Channel Modelling for MBWA

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This short contribution describes a method for simulating ISI channels that allows for MIMO channels to collapse to the ITU channel models. This method is based on the correlation approach. We start by describing the basic steps in simulating a SISO link and then describe the correlation approach in generating the MIMO channel.

1 SISO Channel Modeling

We assume a time-invariant channel model

$$h(t) = \sum_{i=1}^{P} \alpha_i(t)\delta(t - \tau_i) \tag{1}$$

where $\alpha_i(t)$ is the complex tap gain which is assumed to be a complex Gaussian random variable with zero mean and variance σ_i^2 and τ_i is the corresponding tap delay. We assume that the complex tap gains will have an autocorrelation function $R_\alpha(\tau)$ which will be a function of the scattering process and the mobility condition. In case of a uniform scattering, this will be characterized by the classical Jake's spectrum with $R_\alpha(\tau) = J_o(2\pi f_d \tau)$, where f_d is the maximum Doppler shift.

Let $g(t)$ be the transmit filter pulse shape and $f(t)$ be the receive filter impulse response. The transmitted signal is then

$$s(t) = \sum_n s_n g(t - nT) \tag{2}$$

The corresponding received signal is

$$r(t) = h(t) \star s(t) \star f(t) = h(t) \star x(t) \tag{3}$$

where

$$x(t) = f(t) \star s(t) = \sum_n s_n \tilde{g}(t - nT) \tag{4}$$

is the overall transmitted signal which includes the effects of pulse shaping, transmit filtering, and receiver filtering. Hence, the received signal is

$$\begin{aligned} r(t) &= h(t) \star x(t) && (5)\\ &= \left\{ \sum_{i=1}^{P} \alpha_i \delta(t-\tau_i) \right\} \star \left\{ \sum_n s_n \tilde{g}(t-nT) \right\} && (6)\\ &= \int_\beta \sum_{i=1}^{P} \alpha_i \delta(\beta-\tau_i) \sum_n s_n \tilde{g}(t-\beta-nT) d\beta && (7)\\ &\approx \sum_n s_n \sum_{i=1}^{P} \alpha_i \int_\beta \delta(\beta-\tau_i)\tilde{g}(t-\beta-nT)d\beta && (8)\\ &= \sum_n s_n \sum_{i=1}^{P} \alpha_i \tilde{g}(t-\tau_i-nT) && (9) \end{aligned}$$

Let us assume that the received signal is over-sampled by a factor of Q, then the sampling times are

$$t = kT + \frac{qT}{Q} \qquad k = 0, 1, 2, \cdots \quad q = 0, 1, 2, \cdots, Q-1 \tag{10}$$

Then we have

$$\begin{aligned} r(kT + \frac{qT}{Q}) &= \sum_n s_n \sum_{i=1}^{P} \alpha_i \tilde{g}(kT + \frac{qT}{Q} - nT - \tau_i) && (11)\\ &= \sum_n s_n \sum_{i=1}^{P} \alpha_i \tilde{g}((k-n)T + \frac{qT}{Q} - \tau_i) && (12) \end{aligned}$$

Let $k - n = m$. Also, a reasonable assumption is that the overall pulse response (due both the transmit and receiver filters) will have a finite duration. Hence we will have

$$\begin{aligned} r(kT + \frac{qT}{Q}) &= \sum_{m=0}^{L} s_{k-m} \sum_{i=1}^{P} \alpha_i \tilde{g}(mT + \frac{qT}{Q} - \tau_i) && (13)\\ &= \sum_{m=0}^{L} s_{k-m} h_m(q) && (14) \end{aligned}$$

where

$$h_m(q) = \sum_{i=1}^{P} \alpha_i \tilde{g}(mT + \frac{qT}{Q} - \tau_i) \tag{15}$$

