



## **IEEE802.3 4P Task Force**

**Derivation of PSE and PD PI Rmax, Rmin in  
order to meet system worst case End to End  
Pair To pair effective resistance unbalance**

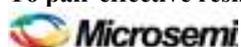
**March 2015**

**Rev 009**

**Yair Darshan / Microsemi** [yadarshan@microsemi.com](mailto:yadarshan@microsemi.com)

**Ken Bennet / Sifos**

Derivation of PSE and PD PI Rmax, Rmin Equation limits that meets system worst case End to End Pair To pair effective resistance unbalance. March 2015 Rev 009.



## Derivation of PSE and PD PI Requirements

The following is the End to End Pair to Pair Resistance Unbalance (E2EP2PRUNB or “ $\alpha$ ”) equation.

$$(1) \quad \alpha = E2E\_P2PRunb = \frac{\left(\sum_{R_{max}} PSE - \sum_{R_{min}} PSE\right) + \left(\sum_{R_{max}} PD - \sum_{R_{min}} PD\right) + \left(\sum_{R_{max}} CH - \sum_{R_{min}} CH\right)}{\left(\sum_{R_{max}} PSE + \sum_{R_{min}} PSE\right) + \left(\sum_{R_{max}} PD + \sum_{R_{min}} PD\right) + \left(\sum_{R_{min}} CH + \sum_{R_{max}} CH\right)}$$

All resistance values are effective resistances which mean that the values include the effects of non-linear components at their operating point and the effects of Pair to pair voltage differences both in the PSE and PD.

$$(2) \text{ The PSE PI P2PRUNB is: } PSE\_P2PRunb = \frac{\left(\sum_{R_{max}} PSE - \sum_{R_{min}} PSE\right)}{\left(\sum_{R_{max}} PSE + \sum_{R_{min}} PSE\right)}$$

(3) The PSE PI P2PRUNB contribution to the E2EP2PRunb of the system is

$$PSE\_P2PRUNB\_contribution = \frac{\left(\sum_{R_{max}} PSE - \sum_{R_{min}} PSE\right)}{\left(\sum_{R_{max}} PSE + \sum_{R_{min}} PSE\right) + \left(\sum_{R_{max}} PD + \sum_{R_{min}} PD\right) + \left(\sum_{R_{min}} CH + \sum_{R_{max}} CH\right)}$$

We can see that PSE contribution (3) is not equal to PSE PI P2PRUNB (2).

As a result we need to transform PSE\_P2PRUNB (2) to PSE\_P2PRUNB\_contribution (3) in order to have the correct weight of the PSE in the whole system defined by (1) i.e. to find the function  $F_x$  that satisfies the worst case E2EP2PRUNB at the maximum PSE Type operating power (which is not necessarily the point of maximum pair current).

$$\frac{\left(\sum_{R_{max}} PSE - \sum_{R_{min}} PSE\right)}{\left(\sum_{R_{max}} PSE + \sum_{R_{min}} PSE\right)} \cdot F_x = \frac{\left(\sum_{R_{max}} PSE - \sum_{R_{min}} PSE\right)}{\left(\sum_{R_{max}} PSE + \sum_{R_{min}} PSE\right) + \left(\sum_{R_{max}} PD + \sum_{R_{min}} PD\right) + \left(\sum_{R_{min}} CH + \sum_{R_{max}} CH\right)}$$

The results of this step will supply the equation that defines the relationship between RPSEmin and RPSEmax in terms of resistance values. e.g. if Rmin is selected by the designer, then what is the corresponding maximum allowable value of Rmax that will not exceed the E2EP2PRunb limit. (Just specifying Rmax/Rmin ratio will not work for our objective above).

**Derivation of PSE and PD PI Rmax, Rmin Equation limits that meets system worst case End to End Pair To pair effective resistance unbalance. March 2015 Rev 009.**

There are few ways to do this task. The following is a simple analytical transformation process:

Describing (1) as a system that includes all parts:

$$(4) \quad E2EP2PRunb = \frac{\left( \sum R_{max} - \sum R_{min} \right)}{\left( \sum R_{max} + \sum R_{min} \right)} = \alpha$$

Opening and solving for Rmax/Rmin in terms of  $\alpha$ .

$$(5) \quad \begin{aligned} \left( \sum R_{max} - \sum R_{min} \right) &= \alpha \cdot \left( \sum R_{max} + \sum R_{min} \right) \\ \sum R_{max} - \sum R_{min} &= \alpha \cdot \sum R_{max} + \alpha \cdot \sum R_{min} \\ \sum R_{max} - \alpha \cdot \sum R_{max} &= +\alpha \cdot \sum R_{min} + \sum R_{min} \\ (1 - \alpha) \cdot \sum R_{max} &= (1 + \alpha) \cdot \sum R_{min} \\ \frac{\sum R_{max}}{\sum R_{min}} &= \frac{(1 + \alpha)}{(1 - \alpha)} = u \end{aligned}$$

