebook dimension	4x1 and 4x2	4x4
Computational Complexity	Low	High
No. of codebooks	One small codebook for each rank	One large codebook for all ranks
Codeword distribution	Even distribution on Grassmannian manifold	Uneven distribution on Grassmannian manifold
Rank adaptation	Rank can be changed at anytime.	Rank changes only after reset.
Correlation adaptation	Two predefined codebooks for small and large correlation scenarios, respectively.	Adaptive codebooks with online SVD computation for different correlation scenarios.
Support of 8 antennas	Easy	Difficult because of wasted codewords and high complexity.

Comparison of throughput and reliability

- System level simulation
- Single-user MIMO
- Implementation losses are included
  - Feedback error and error propagation
  - Feedback delay
  - Quantized reset feedback

General SLS parameters

Parameter Names	Parameter Values
Network Topology	57 sectors wrap around, 10 MS/sector
MS channel	ITU PB3km/h
Frame structure	TDD, 5DL, 3 UL
Feedback delay	5 ms
Inter cell interference modeling	Channel is modeled as one tap wide band
Antenna configuration	4Tx, 2 Rx
Codebook configuration	Baseline 6 bits, Diff 4 bits or 3 bits
Q matrix reset frequency	Once every 4 frames
PMI error	free
PMI calculation	Maximize post SINR
System bandwidth	10MHz, 864 data subcarriers
Permutation type	AMC, 48 LRU
CQI feedback	1Subband=4 LRU, ideal feedback

Codebook related parameters

	Scheme II codebook	Scheme I 4-bit codebook	Scheme I 3-bit codebook
Codebook size	4 bits i.e. 16 codewords	4 bits i.e. 16 codewords	3 bits i.e. 8 codewords
Feedback overhead	18 bits / 4 frames / Subband including 6-bit reset	18 bits / 4 frames / Subband including 6-bit reset	15 bits / 4 frames / Subband including 6-bit reset
CQI erasure rate	10%		

3-bit Scheme I vs. 4-bit Scheme II

4 Tx, 2 Rx, 1 stream

	0.5 λ ant. Spacing, Scheme I 5^o, Scheme II 0.9		4 λ ant. Spacing, Scheme I 20^o, Scheme II 0.9		Uncorrelated, Scheme I 20^o, Scheme II 0.9	
	SE gain over 16e	5%-ile SE gain over 16e	SE gain over 16e	5%-ile SE gain over 16e	SE gain over 16e	5%-ile SE gain over 16e
Scheme I: 3-bit	11.6%	36.3%	3.7%	21.9%	2.5%	16.2%
Scheme II: 4-bit	10.7%	35.3%	1.3%	19%	-1%	7.7%
Scheme I over Scheme II	0.8%	0.7%	2.7%	2.4%	3.5%	7.9%

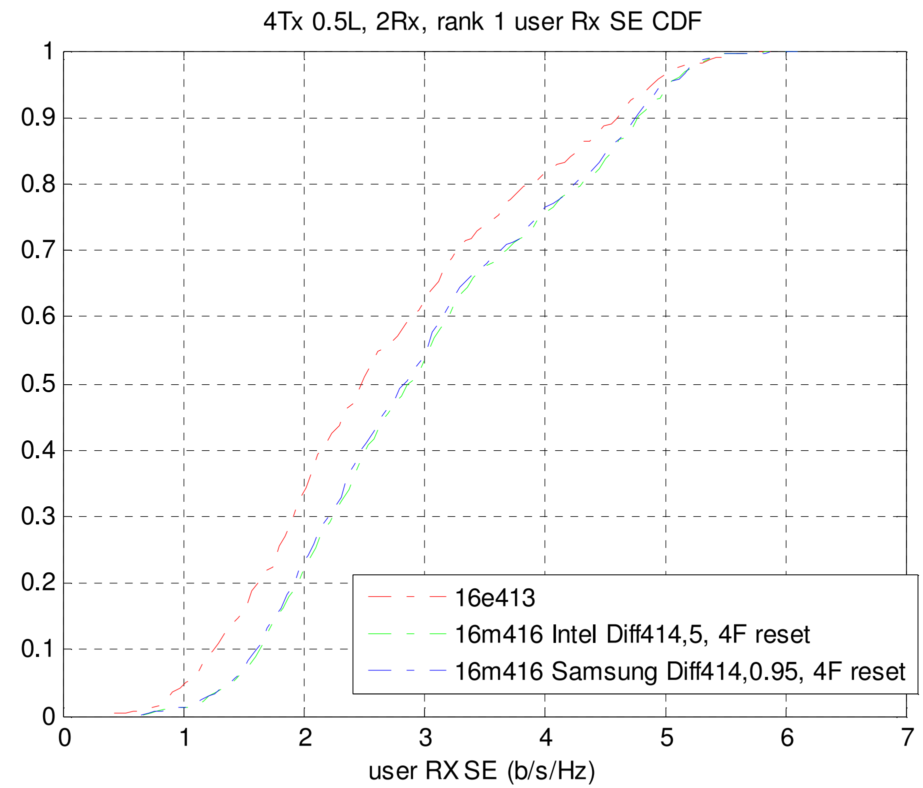
Observations

- Scheme I's 3-bit has higher throughput and reliability than Scheme II's 4-bit. In addition, Scheme I's feedback overhead and complexity are lower than Scheme II's.
- Scheme II's codebook is optimized for highly correlated channels and scarifies uncorrelated/lowly correlated channels.
- Scheme II's codebook can not track channel variation in uncorrelated channel and performs even poorer than 16e codebook.

