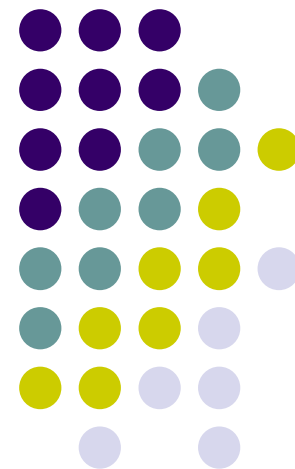


# PAM12 LDPC-CRC Framing

Dariusz Dabiri, Teranetics  
Brett McClellan, Solarflare





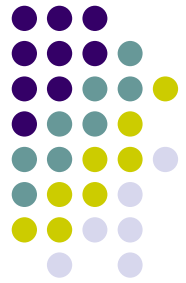
# Supporters

- Katsutoshi Seki, NEC Electronics.
- Chine-Hsin Lee, KeyEye Communications.



# Agenda

- A Complete Specification for the Framing:
  - CRC
  - LDPC
  - Bit ordering
- A Complete Specification of the Mapping Algorithm.
- Mean Time to False Packet Acceptance Analysis.

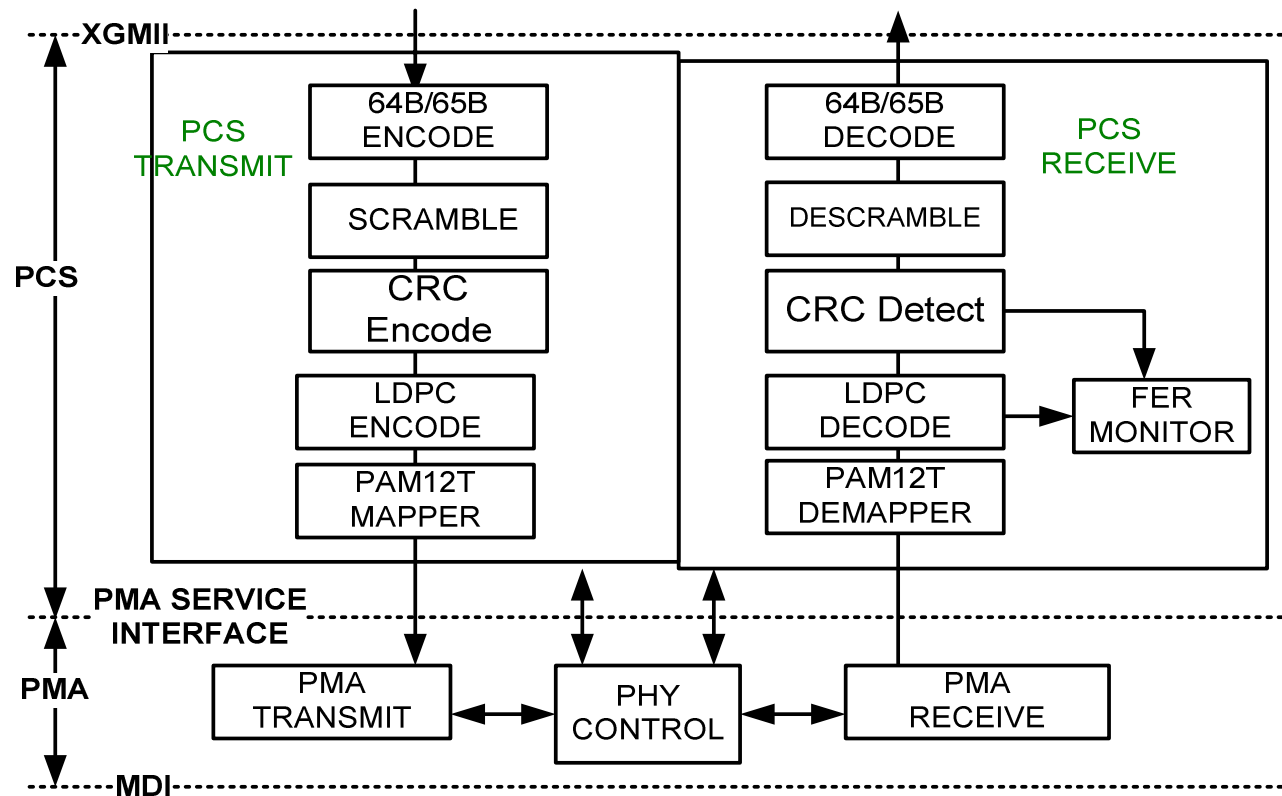


# Framing Highlights

- Data payload of  $25 \times 64 = 1600$  bits.
- 16 bits of the Frame CRC to reduce the undetectable frame error rate.
- LDPC payload:
  - 1625 bits = 25 (blocks) x 65 (bits/blocks)
  - 2 bits of the control bits, usable for synchronization or back channel.
  - 16 bits of the CRC's.
- Frame has single LDPC (1024, 833) block
- Symbol rate:  $10\text{G}/(1600/128) = 800\text{MHz}$
- Symbol rate of  $800\text{MHz} = 32 \times 25\text{MHz}$ 
  - Easy to generate with standard xtals/oscillators with simple  $N/M$  PLL multiplication
  - Results in low jitter PLL

