An Introduction to Jim Hamstra's DFE Error Propagation Spreadsheet

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Supporters

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Agenda

- Purpose of the Spreadsheet
- Organization
- The 4 tap weight constraint methods explained
- How to enter data
- How it works
- How to read the results

Purpose

- **Calculate the probability of DFE error propagation for a given set of DFE tap weights, BER, and post-DFE eye opening.**
- **Evaluate methods of constraining tap weights to limit error propagation.**
  - Four constraint methods are available
    - Harmonic Decay
    - Exponential Decay
    - Cumulative Harmonic Decay
    - Cumulative Exponential Decay
  - There is a separate spreadsheet page for each model

The Problem

- Since the DFE is an Infinite Impulse Response (IIR) structure, if both the total number of feedback taps and the tap weights are not carefully constrained then a single erroneous data sample will produce a very long burst of errors with a high probability, effectively defeating the FEC.

Spreadsheet Organization

- **The spreadsheet contains 8 pages (tabs)**
  - An OIF coversheet
  - A read-me section
    - Basic explanation of spreadsheet operation
    - Results of Jim's analysis and his conclusions from it
  - Separate spreadsheets for each type of tap constraint
    - Individual Harm: Harmonic Decay constraint
    - Individual Exp: Exponential Decay constraint
    - Cumulative Harm: Cumulative Harmonic Decay constraint
    - Cumulative Exp: Cumulative Exponential Decay constraint
  - 2 example sheets applying "Cumulative Exp" to example channels supplied by Graeme Boyd of PMC-Sierra
    - Graeme 6G: CEI-6G channel Tap weights provided by Graeme
    - Graeme 10G: CEI-10G channel Tap weights provided by Graeme

Constraints #1 : Harmonic Decay

Harmonic Constraint

- **For any tap *n* : W(*n*) ≤ 1/(*n* + X)**
  - Where Parameter X controls the strength of the Harmonic decay

Total tap weight Constraint for m-tap DFE

- **For any tap *n :* W(*n*) ≤ Y - Sum(W(*n+1*) + W(*n+2*) .. + W(*m*))**
  - Maximum cumulative tap weight Parameter Y. (0 ≤ Y ≤ 1)
  - Note constraint is applied in **descending** tap order

- ***Note: constraint parameters X, Y & Z appear as cells in the spreadsheet***

Constraints #2 : Exponential Decay

Exponential Constraint

- **For any tap *n* : W(*n*) $\leq$ X * $Z^{(n-1)}$**
    - Where Parameters X & Z control the strength of the Exponential decay

Total tap weight Constraint for m-tap DFE

- **For any tap *n* : W(*n*) $\leq$ Y – Sum(W(*n+1*) + W(*n+2*) .. + W(*m*))**
    - Maximum cumulative tap weight Parameter Y. ($0 \leq Y \leq 1$)
    - Note constraint is applied in **descending** tap order

Constraint #3 : Cumulative Harmonic Decay

Cumulative Harmonic Constraint for an *m* tap DFE

- **For any tap *n* : Sum(W(*n*) + W(*n+1*) .. + W(*m*)) ≤ 1/(*n* + X)**
  - Where Parameter X controls the strength of the Harmonic decay
- **Therefore**
  - W(*n*) ≤ 1/(*n* + X) – Sum(W(*n*+*1*) + W(*n*+*2*) .. + W(*m*))

Constraints #4 : Cumulative Exponential Decay

Cumulative Exponential Constraint for an *m* tap DFE

- **For any tap *n* : Sum(W(*n*) + W(*n+1*) .. + W(*m*)) ≤ X * $Z^{(n-1)}$**
    - Where Parameters X & Z control the strength of the Exponential decay
  - **Therefore**
    - W(*n*) ≤ X * $Z^{(n-1)}$ – Sum(W(*n*+*1*) + W(*n*+*2*) .. + W(*m*))

