High spectrally efficient coded modulation schemes for GEPOF technical feasibility

Rubén Pérez-Aranda
rubenpda@kdpof.com

Agenda

- Background and objectives
- Channel model
- General consideration for spectral efficiency selection
- Shannon limit and spectral efficiency review
- Coded modulation schemes
- Conclusions

Background & Objectives

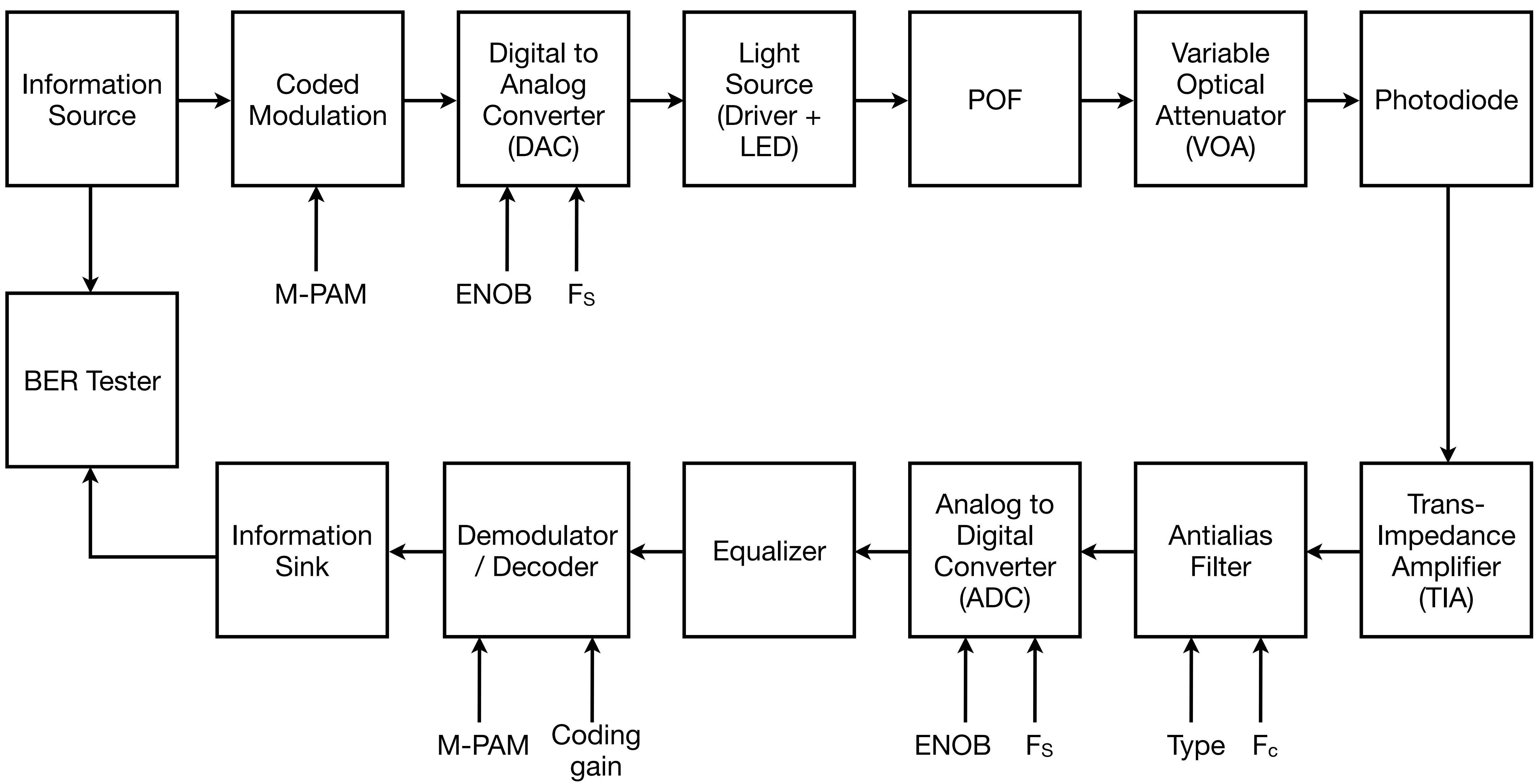
- In May 2014 Interim was demonstrated the necessity of high spectrally efficient coding schemes to approach the POF channel capacity and therefore to meet the link budget requirements (see [1] and [2])
- The analysis based on Information Theory suggested that the combination of high spectrally efficient coded Pulse Amplitude Modulation (PAM) with Tomlinson-Harashima Precoding (THP) is a feasible solution
- This presentation studies several coded modulation schemes providing a comparison between them in terms of coding gain and complexity

Disclaimer

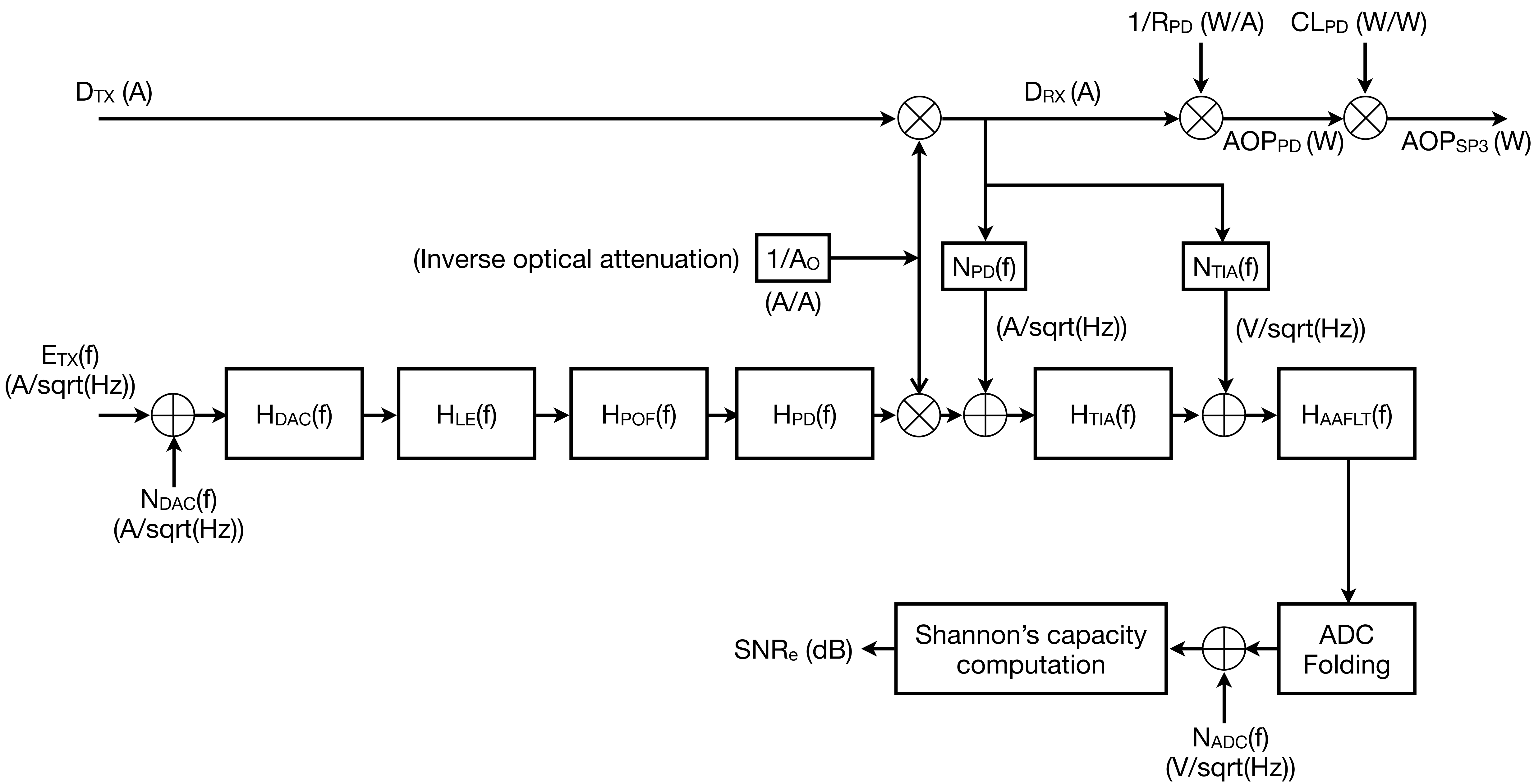
- This presentation concentrates on baseband time-domain modulation schemes, and specifically on coded M-PAM schemes, as it was done in [1].
- Other schemes, like ones operating in frequency domain (e.g. DMT, OFDM, …) are intentionally left outside this presentation, since these kind of schemes incur, by their nature, in additional latency and implementation complexity.

