Root Power Sum of Energy Integrals

Charles Moore
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Agilent Technologies

Outline:

1. What is “Root Power Sum of Energy Integrals”?
2. Why use RPSEI?
3. What assumptions does RPSEI make? Are they valid or reasonable?
4. Conclusions

What is Root Power Sum of Energy Integrals?

RPSEI is a way of computing RMS crosstalk directly from crosstalk channel S parameters. As such it is:

2. Easy to compute.
3. Mathematically, rigorously correct.

The actual sum is:

$$\mathrm{RPSEI} = \sqrt{\sum_{i} \int \mathrm{XT}_{i}(f) \cdot df}$$

(integral and sum are both linear operator and their order can be interchanged freely

What is Root Power Sum of Energy Integrals?

$$\mathrm{RPSEI} = \sqrt{\sum_i \int XT_i(f) \cdot df}$$

Where:

$$XT_i(f) = A^2 \cdot T \cdot \mathrm{sinc}^2(T \cdot f) \cdot |G_i(f)|^2$$

A is the Tx amplitude

T is baud time

f is frequency

G_i is Crosstalk channel gain for i^th^ crosstalk channel

$$G_i(f) = H_t(f) \cdot SDD21_i(f) \cdot H_r(f)$$

$H_t(f)$ is Transmitter transfer function

$SDD21_i(f)$ is for i^{th} crosstalk channel

$H_r(f)$ is Receiver transfer function

Why use RPSEI?

1. It is easy to compute
2. It gives a useful measure of crosstalk
3. It can be used in Link Budget as a Gaussian

What assumptions does RPSEI make? Are they valid or reasonable?

1. Data is an array of uncorrelated random bits.
2. Either all channels are running at (slightly) different rates, or that crosstalk does not vary significantly across a bit period.
3. Enough different uncorrelated bits contribute to the cross talk at any one time that the PDF can be treated as Gaussian via the Central Limit Theorem (law of large numbers).
4. A somewhat pessimistic measure is acceptable.

What assumptions does RPSEI make? Are they valid or reasonable?

1. Data is a stream of uncorrelated random bits.

This is valid for the data in any single channel since we are using scrambled data. For multiple aggressors the data should differ or should have differed in the past in such a way that the scrambler will produce un-correlated data.

It is no longer valid for training sequence but training sequence will have lower high frequency component, which are of most importance in crosstalk, so assumption is at worst pessimistic for training sequence.

What assumptions does RPSEI make? Are they valid or reasonable?

2. **Either all channels are running at (slightly) different rates, or that crosstalk does not vary significantly across a bit period.**

   Another way of saying this is that either:

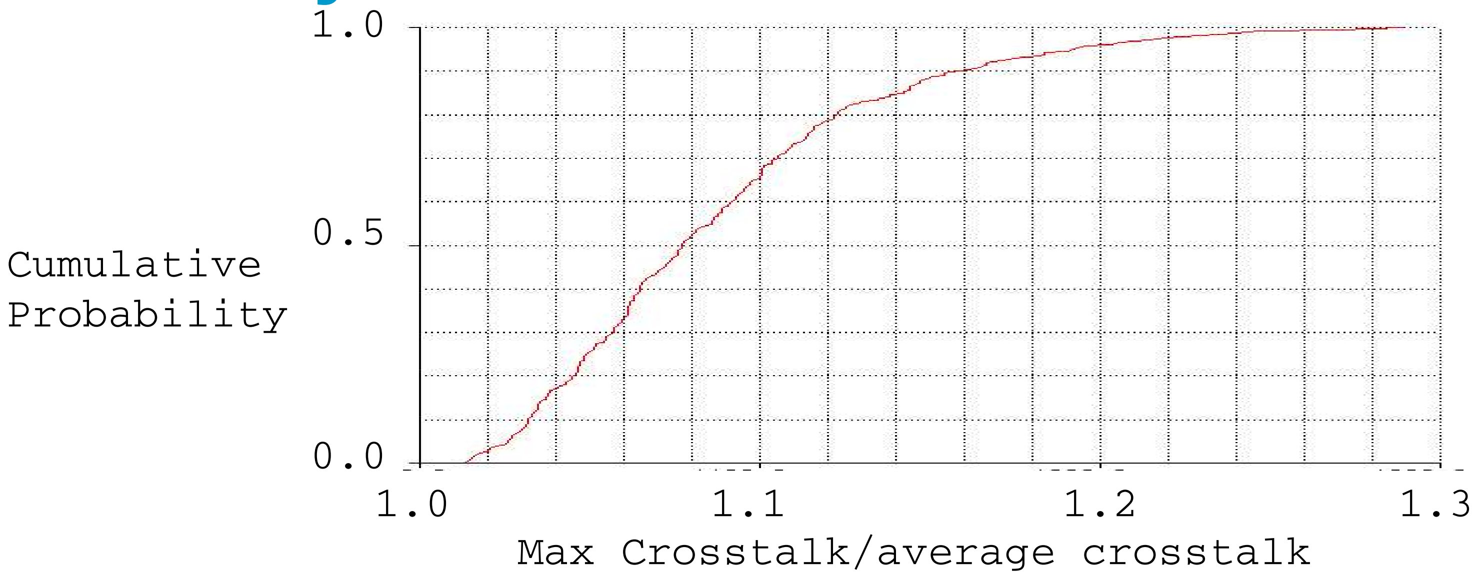
   1. **The crosstalkers are uncorrelated with each other and the thru path or**
   2. **The crosstalk pulse response is sufficiently complex that the correlation does not matter**