How to Enter Data

Enter Unconstrained Tap values on Row 1

A1 contains W(1), B1 contains W(2) etc

Row 4 shows constrained tap weight values

	A	B	C	D	E	F	G	H	I	J	K	L
1	1	0	0	0	0	0	0	0	1	1		
2	0.2625	0.175	0.1166667	0.0777778	0.0518519	0.0345679	0.0230453	0.0153635	0.0102423	0.0068282		
3	0.2625	0.175	0.1166667	0.0777778	0.0518519	0.0345679	0.0230453	0.0153635	0.0102423	0.0068282		
4	0.2522577	0	0	0	0	0	0	0	0.0034141	0.0068282		
5	0.2625	0.0102423	0.0102423	0.0102423	0.0102423	0.0102423	0.0102423	0.0102423	0.0102423	0.0068282		
6		0.2625	0.6666667				7.04	0.475				
7	1	[illegible]612E-13	9.61[illegible]E-13	9.612E-13	9.612E-13	9.612E-13	[illegible]12E-13	9.61[illegible]13	9.612E-13	9.612E-13		
8	0.3917773	1	9.61[illegible]E-13	9.612E-13	9.612E-13	9.612E-13	9.[illegible]12E-13	9.612[illegible]13	9.612E-13	9.612E-13	2.5524707	L > 1
[illegible]	5.68E-06	[illegible].3917773	1	9.612E-13	9.612E-13	9.612E-13	9.[illegible]12E-13	9.612E-[illegible]	9.612E-13	9.612E-13	176042.64	L > 2
10	9.615E-13	5.68E-06	0.3917[illegible]73	1	9.612E-13	9.612E-13	9.[illegible]12E-13	9.612E-1[illegible]	9.612E-13	9.612E-13	2.212E+11	L > 3
11	9.612E-13	9.615E-13	5.68E[illegible]6	0.3917773	1	9.612E-13	9.[illegible]12E-13	9.612E-13	[illegible].612E-13	9.612E-13	2.212E+11	L > 4
12	9.612E-13	9.612E-13	9.615E-[illegible]3	5.68E-06	0.3917773	1	9.[illegible]12E-13	9.612E-13	[illegible]612E-13	9.612E-13	2.212E+11	L > 5
13	9.612E-1[illegible]	9.612E-13	9.612E-1[illegible]	9.615E-13	5.68E-06	0.3917773	1	9.612E-13	9.[illegible]12E-13	9.612E-13	2.212E+11	L > 6
14	9.612E-1[illegible]	9.612E-13	9.612E-1[illegible]	9.612E-13	9.615E-13	5.68E-06	0.3[illegible]17773	1	9.61[illegible]E-13	9.612E-13	2.212E+11	L > 7
15	9.612E-1[illegible]	9.612E-13	9.612E-13	9.612E-13	9.612E-13	9.615E-13	5.[illegible]8E-06	0.3917773	1	9.612E-13	2.212E+11	L > 8
16	1.847E-[illegible]	9.612E-13	9.612E-13	[illegible].612E-13	9.612E-13	9.612E-13	9.6[illegible]5E-13	5.68E-06	0.3917[illegible]73	1	2.212E+11	L > 9

Cell A6 holds constraint parameter **X**

Cell B6 holds constraint parameter **Y**

Cell C6 holds constraint parameter **Z**

Cell G6 holds Q value of the statistical eye (function of BER)

Cell H6 holds value of normalized min Eye opening after DFE summing function

How it works

Initial State : An error is received and tap 1 error probability is set to one
All other taps have the background error probability set by Q
(here Q=7.04 giving a BER of $\sim E^{-12}$)

	A	B	C	D	E	F	G	H	I	J	K	L
1	1	0	0	0	0	0	0	0	1	1		
2	0.2625	0.175	0.1166667	0.0777778	0.0518519	0.0345679	0.0230453	0.0153635	0.0102423	0.0068282		
3	0.2625	0.175	0.1166667	0.0777778	0.0518519	0.0345679	0.0230453	0.0153635	0.0102423	0.0068282		
4	0.2522577	0	0	0	0	0	0	0	0.0034141	0.0068282		
5	0.2625	0.0102423	0.0102423	0.0102423	0.0102423	0.0102423	0.0102423	0.0102423	0.0102423	0.0068282		
6		0.2625	0.6666667				7.04	0.475				
7	1	9.612E-13	9.612E-13	9.612E-13	9.612E-13	9.612E-13	9.612E-13	9.612E-13	9.612E-13	9.612E-13		
8	0.3917773	1	9.612E-13	9.612E-13	9.612E-13	9.612E-13	9.612E-13	9.612E-13	9.612E-13	9.612E-13	2.5524707	L > 1
9	5.68E-06	0.3917773	1	9.612E-13	9.612E-13	9.612E-13	9.612E-13	9.612E-13	9.612E-13	9.612E-13	176042.64	L > 2
10	9.61[illegible]E-13	5.68E-06	0.3917773	1	9.612E-13	9.612E-13	9.612E-13	9.612E-13	9.612E-13	9.612E-13	2.212E+11	L > 3
11	9.61[illegible]E-13	9.615E-13	5.68E-06	0.3917773	1	9.612E-13	9.612E-13	9.612E-13	9.612E-13	9.612E-13	2.212E+11	L > 4
12	9.61[illegible]E-13	9.612E-13	9.615E-13	5.68E-06	0.3917773	1	9.612E-13	9.612E-13	9.612E-13	9.612E-13	2.212E+11	L > 5
13	9.61[illegible]E-13	9.612E-13	9.612E-13	9.615E-13	5.68E-06	0.3917773	1	9.612E-13	9.612E-13	9.612E-13	2.212E+11	L > 6
14	9.61[illegible]E-13	9.612E-13	9.612E-13	9.612E-13	9.615E-13	5.68E-06	0.3917773	1	9.612E-13	9.612E-13	2.212E+11	L > 7
15	9.61[illegible]E-13	9.612E-13	9.612E-13	9.612E-13	9.612E-13	9.615E-13	5.68E-06	0.3917773	1	9.612E-13	2.212E+11	L > 8
16	1.84[illegible]E-12	9.612E-13	9.612E-13	9.612E-13	9.612E-13	9.612E-13	9.615E-13	5.68E-06	0.3917773	1	2.212E+11	L > 9

Column A holds the probability of an error in tap 1. This is calculated using the cumulative density function of the sum of the weighted tap bits in the previous cycle.