Channel model

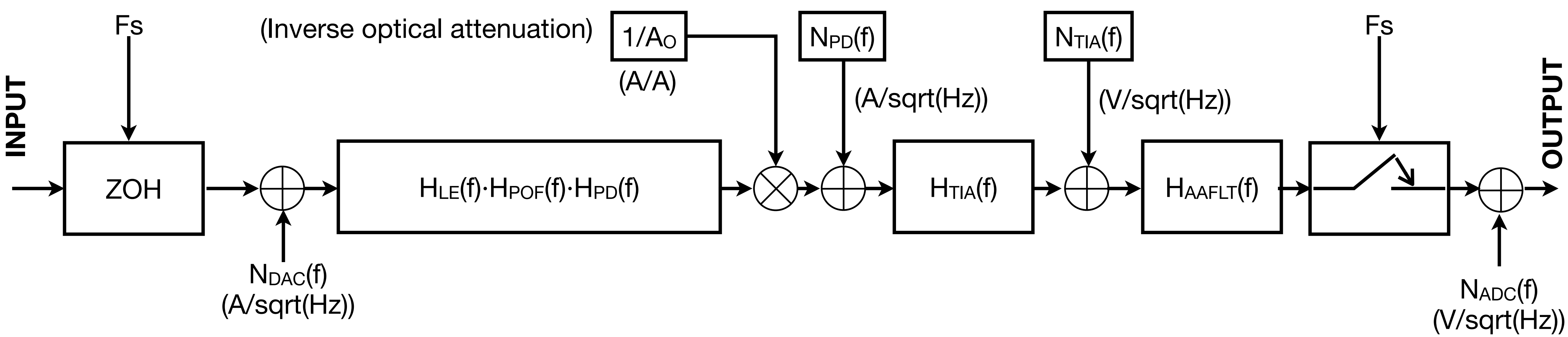
Reference model



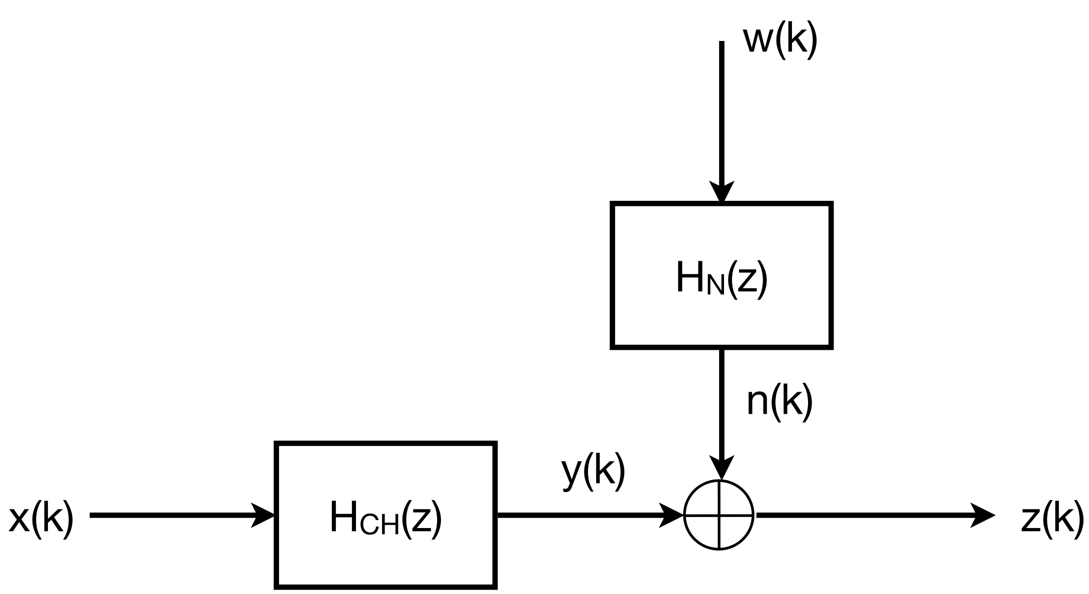
Information theory model



Linear discrete-time channel model

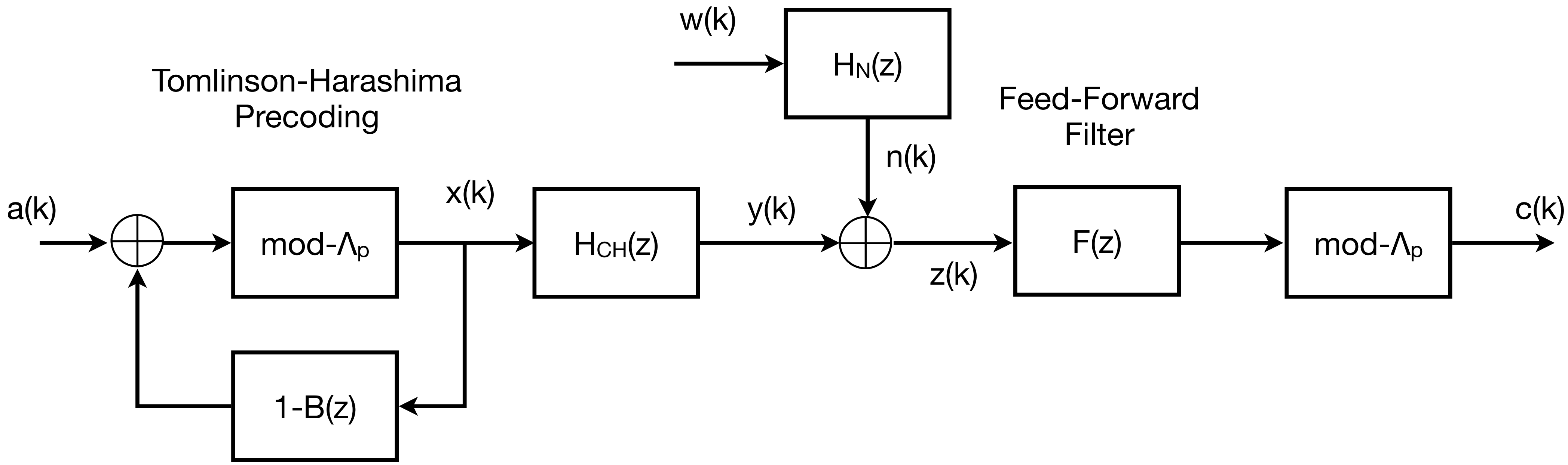


Discrete-time equivalent

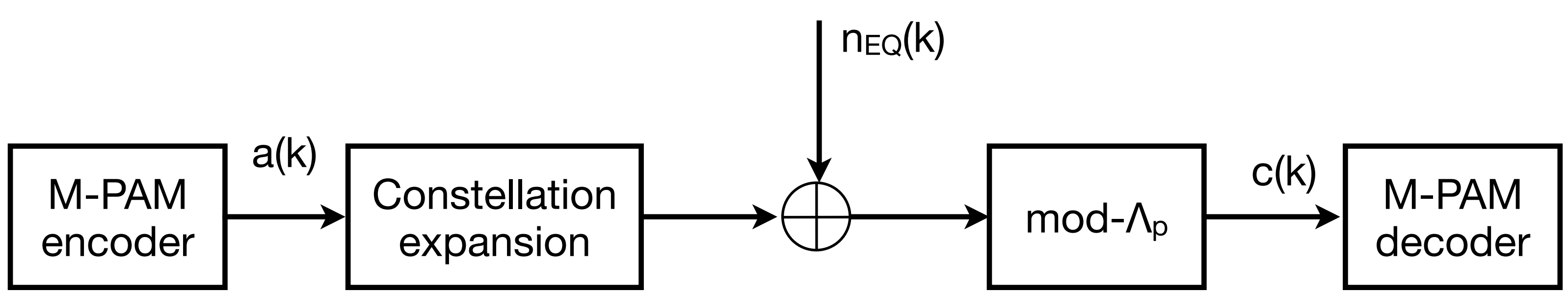


- x(k): channel input signal
- y(k): input affected by the channel impulse response H_{CH} (convolution)
- w(k): white gaussian noise
- n(k): colored noise
- z(k): channel output signal, including ISI and additive colored noise
- It is assumed the timing-recovery is optimal, therefore it works providing optimal sampling phase to ADC

Equalized channel model



Equivalent



$$SNR \triangleq \frac{\sigma_a^2}{\sigma_{n_{EQ}}^2}$$

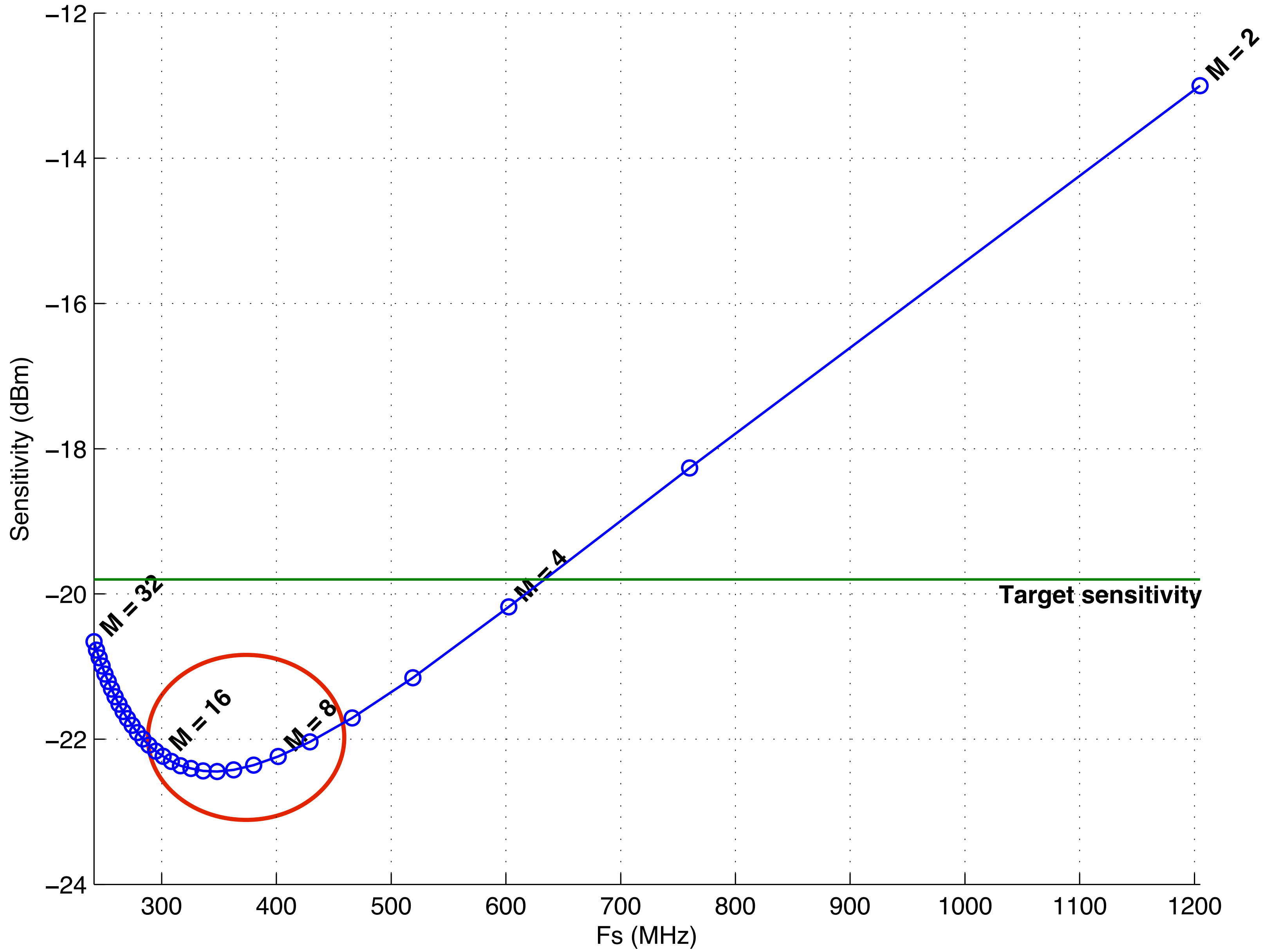
Channel model - general considerations

- In order to make a fair comparison of coded modulation schemes, a channel model has to be defined
- Channel model is defined at the output of equalizer, under following assumptions:
  - ISI has been fully compensated by the THP plus feed-forward equalizer
  - Feed-forward equalizer fully whitens the channel noise
  - The equivalent channel is a memoryless channel with additive white gaussian noise ➤ AWGN channel
  - Therefore, it is assumed neither residual ISI nor colored noise exist
- Modulo operation may be advantageously embedded into the M-PAM decoder to avoid symbol flipping
- The THP power capacity losses due to crest-factor and precoding loss (see [1]) are included in the SNR of equivalent memoryless AWGN channel
- The THP coding penalty due to constellation expansion and modulo operation is going to be considered independently in the evaluation of each coded modulation scheme

General considerations for the spectral efficiency selection

Spectral efficiency selection

- The Shannon capacity analysis of [1] suggests the optimal scheme should be a M-PAM with M between 8 and 16, for THP channel and code-rate ~0.83



Spectral efficiency selection

- The question: what choice is better, 8 or 16 PAM?

- The right answer depends on many implementation aspects:
  1. Equalization complexity ➤ DSP complexity
  2. Power consumption
  3. Optoelectronics design limits
  4. Harmonic distortion of light emitter
  5. DAC and ADC implementation complexity

Spectral efficiency selection

- Equalization complexity - numerical precision
  - Ultimate Shannon limit for M = 16, cr ~0.83, is 19.9 dB
  - Ultimate Shannon limit for M = 8, cr ~0.83, is 14.9 dB
  - Based on this small difference (less than 1 quantization bit), equalizers for M = 8 and M = 16 are going to require very similar numerical precision
- Equalization complexity - number of MACs
  - Symbol rate for M = 16 is F_S = ~300 MHz
  - Symbol rate for M = 8 is F_S = ~400 MHz
  - The number of taps N_B and N_F, of B(z) and F(z), respectively are proportional to the symbol rate
  - Based on that, the number of MACs for M = 8 is **34%** higher than for M = 16.
- Equalization complexity - number of MAC operations per time
  - Based on previous point, the MACs / time is proportional to Fs^2
  - Therefore, the equalization (DSP) complexity for M = 8 is **78%** higher than for M = 16.
- Power consumption due to DSP is also **78%** higher for M = 8 than for M = 16.

Spectral efficiency selection

- Optoelectronics design limits - TIA:
  - According to [3], the maximum trans-impedance provided by a TIA depends on the required bandwidth, and hence on the symbol rate
  - For a PIN diode with capacitance C_T, and a gain-bandwidth product GBW provided by the technology, the maximum achievable trans-impedance is given by

$$R_{F\max} \approx \frac{GBW}{C_T \alpha^2 \pi^2} \left(\frac{1}{F_S} \right)^2$$

  - Therefore, max. R_F for M = 8 is around 1/2 of max. R_F that is possible with M = 16
  - This may impose additional gain stages for M = 8 compared to M = 16 to get the right signal levels in the ADC input
- Optoelectronics design limits - PIN photodiode:
  - Typically, as larger is the PD, smaller is the bandwidth to keep as low as possible the capacitance
  - Therefore, smaller symbol rate could take advantage of larger photodiodes in order to reduce optical coupling losses, which may improve link budget

Spectral efficiency selection

- Capacity penalties due to harmonic distortion of light emitter
  - According to [4], the capacity penalties due to non-linear distortion is < 1 dB for SNR_e < 30 dB
  - Both, M = 8 and M = 16, are going to incur in similar penalties, therefore no difference
- DAC and ADC complexity
  - Resolutions required for M = 8 and M = 16 are very similar (< 1 bit difference), however let assume M = 8 requires 1 bit less than 16
  - Power consumption for M = 8 is 50% of M = 16 for equal sampling frequency
  - Because the power consumption can be considered proportional to sampling frequency, the final advantage of M = 8 respect to M = 16 is calculated 34%
  - Advantage in ADC and DAC of M = 8 does not compensate the DSP power consumption disadvantage
  - On the other hand, it is not exactly true the area of DAC/ADC for M = 8 @ 400 MHz is going to be 50% of M = 16 @ 300 MHz, since they operates at quite different frequencies; the advantage would be less than 20 % in real implementations
- Based on previous criteria, it seems that M = 16 is the winner choice

Shannon limit and spectral efficiency review

Normalized SNR concept

- Shannon showed that for an AWGN channel with signal to noise ratio SNR and bandwidth B (Hz), the data rate R (bits/s) of a reliable (arbitrary low error rate) transmission is upper bounded according to:

$$R < B\log_2\left(1+SNR\right)$$

- Equivalently, Shannon's result shows that spectral efficiency (bits/s/Hz) is upper bounded by:

$$\eta < \log_2\left(1+SNR\right)$$

- Reformulating the equation, the SNR needed for reliable transmission with spectral efficiency η is lower bounded by:

$$SNR > 2^{\eta} - 1$$

- These bounds suggest that we can define a normalized SNR as:

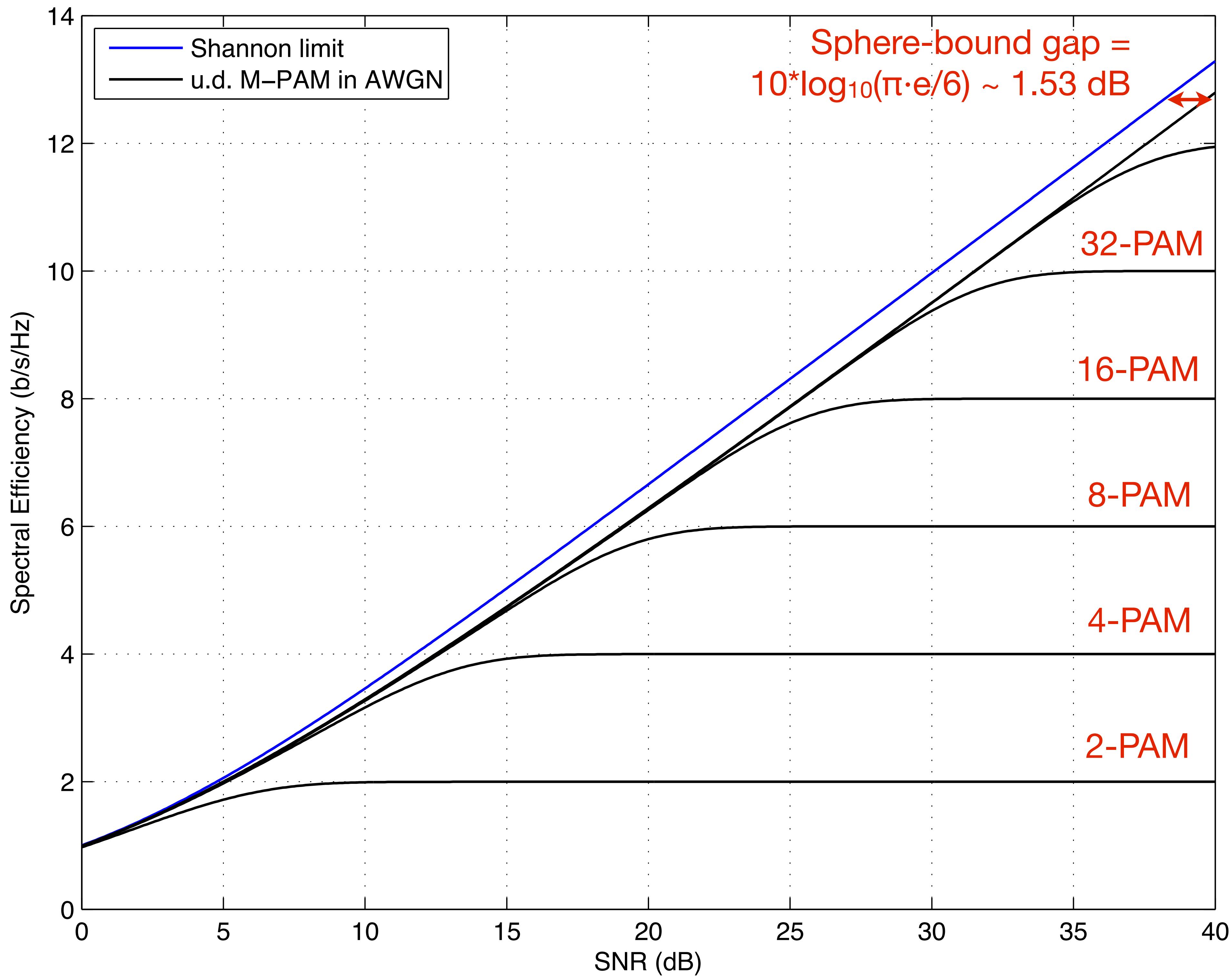
$$SNR_n \triangleq \frac{SNR}{2^{\eta}-1}$$

- Then, for any reliable coding scheme, the ultimate Shannon limit is $SNR_n = 1$ (0dB); therefore, SNR_n measures the gap to capacity

Spectral efficiency review

- As example, let consider PAM16 scheme prepended by a binary FEC with a code-rate $c_r = 0.83$; the following definitions are equivalent:
  - Spectral efficiency = $2 \cdot \log_2(16) \cdot 0.83$ = 6.64 bits/s/Hz
  - Spectral efficiency = 3.32 bits/s/Hz/dim
  - Spectral efficiency = 3.32 bits/Symb, since the number of dimensions for each PAM symbol is 1
- Let consider symbol rate F_S = 312.5 MSymb/s, then:
  - Bandwidth B = 156.25 MHz
  - Data-rate DR = $B \cdot \eta$ = 156.25 MHz x 6.64 bits/s/Hz = 1037.5 Mbps
  - Data-rate DR = $F_S \cdot \eta$ = 312.5 MSymb/s x 3.32 bits/Symb = 1037.5 Mbps
- Ultimate Shannon limit = $10 \cdot \log_{10}(2^{6.64} - 1)$ = 19.94 dB

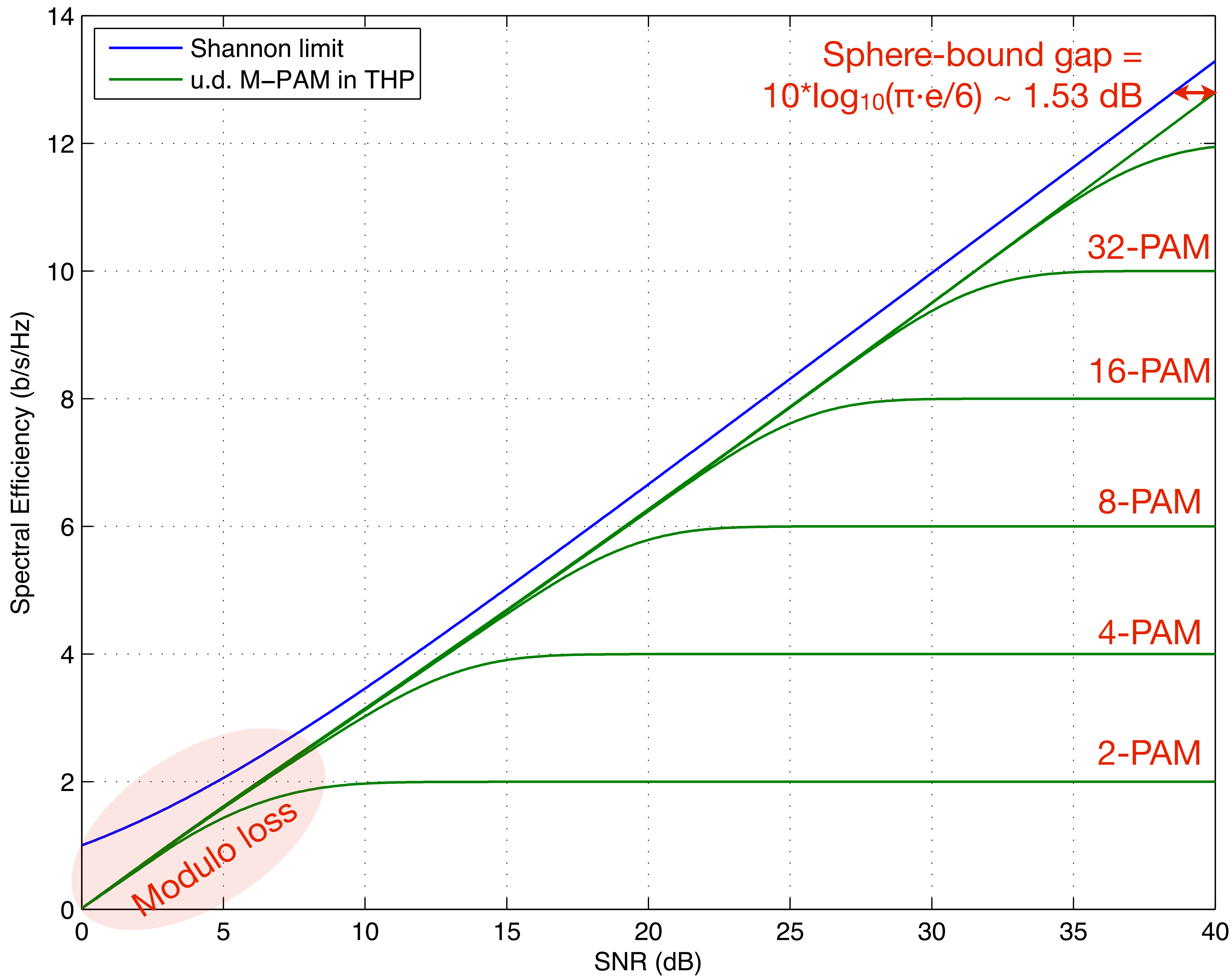
Achievable capacity of u.d. M-PAM in AWGN



Achievable capacity of u.d. M-PAM in AWGN

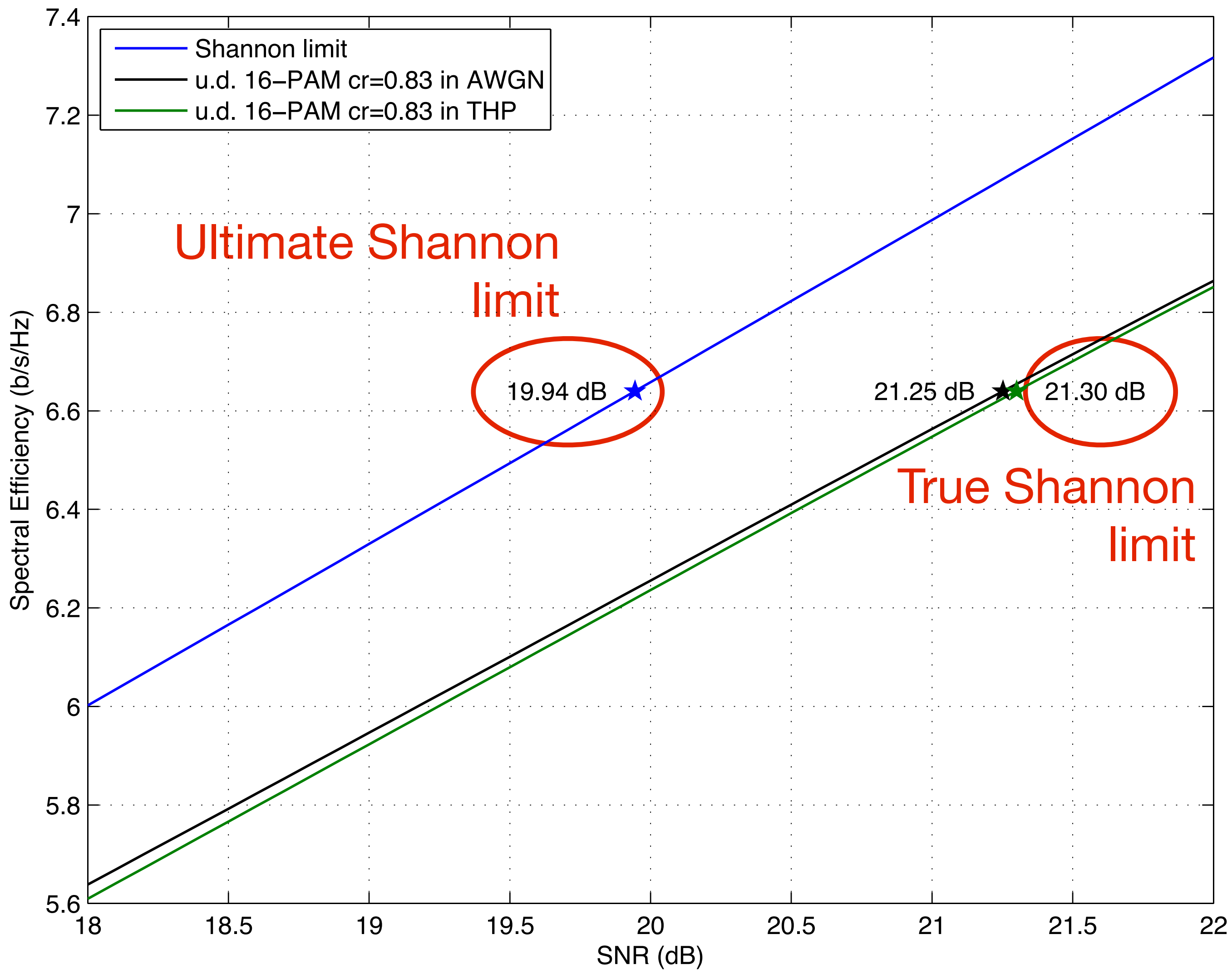
- We must be aware that high spectrally efficient uniformly distributed M-PAM schemes, by their nature, operates with a gap to Shannon
- The experienced capacity loss in AWGN is due to the statistical distribution of PAM levels is not gaussian
- However, under non-negative optical power constraint, the optical channel input is limited in power peak, instead of average power
- Therefore, any constellation shaping intended to make more gaussian the input signal to channel, is going to produce crest-factor increase, reducing the variance of input signal to channel, then reducing the capacity

