Project	**IEEE 802.20 Working Group on Mobile Broadband Wireless Access** **<http://grouper.ieee.org/groups/802/20/>**
Title	**Comments on Modeling MIMO Channels for MBWA**
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Re:	Evaluation Criteria
Abstract	This document clarifies the idea of characterization of the MIMO channel correlation matrix and proposes an action for the group.
Purpose	Discuss and adopt
Notice	This document has been prepared to assist the IEEE 802.20 Working Group to accomplish the simulator calibration process.
Release	
Patent Policy	

1 MIMO Channel

A channel is a stochastic process describing the impact of the propogation environment on the transmitted radio signal. More precisely

Definition 1 *Assume the transmitted signal can be described by the $\delta(t)$ distribution. Then, the corresponding received signal*

$$\int_0^t h(t-\tau)\delta(\tau)d\tau = h(t) \tag{1}$$

is called the radio channel between the given transmitter and the given receiver

A common assumption about the function $h(t)$ is its square integrability $h \in L^2$. Being a function of a Hilbert space, there is a system of orthonormal bases $\{u_i\} \subset L^2$, such that

$$h(t) = \sum_{i=0}^{\infty} a_i u_i(t) \tag{2}$$

In the sense of stochastic process, the orthonormal bases are replaced by identical indepedent random variables with unit variance. The space of valid channel functions is spanned by a system of infinite random process with iid at any time instance.

$$E\{||h(t) - \sum_{i=0}^{n-1} a_i u_i(t)||_{L^2}\} \to 0 \ \text{ for } \ n \to \infty \tag{3}$$

for some iid $\{u_i\}$. This motivates the moving average model for the MIMO channel: Let $f_i(t)$ be a coherent independent channel realization with given Doppler spectrum for $i = 0, 1, ..., n-1$. This implies $u_i = f_i/\sigma$ and

$$a_i =< h, u_i > \tag{4}$$

Then, the channel between a receiver and a transmitter $h(t)$ is assumed to be

$$h(t) = \sum_{i=0}^{n-1} a_i f_i(t) \tag{5}$$

for sufficiently large n. This ansatz implies the assumption that $\{f_i\}_{i=0}^{n-1}$ spans approximately the space of h at any given time instance.
Physically, scatterers are assumed independent with independent cross section and phase shift. The number of the scatters is finite. By these assumptions, the above MA ansatz is approporiate.
From the linear algebra, we know that n linear independent functions can be mapped to another n linear independent functions. In terms of channel functions, we can write the vector of channels as

$$\mathbf{h} = \mathbf{A}\mathbf{f} \tag{6}$$

for $n \leq m$, where $\mathbf{h} = (h_i)_{i=0}^{m-1}$, $\mathbf{A} = (a_{ij})_{i=0,j=0}^{n-1,m-1}$ and $\mathbf{f} = (f_i)_{i=0}^{m-1}$, and $f_i(t)$ is Gaussian random process with

$$< f_i, f_j >= \sigma^2 \delta_{ij} \tag{7}$$

and variance σ^2. Here $\mathbf{A}$ is still unknown and can be determined as following:

Let $\mathbf{h}^\dagger = (\mathbf{h}^*)^t$, i.e. the complex transposition of $\mathbf{h}$, and $\mathbf{A}^\dagger = (\mathbf{A}^*)^t$. Then, the correlation matrix of the n channels is given by

$$< \mathbf{h}, \mathbf{h}^t >= \mathbf{A} < \mathbf{f}, \mathbf{f}^t > \mathbf{A}^\dagger \tag{8}$$

Using 7 we can define

$$\mathbf{H} : \ = \ < \mathbf{h}, \mathbf{h}^t > /\sigma^2 \tag{9}$$
$$= \ \mathbf{A}\mathbf{A}^\dagger \tag{10}$$

Thus, given the coefficients of a MA model, the correlation matrix can be computed as 10. The problem in the reality occurs in the reversed order: The correlation matrix of multiple channels is measured and one wants to determine the coeffients of an approporiate MA model. Since $\mathbf{H}$ contains the correlation coefficients, we have the necessary condition for $\mathbf{A}$ to be MA coefficients of $m < n$ channels

$$|\sum_k a_{i,k} a^*_{j,k}| < 1 \text{ for } i \neq j \tag{11}$$

and

$$\sum_k |a_{i,k}|^2 = 1 \tag{12}$$

There still remains the question about the uniqueness of the decomposition 10. It is readily shown that the decomposition is not unique: Let $\mathbf{A}$ be a decomposition of a given correlation matrix $\mathbf{H}$, and $\mathbf{U}$ be an arbitrary unitary matrix of the same dimension, i.e. $\mathbf{U}\mathbf{U}^\dagger = \mathbf{I}$, where $\mathbf{I}$ is the identity matrix. Then $\mathbf{AU}$ is also a valid decomposition:

$$\mathbf{AU}(\mathbf{AU})^\dagger = \mathbf{AU}\mathbf{U}^\dagger\mathbf{A} = \mathbf{H} \tag{13}$$

Fortunately, the non-uniqueness does not undermine the applicability of the MA model. As long as $\mathbf{H}$ is the only measurable metric, it will allow for any coefficient matrix, for that 10 holds. In the context of simulation of correlated channels, one can use the measured/designated correlation matrix of m channels to determine the coefficients of a generating MA models of order $\leq m$.

Since $\mathbf{H}$ is square, by using Cholesky decomposition, $\mathbf{A}$ can be determined as a matrix of lower triangle. We notice that the n channels given by $\mathbf{h} = \mathbf{Af}$ is in fact constructed consecutively similar to the Gram-Schmidt method. The difference to the Gram-Schmidt is the orthogonality being replaced by given correlations between channels.

2 One-Step Construction

By definition, each channel is associated with a transmitter and a receiver. Thus, the correlation of two channels involves either two receiver and one transmitter, or one receiver and two transmitter, or two transmitter-receiver pairs. A MIMO system with n inputs and m outputs is associated with nm channels. When a correlation matrix of $nm \times nm$ is given, a MA model for each channel with nm coefficients and as many iid processes can be determined that correspond to the given correlation matrix.

3 Recommendation

For the determination of the MIMO channel using the MA model, the correlation matrix of order $mn \times mn$ is required, where m and n are the number of the transmit and receive antennas, respectively. We recommend that the group assign one person or an ad-hoc group the responsibility to determine the specific values of the correlation matrix for each needed combinations of $mn \times mn$ to be used by all parties that deploy MIMO in their proposals. It should be noted that the method of generating the correlation matrix should not be mandated by the group, as the goal is to have a common matrix that can be used by the simulation, rather than a single method of channel modeling.