Effect of nonlinearities in a 10 Gbit/s EPON system

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After the standardization of the Ethernet Passive Optical Network (EPON) system in 2004 (IEEE 802.3-2005), its penetration in the global market has been quite remarkable. This success incited the study of an EPON system operating at 10 Gbit/s. The work group responsible for this task (IEEE P802.3av Task Force) has been analyzing several physical issues, in order to produce a standard. Among them is the analysis of higher splitting ratios, namely 64 and 128. This demands higher power launched into the fiber, to mitigate the higher splitter losses. This report aims to assess the degradation in such a system, caused by the fiber nonlinearities that arise with these higher powers.

Besides the issues related with the splitting ratio, a 10 Gbit/s EPON system will suffer signal degradation due to the interaction between group velocity dispersion and intensity-dependent self-phase modulation (SPM). Since this high data rate system requires greater received power for error-free detection, the SPM, caused by the nonlinear dependence of the refractive index on intensity, should be considered.

Regarding the nonlinearities of the system, the following discussion is based on a theoretical analysis, where light scattering effects are considered, and a simulation analysis, where SPM is considered. Other nonlinearities, such as cross-phase modulation and four-wave mixing, are not relevant in the EPON system, since only one channel is transmitted through the optical fiber.

Theoretical limitations

Stimulated Brillouin Scattering (SBS) and Stimulated Raman Scattering (SRS) are inelastic processes in which part of the power is lost from an optical wave and absorbed by the transmission medium. The remaining energy is then re-emitted as a wave at lower frequency. SBS and SRS processes can become nonlinear in optical fibers due to the high optical intensity in the core and the long interaction lengths afforded by these waveguides. These nonlinear effects occur when the light launched into the fiber exceeds a threshold power level for each process [1].

Besides the stimulated light scattering effects, the nonlinear phase modulation should also be considered. In reality, all materials behave nonlinearly at high intensities and their refractive index varies with intensity. The physical origin of this effect lies in the non-harmonic response of electrons to optical fields, resulting in a nonlinear susceptibility.

Stimulated Brillouin Scattering

The physical process behind SBS is the tendency of materials to become compressed in the presence of an electric field. For an oscillating electric field at a specific pump frequency ω_P, this compression process generates an acoustic wave at some frequency Ω equal to the Stokes shift. The SBS can be viewed as scattering of the pump wave from this acoustic wave, resulting in the creation of a new wave at the Stokes frequency ω_S. The scattering process must conserve both the energy and the momentum. Once the scattered wave is generated spontaneously, it beats with the pump and creates a frequency component at the acoustic frequency. As a result, the beating term acts as source that increases the amplitude of the sound wave, which in turn increases the amplitude of the scattered wave, resulting in a positive feedback loop [1].

In single-mode fibers, light can travel only in the forward and backward directions. As a result, SBS occurs in the backward direction with a specific Brillouin frequency shift Ω_B. The SBS gain g_B is frequency dependent because of the finite damping time T_B of acoustic waves, and has a Lorentzian spectral profile, given by [1]:

$$g_B(\Omega) = \frac{g_B(\Omega_B)}{1 + (\Omega - \Omega_B)^2 T_B^{\,2}}\,. \tag{1}$$

The peak value of the Brillouin gain occurs for $\Omega = \Omega_B$, (that is typically 10.7 GHz) and depends on various material parameters such as the density and the elasto-optic coefficient.

One criterion for determining at what point SBS becomes a limiting factor, is to consider the SBS threshold power, P_{SBS}. In the early research paper on SBS phenomenon the threshold power P_{SBS} was defined as the signal power at which the backscattered light equals the fiber input power. For example based on this definition the SBS threshold for CW pump light, assuming Lorentzian linewidth profiles, is approximated by [2]:

$$P_{SBS}^{CW} \approx 21\frac{A_{eff}\,k_{SBS}}{g_B L_{eff}}\left(\frac{\Delta\nu_{SBS} + \Delta\nu_P}{\Delta\nu_{SBS}}\right), \tag{2}$$

where $\Delta\nu_{SBS}$ is the spontaneous Brillouin bandwidth, $\Delta\nu_P$ is the pump light bandwidth, A_{eff} is effective area defined by transverse distribution of electric field $f(r)$ of the optical mode in a fibre and is given by formula from [3].