Scheme I 4-bit vs. Scheme II 4-bit

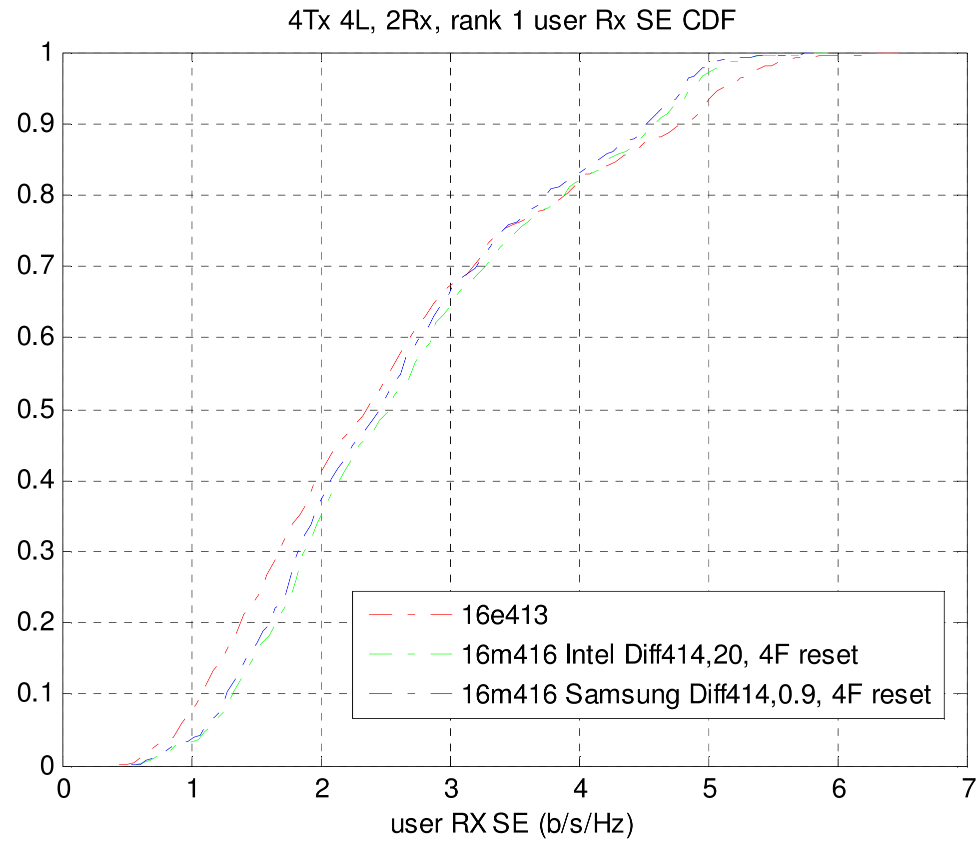
4 Tx (0.5 λ), 2 Rx, 1 stream

	SE gain over 16e	5%-ile SE gain over 16e
Scheme I: 4-bit, 5°	11.7%	37.7%
Scheme II: 4-bit, 0.95 ρ	10.6%	35.4%
Scheme I over Scheme II	1.06%	1.37%



