Project	**IEEE 802.20 Working Group on Mobile Broadband Wireless Access** **<http://grouper.ieee.org/groups/802/20/>**
Title	**Proposal of the Basic Fading Channel Model for the Link Level Simulation**
Date Submitted	**2004-04-26**
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Re:	Link level simulation and evaluation criteria.
Abstract	This document proposes a detailed model for the basic fading channels to be used in the link level simulation.
Purpose	Discuss and adopt.
Notice	
Release	
Patent Policy	

1 Introduction

In order for different proposals for MBWA to be compared fairly, the performance of each proposed system has to be simulated using the the same procedure. It is the common understanding of IEEE802.20 community that the simulation shall be performed in two parts, as link level simulatoin and system level simulations, where the system level simulation make use of the statistics obtained by the link level simulation. In order for the simulation following this approach to be feasible and affordable, we proposed in [1] to simplify the channel model developed in [2]for both the link-level simulation and the system level simulation. This contributoin focuses on the link level channel model, and we look for an adequate model in the following aspects:

1. Power-Delay Profile: It specifies the power fraction and the associated time delay of each single path fading process.
2. Power Spectrum Density: It characterizes the frequency response of the fading channel for each single path
3. Cross Correlation: It quantifies the quality of the implementation of the theoretically independent fading processes.
4. Physical Background: Variables within the model should have physical meaning and either directly or inderectly measurable.

To satisfy all these requirements is in fact very tough, given the feaibility of a computer simulation. Thus, the result oughts to be a compromise. In the rest of this contribution, we are going to present a detailed fading model to be used by the link level simulation.

2 Power-Delay Profile

The power-delay profile includes the power distribution and the associated delay of a multipath channel, so that it captures the frequency selectivity of the channel. For a simulation to be useful, power-delay profiles should be used that are considered representative for the given topographical environment, such as those summarized in documents of different standard development organizations[ITU, 3GPP, 3GPP2]. Therefore, we will follow and consider the power-delay profile recommended by [2]. In particular, we assume for the link level simulation in this contribution the power-delay profile chosen by [1] , so that the channel model has the following expression

$$h(t,\tau) = \sum_{k=0}^{M-1} b_k h_k(t) \delta(\tau - \tau_k) \quad (1)$$

where

- M: number of paths of the channel

- b_k : given path amplitude satisfying the power normalization condition

$$\sum_{k=0}^{M-1} b_k^2 = 1 \tag{2}$$

  This requirement allows the channel to be scaled by the transmit power in the simulation.

- τ_k : time delay of the path relative to the first path; normally $\tau_0 = 0$ is assumed.

- $h_k(t)$: the fading model of the k-path, can be Rayleigh or Rician.

- $\delta(t)$: the delta distribution charcterizing the ideal channel response

The physical interpretation of this channel model is that the fading process is a linear combination of identical fading process with different powers and time delays. Each fading process is referred to as a propagation path.

3 Fading Model

The fading process of a propagation path is assumed as a linear combination of rays, referred to as sub-paths. The model we propose for such a fading process is based on the idea of S.O.Rice[4], in which the fading response of a channel in time is a linear combination of independent sine wave oscillators. What differs this model from that proposed by W.C.Jakes[5] is that we do not assume deterministic coefficients and doppler shifts for each oscillator. Rather we assume only the power normalization of i.i.d. random amplitude of each oscillator and uniformly distributed incident angles. For simplicity of the simulation, we assume further a fixed, but sufficiently large, number of such oszillators for each fading process. Depending on whether the line-of-sight is present or not, the model can have different expressions, as will be shown in the following.

3.1 Rayleigh Fading

This applies to the situation where there is no line-of-sight. A propagation path with index k is computed as funtion of time t by

$$h_{1,k}(t) = \sum_{i=0}^{N-1} a_{k,i} e^{j[(\omega \cdot \cos\theta_{k,i})t + \phi_{k,i}]} \tag{3}$$

with

$$\omega = \omega_c \frac{v}{c} \tag{4}$$

where

- N: the number of sub-paths

- ω, ω_c: angular frequency of Doppler shift and the RF carrier, respectively
- v, c: mobile speed and light speed, respectively
- $\phi_{k,i}, \theta_{k,i}$: a uniformly distributed random number in $[0, 2\pi)$, where $\phi_{k,i}$ accounts for the random phase introduced by the scatter and $\theta_{k,i}$ accounts for the random incident angle due to the scatter.
- $a_{k,i}$: a uniformly distributed random number in $[0, 1)$ satisfying the normalization condition; it accounts for different attenutations of the different scatters and the normalization condition

$$\sum_{i=0}^{N-1} a_{k,i}^2 = 1 \tag{5}$$

  is required so that the generated fading response for each k has an expected unit power.