Achievable capacity of u.d. M-PAM in THP



Achievable capacity of u.d. M-PAM in THP

- THP produces additional capacity penalty due to modulo loss and detection over an infinite lattice, which is specially relevant for low spectral efficiencies
- This was already evaluated in [1] as THP coding loss vs. ideal DFE
- Therefore, let us consider the previous example for illustration of the true capacity limit



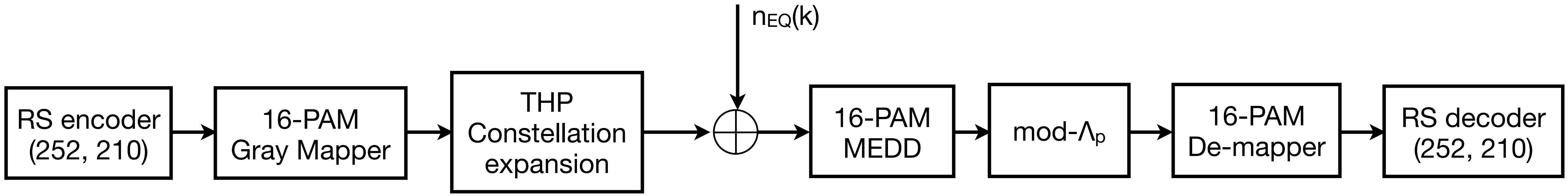
- The true capacity limit is ~1.4 dB higher in SNR than ultimate Shannon limit
- This represents a lower bound for any proposed FEC scheme
- This has been calculated considering u.d. M-PAM scheme and THP channel

Coded modulation schemes

FEC schemes: general considerations

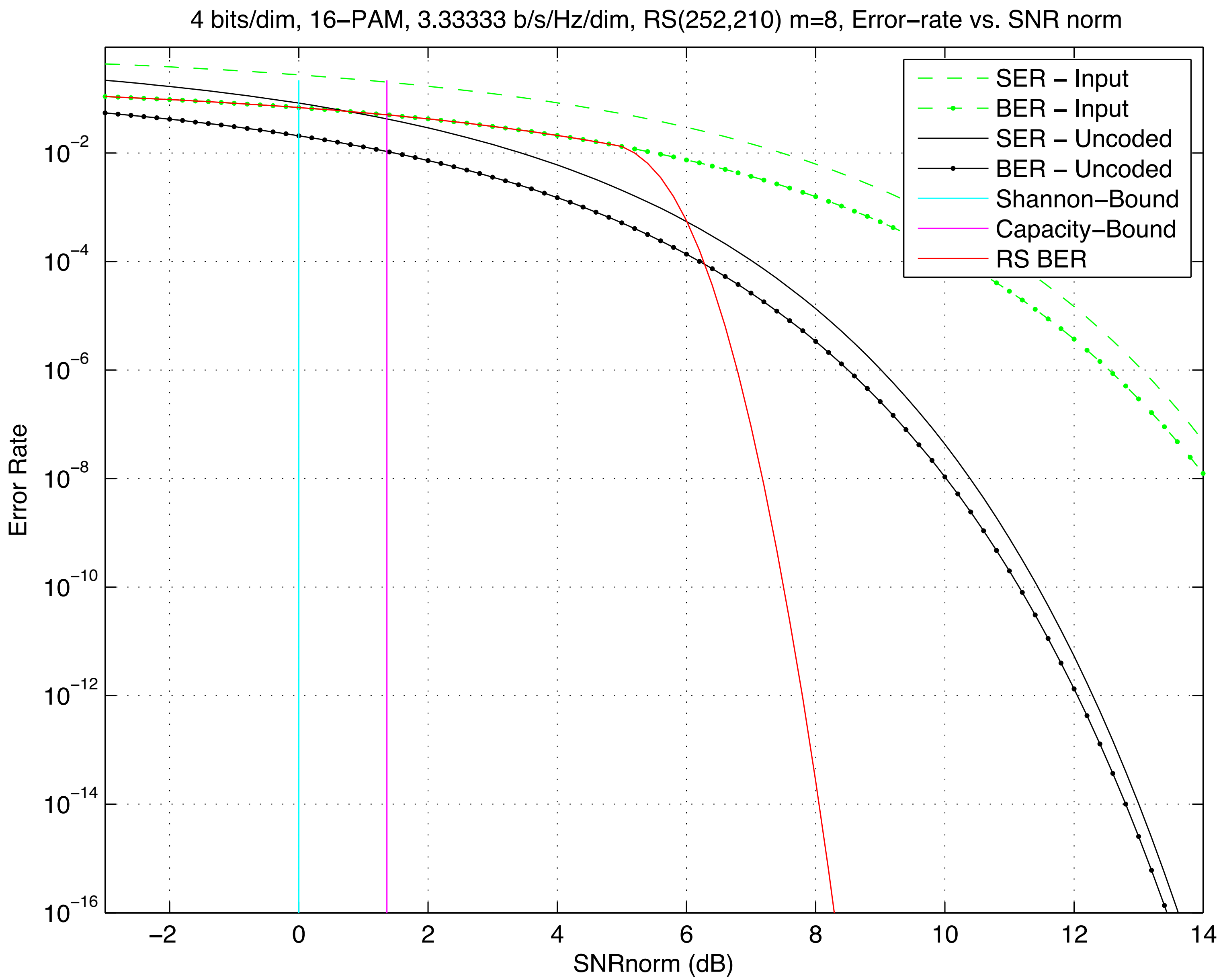
- Neither impulse noise nor RFI noise are considered, since it is an optical communication system
- Only algebraic codes like Reed-Solomon and BCH are considered
- Equal code-word lengths and similar code-rates are considered for fair comparison of several FEC schemes
- Only hard-decoding is considered, because:
  - Soft-decoding algorithms applied to algebraic codes are prone to error floor
  - Soft-decoding algorithms applied to algebraic codes are prohibitive in terms of implementation complexity,
  - Soft-decoding only provides about 0.6 dB additional coding gain vs. hard-decoding
- Target BER < 10^{-12}, therefore any error floor should be below it
- LDPC codes are left outside the scope, because their high design and encoding/decoding complexities
  - Silicon area of LDPC > 10x silicon area of BCH with similar code-rate, codeword length, bit-rate and system clock frequency (2k codes considered)
  - LDPC design is a hard task, requiring dedicated HW for very long simulations to get a design free of error floor and trapping-sets
  - Error floor and coding gain is always dependent on the soft-decoding implementation (BPA approximation algorithms, numerical precision of message passing, scheduling, # of iterations, etc)

FEC #1: scheme



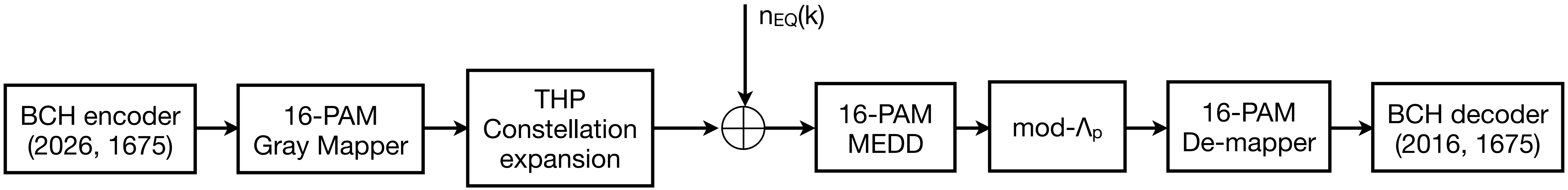
- Minimum Euclidian Distance Detector (MEDD) is considered, which is equivalent to optimal MAP and ML criteria, since PAM levels are equiprobable and it is a memoryless AWGN channel
- MEDD operates over the infinite lattice to wich constellation belongs, therefore no symbol flipping happens
- Congruent modulo reduction for THP lattice is performed after detector
- Gray mapper ➤ single symbol error produces a single bit error
- Reed-Solomon (252, 210) over GF(2^m) with m = 8 and t = 21
  - Code-word length = 2016 bits
  - cr = 0.8333
  - Spectral efficiency = 3.33333 bits/s/Hz/dim
  - Encoder complexity is negligible ➤ simple shift register
  - Decoder complexity is proportional to **2·m·t**, assuming Berlekamp-Massey Algorithm (BMA)
  - Complexity figure of merit = 2·m·t·BR/cr = 2·8·21·1000/0.8333 = **403200**

FEC #1: performance




```
Channel: THP
RS(252, 210) m = 8
Spect. Eff.: 3.33333 b/s/Hz/dim
Shannon gap (BER = 1e-12):          7.79 dB
Capacity bound gap (BER = 1e-12):   6.43 dB
SNR (BER = 1e-12):                  27.82 dB
Coding gain (BER = 1e-12):          4.26 dB
Input SER (BER = 1e-12):            0.00761718
```

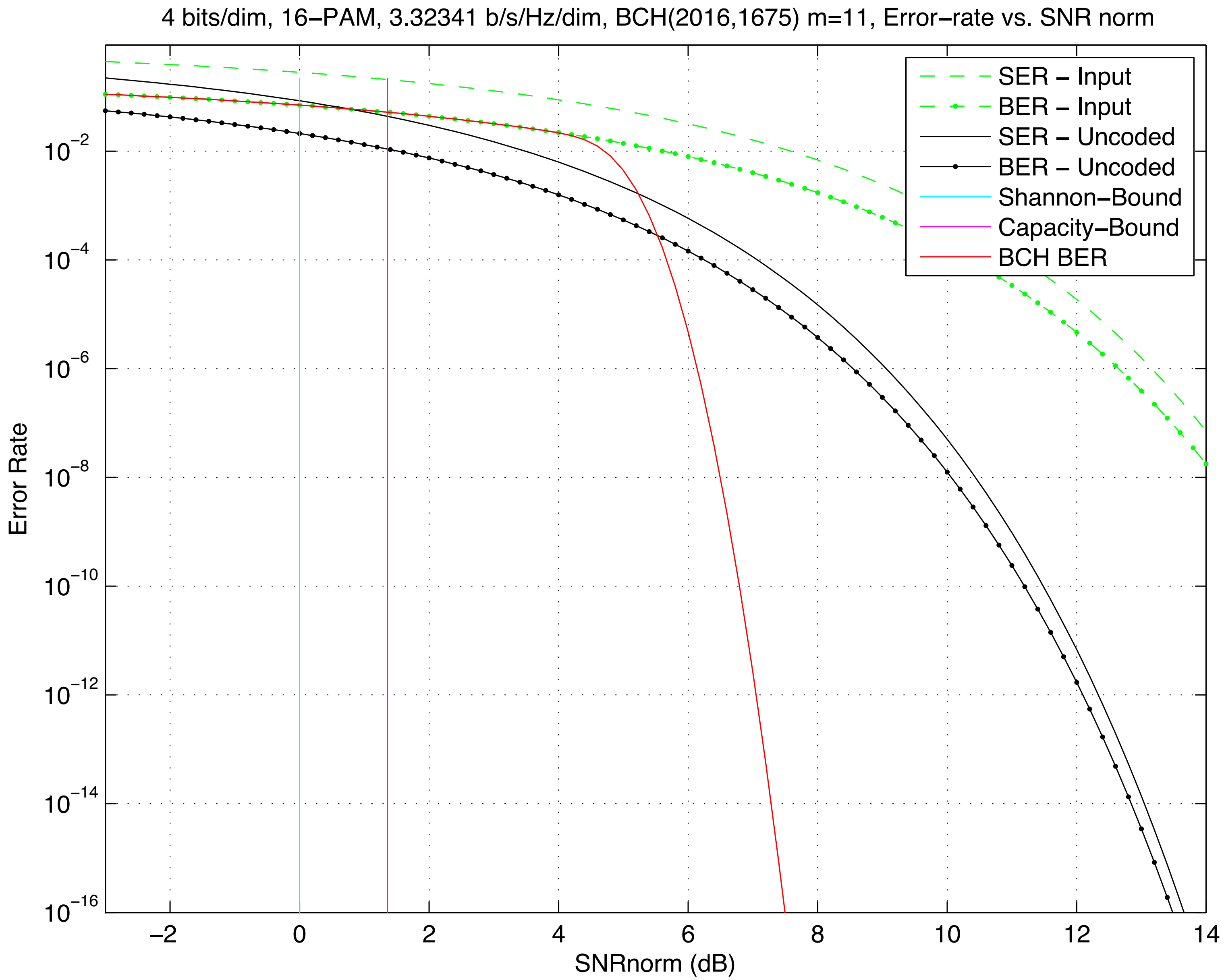
FEC #2: scheme



- BCH (2016, 1675) over GF(2^m) with m = 11 and t = 31
  - cr = 0.83085
  - Spectral efficiency = 3.32341 bits/s/Hz/dim
  - Encoder complexity is negligible ➤ simple shift register
  - Decoder complexity is proportional to **m·t**, assuming Berlekamp-Massey Algorithm (BMA)
  - Complexity figure of merit = m·t·BR/cr = **410421**