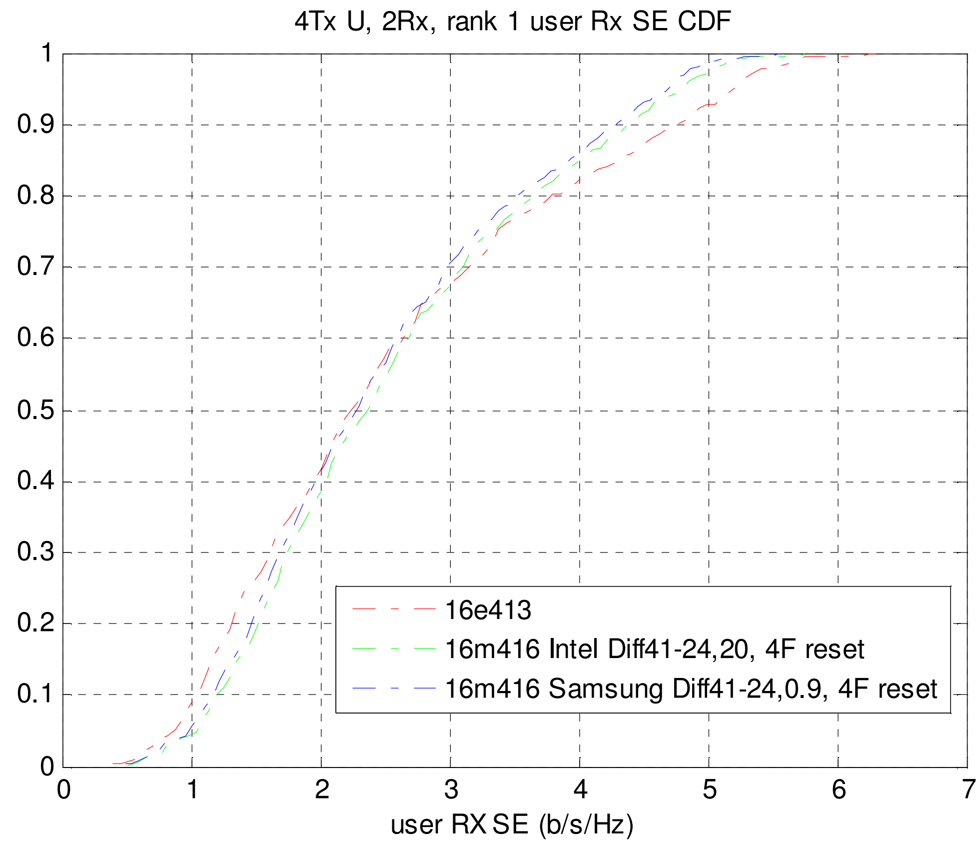
4 Tx(4 λ), 2Rx, 1 stream

	SE gain over 16e	5%-ile SE gain over 16e
Scheme I: 4-bit, 20º	3.9%	21.5%
Scheme II: 4-bit, 0.9 ρ	1.36%	19%
Scheme I over Scheme II	2.5%	2.1%



4 Tx (uncorrelated), 2 Rx, 1 stream

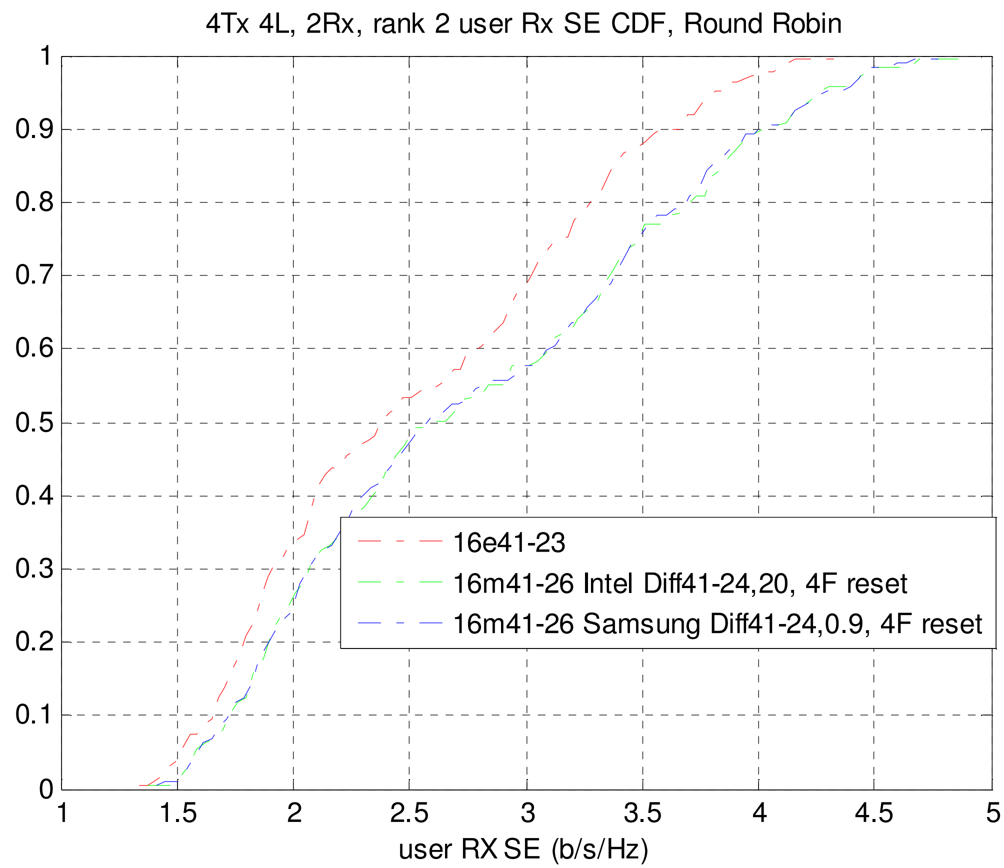
	SE gain over 16e	5%-ile SE gain over 16e
Scheme I: 4-bit, 20°	0.2%	17.4%
Scheme II: 4-bit, 0.9 ρ	-2.9%	13%
Scheme I over Scheme II	3.1%	3.9%