   In at least some cases all channels will use a common reference clock so first part is not valid.

   The plot below shows cumulative probability of ratio of maximum crosstalk to average crosstalk, measured across a bit period for 259 crosstalk channels in the 802.3ap data base. It shows that in >95% of the cases that the max crosstalk is less than 20% greater than the average, and I could not find any case where the max was more than 30% greater.

   Assumption is not fully valid but reasonably good.

What assumptions does RPSEI make? Are they valid or reasonable?



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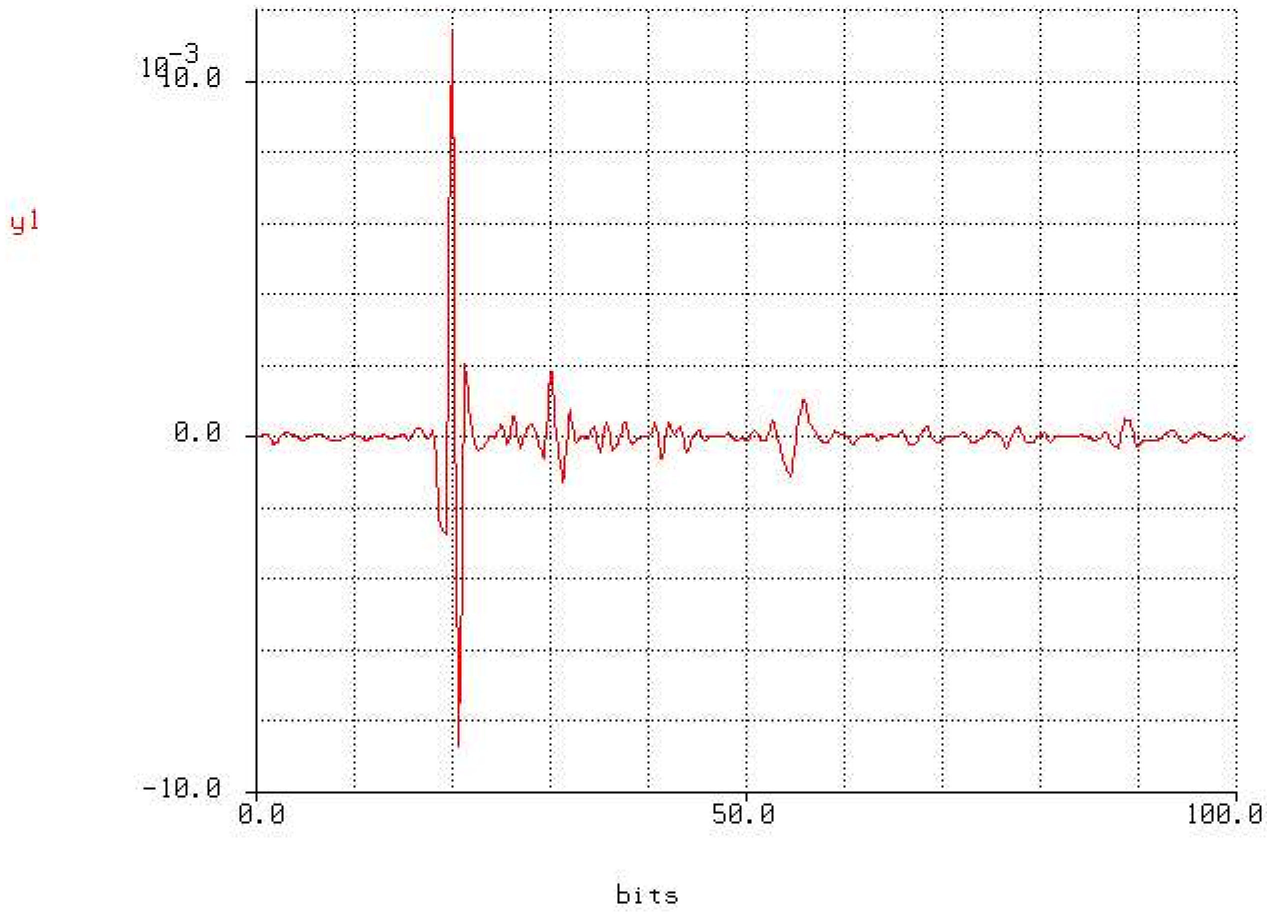
What assumptions does RPSEI make? Are they valid or reasonable?