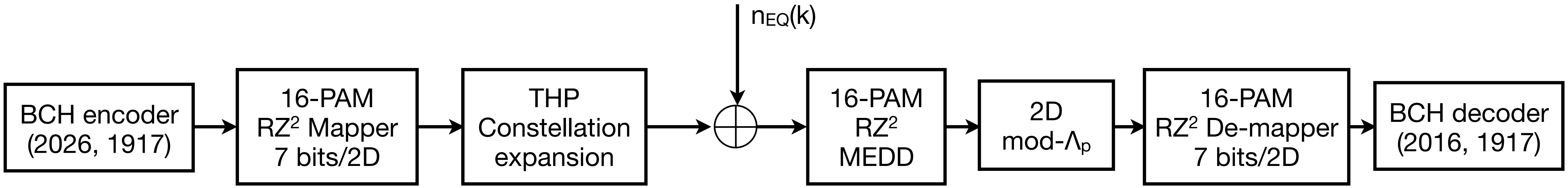
- BCH codes are more efficient than RS codes in AWGN and assuming gray mapping, since errors affect to single bits, and all the correction capability of RS able to correct groups of m bits is underutilized

FEC #2: performance



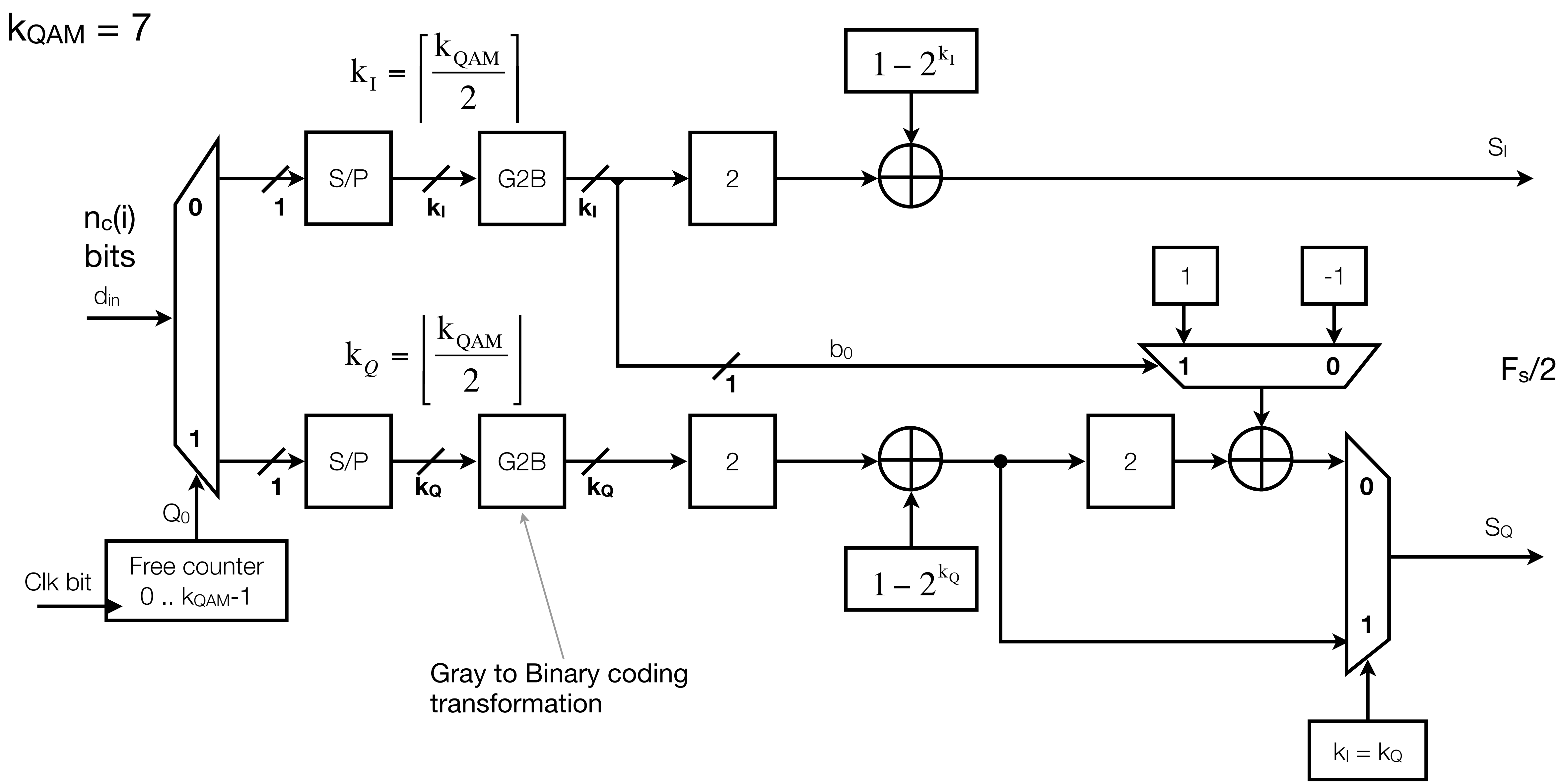

```
Channel: THP
BCH(2016, 1675, 31) m = 11
Spect. Eff.: 3.32341 b/s/Hz/dim
Shannon gap (BER = 1e-12):          7.05 dB
Capacity bound gap (BER = 1e-12):   5.69 dB
SNR (BER = 1e-12):                  27 dB
Coding gain (BER = 1e-12):          5.04 dB
Uncoded reference (BER = 1e-12):    12.09 dB
Input SER (BER = 1e-12):            0.0154889
Input BER (BER = 1e-12):            0.00387223
```

FEC #3: scheme



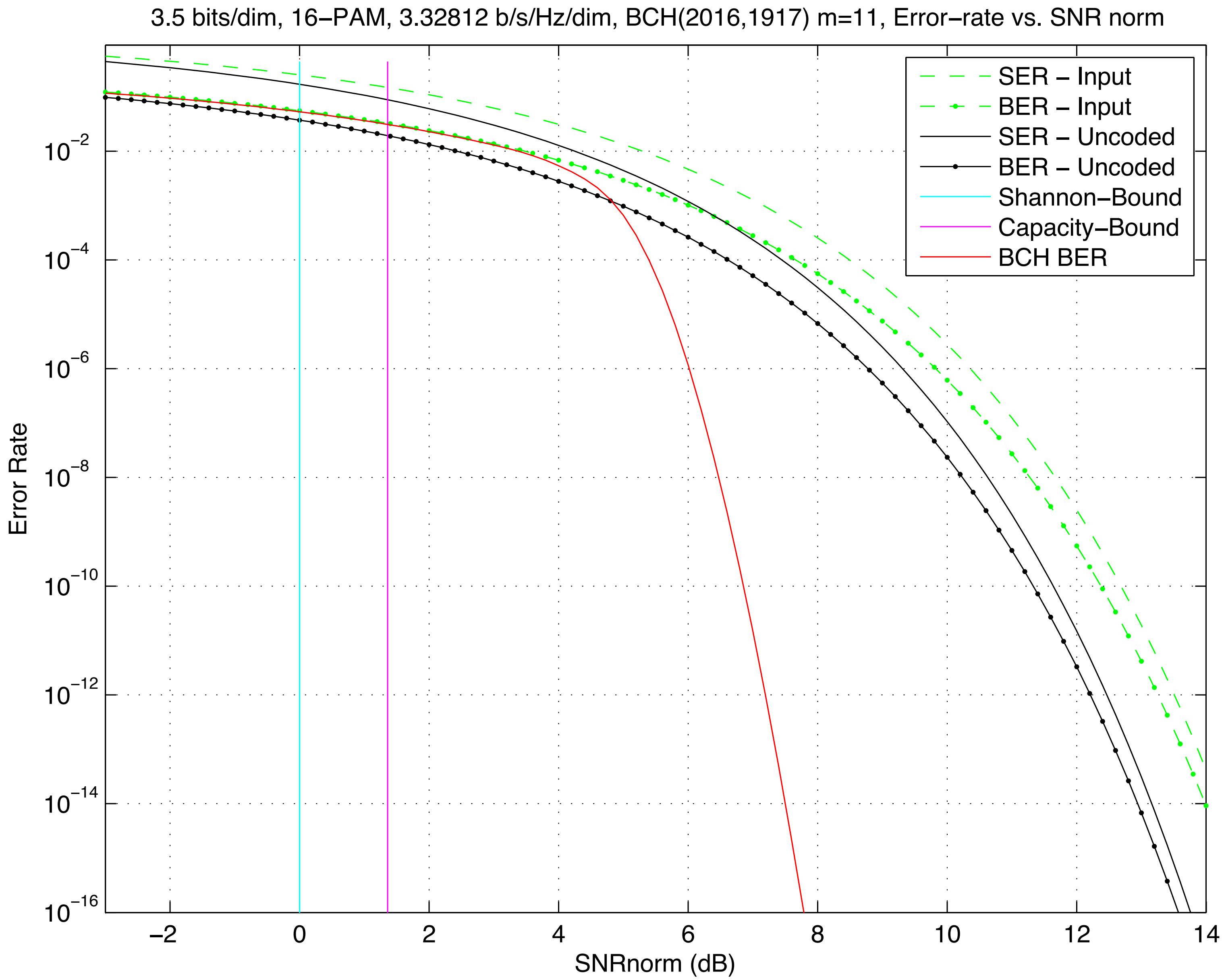
- BCH (2016, 1917) over GF(2^m) with m = 11 and t = 9
    - cr = 0.9509
    - Spectra efficiency = 3.328125 bits/s/Hz/dim
    - Complexity figure of merit = BR/cr·m·t = 99 = **104112 ➤ 75% complexity reduction**
- The complexity of binary code is reduced because 2 dimensions lattice is used for constellation, which provide 3.5 bits/dim
- 2D mapper, detector, modulo and de-mapper are trivial in terms of computational complexity
- Only an small coding gain penalty ~0.15 dB