The random numbers are generated once for every i and every k. Details of this fading model can be found in the attached document.

3.2 Rician Fading

This applies to the situation where there is a line-of-sight. Based on the model for Rayleigh fading of (3), the Rician fading is obtained as a linear combination

$$h_{2,k}(t) = \frac{\sqrt{R} \cdot e^{j(\omega \cdot \cos\theta_k)t} + h_{1,k}(t)}{\sqrt{1+R}} \tag{6}$$

with

$$\omega = \omega_c \frac{v}{c} \tag{7}$$

where

- R: the power ratio between the line-of-sight component and the non-line-of-sight components.
- θ_k: a uniformly distributed random number in $[0, 2\pi)$

4 Conclusion

For a fair comparison of different proposals for MBWA, performance simuation for each proposal should be based on the same procedure and using the same basic models. For the link level simulation we propose to use the channel model (1), in which the fading process $h_k(t)$ is either modeled by (3) for non-line-of-sight or modeled by (6) for line-of-sight. Since both (3) and (6) are designed to be independent, a model in which multiplathes are correlated can be readily derived from the basic model (1) for given correlation coefficients.

5 Recommendation

We recommend to adopt the proposed fading model for the link level simulation and incorporate the formula given here into the evaluation criteria document [3].

References

[1] IEEE C802.20-04/39: Proposal for Link-System Interface

[2] IEEE C802.20-03/92: Channel Models for IEEE 802.20 MBWA System Simulations

[3] IEEE C802.20-04/21: Evaluation Document for IEEE 802.20 MBWA System Simulations

[4] S.O.Rice, "Statistical Properties of a Sine Wave Plus Random Noise", Bell System Technical Journal, Vol. XXXVII, No.3, 1958.

[5] W.C.Jakes,"Microwave Mobile Communications", New York Plenum, 1974.

A Appendix: Deterministic Fading Model

A.1 Formulation

A Rayleigh fading process can be formulated for the vertical electrical field as follows

$$Z_t(\mathbf{c}, \mathbf{a}, \mathbf{b}, N) = \sum_{i=0}^{N-1} c_i \cdot e^{j[(w \cdot \cos a_i)t + b_i]} \tag{8}$$

where $\mathbf{c} = (c_0, c_1, ..., c_{N-1}), \mathbf{a} = (a_0, a_1, ..., a_{N-1}), \mathbf{b} = (b_0, b_1, ..., b_{N-1})$ are arrays of non-negative real random numbers, and $t \in \mathbf{N}$ refers to the discrete time and w is the maximum radian Doppler frequency. In terms of mathematics, Z_t is a sequence of random variables, i.e.

$$Z_t : (0, 1]^N \times (0, 2\pi]^{2N} \mapsto \mathbf{C} \tag{9}$$

for each t, where

$$Z_t^{-1} : \mathcal{B}(\mathbf{C}) \mapsto \mathcal{F}_t \tag{10}$$

where $\mathcal{F}_t$ is the σ-algebra on $(0, 1]^N \times (0, 2\pi]^{2N}$ at time t. Thus, the process is defined on $\{(\Omega, \mathcal{F}_t, P), t \in \mathbf{N}\}$ with the event set Ω, the adapted filter $\mathcal{F}_t$ and the probability measure P.[1] In specifics, a_i is the incident angle, b_i and c_i are the

[1] we keep the notion of process in view of its application to simulaiton of the stochastic process, although the outcome is actually deterministic by the definition.