3. Enough different uncorrelated bits contribute to the cross talk at any one time that the PDF can be treated as Gaussian via the Central Limit Theorem (law of large numbers).

This often will not be fully met but the Gaussian approximation will always be on the pessimistic side.

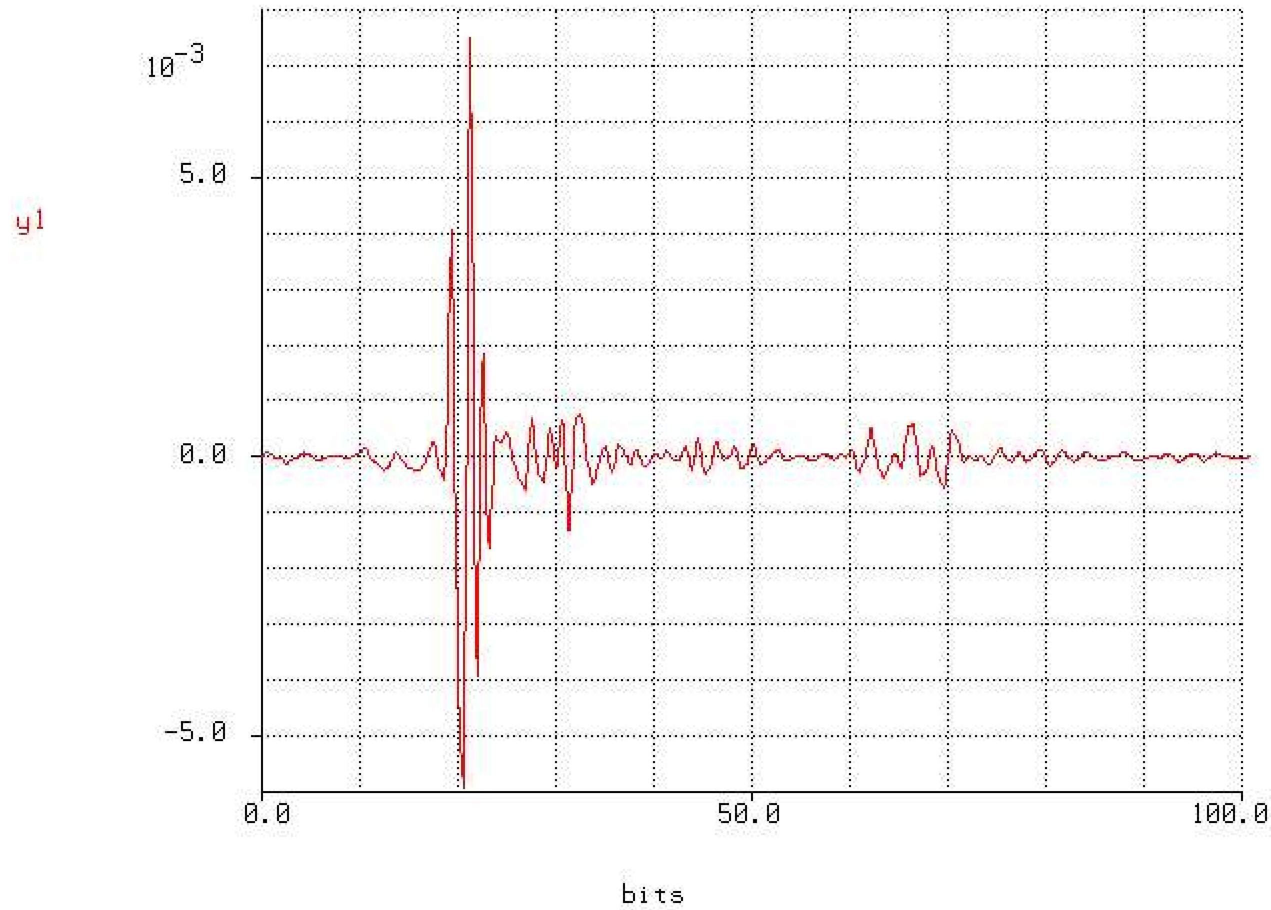
First look at the range of complexity of crosstalk pulse responses:

Goergen_ch77_10_7next



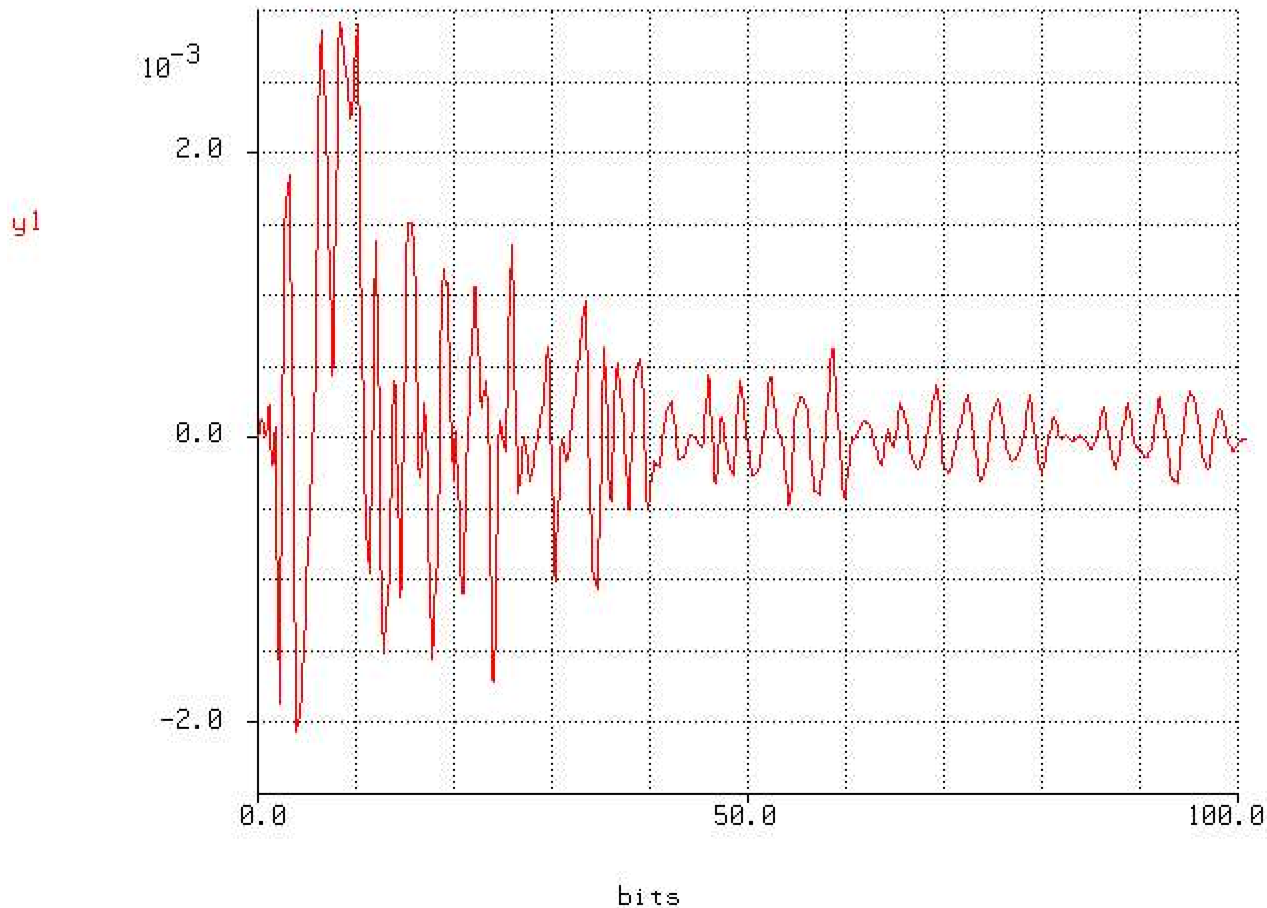
Short crosstalk pulse response for connectorless channel

Peters_M8_fext1



Short crosstalk pulse response for channel with connector

Peters_T20_next6



long crosstalk pulse response for channel with connector

Is the Gaussian approximation OK?