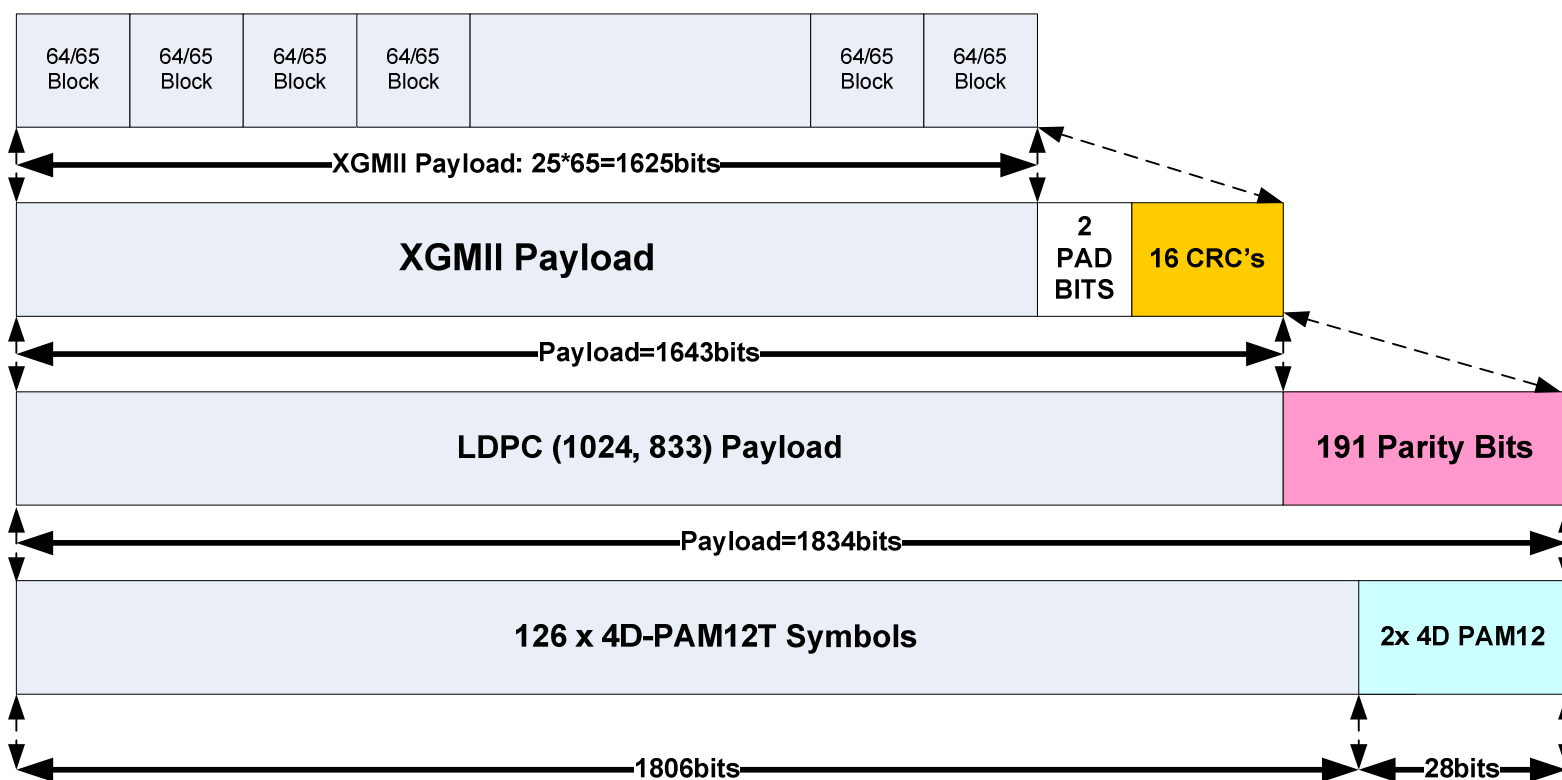


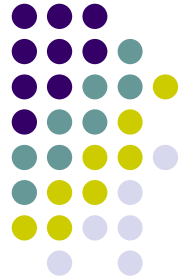
# PCS Functional Block



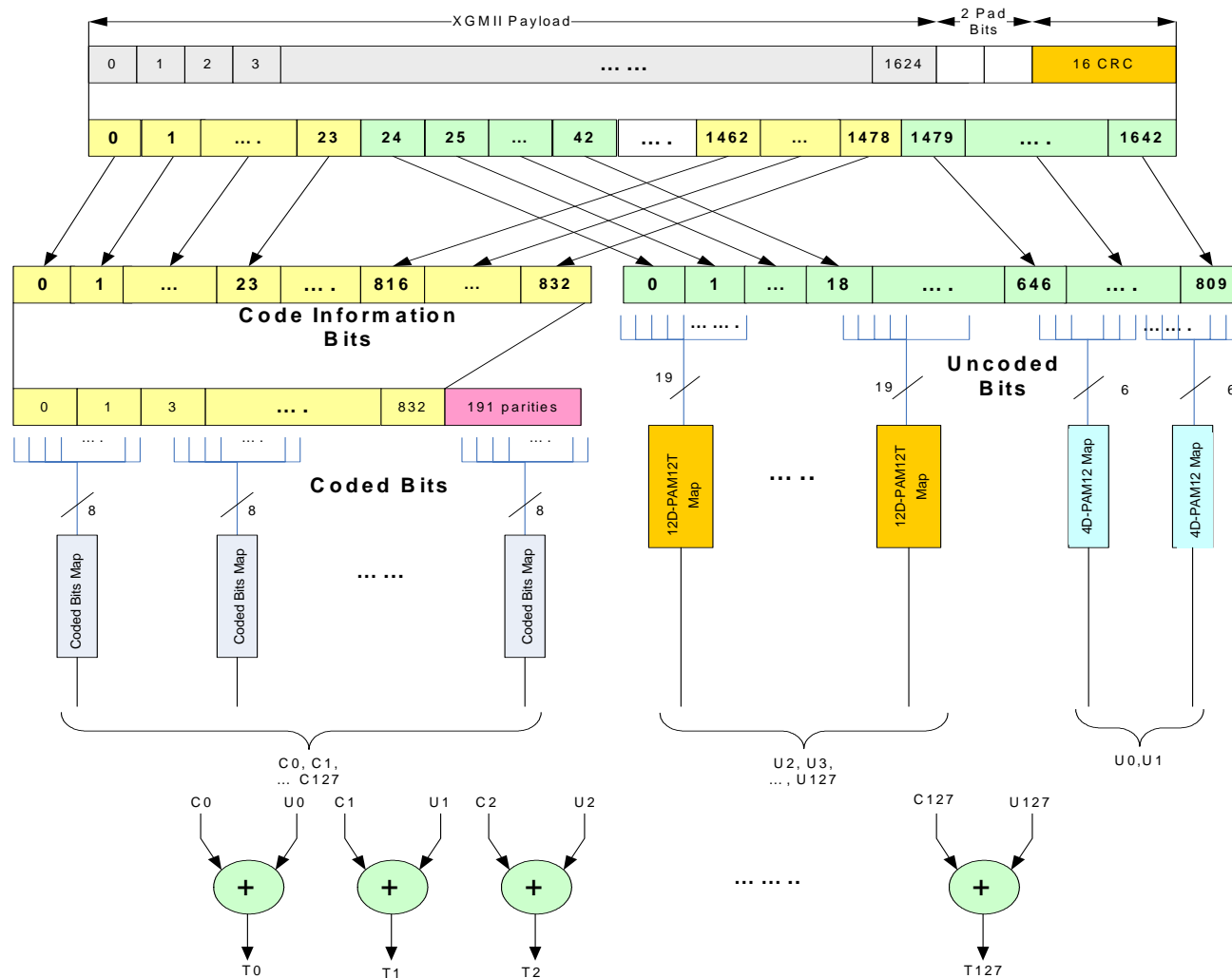


# Framing Overview





# A More Detailed View

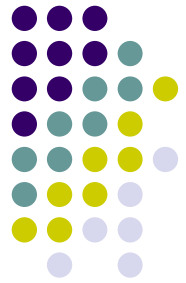




# XGMII to LDPC

- The XGMII payload consists of 1625 bits.
- The LDPC payload has 16 additional bits of CRC's and 2 pad bits.
- The LDPC payload is split between:
  - information bits for the LDPC(1024, 833).
  - uncoded payload bits.





