

Modal Excitation of Optical Fibers

Estimating the Modal Power Distribution

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1. Summary

This note summarizes one approach to estimating the modal power P_m in a multimode fiber from the measured near field intensity $I(r)$, measured at the end of a length of fiber.

The goal of the method is to give a robust estimate of P_m which is not over-sensitive to noise in the measurement, but which is internally consistent and a clear improvement over existing techniques (references [1]-[3]).

The purpose of estimating P_m is to better explain the performance of different laser sources in high bit rate MM applications, and to improve the design of the sources.

2. Background/Assumptions

The measured near field intensity $I(r)$ of a multimode optical fiber which has been excited by a laser source is assumed to be given by

$$I(r) = \sum_m^M P_m \psi_m^2(r) \quad [1]$$

Here

1. $I(r)$ is the near field intensity.
2. $\psi_m(r)$ is the modal function for mode m .
3. P_m is the power in mode m .

It is assumed to be adequate to calculate $\psi_m(r)$ for a reference profile and not for the exact profile in the measurement. This is because the propagation parameters and mode delays β_m and τ_m vary to first order with any index perturbation, while the eigenfunctions $\psi_m(r)$ vary to 2nd order. It is further assumed, for computational convenience, that the individual modes in a so-called *mode group* can be combined, so that $\psi_m^2(r)$ represents the sum of

the squares of all individual modal functions in group m . This assumption is most valid if the length of fiber is long enough for full coupling within a mode group or if the launch puts nearly equal power into all modes within a mode group.

It is assumed that the measured $I(r)$ is indeed the intensity of light in the fiber, which is related to the electric field as outlined in Snyder and Love **Optical Waveguide Theory**[4] pp. 210-217. Although it is true that $I(r) = P_m \psi_m^2(r)$ if there is power in only a *single* mode, when there are multiple modes one must assume the interference or cross terms are zero in order for equation [1] to be valid. This is rather rigorously true if the source is incoherent, and becomes increasingly suspect under some conditions.

Equation [1] is consistent with the historic conceptual picture of the mode power distribution (MPD), and if the weight of all individual modes is equal (giving twice the weight to azimuthal modes with $\nu > 0$ to account for both sine and cosine modes), then $I(r)$ will sum to a parabola. Figures 1 and 2 demonstrate this for the standard 62.5um 2% Δ MM fiber at 1300nm and 850nm respectively. Note that it is a wiggly parabola and becomes smoother as the number of modes increases. For 850nm there are approximately 289 modes and 33 mode groups (the outer groups likely having negligible power because of bend losses due to fiber perturbations).

Finally, it should be noted that the power in each mode P_m must be greater than or equal to zero, and cannot be negative. Note that even if equation [1] is rigorously true, one can obtain the same $I(r)$ from more than one source because each source determines a unique electric field given by

$$E(r) = \sum_m^M a_m \psi_m(r) \quad [2]$$

where m denotes an individual mode (not a mode group). $P_m = a_m^2$ and hence even if one knows P_m exactly one cannot determine a_m , since it can be positive or negative. It is not yet clear whether this presents any difficulty for us.

3. Approach for Estimating P_m

There are a number of approaches to estimating P_m . A preliminary step which applies to any proposed approach is to gain familiarity with equation

[1] and calculate the predicted $I_{pred}(r)$ for various P_m 's. One example is the uniform power mentioned above; another is to calculate $I_{pred}(r)$ for an offset gaussian spot [5] and compare that to the measured $I_{meas}(r)$ after propagation down a fiber of significant length to allow mode coupling.

The approach which we will use will be to identify numerical procedures for solving for P_m such that $I_{pred}(r)$ is as close to $I_{meas}(r)$ as possible. That is, we want to minimize χ_1^2 where

$$\chi_1^2 = \sum_r (I_{meas}(r) - I_{pred}(r))^2 \quad [3]$$

Here $I_{meas}(r)$ is the $I(r)$ in equation [1] and $I_{pred}(r)$ is simply $\sum P_m \psi_m^2(r)$

We might ask that this least squares criteria be modified to make the estimate of P_m more robust to measurement variation in $I_{meas}(r)$. One way to do this is to append a smoothness criterion (as is done with splines) and to simultaneously minimize χ_1^2 and

$$\chi_2^2 = \sum_m \left(\frac{d^2 P}{dm^2} \right)^2 \quad [4]$$

(One could choose other figures of merit as well, but this is computationally convenient).