Columns B-J hold the probability of an error in corresponding tap bits. They shift down and right with time

The maximum value in Column A for this and all higher rows - inverted

Reading the Results

Rows in column K indicate the inverse probability of an error burst greater than ROW() - 7 bits

	A	B	C	D	E	F	G	H	I	J	K	L
1	1	0	0	0	0	0	0	0	1	1		
2	0.2625	0.175	0.1166667	0.0777778	0.0518519	0.0345679	0.0230453	0.0153635	0.0102423	0.0068282		
3	0.2625	0.175	0.1166667	0.0777778	0.0518519	0.0345679	0.0230453	0.0153635	0.0102423	0.0068282		
4	0.2522577	0	0	0	0	0	0	0	0.0034141	0.0068282		
5	0.2625	0.0102423	0.0102423	0.0102423	0.0102423	0.0102423	0.0102423	0.0102423	0.0102423	0.0068282		
6		0.2625	0.6666667				7.04	0.475				
7	1	9.612E-13	9.612E-13	9.612E-13	9.612E-13	9.612E-13	9.612E-13	9.612E-13	9.612E-13	9.612E-13		
8	0.3917773	1	9.612E-13	9.612E-13	9.612E-13	9.612E-13	9.612E-13	9.612E-13	9.612E-13	9.612E-13	2.5524707	L > 1
9	5.68E-06	0.3917773	1	9.612E-13	9.612E-13	9.612E-13	9.612E-13	9.612E-13	9.612E-13	9.612E-13	176042.64	L > 2
10	9.615E-13	5.68E-06	0.3917773	1	9.612E-13	9.612E-13	9.612E-13	9.612E-13	9.612E-13	9.612E-13	2.212E+11	L > 3
11	9.612E-13	9.615E-13	5.68E-06	0.3917773	1	9.612E-13	9.612E-13	9.612E-13	9.612E-13	9.612E-13	2.212E+11	L > 4
12	9.612E-13	9.612E-13	9.615E-13	5.68E-06	0.3917773	1	9.612E-13	9.612E-13	9.612E-13	9.612E-13	2.212E+11	L > 5
13	9.612E-13	9.612E-13	9.612E-13	9.615E-13	5.68E-06	0.3917773	1	9.612E-13	9.612E-13	9.612E-13	2.212E+11	L > 6
14	9.612E-13	9.612E-13	9.612E-13	9.612E-13	9.615E-13	5.68E-06	0.3917773	1	9.612E-13	9.612E-13	2.212E+11	L > 7
15	9.612E-13	9.612E-13	9.612E-13	9.612E-13	9.612E-13	9.615E-13	5.68E-06	0.3917773	1	9.612E-13	2.212E+11	L > 8
16	1.847E-12	9.612E-13	9.612E-13	9.612E-13	9.612E-13	9.612E-13	9.615E-13	5.68E-06	0.3917773	1	2.212E+11	L > 9
17	4.52E-12	1.847E-12	9.612E-13	9.612E-13	9.612E-13	9.612E-13	9.612E-13	9.615E-13	5.68E-06	0.3917773	2.212E+11	L > 10

Column L is labelled with the burst length associated with the result in Column K. (Eg. L > 3 indicates a burst longer than 3 bits)

Jim's conclusions

1. Achieving a significant BER reduction factor is feasible given suitable DFE constraints.
2. Cumulative Exponential Decay is the most stable way to constrain error propagation under a variety of pessimistic conditions. It is stable independent of the total number of taps in the DFE, the distribution of tap weights, and the raw BER of the data link.
3. All channel models in oif2003.260.00 (PMCS CEI channels) can be accommodated with ample headroom using the following Cumulative Exponential Decay constraints
   - Maximum cumulative weight Y = (1 - eye opening)/2 for sum of all taps N through M
   - Exponential decay factor Z = 2/3

Jim’s conclusions

4. Assuming near worst case legal tap weights within the recommended constraints the following DFE-induced error burst properties are predicted:
   - The average burst length produced by the DFE is 1.5 bits at Q > 4
   - **A DFE with >= 2-taps can generate a burst of errors = 3 bits with p < BER/100 at Q = 7 (10^-12 BER).**
   - A DFE with >= 4-taps can generate a burst of errors > 4 bits with p < BER/2000000 at Q = 7 (10^-12 BER).
   - A DFE with >= 5-taps can generate a burst of errors > 5 bits with p < BER/1000000 at Q = 6 (10^-9 BER).
   - A DFE with > 8-taps does not produce longer bursts of errors than a shorter DFE.