As a result from (5):

$$(6) \quad \begin{aligned} \frac{\sum R_{max}}{\sum R_{min}} &= u \\ u \cdot \sum R_{min} - \sum R_{max} &= 0 \end{aligned}$$

The E2EP2PRunb equation from (1) or it simpler form (4) can be expressed in the following form:

$$(7) \quad U \cdot \sum R_{min} - \sum R_{max} = 0, \quad \text{Where } U = \frac{1+\alpha}{1-\alpha}$$

Separating the contributors PSE , PD and Channel results in:

$$(8) \quad \begin{aligned} (U \cdot R_{PSEmin} - R_{PSEmax}) + (U \cdot R_{CHmin} - R_{CHmax}) + (U \cdot R_{PDmin} - R_{PDmax}) \\ = Cont\_PSE + Cont\_CH + Cont\_PD = 0 \end{aligned}$$

**Derivation of PSE and PD PI Rmax, Rmin Equation limits that meets system worst case End to End Pair To pair effective resistance unbalance. March 2015 Rev 009.**



Each contributor is a constant in the worst case model:

$$Cont\_PSE + Cont\_CH + Cont\_PD = 0$$

And a contributor can be solved independently to meet an E2ERunb limit, given the worst case scenario:

(10)

$$(U \cdot R_{PSEmin} - R_{PSEmax}) + (U \cdot R_{CHmin} - R_{CHmax}) + (U \cdot R_{PDmin} - R_{PDmax}) = Cont\_pse + Cont\_CH + Cont\_PD = 0$$

$$(11) \quad (U \cdot R_{PSEmin} - R_{PSEmax}) + Cont\_CH + Cont\_PD = 0$$

Simplifying further by combining the constants:

$$K_{pse} = Cont\_CH + Cont\_PD$$

$$(12) \quad (U \cdot R_{PSEmin} - R_{PSEmax}) + K_{pse} = 0$$

Solving for Rmax expressed as a range with a worst case limit results in:

$$(13) \quad R_{PSEmax} \leq U \cdot R_{PSEmin} + K_{PSE}$$

Where:

U is a constant determined by the target balance, an

$K_{pse}$  is a constant derived for the PSE contribution to the worst case E2ERunb.

[See Example next page.](#)

**Example Usage:**

Assuming that the following represents a system that is considered to be a worst case system in terms of the components it uses, operation at maximum operating power, and at the practical shortest channel length[m] (ie. the minimum Channel Rmax, Rmin.).

$$\text{PSE: } R_{\text{pair\_max\_pse}} = 0.09 \quad R_{\text{pair\_min\_pse}} = 0.075$$

$$\text{Ch + PD: } R_{\text{pair\_max\_ch+pd}} = 1.253 \quad R_{\text{pair\_min\_ch+pd}} = 0.634$$

Initial Determination of E2ERunb and Kpse using worst case simulation values of example system:

$$\frac{\Sigma R_{\text{max}} - \Sigma R_{\text{min}}}{\Sigma R_{\text{max}} + \Sigma R_{\text{min}}} = E2ER_{\text{unb}} = 0.3089$$

**Derive PSE Specification:**

$$U = \frac{1 + E2ER_{\text{unb}}}{1 - E2ER_{\text{unb}}} = 1.894 .$$

$$K_{\text{pse}} = [R_{\text{pair\_min\_ch+pd}} * U - R_{\text{pair\_max\_ch+pd}}] = 0.634 * 1.894 - 1.253 = -0.052$$

**Specification becomes:**

$$R_{\text{pair\_max\_pse}} = 1.894 * R_{\text{pair\_min\_pse}} - 0.052.$$

**Cross-Check:**

Arbitrary PSE  $R_{\text{pair\_min}} = 0.2$

PSE  $R_{\text{pair\_max}}$  limit Calculated with above equation: 0.326

(The above values of PSE  $R_{\text{pair\_max}}$  and  $R_{\text{pair\_min}}$  should meet the E2ERunb limit with the worst case ch+pd values represented by U and Kpse above):

Any PSE that uses  $R_{\text{pair\_max\_pse}} \leq 0.326$  and  $R_{\text{pair\_min\_pse}} = 0.2$

With the worst case load:  $R_{\text{pair\_max\_ch+pd}} = 1.253$  and  $R_{\text{pair\_min\_ch+pd}} = 0.634$

will meet the E2ERunb Limit:

$$E2ER_{\text{unb}} = [(0.326 + 1.253) - (0.2 + 0.634)] / [(0.326 + 1.253) + (0.2 + 0.634)] = 0.3087$$

Derivation of PSE and PD PI Rmax, Rmin Equation limits that meets system worst case End to End Pair To pair effective resistance unbalance. March 2015 Rev 009.