Comparison on rank 2 and 3 codebooks

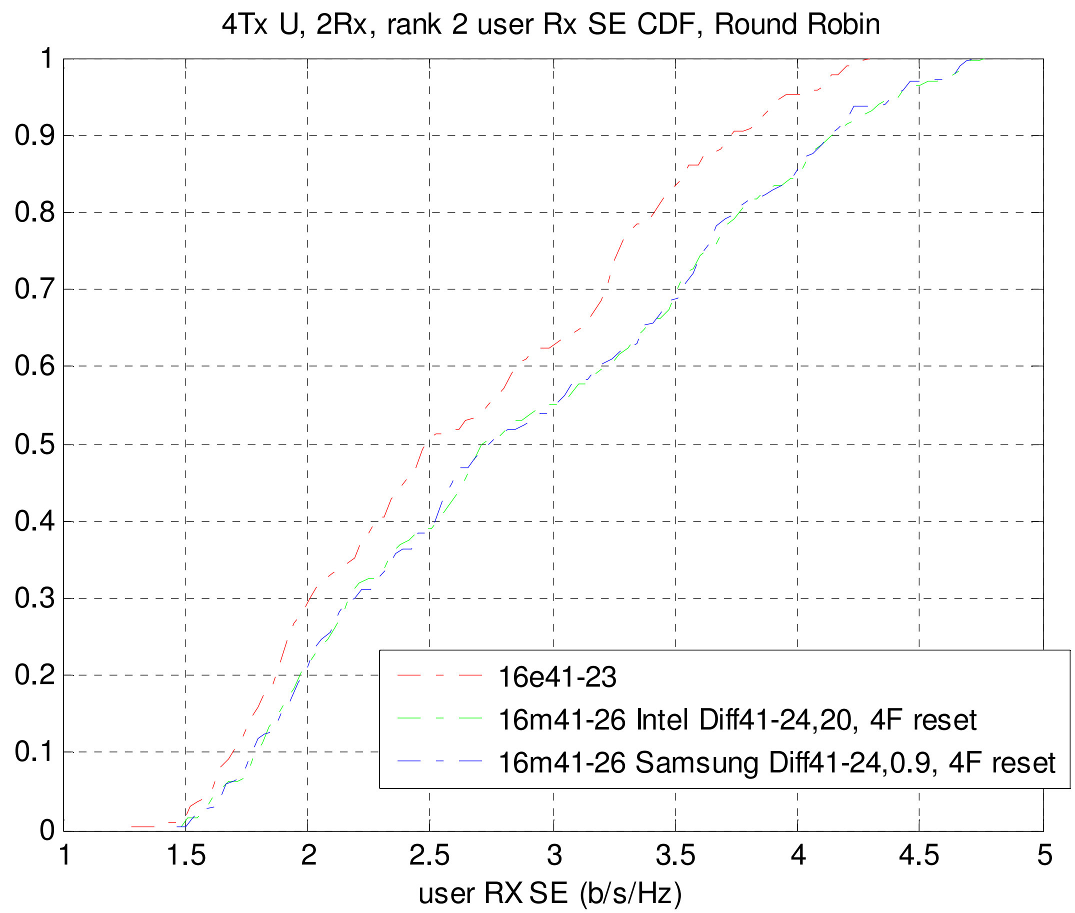
4 Tx (4 λ), 2 Rx, 2 streams

	SE gain over 16e
Scheme I: 4-bit, 20°	10.29%
Scheme II: 4-bit, 0.9 ρ	10%
Scheme I over Scheme II	0.27%