$$A_{eff} = 2\pi\frac{\left[\int_0^\infty f^2(r)rdr\right]^2}{\int_0^\infty f^4(r)rdr}, \tag{3}$$

A_{eff} if often approximated by $Aeff=\pi w^2$ where $2w$ is mode field diameter. k_{SBS} is the polarization factor, varying between 1 and 2 depending on the relative polarizations of the pump and Stokes waves ($K = 3/2$ for the complete polarization scrambling [4]), and L_{eff} is the effective interaction length given by [2]:

$$L_{eff} = \frac{1 - exp(-\alpha L)}{\alpha}, \tag{4}$$

where α is the attenuation coefficient of the fiber in km^{-1}, and L is the fiber length.

Formula (2) implies that SBS threshold is increasing with the linewidth of the CW pump $\Delta\nu_{P,}$ scales with length of the fiber according to equitation (4) and is directly proportional to the *Aeff*. Whereas the first and second dependencies are confirmed in a number of experiments, recent studies of SBS effect in optical fibers reveal that the relation between SBS threshold and *Aeff* is more complex and involves not only the intensity of the *optical mode* (see equations (2) and (3) but also the structure of the acoustical wave that was ignored in the early SBS studies (plane acoustic waves were assumed in those studies).

To demonstrate that SBS does not depend on *Aeff* as prescribed by equation (2) we show the dependence of reflected and transmitted powers on launched power for three fibers with the identical length of 25 km (see Figure 1). Fiber A is G.652 compliant fiber with Aeff≈85 μm^2, Fiber B is G.655 compliant fiber with Aeff≈72 μm^2 and Fiber C is another G.652 compliant fiber with Aeff≈88 μm^2.