Let us consider symbol rate equivalent channel ($q = 0$, the case when the output of the receive filter is sampled at the symbol rate):

$$\begin{bmatrix} h_0(0) \\ h_1(0) \\ h_2(0) \\ \vdots \\ h_L(0) \end{bmatrix} = \begin{bmatrix} \tilde{g}(-\tau_1) & \tilde{g}(-\tau_2) & \tilde{g}(-\tau_3) & \cdots & \tilde{g}(-\tau_P) \\ \tilde{g}(T-\tau_1) & \tilde{g}(T-\tau_2) & \tilde{g}(T-\tau_3) & \cdots & \tilde{g}(T-\tau_P) \\ \tilde{g}(2T-\tau_1) & \tilde{g}(2T-\tau_2) & \tilde{g}(2T-\tau_3) & \cdots & \tilde{2}g(2T-\tau_P) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \tilde{g}(LT-\tau_1) & \tilde{g}(LT-\tau_2) & \tilde{g}(LT-\tau_3) & \cdots & \tilde{g}(LT-\tau_P) \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \vdots \\ \alpha_P \end{bmatrix} \tag{16}$$

Or in a vector form

$$\mathbf{h}(t) = \mathbf{G}(\boldsymbol{\tau}) \cdot \boldsymbol{\alpha}(t) \tag{17}$$

The model given in (17) describes a SISO ISI channel. The time correlation behavior of this channel is reflected in the time correlation behavior (which depends on the Doppler spread of the channel) of the α_l's. The frequency correlation behavior of the channel is reflected in the pulse shaping matrix $\mathbf{G}(\boldsymbol{\tau})$ and its dependency on the delays τ_l's (and hence its dependency on the channel delay spread). Given the model in (17), we can summarize the step for generating a SISO link according the ITU channel models as follows:

1. Let $A_1, A_2, \cdots, A_P$ and $\tau_1, \tau_2, \cdots, \tau_P$ represent the power-delay profile for the specified ITU channel model

2. Generate P independent Rayleigh fading processes each having a Doppler spread f_d.

3. Scale the p-th Rayleigh process by σ_p where

$$\sigma_p^2 = \frac{A_p}{\sum_{p=1}^{P} A_P}$$

4. Generate the pulse shaping matrix $\mathbf{G}(\boldsymbol{\tau})$ as specified in (16)

5. Compute the channel taps' processes according to (17)

2 Antenna Correlation with MIMO Links

In the MIMO channel models developed below, only azimuth angles are considered in the propagation geometry, but the results can be generalized to three dimensions. [Focus on SIMO ZZZ]. The propagation environment under consideration is densely populated with both natural and man-made structures. An illustration of the vector channel propagation environment is shown in Figure 1. The transmitter radiation pattern illuminates all local scattering structures, or *local reflectors*, surrounding the transmitter that are within few hundred wavelengths. Radiation from these local reflectors and/or the transmitter reaches the receiver either directly or by reflection from large reflecting objects in the environment such as large buildings and hills. These objects are termed *dominant reflectors*. Scalar SISO channel models do not explicitly include the effects of these dominant reflectors. For the vector or MIMO channels, the angle of arrival of each reflected wave with respect to the receiver coordinate system is determined by the physical position of the dominant reflector with respect to the receiver. An illustration of the MIMO multipath channel is shown in Figure 1. Let $s(t)$ be the complex baseband transmitted signal. The complex baseband received signal vector at the base station antenna array can be written as

$$\mathbf{x}(t) = \sum_{i=1}^{L} \mathbf{v}(\theta_i) \cdot R_i e^{j2\pi(f_d \cos\psi_i t - f\tau_i)} \cdot s(t-\tau_i) \tag{18}$$

where ψ_i is the direction of the local scatterer with respect to the mobile velocity vector and θ_i is the angle of arrival of the ith signal path which is also the angular position of the ith dominant reflector (or local scatterer) with respect to the base station coordinate system. $\mathbf{v}(\theta_i)$ is the $M \times 1$ receiver array response vector for signals arriving in the ith wavefront and it depends on the angle of arrival θ_i (measured with respect to the receive antenna array geometry) . R_i^2 is the fraction of the incoming power in the ith path. In, SISO channel models, the effect of $\mathbf{v}(\theta_i)$ is not present since with a single receive antenna, the concept of angle of arrival is meaningless.

In general, signals arrive at the receive antennas mainly from one given direction. For example, in rural or suburban areas, a high receiver antenna array typically has a line-of-sight path to the mobile, with local scattering around the transmitter generating signals that arrive mainly within a given range of angles or beamwidth. We assume that all signals from the transmitter arrive at the base station antenna array uniformly $\pm\Delta$ of the mean angle of arrival θ according to some distribution $f_\Theta(\theta_i)$. The value of Δ is called the *angle spread* (around the mean angle of arrival).