random phase and amplitude associated with a scatter, respectively, accounting for the physical property of the scatterer. The physical model is based on the observation that the Rayleigh fading of elecmagnetic waves is caused by the scattering of the field, causing random phase and attenuation: Each index i refers to a ray with a Doppler frequency $\omega \cdot \cos a_i$, phase b_i and amplitude c_i. Phase, attenuation and incident angle are all assumed independent. Then, it is readily verified that

$$E\{Z_t \cdot Z^*_{t+\tau}\} = E\{\sum_{i=0}^{N-1} c_i^2\} J_0(w \cdot \tau) \tag{11}$$

with J_0 the Bessel function of 0-th order. This is the autocorrelation function. Invoking the central limit theorem, one find readily, that the amplitude of $Z_t = Z(\mathbf{c}, \mathbf{a}, \mathbf{b}, N, t)$ has the Rayleigh distribution for sufficient large N.
While using this model to simulate the fading process, each sample path can be generated by $3N$ uniformly distributed random numbers. The quality of the numerical implementatoin of this model can be evaluated in 4 aspects:

- Rayleigh Distribution:

$$Pr(|Z_t| < x) = 1 - e^{-\frac{x^2}{2\sigma^2}} \tag{12}$$

  [2] The expectation and variance are

$$\begin{aligned} E\{|Z_t|\} &= \sigma\sqrt{\pi/2} \\ Var\{|Z_t|\} &= \sigma^2 2(1 - \frac{\pi}{4}) \end{aligned}$$

  respectively.

- Autocorrelation:

$$\Re E\{Z_t Z^*_{t+\tau}) = J_0(w_c \tau) \tag{14}$$

- Cross Correlation:

$$E\{Z_t(\omega) Z^*_{t+\tau}(\omega')\} = 0. \tag{15}$$

- Fading Duration: The expected value of $\tau_f := t|_{\Re Z_t < 0}$ is

$$E\{\tau_f\} = \frac{2\pi}{\omega_c}\sqrt{\pi/2} \tag{16}$$

  The distribution of the real and imaginary parts of Z_t under zero shall be identical.

[2] For (8) the relation

$$\sigma^2 = E\{\sum_{i=0}^{N-1} c_i^2\}/2 \tag{13}$$

can be found.

A.2 Statistic Inference

By ergodicity and large number theorem

$$\lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}|Z_t| = E\{|Z_t|\} \tag{17}$$

The autocorrelaiton function of a sample path with $\omega \in \Omega$ is

$$\rho(\tau|\omega) = E\{X_t(\omega)X^*_{t+\tau}(\omega)|\omega\} \tag{18}$$

and the cross correlation of the sample path for two independent events is

$$\rho 1(\tau|\omega,\omega') = E\{X_t(\omega)X^*_{t+\tau}(\omega')|\omega,\omega'\} \tag{19}$$

The expected autocorrelation and cross correlation are

$$\rho(\tau) = E\{\rho(\tau|\omega)\} \tag{20}$$

and

$$\rho 1(\tau) = E\{\rho(\tau|\omega,\omega')\}, \tag{21}$$

respectively. Let $z_{s,t}$ denote the s-th sample at time t. Then, s represents an independent initialization of the process, so that $z_{s,t}$ and $z_{s',t}$ are independent for $s \neq s'$. The autocorrelation function is evaluated in the sense of

$$\lim_{S\to\infty}\frac{1}{S}\sum_{s}\sum_{t=1}^{W} z_{s,t}z^*_{s,t+\tau} =: \lim_{S\to\infty}\rho(\tau|S) \tag{22}$$

where W is the window size for the computation of the convolution operation. The cross correlation function is evaluated in the sense of

$$\lim_{S\to\infty}\frac{1}{S}\sum_{s,s'=1}^{S}\sum_{t=1}^{W} z_{s',t}z^*_{s,t+\tau} =: \lim_{S\to\infty}\rho 1(\tau|S) \tag{23}$$

A.3 Numerical Error

By simulation using a digital computer, the statistic inference can only be performed with finite samples. Thus, every implementation has error. For the quantities concerned here the following metrics will be used to estimate the error

- For Rayleigh Distribution:

$$|\frac{1}{T}\sum_{t=1}^{T}|z_{s,t}| - \sigma\sqrt{\pi/2}| \tag{24}$$

  for mean and

$$|\frac{1}{T-1}\sum_{t=1}^{T}(|z_{s,t}| - \frac{1}{T}\sum_{t=1}^{T}|Z_t|)^2 - \sigma^2 2(1-\pi/4)| \tag{25}$$

  for variance.