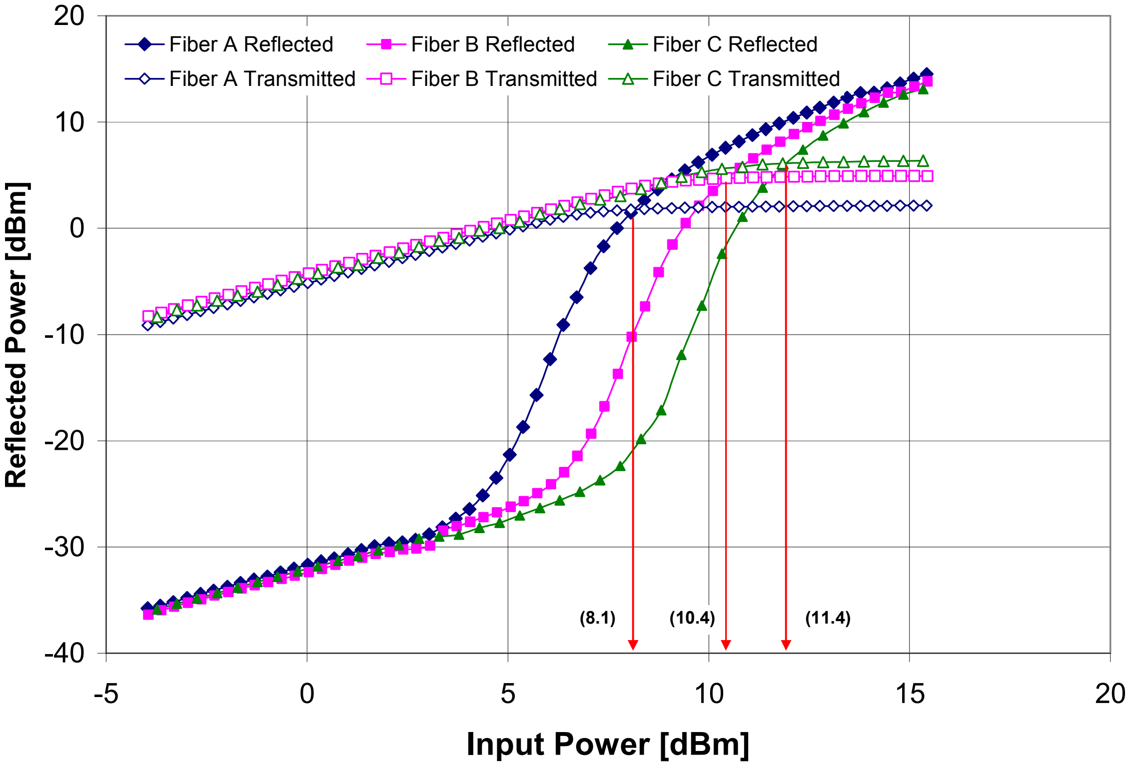


Fig. 1 Reflected and transmitted powers as a function of input power for three fibers A, B and C. Fiber A is a representative of typical G.652 fibers (mostly used in FTTx deployment), fiber B is a representative of G.655 fibers (non-zero-dispersion shifted fiber) and fiber C is another G.652 fiber with enhanced SBS threshold.

We also plot transmitted powers for Fibers A, B and C and use definition of SBS threshold specified in the ITU-T Recommendation G.650.2 (2005) as input power at which transmitted power equals to reflected power. Resulting SBS thresholds in dBm are given in the graph for reference. If equation (2) were correct Fiber B should have SBS threshold lower than that of Fiber A but the measured SBS threshold is 2 dB higher than that of Fiber A. Effective areas Aeff of Fibers A and C are very similar and moreover they belong to the same ITU-T G.652 fiber type (so called standard single mode fibers), yet their SBS thresholds are more than 3 dB different! We will refer to fibers of type C as G.652 fibers with enhanced SBS threshold. Next we explain the origin of this enhancement.

The treatment based on the plane wave analysis [5] is somewhat outdated and strictly speaking not applicable to optical fibers where *both optical and acoustic waves* are guided. To describe the effect properly one has to solve the acousto-optic equation (see Appendix A; a more detailed derivation can be found in [6]. One can therefore show that the back-reflected (Stokes) power is described by the evolutional equation

$$\frac{dP_S}{dz} + \alpha P_S + \frac{g_B}{A_m^{ao}} \bar{\gamma}(\nu) P_S P_p = 0 \tag{5}$$

where the nonlinear term is inversely proportional to the *acousto-optic effective area* rather than the optical effective area A_{eff}

$$A^{ao} = 2\pi \left[\frac{\int_0^\infty f^2(r) r dr}{\int_0^\infty \xi(r) f^2(r) r dr} \right]^2 \int_0^\infty \xi^2(r) r dr \tag{6}$$

The acousto-optic effective area depends on the overlap between the acoustic $\xi(r)$ and optical $f(r)$ modes.

Now the experimental results of Fig. 1 can be readily explained. Although fiber B has a smaller optical effective area, its acousto-optic effective area is larger that entails higher SBS threshold. The highest A_{ao} is for fiber C that has about 3 dB higher SBS threshold compared to fiber A.

This finding has the following important consequence: Aeff in formula (2) should be replaced with Aao. Of a lesser importance, the numerical coefficient 18 in (2) gives better accuracy than coefficient 21 which was obtained for the fibers with the loss of 2 dB/km. This observation was already mentioned in several previous works [7-9].

It is also worth mentioning that for fiber A (standard single mode G.652 fiber) $A_1^{ao} \approx A_{eff}$ that explains the fact why formula (2) was used over years to describe SBS threshold in optical fibers. As is obvious from the above analysis, the SBS threshold depends on a large number of parameters (see the exact expression for gB in Appendix A) which are difficult to determine accurately from the first principles. Therefore, the most reliable method to obtain the SBS threshold is to measure it for a reference fiber and then calculate it for different fiber length or fiber type using the modified version of equation (2).

In digital systems employing intensity modulation, for the special case of a non-return-to-zero (NRZ) data format, and where the average pump power of the modulated laser is equal to the CW power when the laser is not modulated, the SBS threshold is given by [10]:

$$P_{SBS}^{NRZ} = \frac{P_{SBS}^{CW}}{1 - \frac{B}{2\Delta\nu_B}\left(1 - e^{-\Delta\nu_B/f_0}\right)}, \quad (7)$$

where B is the bit rate.