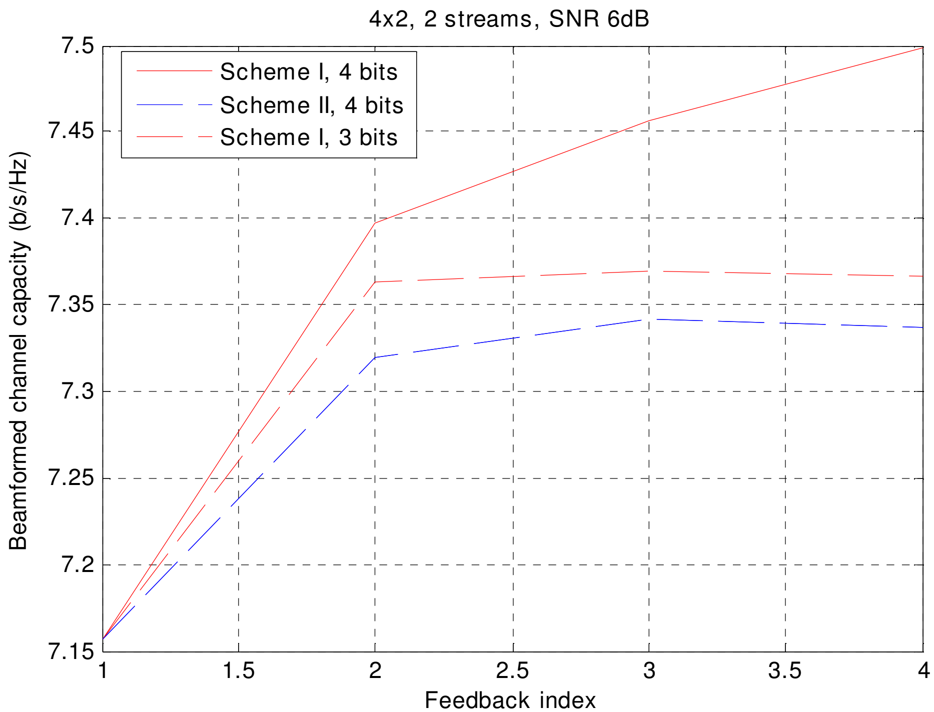
4 Tx (uncorrelated), 2 Rx, 2 streams

	SE gain over 16e
Scheme I: 4-bit, 20°	9.45%
Scheme II: 4-bit, 0.9 ρ	9.28%
Scheme I over Scheme II	0.15%



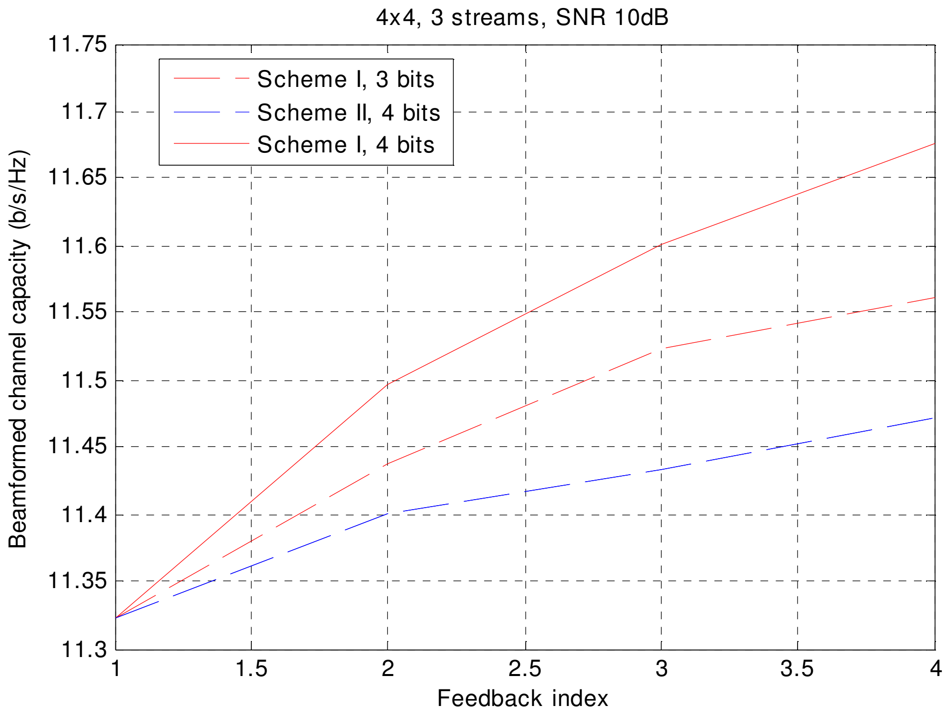
4 Tx (uncorrelated), 2 Rx, 2 streams

- Link level channel capacity at SNR 6 dB.
- Scheme I 4-bit outperforms Scheme II 4-bit by 1.21%.
- Scheme I 3-bit outperforms Scheme II 4-bit by 0.34%.



4 Tx (uncorrelated), 4 Rx, 3 streams

- Link level channel capacity at SNR 10 dB.
- Scheme I 4-bit outperforms Scheme II 4-bit by 1.02%.
- Scheme I 3-bit outperforms Scheme II 4-bit by 0.47%.



Observations

- Scheme I's 4-bit has higher throughput and reliability than Scheme II's 4-bit.
- Scheme I's complexity is lower than Scheme II's.
- Scheme I's 4-bit has 0.3% higher throughput than Scheme I's 3-bit.

Conclusions

- Scheme I's 3-bit outperforms Scheme II's 4-bit in all cases in terms of throughput and reliability.
- Scheme I's 3-bit scheme has feedback overhead and computational complexity lower than Scheme II's 4-bit by 17% and 60%, respectively.
- Scheme I's 4-bit has even higher throughput than Scheme I's 3-bit.
- Scheme II's new design solves vibration problem by adding identity matrix but it can not track channel variation in uncorrelated channels.
- Scheme I is proposed for adoption.

Proposed text

Add proposed text to line 63, page 91, section **15.3.7.2.6.6.4.**

The differential feedbacks exploit the correlation between precoding matrixes adjacent in time or frequencies. The feedback shall start initially and restart periodically by sending a one-shot feedback that fully depicts the precoder by itself. The codebook for the one-shot feedback is defined for the base mode.