- $$max|\rho(\tau|S) - J_0(\omega \cdot \tau)| \tag{26}$$

  for autocorrelation, and

- $$max|\rho 1(\tau|S)| \tag{27}$$

  for cross correlation.

- The variances of the correlation function:

  $$\sum_{s=1}^{S}[\rho(\tau|s) - \rho(\tau|S)]^2 \tag{28}$$

  for autocorrelation and

  $$\sum_{s=1, s\neq s'}^{S}[\rho 1(\tau|s, s') - \rho 1(\tau|S)]^2 \tag{29}$$

  for cross correlation.

A.4 Example

Using a samples size of $S = 500$, each of length $T = 10000$, computer simulation is conducted with the results shown in Fig.1 and Fig.2. The variance of the autocorrelation is within 0.3 and cross correlation has the maximum 0.2, as shown in Fig.3.

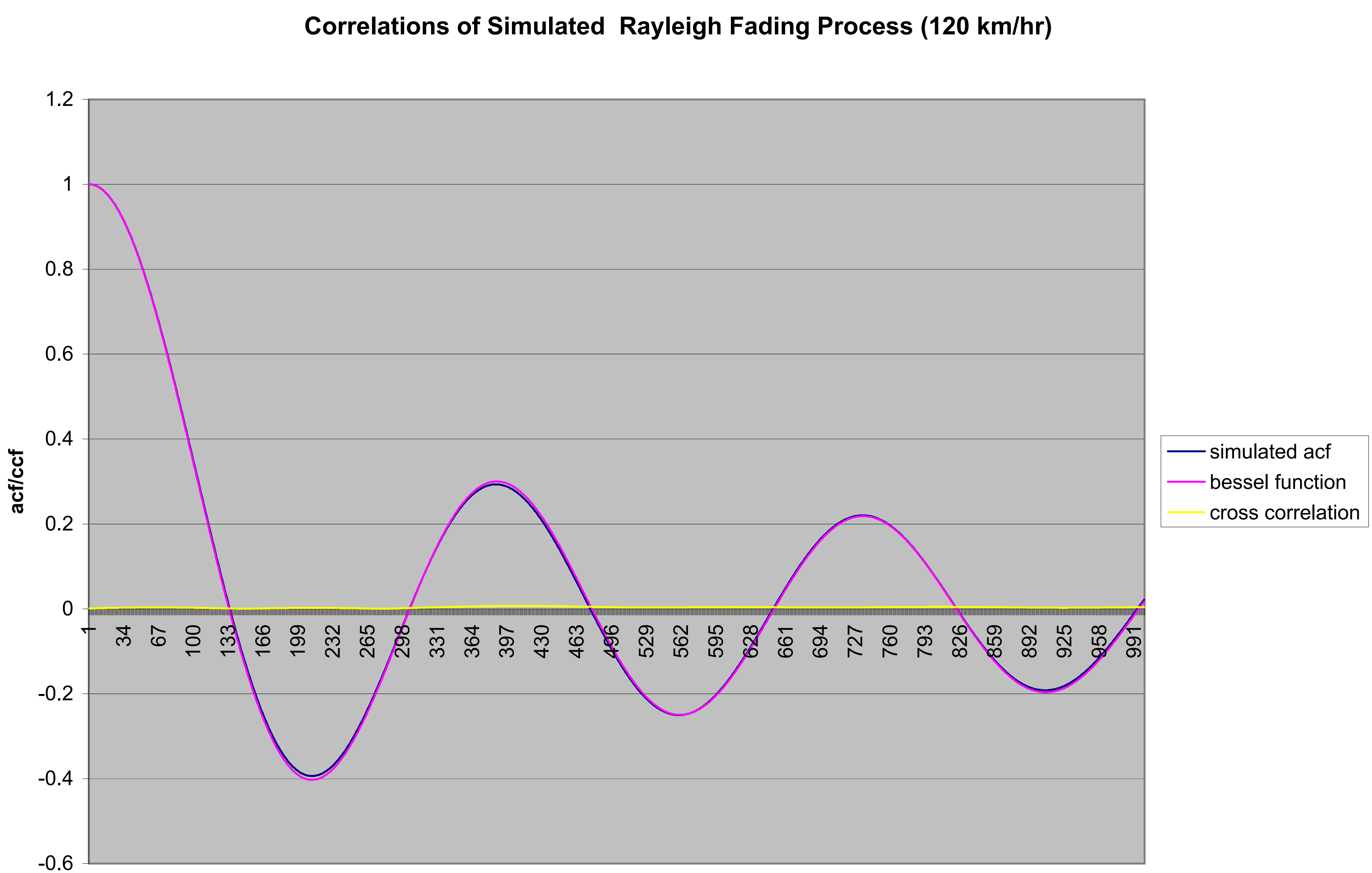

Figure 1: Function of time lag (unit :16 chips=16x 813.8 ns)