Once the power launched into the fiber exceeds the threshold level, the exceeding light is reflected backwards through SBS. Therefore, the SBS limits the launched power to a few mW, because of its low threshold.

Several schemes are available for reducing the effect of SBS [11]. Near total suppression of SBS can be achieved by providing a low frequency dither to the laser. In other words, the SBS suppression can be achieved by directly modulating the laser with a sinusoidal current at a frequency very much lower than the low-frequency cutoff of the receiver. This will cause the laser to be FM modulated at a frequency that is outside the receiver bandwidth, but will accomplish a large effective bandwidth. Since the dither frequency is outside the receiver bandwidth, it will not degrade the signal in the presence of dispersion [10]. This dithering method is efficient since the SBS is a narrowband effect. Note that the dithering frequency should scale as the ratio of the injected power to the SBS threshold. The magnitude of the dither necessary for SBS suppression depends on the FM response of the laser. A small percent of dither amplitude corresponds to a very large effective laser linewidth [10]. However it is clear that dithering cannot infinitely increase SBS threshold without additional penalties since at some point induced phase modulation of the signal will be converted in the amplitude modulation by fiber dispersion.

Stimulated Raman Scattering

Spontaneous Raman scattering occurs in optical fibers when a pump wave is scattered by the silica molecules. An important difference from SBS is that the vibrational energy levels of silica dictate the value of the Raman shift Ω_R. As an acoustic wave is not involved, spontaneous Raman scattering is an isotropic process and occurs in all directions.

The spectrum of the Raman gain depends on the decay time associated with the excited vibrational state. In the case of optical fibers, the bandwidth of the Raman gain exceeds10 THz. The maximum gain occurs when the Raman shift $\Omega_R = \omega_p$-ω_s is about 13 THz [1].

Similar to the SBS case, the Raman scattering becomes stimulated if the pump power exceeds a threshold value. The threshold power P_{SRS} is defined as the incident power at which half of the pump power is transferred to the Stokes field at the output end of a fiber of length L, and is given by [12]:

$$P_{SRS} \approx 16 \frac{A_{eff} k_{SRS}}{g_R L_{eff}}, \tag{8}$$

where g_R is the peak Raman gain coefficient for co-polarized pump and Stokes waves, and k_{SRS} is a factor that depends on the relative polarizations of the pump and Stokes waves. Raman gain is maximized when the pump and Stokes waves maintain identical polarization along the fiber. For conventional single-mode fiber, there is a degree of polarization scrambling and a value of $k_{SRS} = 2$ has been suggested [10]. Note that equation (8) provides an order-of-magnitude estimate only, as many approximations are made in its derivation.

In any system with a single pump wavelength or number of pump wavelengths detuned by much more than the width of Raman gain (~13 THz) the onset of SBS will prevent conversion of pump wavelength in the Stokes component. However, when two high power wavelengths are present in the same fiber detuned by less than 13 THz they will interact through stimulated Raman scattering that will result in energy transfer form the shorted wavelength signal to the longer wavelength signal.

Self-Phase Modulation

The refractive index n of many optical materials has a weak dependence on optical intensity, given by [13, 14]:

$$n = n_0 + n_2 \frac{P}{A_{eff}}, \tag{9}$$

where n_0 is the ordinary refractive index of the material and n_2 is the nonlinear index coefficient. The nonlinearity in the refractive index is known as the Kerr nonlinearity. This nonlinearity produces a carrier-induced phase modulation of the propagating signal, which is called Kerr effect. In single-wavelength links, this gives rise to SPM, which converts optical power fluctuations in a propagating light wave to spurious phase fluctuations in the same wave.