Then the full minimization equation we will use is

$$\chi_{tot}^2 = \chi_1^2 + \lambda_a \chi_2^2 = \sum_r (I_{meas}(r) - I_{pred}(r))^2 + \lambda_a \sum_m \left(\frac{d^2 P}{dm^2} \right)^2 \quad [5]$$

In the limit that λ_a goes to zero, the smoothness criterion is not used at all. In practice, we will make λ_a as small as possible so that the solution does not look excessively noisy. In the limit that λ_a gets large enough, it will force all the P_m 's to be nearly equal and will return an $I_{pred}(r)$ like a parabola.

4. Computation of P_m

We write equation [1] as a matrix equation of the form $b = Ax$:

$$I_r = C_{rm} P_m \quad [6]$$

Here C_{rm} is an $r \times m$ matrix, I_r is a known vector, and P_m is the vector of unknowns. If $r = m$ this is a standard set of linear equations in m variables and can be solved by standard methods. In general what we would like is that $r \gg m$ so that there is more measurement data than parameters which need to be estimated, so that we can solve this in a least squares sense. In the case of 289 modes and even in the case of 33 mode groups and a core radius of 31.25 microns, this is hard to achieve. The portion of the near field pattern $I(r)$ which is repeatable will tend to be smooth and will not consist of 33 useful degrees of freedom. This is why the extra smoothness criterion χ^2_2 in equation [4] is helpful. There is a second matrix equation

$$0_m = \lambda_a D_{mm} P_m \quad [7]$$

where D_{mm} is a matrix with -2 on the diagonal and 1 on the offdiagonal so that $D_{mm} P_m$ gives an approximation to $d^2 P_m / dm^2$. Then we can augment equation [6] by extending both the left hand size vector I_r to include an m -vector of zeros 0_m , and augment the matrix C_{rm} to include the matrix $\lambda_a D_{mm}$. This gives the matrix equation

$$b_{r+m} = F_{r+m,m} P_m \quad [8]$$

This matrix equation is equivalent to the least squares statement in equation [5]. One can now invert (in a least squares sense) equation [8] using singular value decomposition techniques (See for example, sections in *Numerical Recipes*) [6] to get the equation

$$P_m = G_{m,r+m} b_{r+m} = A_{mr} I_r$$

where the final formula of course needs only the first r non-zero entries of b_{r+m} , which is I_r . Note A_{mr} depends on λ_a and must be calculated for a few λ_a 's to see how it works.

5. Results/Discussion

6. References

- [1] Piazzola, S., and De Marchis, G., "Analytical Relations between Modal Power Distribution and Near Field Intensity in Graded-Index Fibres", *Electronic Letters* **15** no. 22 (25 October 1979) pp.721-722.

- [2] Calzavara, M., et al., "Mode Power Distribution Measurements in Optical Fibres", *CSELF Report IX* no. 5 (October 1981) pp.447-451.
- [3] Daido, et al., "Determination of modal power distribution in graded-index optical waveguides from near-field patterns and its application to differential mode attenuation measurement", *Applied Optics* **18** no. 13 (1 July 1979) pp.2207-2213.
- [4] Snyder, A.W., and Love, J.D., *Optical Waveguide Theory*. New York: Chapman and Hall, 1983.
- [5] Saijonmaa, J., et al., "Selective excitation of parabolic-index optical fibers by Gaussian beams", *Applied Optics* **19** no. 14 (15 July 1980) pp.2442-2452.
- [6] Press, W.H., et al., *Numerical Recipes in FORTRAN: The Art of Scientific Computing. Second Edition*. New York: Cambridge University Press, 1992.