FEC #3: scheme - RZ^2 mapper



S_I and S_Q are time interleaved to generate 16-PAM signal at F_S

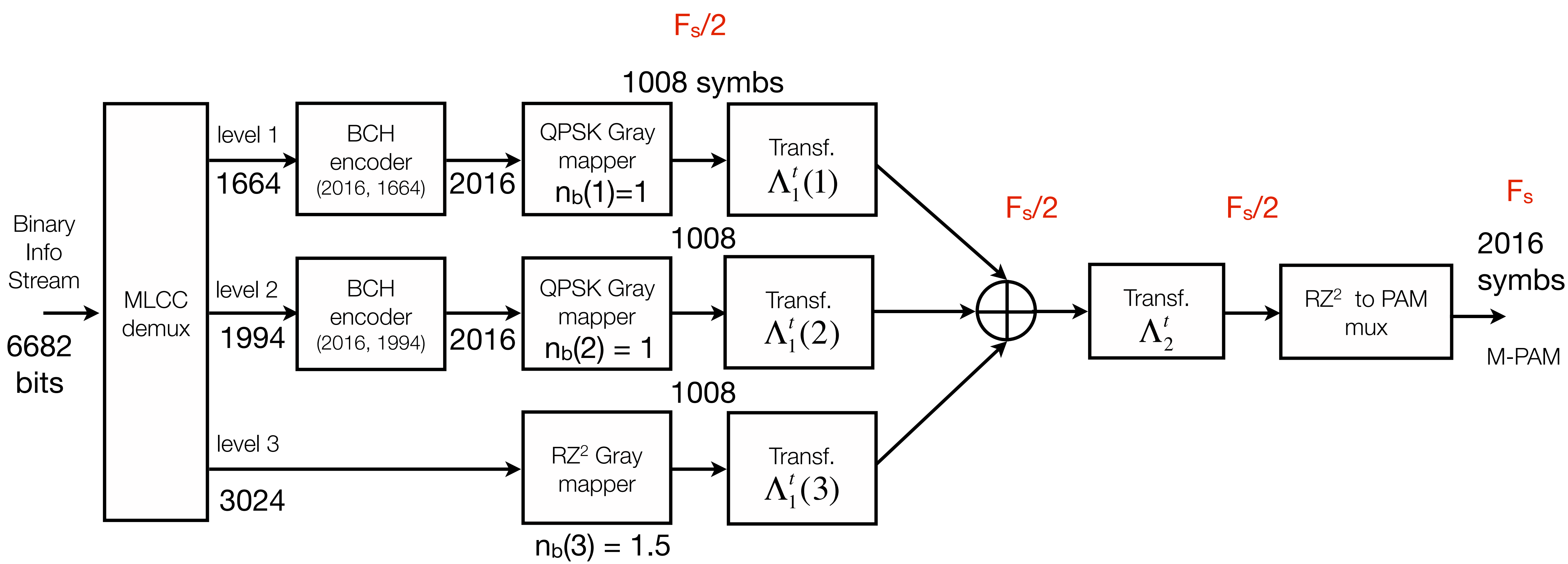
FEC #3: performance



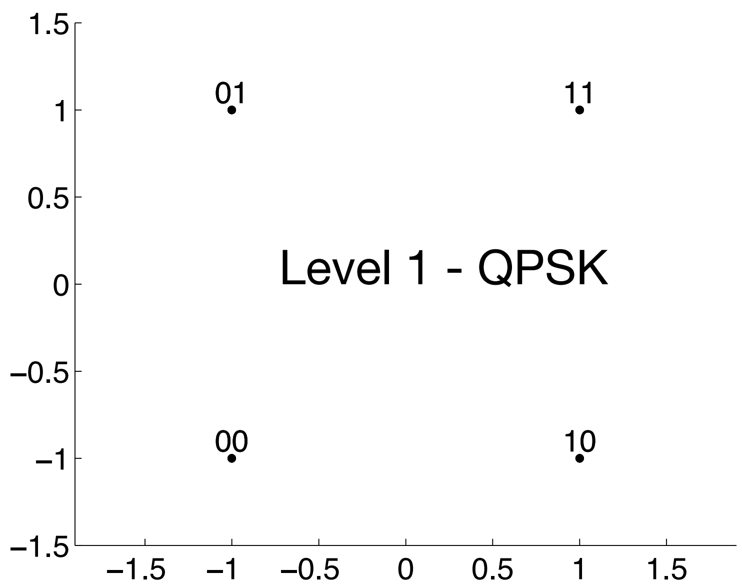

```
Channel: THP
BCH(2016, 1917, 9) m = 11
Spect. Eff.: 3.32812 b/s/Hz/dim
Shannon gap (BER = 1e-12):          7.19 dB
Capacity bound gap (BER = 1e-12):   5.84 dB
SNR (BER = 1e-12):                  27.2 dB
Coding gain (BER = 1e-12):          5.02 dB
Uncoded reference (BER = 1e-12):    12.21 dB
Input SER (BER = 1e-12):            0.000958185
Input BER (BER = 1e-12):            0.000209888
```

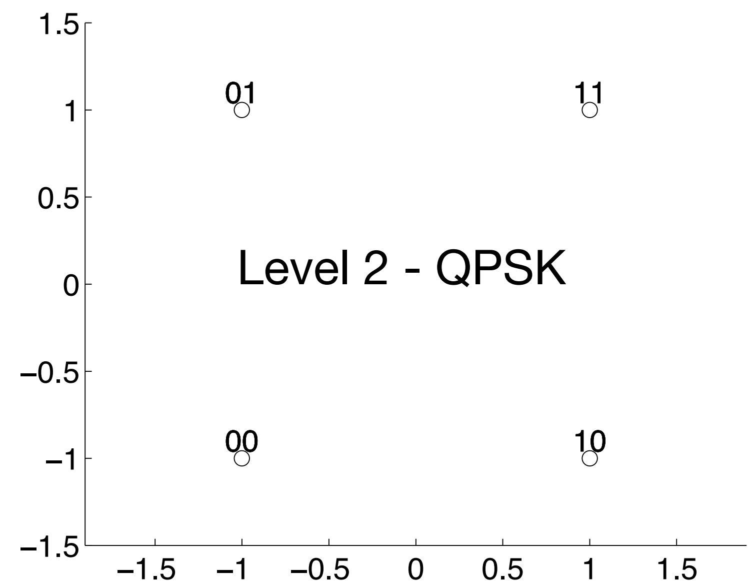
FEC #4: scheme

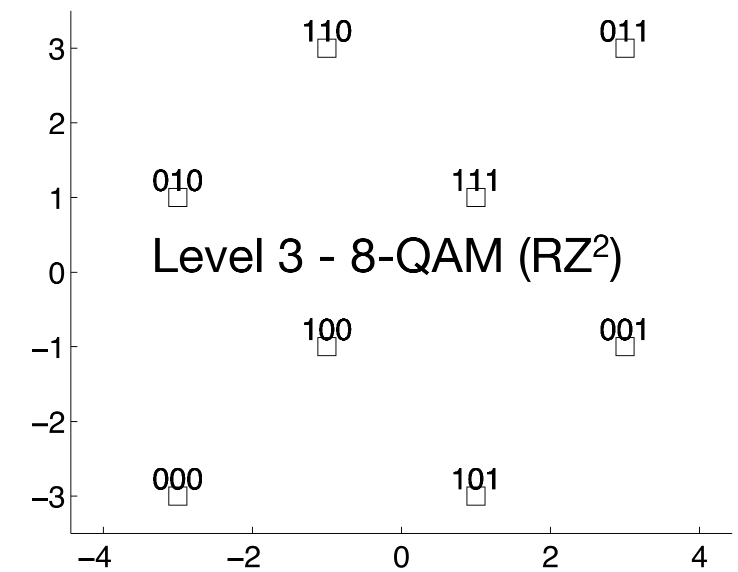
- Multilevel Coset Coding (MLCC) of 3 levels based on Z^2 and RZ^2 lattices to adjust accurately the spectral efficiency with low complexity binary component codes
- The constellation is partitioned in such a way the bits more likely to be corrupted by noise are protected by stronger component code (more parity), and those bits less corrupted are indeed no protected
- Based on [5]
- Theory behind this code is similar to that used for 10GBASE-T FEC.