Because of SPM, the propagation constant becomes power dependent, given by:

$$\beta' = \beta + \gamma P, \tag{10}$$

where $\gamma = 2\pi n_2/(A_{eff}\lambda)$ is a nonlinear parameter. Assuming constant input power, the γ term produces a nonlinear phase shift given by [1]:

$$\phi_{NL} = \int_0^L (\beta' - \beta) dz = \int_0^L \gamma P(z) dz = \gamma P_{in} L_{eff}, \tag{11}$$

where $P(z) = P_{in}\exp(-\alpha z)$ accounts for fiber losses. In practice, P_{in} is time dependent making ϕ_{NL} varying with time. In fact, any changes in the optical power will produce corresponding changes in the phase, and can potentially impact the system performance.

The impact of SPM depends also on the dispersion, which converts the phase in intensity. The use of dispersion compensation techniques is the best way of minimizing the effect of SPM.

Network analysis

The analysis presented in this section is based on an EPON system at 10 Gbit/s, considering 64 or 128 ONUs connected to the network.

The network analysis assumes the use of a specific set of parameters values that is justified in the following. The simulation results were obtained assuming an operating wavelength of 1550 nm. The 1310 nm window was not focused since it is close to the zero-dispersion wavelength, so the dispersive effects are not significant. External modulation is assumed in the transmitter, as previous results [15] demonstrated the poor performance of the 10 Gbit/s EPON system, when directly modulation is used in the 1550 nm window. The NRZ modulation format is employed and a fiber attenuation of 0.22 dB/km is assumed. The model used to characterize the optical fiber includes the effects of attenuation, dispersion, and SPM.

The results presented in Figure 1 were obtained for the downstream of a network with 128 ONUs, and represent the maximum launched power allowed into the fiber, in order to guarantee a normalized eye opening penalty not exceeding 1 dB. Similar results were obtained for a network with 64 ONUs, and are displayed in Figure 2. As expected these results do not differ significantly from those in Figure 1. The distance from the OLT to the optical passive splitter/combiner (PSC), Y axis, is varied from 0 to 40 km, as well as the distance from the PSC to the ONU under analysis. This is justified by the interest on longer network reaches, and on knowing which limitations are significant beyond 20 km network reach.

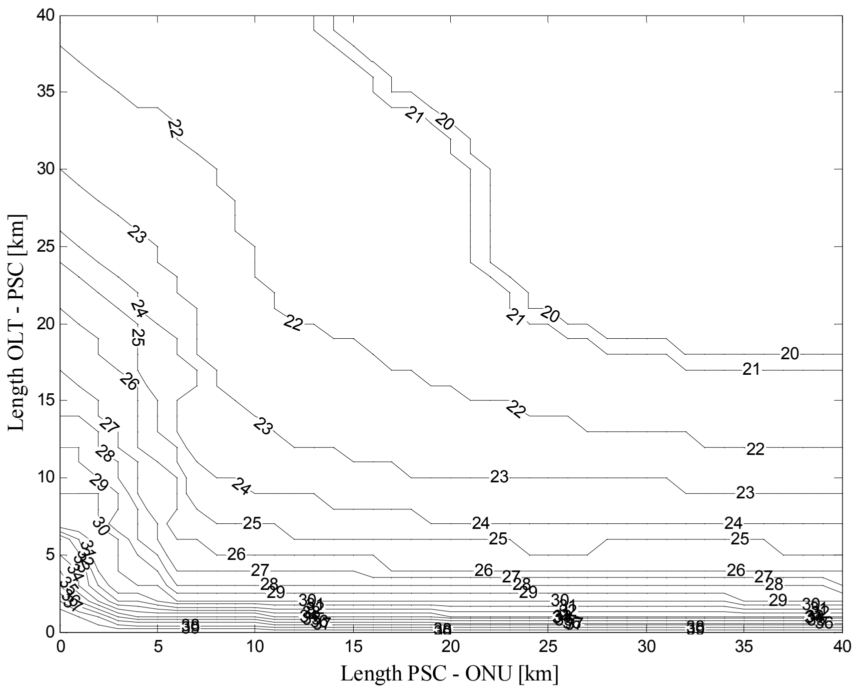


Figure 1 – Maximum launched power into the fiber, in dBm, to guarantee an eye opening penalty not exceeding 1 dB, in a network with 128 ONUs.