Figure 1
 $I(r)$ with $P_m = 1$ (uniform power)

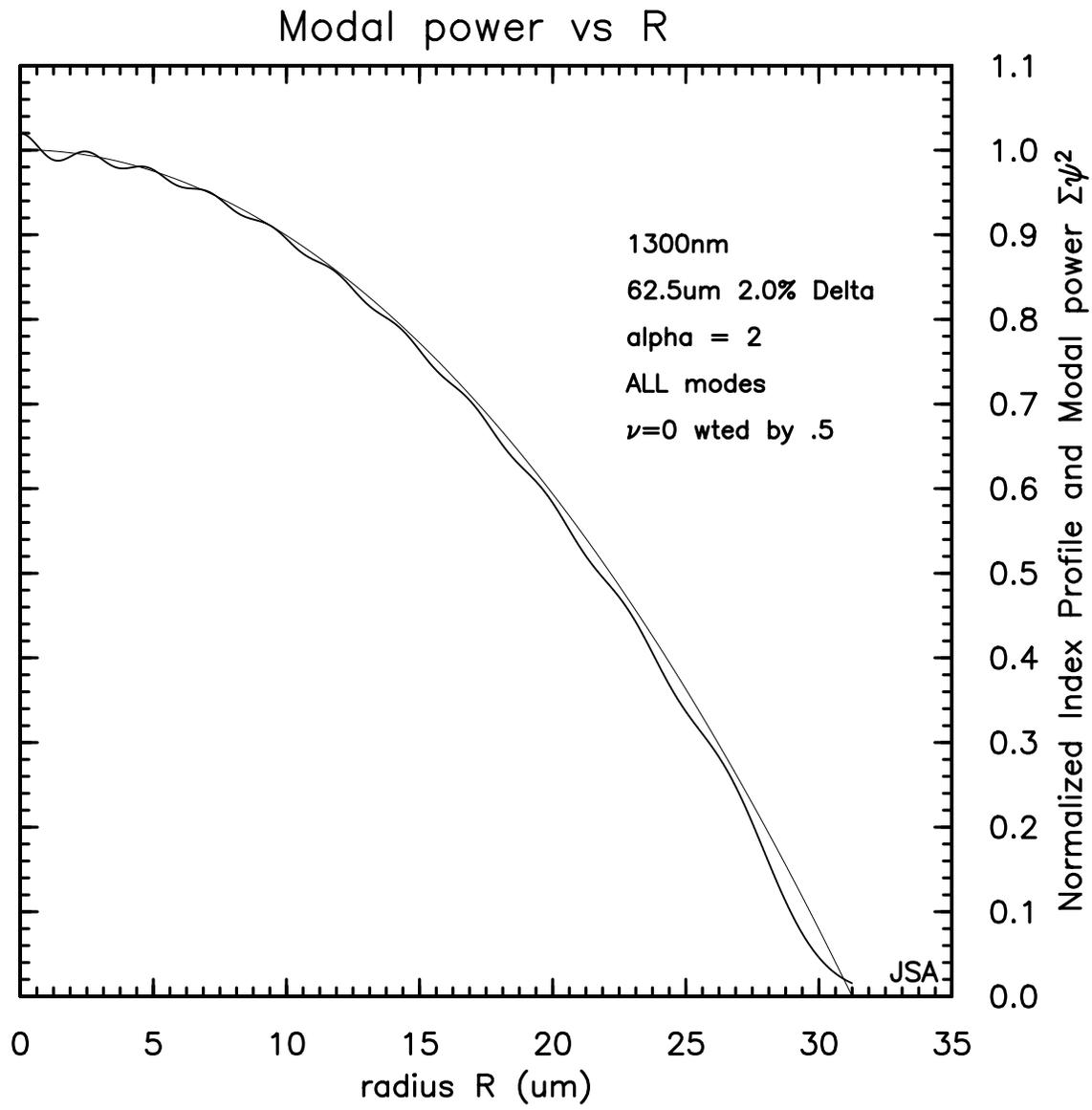


Figure 2

$I(r)$ for case $P_m = 1$ (uniform power)

a. $I(r)$ and ideal parabola

b. cumulative $I(r)$ and ideal parabola case

c. $I_0(r)$: fundamental mode