Denote the feedback index, the correspondingly fed back matrix, and the corresponding precoder by t, $\mathbf{D}(t)$, and $\mathbf{V}(t)$, respectively. The sequential index is reset to 0 at $T_{\max}+1$. The index for the initial or the restart feedback is 0 and $\mathbf{V}(0)=\mathbf{D}(0)$. The indexes of the subsequent differential feedback are $1,2,\cdots,T_{\max}$ and the corresponding precoders are $\mathbf{V}(t)=\mathbf{Q}_{\mathbf{V}(t-1)}\mathbf{D}(t)$, where $\mathbf{Q}_{\mathbf{V}(t-1)}$ is a unitary $N_t \times N_t$ matrix computed from the previous precoder $\mathbf{V}(t-1)$; N_t is the number of transmit antennas. The dimension of the fed back matrix $\mathbf{D}(t)$ is $N_t \times N_s$ for $t=0,1,2,\cdots,T_{\max}$, where N_s is the number of spatial streams.

The rotation matrix $\mathbf{Q}_{\mathbf{V}(t-1)}$ of $\mathbf{V}(t-1)$ has the form $\mathbf{Q}_{\mathbf{V}(t-1)}=\begin{bmatrix}\mathbf{V}(t-1) & \mathbf{V}^{\perp}(t-1)\end{bmatrix}$, where $\mathbf{V}^{\perp}(t-1)$ consists of columns each of which has a unit norm and is orthogonal to the other columns of $\mathbf{Q}_{\mathbf{V}(t-1)}$. Define the Householder matrix $\mathbf{\Omega}_{\mathbf{x}}$ of unit vector $\mathbf{x}$ as

$$\mathbf{\Omega}_{\mathbf{x}}=\begin{cases}\mathbf{I}-\dfrac{2}{\|\mathbf{w}\|^2}\mathbf{w}\mathbf{w}^H & \text{for } \|\mathbf{w}\|,\|\mathbf{x}\|>0\\ \mathbf{I} & \text{otherwise}\end{cases}$$

where $\|\mathbf{x}\|=1$ and $\mathbf{w}=e^{-j\theta}\mathbf{x}-\mathbf{e}_1$; θ is the phase of the first entry of $\mathbf{x}$; $\mathbf{e}_1=\begin{bmatrix}1 & 0 & \cdots & 0\end{bmatrix}^T$. For $N_s=1$, $\mathbf{V}(t-1)$ is an $N_t \times 1$ vector and $\mathbf{Q}_{\mathbf{V}(t-1)}=\mathbf{\Omega}_{\mathbf{V}(t-1)}$. For $N_S=2$ and $N_t=4$, $\mathbf{V}(t-1)$ is 4×2. Denote $\mathbf{V}(t-1)$ as $\mathbf{B}=\begin{bmatrix}\mathbf{b}_1 & \mathbf{b}_2\end{bmatrix}$. Two columns are appended to $\mathbf{B}$ as $\mathbf{M}=\begin{bmatrix}\mathbf{B} & \mathbf{e}_i & \mathbf{e}_j\end{bmatrix}$, where $\mathbf{e}_i$ and $\mathbf{e}_j$ are vectors with all zeros except that the i-th and j-th entries are ones, respectively. The index i and j are selected. Let the i-th and j-th entries of $\mathbf{g}=\left(|\mathrm{Re}(\mathbf{B})|+|\mathrm{Im}(\mathbf{B})|\right)\begin{bmatrix}1\\1\end{bmatrix}$ be the smallest and the second smallest, respectively, where $|\mathbf{A}|$ converts $\mathbf{A}$'s entries to their absolute values; $|\mathrm{Re}(\mathbf{B})|$ and $|\mathrm{Im}(\mathbf{B})|$ are the real and imaginary parts of $\mathbf{B}$, respectively. Gram-Schmidt orthogonalization is applied on $\mathbf{e}_i$ as $\mathbf{m}_3=\mathbf{e}_i-b_{i,1}^*\mathbf{b}_1-b_{i,2}^*\mathbf{b}_2$, where $b_{k,l}^*$ is the conjugate of $\mathbf{B}$'s entry of on the k-th row and l-th column. Normalization follows the orthogonalization as

$\mathbf{b}_3 = \frac{\mathbf{m}_3}{\|\mathbf{m}_3\|}$. The matrix $\mathbf{B}$ is extended by one column as $\mathbf{B} = [\mathbf{b}_1 \quad \mathbf{b}_2 \quad \mathbf{b}_3]$. The Gram-Schmidt process on $\mathbf{e}_j$ is

$\mathbf{m}_4 = \mathbf{e}_j - b_{j,1}^*\mathbf{b}_1 - b_{j,2}^*\mathbf{b}_2 - b_{j,3}^*\mathbf{b}_3$. The followed normalization is $\mathbf{b}_4 = \frac{\mathbf{m}_4}{\|\mathbf{m}_4\|}$. Finally, $\mathbf{Q}_{\mathbf{V}(t-1)} = [\mathbf{V}(t-1) \quad \mathbf{b}_3 \quad \mathbf{b}_4]$. The Gram-Schmidt orthogonalization is the same as the one applied in the transformed codebook. An illustration of the computation of $\mathbf{Q}_{\mathbf{V}(t-1)}$ is shown in Figure xxx. Let $\mathbf{A}$ be a vector or a matrix with two columns. Denote $\mathbf{Q}_{\mathbf{A}}$ the rotation matrix of $\mathbf{A}$.