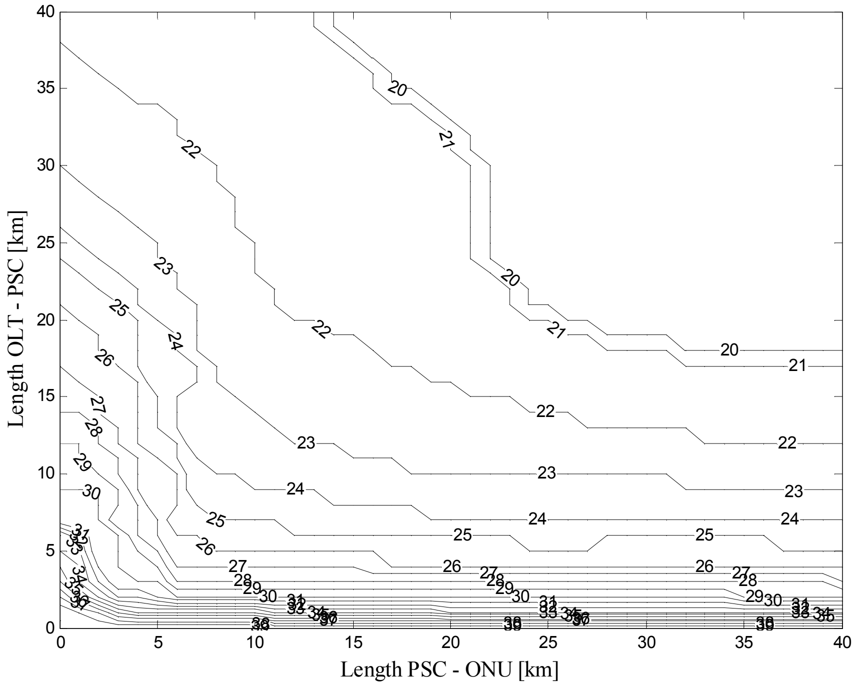


Figure 2 – Maximum launched power into the fiber, in dBm, to guarantee an eye opening penalty not exceeding 1 dB, in a network with 64 ONUs,

The results presented in Figure 1 and Figure 2 show that the maximum allowed launched power may be considerably high, with no significant degradation in the system performance. Note that the transmission distance between the PSC and the ONU does not have the same influence in the final results as the distance between the OLT and the PSC. This is justified by the large difference between the power levels of the signals in the feeder and in the distribution networks.

This large difference between the power levels of the signal in the two parts of the network suggests that, in the span between the PSC and the ONUs, where the power level is quite low, the nonlinear effects may be neglected. Therefore, simulation results were obtained considering linear transmission between the PSC and the ONUs. These results are presented in Figure 3 and Figure 4 for 128 and 64 ONUs, respectively. From the comparison of these results with those presented in Figure 1 and Figure 2, it can be concluded that the span between the PSC and the ONUs may be treated as a linear medium, since the results do not present significant differences, both for 64 and 128 ONUs.