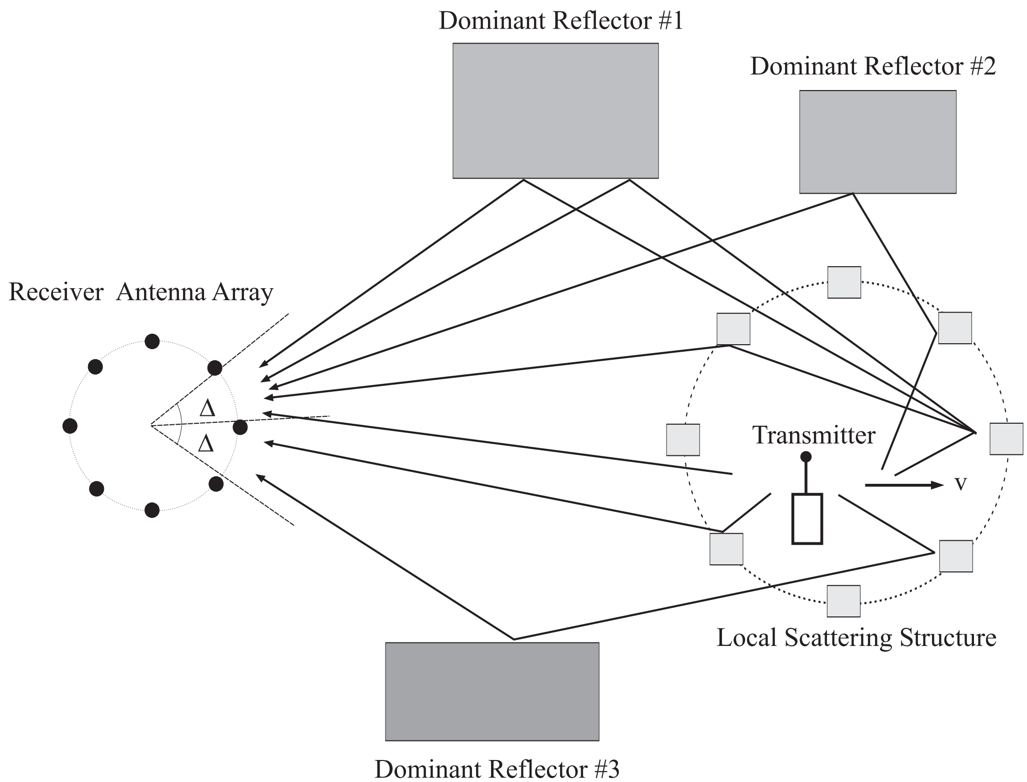

Figure 1: An illustration of the vector channel.

Assuming that $s(t)$ is a narrowband signal (wrt the carrier frequency), then $s(t-\tau_i) \approx s(t-\tau_o)$ where $\tau_o \in [\min_i \tau_i, \max_i \tau_i]$ and hence we can write the lowpass received signal vector as

$$\mathbf{x}(t) \approx s(t-\tau_o) \cdot \left(\sum_{i=1}^{L} \mathbf{v}(\theta_i) \cdot R_i e^{j\phi_i(t)} \right) \tag{19}$$

where $\phi_i = 2\pi(f_d \cos\psi_i t - f\tau_i)$. As before, the ϕ_i modulo 2π are assumed to be i.i.d. over $[0, 2\pi]$. We define the complex channel vector $\mathbf{a}(t)$ as

$$\mathbf{a}(t) = \sum_{i=1}^{L} \mathbf{v}(\theta_i) R_i e^{j\phi_i(t)} \tag{20}$$