The feedback matrix $\mathbf{D}(t)$ is selected from a differential codebook. Denote the codebook by $D(N_t, N_s, N_w)$, where N_w is the number of codewords in the codebook. The codebooks $D(2,1,4)$, $D(2,2,4)$, $D(4,1,16)$, $D(4,2,16)$ are listed in Table xxx, xxx, xxx, xxx. Denote $\mathbf{D}_i(N_t, N_s, N_w)$ the i-th codeword of $D(N_t, N_s, N_w)$. The rotation matrixes $\mathbf{Q}_{\mathbf{D}_i}$ s of the $\mathbf{D}_i(N_t, N_s, N_w)$s comprises a set of N_t by N_t matrixes that is denoted by $\mathcal{Q}_{D(N_t,N_s,N_w)}$.

The differential codebook $D(4,3,N_w)$ is computed from $\mathcal{Q}_{D(4,1,N_w)}$. The i-th codeword of $D(4,3,N_w)$ denoted by $\mathbf{D}_i(4,3,N_w)$ is computed as

$$\mathbf{D}_i(4,3,N_w) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \tilde{\mathbf{Q}}_i(4,1,N_w),$$

where $\tilde{\mathbf{Q}}_i(4,1,N_w)$ consists of the last three columns of the i-th matrix in $\mathcal{Q}_{D(4,1,N_w)}$. The differential codebook $D(4,4,N_w)$ is computed from $\mathcal{Q}_{D(4,2,N_w)}$. The i-th codeword of $D(4,4,N_w)$ is the i-th matrix in $\mathcal{Q}_{D(4,2,N_w)}$. Two sets of differential codebooks are defined. One has a large step size for fast tracking capability and the other has a small step size for high tracking accuracy. For $t=1$, the codebook with large step size shall be used. A 1-bit indicator may be fed back for the step size used for $t = 2,\cdots,T_{\max}$.

Table 1. $D(2,1,4)$ **codebook.**

	Index	Codeword	Index	Codeword
Codebook of large step size	1	$[1 \quad 0]^T$	3	$\left[\cos(15^\circ) \quad \sin(15^\circ)e^{j120^\circ}\right]^T$
	2	$\left[\cos(15^\circ) \quad \sin(15^\circ)\right]^T$	4	$\left[\cos(15^\circ) \quad \sin(15^\circ)e^{-j120^\circ}\right]^T$

Table 2. $D(2,2,4)$ codebook.

	Index	Codeword	Index	Codeword
Codebook of large step size	1	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	3	$\begin{bmatrix} \cos(15^\circ) & \sin(15^\circ)e^{j120^\circ} \\ \sin(15^\circ)e^{j120^\circ} & -\cos(15^\circ) \end{bmatrix}$
	2	$\begin{bmatrix} \cos(15^\circ) & \sin(15^\circ) \\ \sin(15^\circ) & -\cos(15^\circ) \end{bmatrix}$	4	$\begin{bmatrix} \cos(15^\circ) & \sin(15^\circ)e^{-j120^\circ} \\ \sin(15^\circ)e^{-j120^\circ} & -\cos(15^\circ) \end{bmatrix}$

Table 3. $D(4,1,16)$ **codebook.**

	Index	Codeword	Index	Codeword
Codebook of large step size	1	$[1 \quad 0 \quad 0 \quad 0]^T$	9	[cos(20°) 0.2553 + 0.1430i 0.0282 + 0.0897i 0.1469 + 0.0308i]^T
	2	[cos(20°) 0.2062 - 0.0657i 0.0485 - 0.2038i -0.0885 + 0.1358i]^T	10	[cos(20°) 0.0507 - 0.3289i 0.0276 + 0.0448i 0.0508 - 0.0297i]^T
	3	[cos(20°) -0.0531 - 0.0765i 0.0806 - 0.1811i -0.1432 - 0.2203i]^T	11	[cos(20°) -0.0352 + 0.2445i 0.0560 + 0.1197i -0.1178 - 0.1569i]^T
	4	[cos(20°) -0.0762 - 0.1024i -0.2492 - 0.1865i 0.0616 + 0.0028i]^T	12	[cos(20°) -0.0505 - 0.0233i -0.1061 + 0.3140i 0.0505 + 0.0382i]^T
	5	[cos(20°) -0.0475 - 0.0535i 0.0266 - 0.0109i 0.1997 + 0.2668i]^T	13	[cos(20°) -0.3407 - 0.0014i 0.0280 + 0.0108i 0.0021 + 0.0020i]^T
	6	[cos(20°) -0.0478 - 0.0010i -0.0229 + 0.0325i 0.2359 - 0.2397i]^T	14	[cos(20°) -0.0180 - 0.0100i 0.3300 + 0.0502i 0.0685 - 0.0205i]^T
	7	[cos(20°) 0.0030 + 0.1854i -0.1733 -0.1136 + 0.1992i]^T	15	[cos(20°) -0.0401 - 0.0885i 0.0946 + 0.1084i -0.2792 + 0.0942i]^T
	8	[cos(20°) 0.1926 - 0.0378i -0.1914 + 0.0534i -0.1467 - 0.1320i]^T	16	[cos(20°) -0.0436 + 0.2160i 0.0596 - 0.2318i 0.1057 + 0.0002i]^T