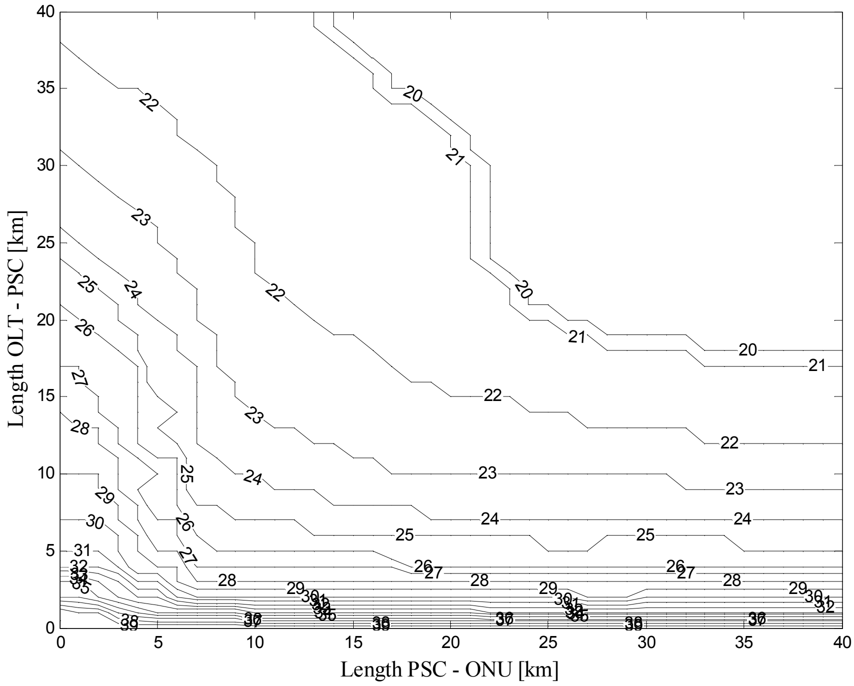


Figure 3 – Maximum launched power into the fiber, in dBm, to guarantee an eye opening penalty not exceeding 1 dB, in a network with 128 ONUs, considering linear transmission between the PSC and the ONUs.

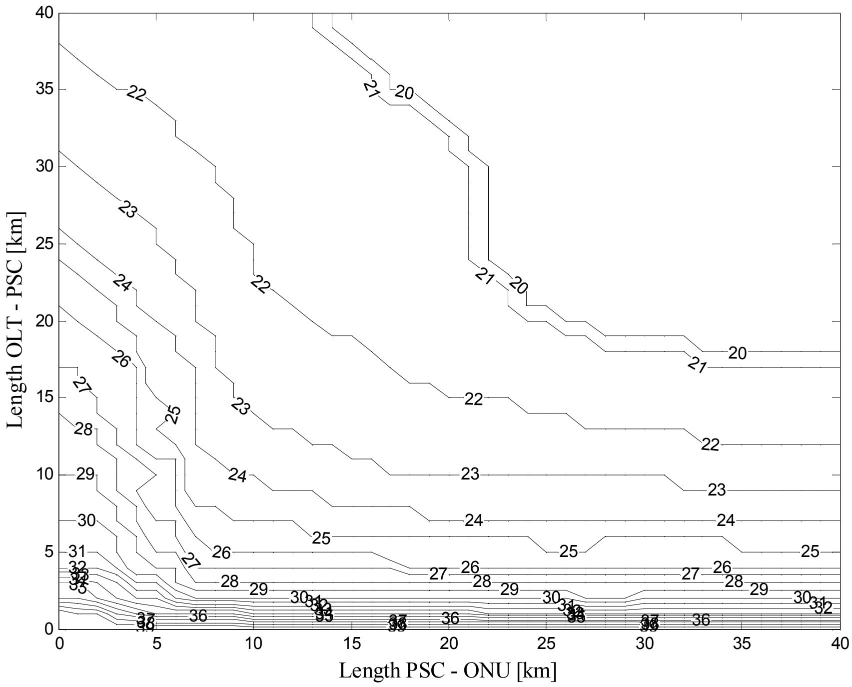


Figure 4 – Maximum launched power into the fiber, in dBm, to guarantee an eye opening penalty not exceeding 1 dB, in a network with 64 ONUs, considering linear transmission between the PSC and the ONUs.

In addition to SPM, there are other nonlinear effects that can occur, when the power launched into the fibers is increased, as referred above. Therefore, the influence of SBS and SRS on the maximum allowed launched power is analytically estimated in the following.