Now look at what happens when only a few bits contribute significantly to the crosstalk.

Is the Gaussian approximation OK?

Assume that the crosstalk has N equal contributions, all of equal amplitude A.

$$\mathrm{PeakCrosstalk} = A \cdot N$$

$$\mathrm{RMScrosstalk} = A \cdot \sqrt{N}$$

$$\mathrm{GaussianPeak}_{BER=1e-12} = A \cdot 7 \cdot \sqrt{N}$$

$$\mathrm{GaussianPeak}_{BER=1e-12} \geq \mathrm{PeakCrosstalk}$$

For:

$$N \leq 7^2 = 49$$

Is the Gaussian approximation OK?

For:

$$N \le 7^2 = 49$$

$$P(\pm \text{PeakCrosstalk}) \le 2^{-48} = 3.6 \cdot 10^{-15} < 10^{-12}$$

So under these conditions, the Gaussian approximation overestimates the Peak at 10^{-12}

If the contributors are unequal the overestimate increases.

Is the Gaussian approximation OK?

The analysis above assumes that that the crosstalk is uniform over the baud time. What if it is not? Let us assume that the crosstalk is concentrated over 1/3 of the baud period. There is no evidence of this occurring in any of the 250+ crosstalk paths I looked at but it has been suggested as a possibility.

Is the Gaussian approximation OK?

Then, again for N equal crosstalk cursors:

$$\mathrm{PeakCrosstalk} = A \cdot N$$

$$\mathrm{RMScrosstalk} = A \cdot \sqrt{N/3}$$

$$\mathrm{GaussianPeak}_{BER=1e-12} = A \cdot 7 \cdot \sqrt{N/3}$$

$$\mathrm{GaussianPeak}_{BER=1e-12} \geq \mathrm{PeakCrosstalk}$$

For:

$$N \leq 7^2/3 = 16.3$$

Is the Gaussian approximation OK?

For:

$$N = 16$$

$$P(\pm \text{PeakCrosstalk}) \leq 2^{-15} \cdot 3^{-16} = 7.0 \cdot 10^{-13} < 10^{-12}$$

So under these conditions, the Gaussian approximation still overestimates the Peak at 10^{-12}

In a random synchronous system the probability that 16 such cursors will align:

$$P(16\,\text{align}) \leq 3^{-15} = 7.0 \cdot 10^{-8}$$

So very few systems will show this level of problem

What assumptions does RPSEI make? Are they valid or reasonable?

4. A somewhat pessimistic measure is acceptable.

This is up to the Task Force to decide, do we

1. Make the specs conservative and risk “leaving stuff on the table”, including perhaps your favorite Backplane?
2. Loosen up the specs to counteract the conservative bias of RPSEI and risk passing unworkable Backplanes?
3. Write a spec so thorough that no one can understand it and no one uses it.

Conclusions:

I recommend:

- **That in a Normative Channel Spec, we use RPSEI as defined above and in healey_c1_0505 as a crosstalk penalty item in the Link Budget.**
- **That in an Informative Channel Spec, we use a crosstalk limit line as justified using RPSEI method, in healey_c1_0505.**

Supporting Slides
Is the Gaussian approximation OK?

Assume that the crosstalk has N unequal contributions, of amplitudes $A+x_i$

Where x_i has zero mean and variance $\langle x^2 \rangle$

$$\mathrm{PeakCrosstalk} = A \cdot N$$

$$\mathrm{RMScrosstalk} = \sqrt{A^2 + \langle x^2 \rangle} \cdot \sqrt{N}$$

$$\mathrm{GaussianPeak}_{\mathrm{BER=1e\text{-}12}} = A \cdot 7 \cdot \sqrt{N}$$

$$\mathrm{GaussianPeak}_{\mathrm{BER=1e\text{-}12}} > \mathrm{PeakCrosstalk}$$

For:

$$N \leq 7^2 = 49$$