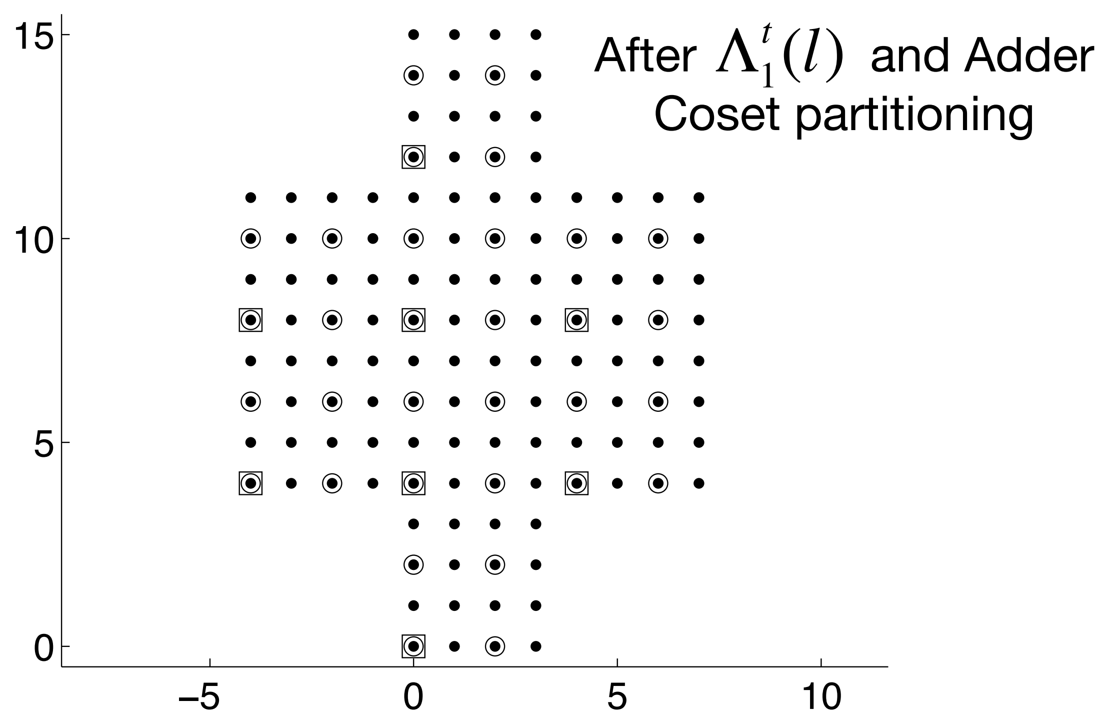


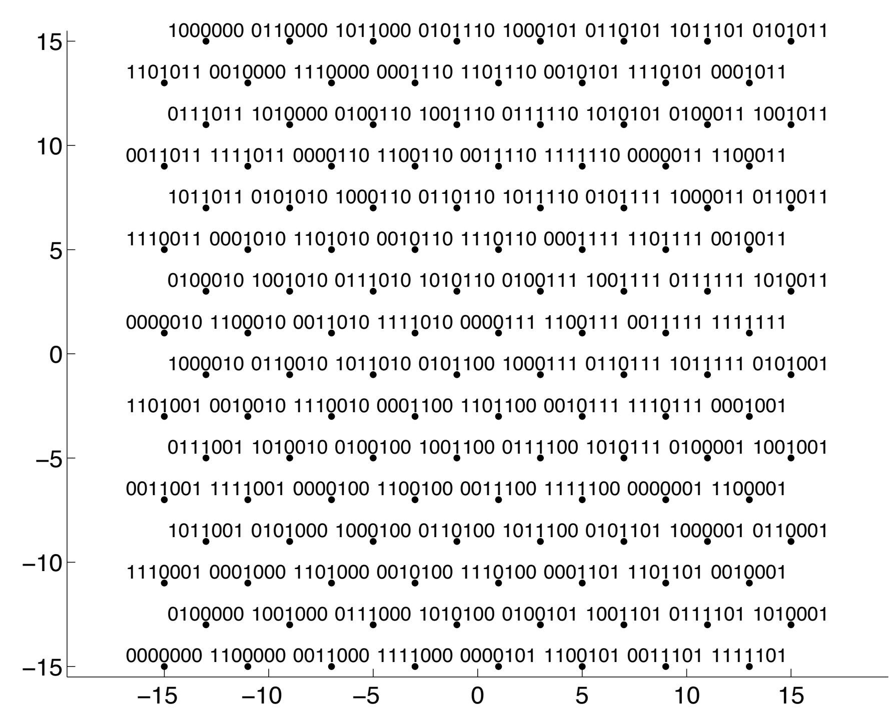
FEC #4: scheme - coset partitioning





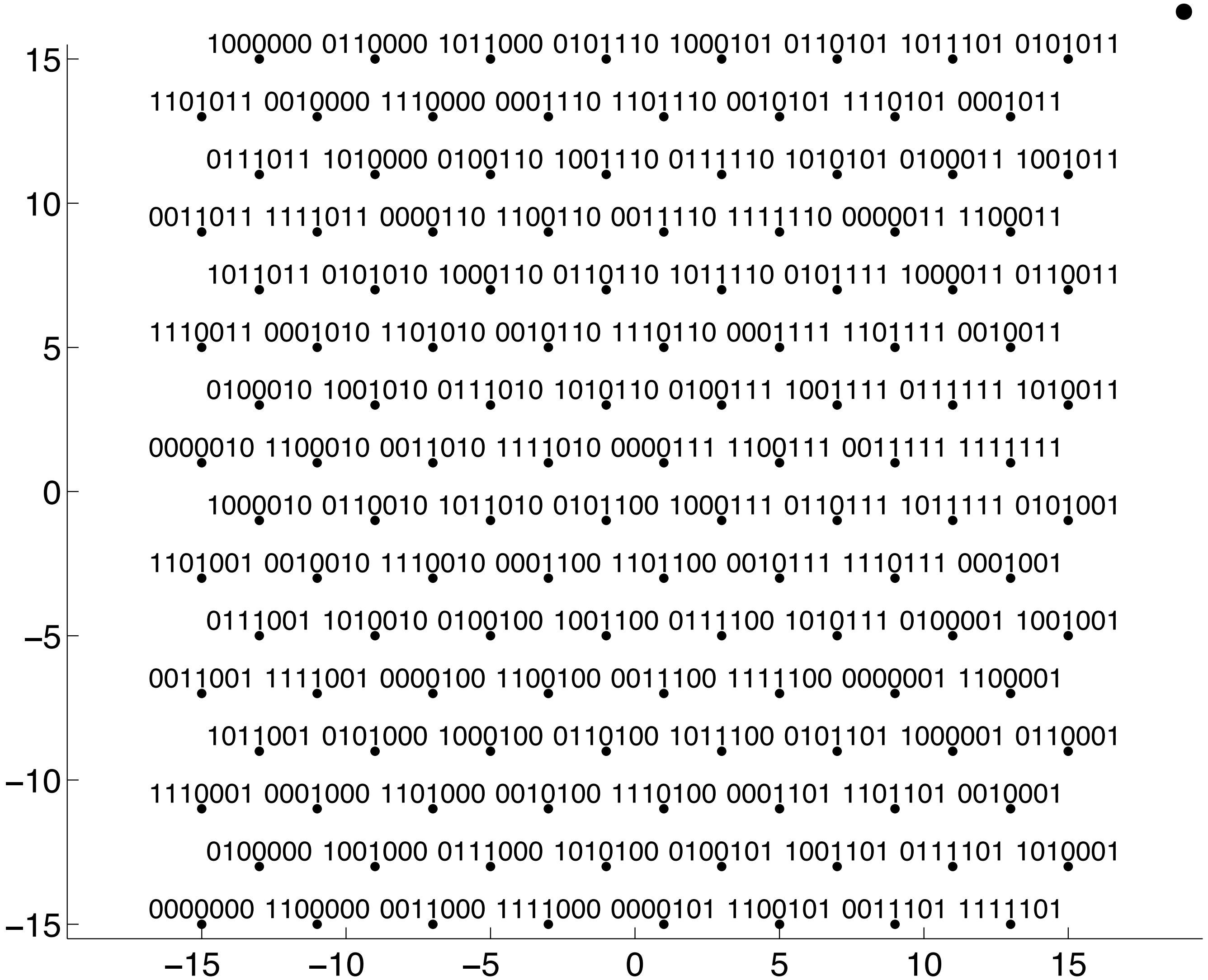






After Λ_2^t
Final labeling

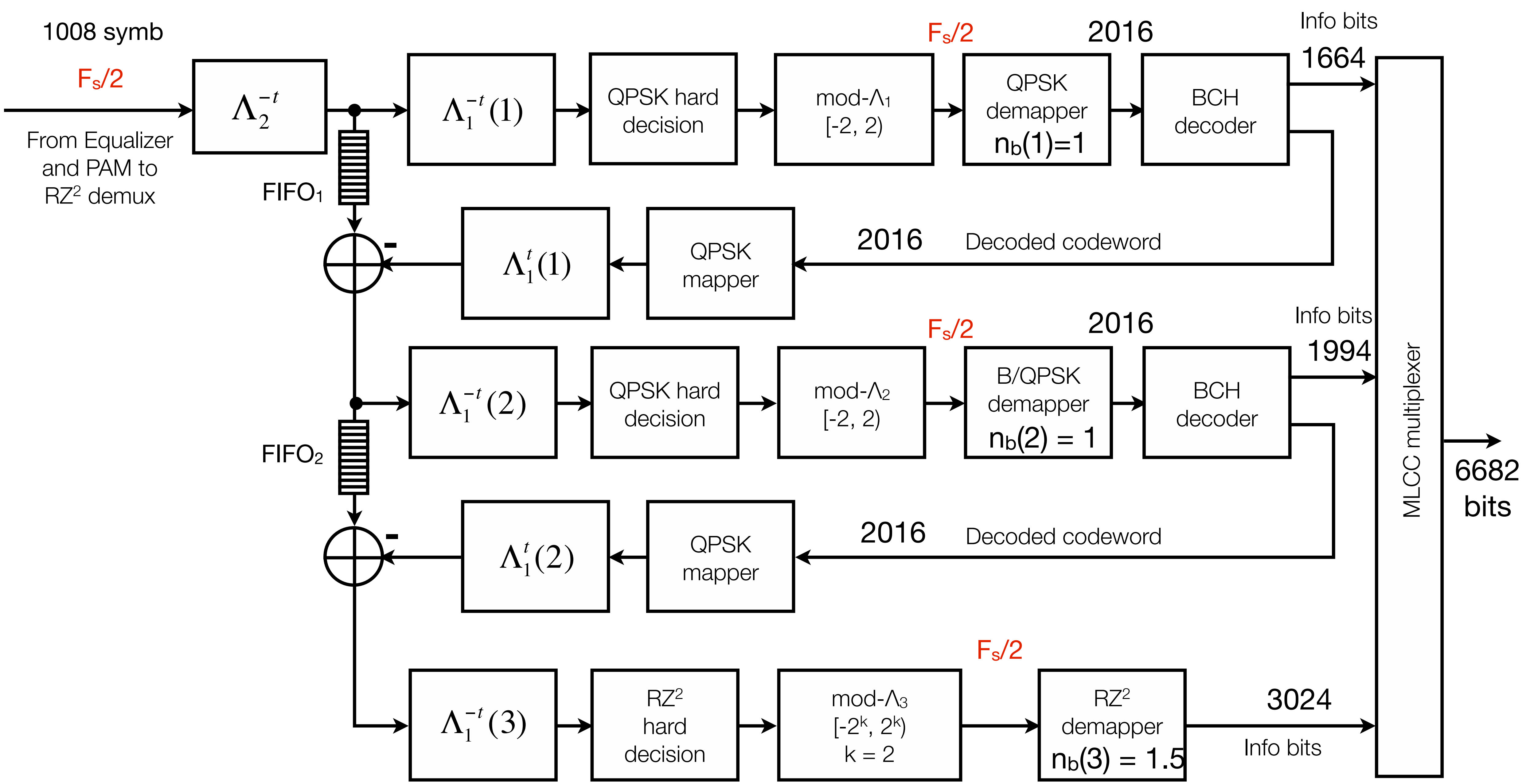
FEC #4: scheme - coset partitioning



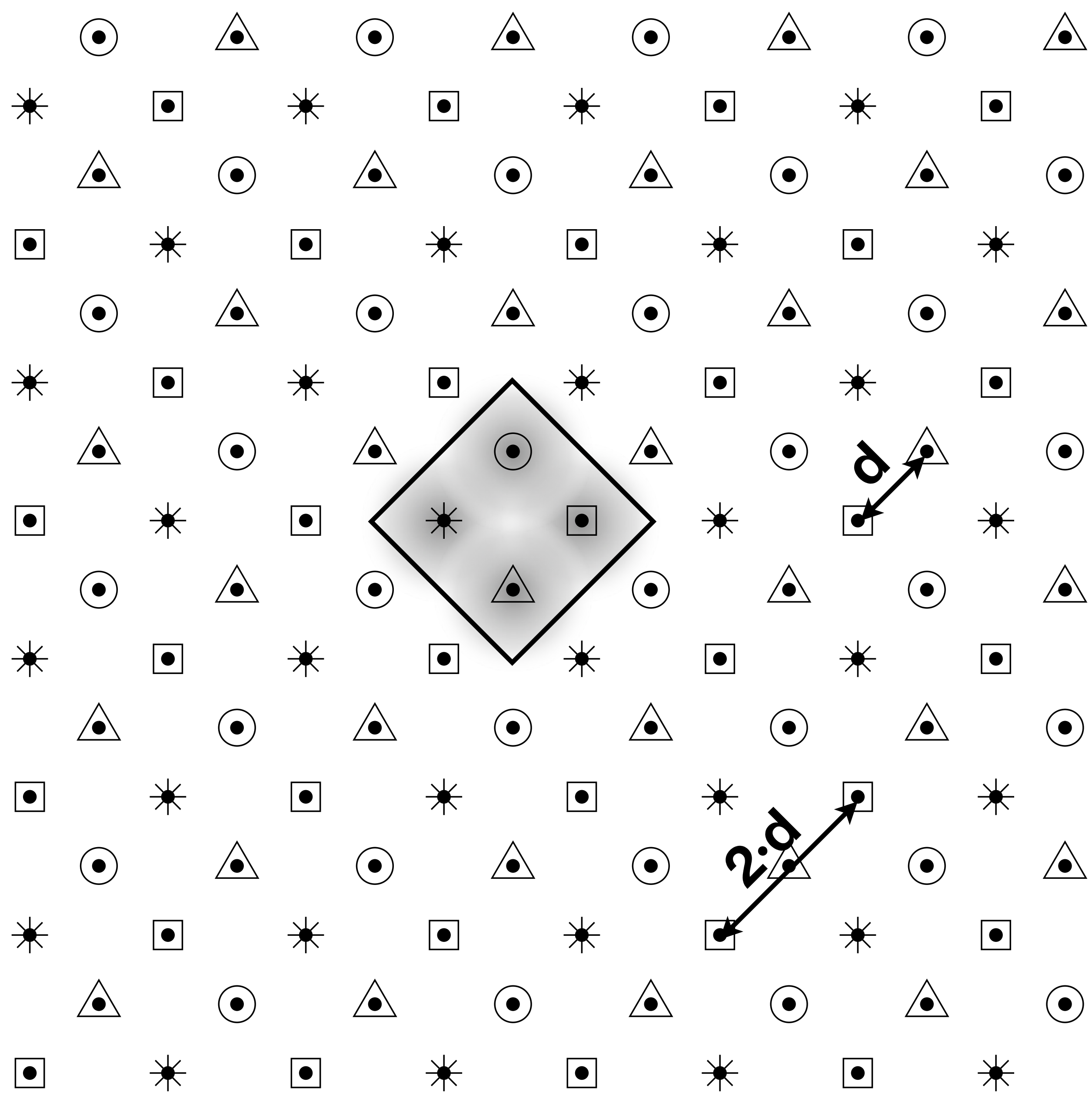
(1664+1994)/2016 + 1.5 =
3.3145 bits/s/Hz/dim

- Basic numbers of constellation:
  - 128 points in a 2D constellation
  - $\log_2(128)$ = 7 bits / 2D symbol
  - 7 bits =
    - 2 bits of 1st MLCC level
    - 2 bits of 2nd MLCC level
    - 3 bits of 3rd MLCC level
  - Each 2D symbol transmitted at a rate of Fs/2
  - To transmit over 1D (i.e. intensity modulation of LED), the system does time interleaving of both coordinates of 2D constellation at double rate, that is Fs
  - Each 2D point can be represented by 2 coordinates that can take 16 different values each one: {-15, -13, … 13, 15} ➤ 16-PAM
  - This is 16-PAM, but encoding by 3.5 bits/1D symbol (i.e. 7bits/2D) instead of 4 bits as usual, since the 1D constellation was generated from odd bits 2D constellation.
  - 3.3145 bits of 3.5 are information bits, the rest is parity for error correction