Table 4. $D(4,2,16)$ **codebook.**

	Index	Codeword	Index	Codeword
Codebook of large step size	**1**	[1 0 0 0 0 1 0 0]^T	**9**	[0.9770 0.1518 + 0.0929i 0.0606 - 0.0773i 0.0063 - 0.0647i -0.0507 - 0.1011i -0.0981 + 0.8703i -0.1618 - 0.0957i -0.3914 + 0.1776i]^T
	2	[0.9571 -0.0238 + 0.0314i -0.0454 - 0.2541i -0.0790 + 0.0977i -0.0965 + 0.0299i 0.9114 + 0.0872i - 0.0431 - 0.3386i -0.1023 + 0.1567i]^T	**10**	[-0.6295 -0.5472 - 0.3123i -0.0136 - 0.1891i 0.2222 - 0.3486i 0.5496 - 0.3201i -0.7539 - 0.0022i 0.0440 + 0.0657i -0.0189 + 0.1434i]^T
	3	[-0.0262 0.7460 - 0.6224i 0.2085 + 0.1061i 0.0104 - 0.0226i 0.6933 + 0.5709i 0.1217 + 0.0055i - 0.1479 - 0.3702i -0.1061 - 0.0917i]^T	**11**	[0.3622 -0.8103 - 0.3554i 0.0797 - 0.2550i -0.1050 + 0.0596i -0.8270 + 0.3289i -0.2410 - 0.0429i -0.1349 - 0.3222i -0.0937 + 0.1311i]^T
	4	[0.9990 0.0386 - 0.0212i 0.0035 - 0.0023i 0.0002 + 0.0019i -0.0343 - 0.0200i 0.8730 + 0.0488i 0.3473 - 0.1714i 0.2857 - 0.0483i]^T	**12**	[-0.4402 -0.8115 - 0.0841i 0.0434 + 0.1299i -0.1636 + 0.3083i -0.7666 + 0.1113i 0.5535 + 0.0180i 0.0170 + 0.1186i -0.1660 + 0.2268i]^T
	5	[0.9556 0.1479 - 0.0806i -0.0215 + 0.1307i -0.0706 - 0.1894i -0.0844 + 0.0610i 0.8284 - 0.3568i 0.1996 - 0.1472i -0.1478 + 0.3037i]^T	**13**	[1 0 0 0 0 -0.8741 + 0.0445i 0.3194 - 0.1760i 0.3172 – 0.0173i]^T
	6	[-0.8726 0.1100 - 0.0735i -0.4250 - 0.1821i -0.0795 + 0.0325i 0.1648 + 0.1221i 0.9722 - 0.0007i - 0.0410 - 0.1039i 0.0018 + 0.0180i]^T	**14**	[-0.8851 -0.3025 + 0.3449i 0.0049 + 0.0437i -0.0340 - 0.0557i 0.2630 + 0.2692i -0.7941 - 0.0049i 0.2671 - 0.0632i -0.2947 + 0.2561i]^T
	7	[-0.6845 -0.0048 - 0.7234i 0.0310	**15**	[0.8990 -0.1582 - 0.1183i 0.1246 -

		- 0.0167i 0.0831 + 0.0006i 0.0085 + 0.6243i 0.6200 + 0.0054i -0.3294 - 0.2343i 0.2292 - 0.0994i]^T		0.0775i -0.3616 - 0.0214i 0.0035 + 0.2203i -0.8650 + 0.3492i -0.0464 + 0.0693i 0.2338 + 0.1398i]^T
	8	[0.5617 0.8043 + 0.1719i -0.0617 - 0.0099i 0.0607 - 0.0241i 0.7006 - 0.1414i -0.5130 - 0.0152i -0.1561 + 0.2422i 0.3191 + 0.2023i]^T	**16**	[0.5212 0.3746 + 0.7570i 0.0670 + 0.1016i -0.0085 - 0.0003i 0.3025 - 0.7018i -0.4381 + 0.0708i -0.2495 - 0.2784i 0.2622 - 0.1028i]^T