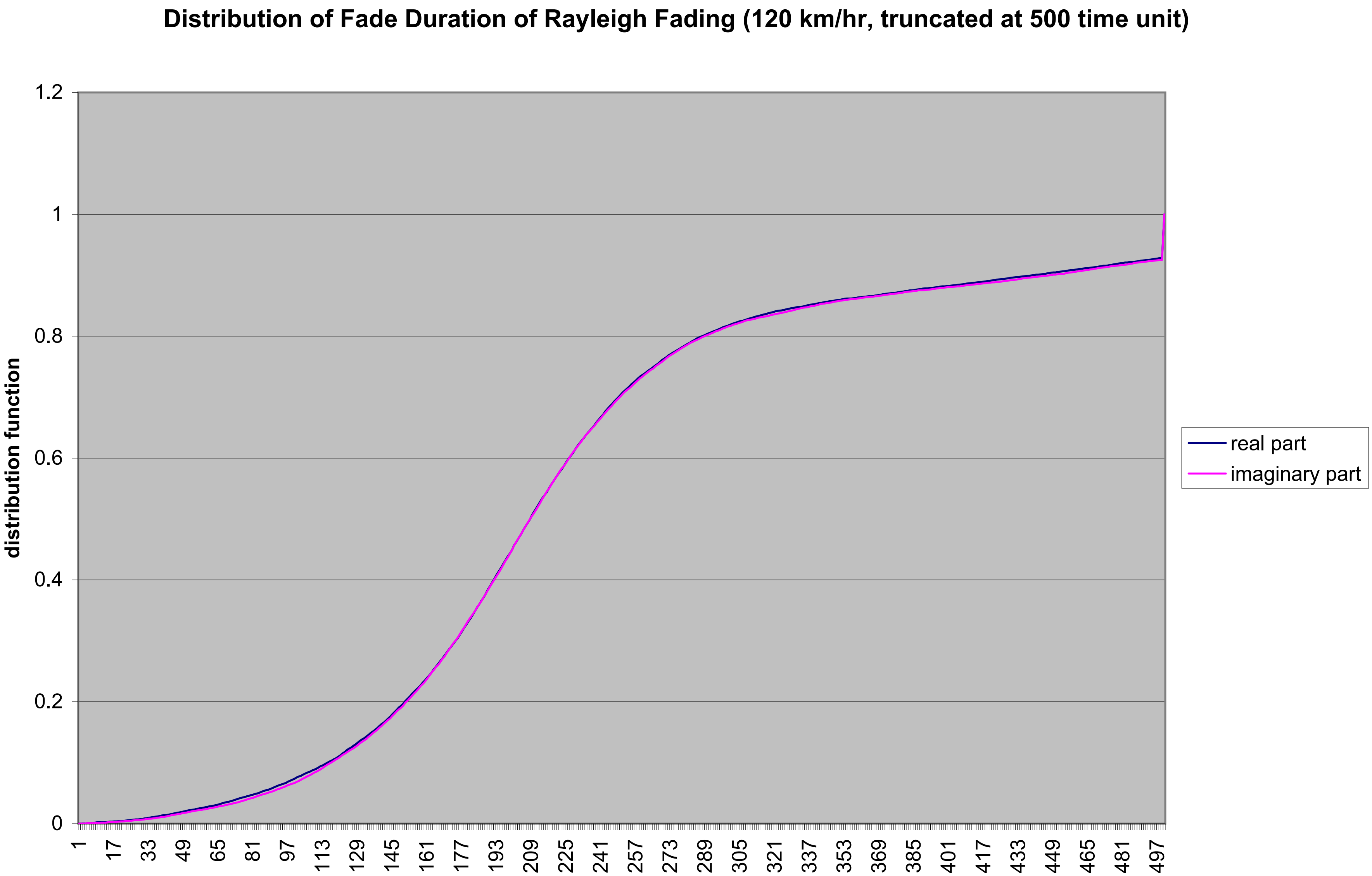

Figure 2: Fade duration (time unit: 16 chips=16x 813.8 ns)