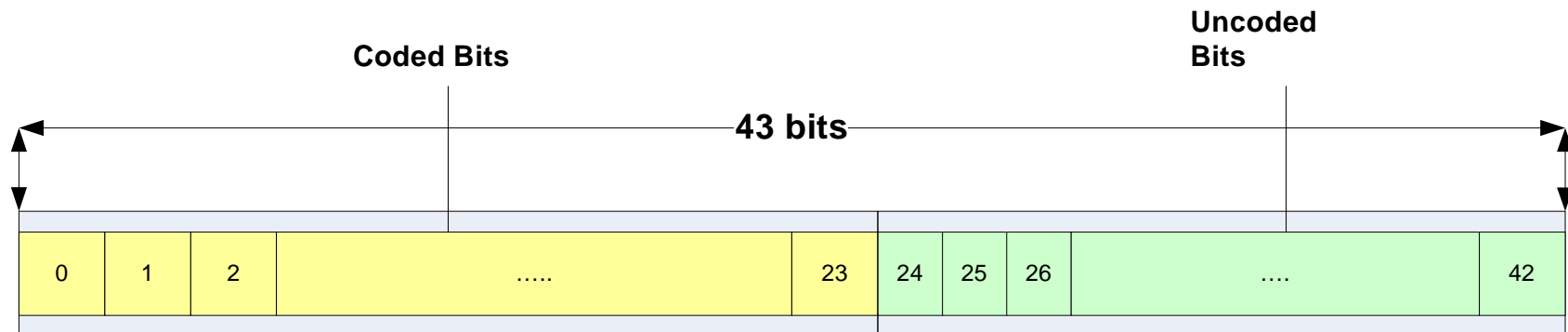
# CRC16

- $G(X) = X^{16} + X^{15} + X^{11} + X^8 + X^6 + X^5 + X^4 + X^3 + X + 1.$
- Introduced in:
  - D. Chun, J. Wolf, “Special Hardware for Computing the Probability of Undetected Error for Certain Binary CRC Codes and Test Results”, IEEE Trans. Commun. Vol. 42, pp2769-2772, Oct. 94.
  - The authors have given a *rigorous proof* of the upper bound  $2^{-16}$  on the probability of undetectable frame error for this code.
- CRC is calculated after scrambling and checked prior to descrambling to prevent error propagation.
- CRC error detected in frame N will cause erasure for all 25 65B blocks in the frame and the first 65B block in frame N+1.



# XGMII to LDPC Split

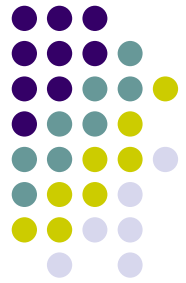
- The LDPC payload split:
  - The first 1462 bits of the LDPC payload is divided into 34 groups of 43 bits.
  - The next 17 bits (bit 1462 to bit 1478) are coded bits.
  - The rest of the bits (bit 1479 to bit 1642) are all uncoded bits.





# LDPC Encoder

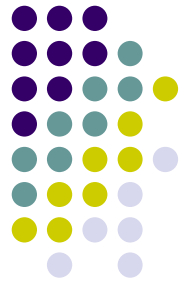
- A systematic encoder that generates 191 parity bits for 833 information bits.
- A systematic generator matrix for this code will be provided.



# Mapping of Coded Bits

- Generates levels  $X_n = \{-3, -1, 1, 3\}$ ,  $n = 0:511$  according the following Gray map table.
- Each 4D coset representative,  $T_m = (X_{4m}, X_{4m+1}, X_{4m+2}, X_{4m+3})$ ,  $m = 0, 1, 2, \dots 127$ .

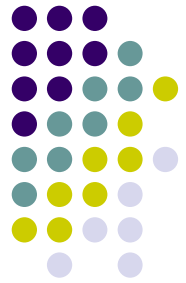
LDPC_data_group <2n:2n+1> n=0:511 (coded bits)	$X_n$ (512 cosets)
00	-3
01	-1
11	1
10	3



## 4D-PAM12 Map

- Converts 3-tuples of uncoded bits to points,  $X_n$  in the 2D constellation:  $\{-8, 0, 8\}^2$  according to the 2D-PAM 12 Table.
- $T_n = (X_{2n}, X_{2n+1})$  defines the 4D coset points.