Similar to the scalar channel case, the lowpass vector channel is described by the time-variant impulse response

$$\mathbf{h}(t;\tau) = \delta(\tau - \tau_o) \cdot \mathbf{a}(t) \tag{21}$$

Note that for a single receive antenna, the above expression for the vector channel reduces exactly to the complex channel gain model for a SISO link [**?**]. For a large number of incoming paths, the complex channel vector will approach a zero mean complex Gaussian random vector, that is

$$\mathbf{a}(t) \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_a) \tag{22}$$

where $\mathbf{R}_a$ is the channel vector covariance defined as

$$\mathbf{R}_a = \mathrm{E}\{\mathbf{a}(t)\mathbf{a}^*(t)\} \tag{23}$$

and is a function of the angular frequency ω, the mean angle of arrival θ, the angle spread Δ, and the spacing between sensors and their geometry. It is worthwhile to understand the behavior of this *correlation matrix* and to characterize and study their structure, how this structure expresses and describes the propagation environment and signals propagating in that environment, and what structure is "naturally" present (induced by stationarity assumptions, for example). It can be shown that the space-time-frequency correlation matrix of $\mathbf{a}(\omega_1, t)$ and $\mathbf{a}(\omega_2, t+\nu)$ is given by

$$\begin{aligned} \mathcal{R}_a(\omega_1, \omega_2, \nu) &= \mathrm{E}\{\mathbf{a}(\omega_1, t)\mathbf{a}^*(\omega_2, t+\nu)\} && (24) \\ &= J_o(\omega_d \nu) \cdot F_T(j\Delta\omega) \cdot \mathbf{R}_s && (25) \end{aligned}$$

where $\mathbf{R}_s$ is the *array spatial correlation matrix* defined as

$$\mathbf{R}_s = \int_{-\Delta+\theta}^{\Delta+\theta} \mathbf{v}(\omega_1, \vartheta)\mathbf{v}^*(\omega_2, \vartheta) f_\Theta(\vartheta) d\vartheta \tag{26}$$

and $\mathbf{v}(\omega, \theta)$ is the array response vector at carrier frequency ω. Similar to $\mathbf{R}_a$, $\mathbf{R}_s$ is a function of the angular frequencies ω_1 and ω_2, the mean angle of arrival θ, the angle spread Δ, and the *spacing* between sensors and their geometry. For $\omega_1 = \omega_2 = \omega$ we will write $\mathcal{R}_a$ as $\mathcal{R}_a(\omega, \nu)$.

Give an mean angle of arrival θ and angle spread distribution $f_\Theta(\theta)$, the spatial correlation matrix can be evaluated. For example, if assume that the angle of arrival is uniformly distributed within $\boldsymbol{\Delta}$ of the mean angle of arrival, the we can show that the real and imaginary parts of $\mathbf{R}_s(m, n)$, the signal correlation between the mth and nth antenna elements, are given by

$$\mathrm{Re}\{\mathbf{R}_s(m,n)\} = J_o(z_{mn}) + 2\sum_{l=1}^{\infty} J_{2l}(z_{mn})\cos(2l(\theta_{mn}+\delta_{mn}))\mathrm{sinc}(2l\Delta) \tag{27}$$

$$\mathrm{Im}\{\mathbf{R}_s(m,n)\} = 2\sum_{l=0}^{\infty} J_{2l+1}(z_{mn})\sin((2l+1)(\theta_{mn}+\delta_{mn}))\mathrm{sinc}((2l+1)\Delta) \tag{28}$$

where $\theta_{m,n}$ is the mean angle of arrival measured with respect to the normal to the line joining the two sensors as shown and $z_{m,n}$ is distance between the two antennas normalized by the wavelength.

Note that even though the above spatial correlation model is described for a single transmit antenna and M receive antennas, extension to N transmit antennas is straight forward. Also the same concept con be used to generate transmit antenna correlation matrices.

Based on the above discussion, we can summarize the step for generating a MIMO channel that have an underlying SISO ITU model as follows:

1. We assume N transmit and M receive antennas. For every transmit antenna, generate M SISO links according to procedure outlined above.

2. For every link, generate M random angle of arrivals.

3. For a given angle spread distribution and specific antenna geometry compute the array correlation matrix $\boldsymbol{R}_s$.

4. Color the M independent SISO links using the array correlation matrix, i.e. at any given time, the l tap of the $M \times 1$ channel vector is generated as

$$\tilde{\boldsymbol{h}}(t,\tau) = \boldsymbol{R}_s^{1/2}\boldsymbol{h}(t,\tau).$$