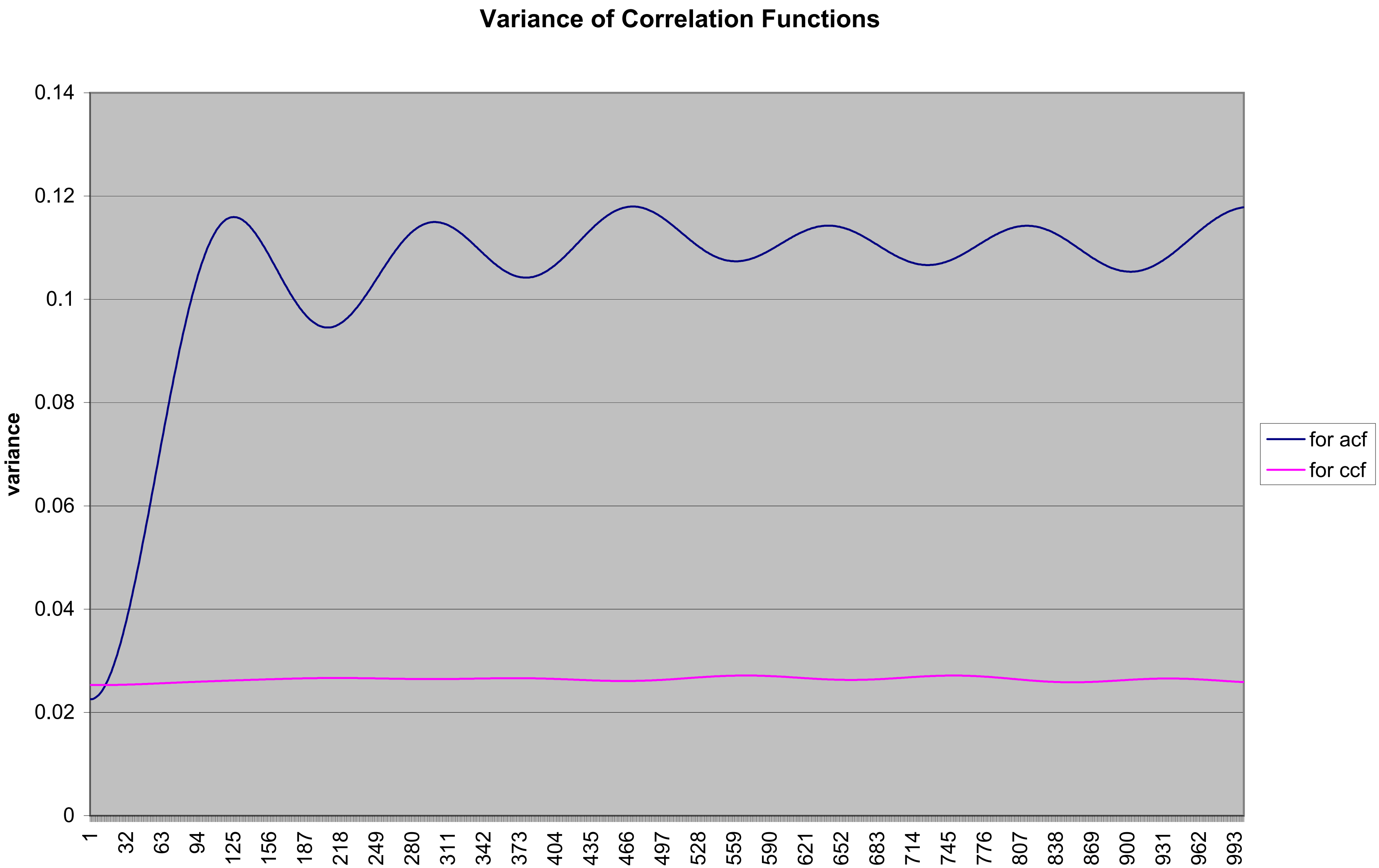

Figure 3: Function of time lag (unit 16 chips=16x 813.8 ns)