$(-8, -8)$	000
$(-8, 0)$	001
$(-8, 8)$	011
$(0, 8)$	010
$(8, 8)$	110
$(8, 0)$	111
$(8, -8)$	101
$(0, -8)$	100



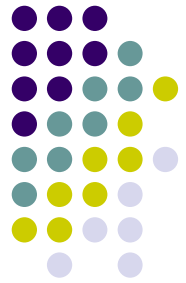
# 12D-PAM12T

- Follows mcclellan\_2\_0904.pdf.
- Each 12D-PAM12T symbol consists of 3 4D-symbols.
- Each 4D symbol is viewed as 2 2D-symbols.
- The 2D-constellation  $\{-8, 0, 8\}^2$  is divided into two sets, the point in the origin, denoted by R, and the rest of the points denoted by B.
- The 4D constellation is divided into 3 subsets:
  - $X = B \times B$ . (64 points)
  - $Y = R \times B + B \times R$ . (16 points)
  - $Z = R \times R$ . (one point).
- The unique prefix of the 19-bit input specifies which subsets should be used for each of the 3 4D-symbols.



# Prefix Map Table

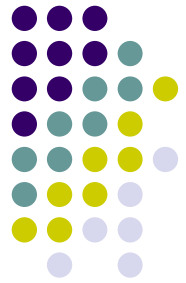
Prefix	Subset	No of Suffix Bits	Total Bits
0*	(X, X, X)	18	19
100*	(X, X, Y)	16	19
101*	(X, Y, X)	16	19
110*	(Y, X, X)	16	19
11100*	(Y, Y, X)	14	19
11101*	(Y, X, Y)	14	19
11110*	(X, Y, Y)	14	19
1111100*	(X, X, Z)	12	19
1111101*	(X, Z, X)	12	19
1111110*	(Z, X, X)	12	19
1111111*	(Y, Y, Y)	12	19



# 12D-PAM12T

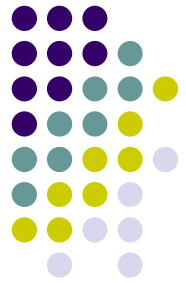
- The suffix bits define which points of the 4D subsets should be used.
- Any point 4D point  $U = (u_0, u_1, u_2, u_4)$  in
  - X, carries 6 bits according to the 4D-PAM12 mapping rules.
  - Y, carries 4 bits ( $b_0, b_1, b_2, b_3$ ):
    - If  $b_0 == 0$ :
      - $u_0 = u_1 = 0$ ;
      - Use ( $b_1, b_2, b_3$ ) to find ( $u_2, u_4$ ) according to the 2D-PAM12 Table.
    - If  $b_0 == 1$ :
      - $u_2 = u_4 = 0$ ;
      - Use ( $b_1, b_2, b_3$ ) to find ( $u_0, u_1$ ) according to the 2D-PAM12 Table.
  - Z, carries no bits.





# Mean Time to False Packet Acceptance

- Since the minimum distance of the (1024, 833) code is lower bounded by 12, the length of a single undetectable error event can never be less than 12 .
- Therefore, at  $\text{BER} = 1\text{e-}12$ , the undetectable  $\text{FER} < 8.33\text{e-}11$ .
- Assuming that all errors made by the SPA algorithm for the (1024, 833) code, are undetectable (*very pessimistic assumption*).
- The probability of undetectable erroneous frame is  $2^{(-32)} * 2^{(-16)} * \text{FER}$ .
- Therefore in average one out of  $3.4\text{e}24$  frames have undetectable errors.
- Therefore Mean Time to False Packet Acceptance is about  $1.7\text{e}10$  years: 1.7 x life of universe.



# Discussions

- Our estimates tend to be pessimistic because:
  - For a good code with a high minimum distance (like (1024, 833)) most errors tend to be pseudo-codewords, not codewords.
  - The  $d_{\min}$  of the (1024, 833) is likely to be larger than 12.
- The upper bound  $2^{-R}$  does *not* necessarily hold for any code:
  - A good counter example is the probability of undetectable error for the widely used CCITT CRC16 code which exceeds  $2^{-16}$  for some code lengths:
    - J. K. Wolf, R.D. Blakeney, “An Exact Evaluation of the Probability of Undetectable Errors for Certain Shortened Binary CRC codes”, MILCOM’ 88, pp. 15.2.1-15.2.6.
- For the CRC 16 presented in this paper, this bound *has been* proved rigorously.
- For the Ethernet CRC-32 this bound *has been* proved rigorously.



# Conclusions

- We have shown a simple framing based on (1024, 833) LDPC code.
- The (1024, 833) LDPC-CRC guarantees "life of the universe" MTTFPA for undetected errors.
- The CRC covers uncoded bits in cases of impulse events.
- The (1024, 833) LDPC-CRC satisfies the 800MHz PAM12 target set by the Task Force.