FEC #4: scheme - Multi-Stage Decoding

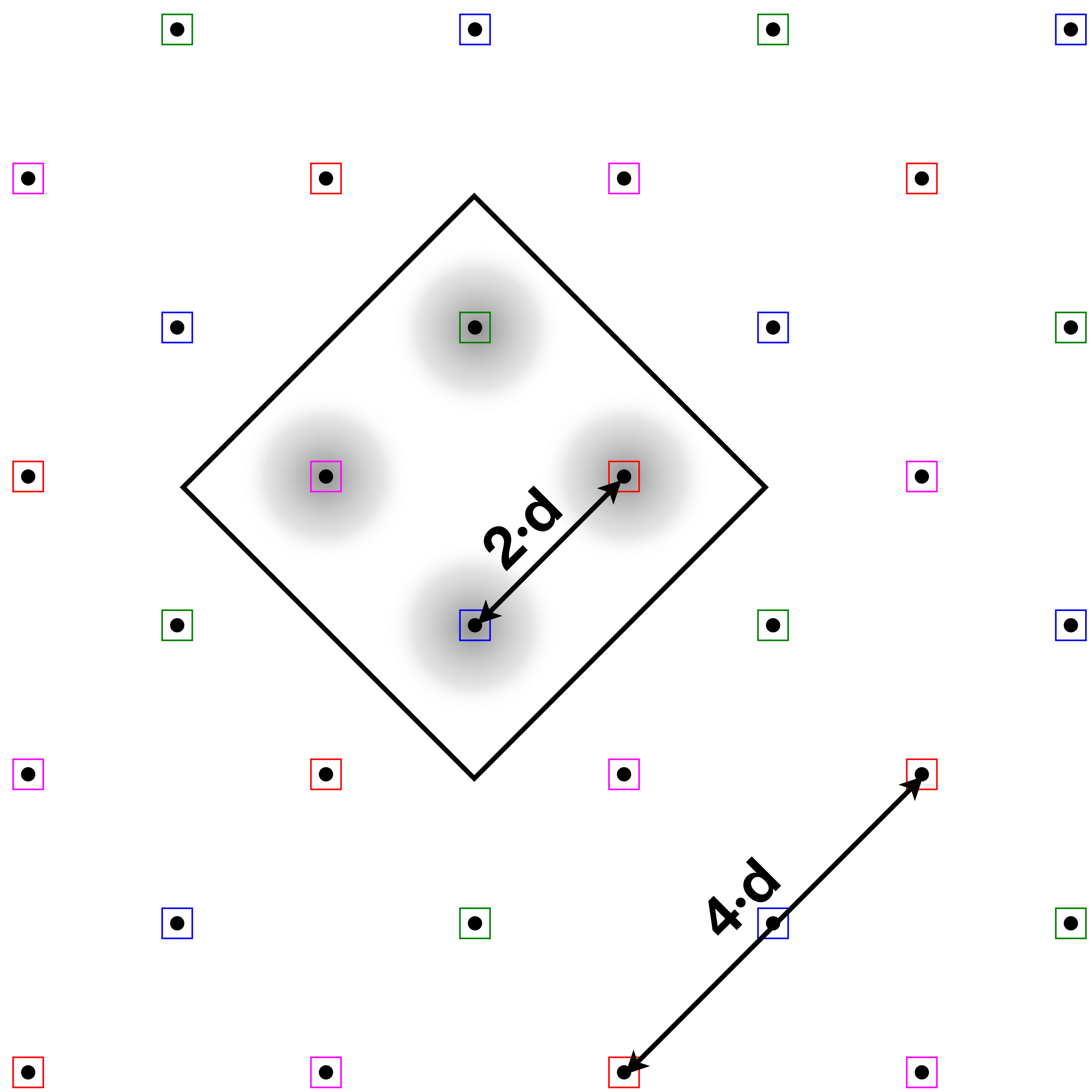


FEC #4: scheme - Intuitive point of view - 1st level



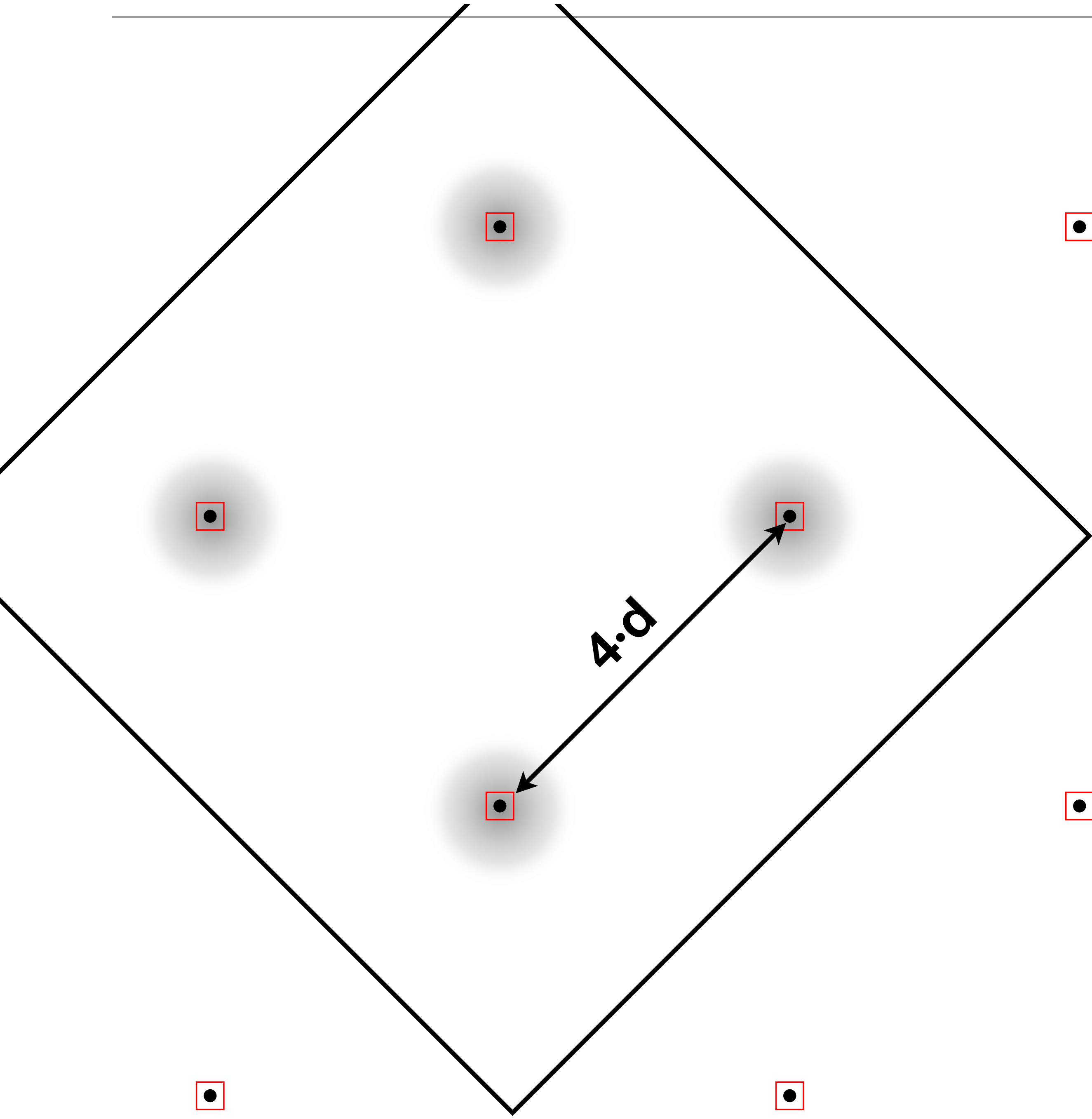
- The 2 bits mapped to 1st level encode 4 co-sets, that is, they divide the constellation into 4 sets of points:
  - □, △, ⁜, and ◯
- The MSD has to decide in the 1st stage for each received symbol to which co-set belongs
- Minimum distance of 1st level constellation is the minimum one, i.e. **d**, so we need the most powerful component code for this level
- Let's assume, for instance, the 1st level decided (after 1st BCH decoding) that the received symbol belongs to the coset of squares

FEC #4: scheme - Intuitive point of view - 2nd level



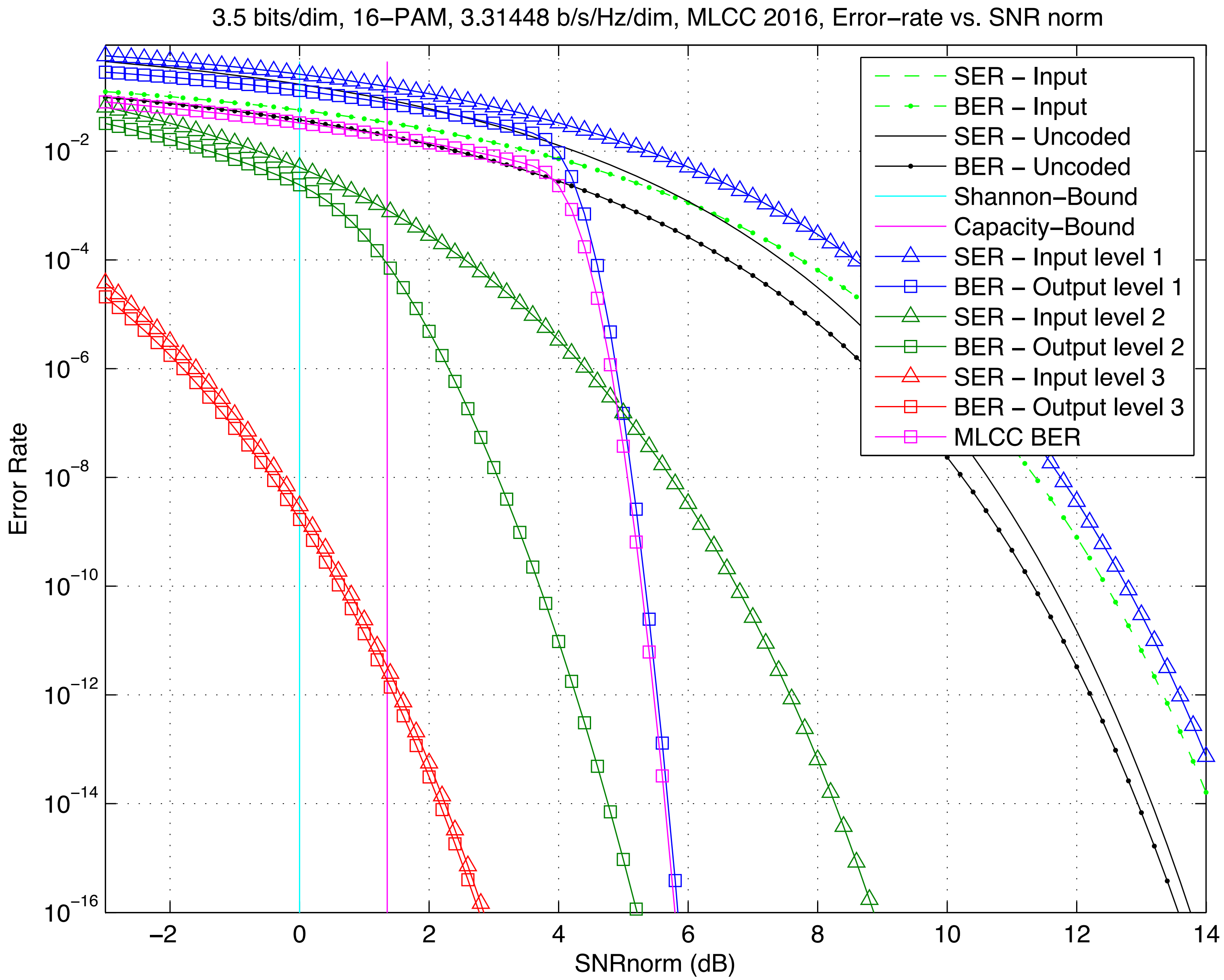
- The 2nd stage of MSD has to decide between magenta, green, blue and red squares.
- The 4 “colors” are encoded by the 2 bits assigned to 2nd level of MLCC encoder, which are mapped in another QPSK constellation.
- The variance of noise affecting to 2nd stage is the same, but the min distance of constellation is **2·d**, the probability of error is much smaller than in the 1st level, therefore the component code can be lighter (less protection).
- Let’s assume the 2nd level decide that the received symbol belongs to the coset of red squares

FEC #4: scheme - Intuitive point of view - 3rd level



- The 3rd stage of MSD has to decide one of the red squares.
- The red squares are directly encoded by the 3rd level of MLCC
- The SNR has been increased 12 dB compared to 1st level, therefore, the probability of symbol error is very low for the operating point, and not coding is required for good operation (BER < 10^{-12})
- And … that's all:
  - The information bits more affected by noise are more protected when the coded modulation is performed in transmission
  - Binary component codes, mapping and constellations are designed all together also considering the THP and MSD

FEC #4: performance




```
Channel: THP
Level 1: BCH(2016, 1664, 33) m = 11
Level 2: BCH(2016, 1994, 2) m = 11
Spect. Eff.: 3.31448 b/s/Hz/dim
Shannon gap (BER = 1e-12):          5.47 dB
Capacity bound gap (BER = 1e-12):   4.12 dB
SNR (BER = 1e-12):                  25.38 dB
Uncoded gap (BER = 1e-12):          12.2 dB
Coding gain (BER = 1e-12):          6.74 dB
Input SER (BER = 1e-12):            0.00914449
Input BER (BER = 1e-12):            0.00200308
Input BER MLC level 1 (BER = 1e-12):  0.00457225
Input BER MLC level 2 (BER = 1e-12):  1.41571e-08
Input BER MLC level 3 (BER = 1e-12):  8.92866e-30
```

FEC #4: performance

- Advantages:
  - Improved coding gain: by using simple algebraic codes we get improvement of ~1.7 dB with small complexity impact (lattice transformations, 2D mappers, etc are very simple to implement)
  - More efficient in terms of computational complexity: the BCH of 1st level run at ~30% of the full information bit rate, i.e. decoding operates at ~300 Mbps
  - m(1)·t(1)·BR/3.5/cr(1) + m(2)·t(2)·BR/3.5/cr(2) = **132008 ➤ 68% complexity reduction**
  - Silicon area of FEC#4 (encoder+decoder) < 17% of the silicon area required for 1Gbps LDPC (2048, 1723) based on shortened RS codes with 2 information symbols

- Disadvantages:
  - Latency: the multistage decoding incurs in additional latency, since the decoded information of 1^{st} level has to be available to begin 2^{nd} level decoding, and the decoded information of 2^{nd} level to begin 3^{rd} level detection and de-mapping
    - As Shannon already advanced, the channel capacity can only be approached with an infinite length code word, therefore, higher coding gain implies higher latency

Conclusions

- In May 2014 Interim was demonstrated the necessity of high spectrally efficient coding schemes and THP equalization to approach the POF channel capacity and therefore to meet the link budget requirements presented in [2]
- TH precoded communication channel model and capacity bounds have been explained and calculated
- Spectral efficiency has been selected and argued
- Several FEC schemes able to operate in THP channel providing the selected spectral efficiency have been proposed and compared
- FEC schemes have been compared in terms of coding gain and computational complexity, resulting the most powerful solution a FEC based on Multilevel Coset Coding, which provides the maximum coding-gain with minimum computational complexity

References

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Questions?