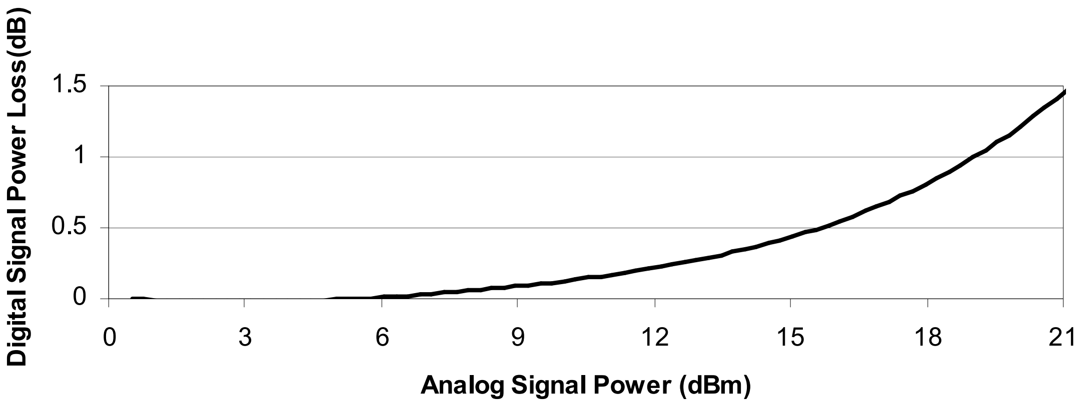
For the estimation of SBS influence, two different network configurations are analyzed. The first corresponds to a worst case SBS scenario where a single PSC is located near the ONUs. In this scenario, the feeder network that is the part of the network with higher power levels, and then more susceptible to SBS effect, is 20 km long. The second network configuration is a best case SBS scenario that considers the PSC located near the OLT, so the feeder length is reduced to 0.5 km.

Therefore, in the worst case situation, the effective interaction length is $L_{eff} \approx 12.6$ km, given by (4), considering $\alpha = 0.22$ dB/km. Fig. 1 (Fiber A) represent typical reflection curve as a function of input power. We found that most of commercially available G.652 fibers that are commonly deployed in the FTTx applications deviate from this curve by less than 0.5 dB. Using definition pictorially shown in Fig. 1 Brillouin threshold power for CW transmission is P_{SBS}^{CW} [dBm] ≈ 8.1 dBm), given by (2). The Brillouin threshold for an intensity modulated NRZ signal at 10 Gbit/s is approximately 3 dB higher i.e. P_{SBS}^{NRZ} [dBm] ≈ 11.1 dBm. Note that the Brillouin threshold increases with the increase of the source linewidth. Note that G.652 compliant SBS enhanced fibers provide additional 3 dB improvement in the SBS threshold, thus in the example shown in Fig.1 SBS threshold enhanced G.652 fiber will have SBS threshold of 14.4 dBm for NRZ modulated signal.

The analysis of the best case scenario assumes the same values for all parameters except for the interaction length. In this situation $L_{eff} \approx 0.49$ km. The estimated Brillouin threshold power for CW transmission is P_{SBS}^{CW} [dBm] ≈ 22.2 dBm, resulting in a Brillouin threshold for an intensity modulated NRZ signal at 10 Gbit/s of P_{SBS}^{NRZ} [dBm] ≈ 25.5 dBm. SBS enhanced threshold fibers will have correspondingly >3 dB higher thresholds.

When SRS occurs, part of the energy is transferred to a different wavelength, but this effect is significant only for very high powers (around 500 mW) [13]. Using (8), and assuming $g_R = 6\text{x}10^{-13}$ m/W at 1550 nm [1], the Raman threshold power is $P_{SRS} \approx 155$ mW (P_{SRS} [dBm] ≈ 22 dBm). The onset of SBS will prevent energy transfer from the signal wave to the Stockes component and thus SRS will not be a significant penalty.

However, if the RF overlay is deployed in the system together with data channel e.g. at analog signal at wavelength 1550 nm and digital at 1490 nm, then SRS will result in the energy transfer from 1490 nm signal to 1550 nm signal. This will result in the additional loss that 1490 nm signal will experience. For powers of 1550 nm signal ranging from 0 to 21 dBm the additional signal loss of 1490 nm signal could be 1.5 dB. In addition, due to propagation of both signals in the same direction intensity modulation from short wavelength signal is transferred to longer signal causing additional penalties. For more information see reference F. Coppinger et.al. in paper TuR5 OFC 2005, vol 1. p.283 and A. Kobyakov et. al. ECOC 2004 paper We.P.086.



Conclusions

In summary, of the three nonlinearities that were analyzed, SBS appears to be the fundamental nonlinear limitation for very high-speed, single-channel, intensity modulated systems. The simulation results have shown that the effect of SPM is not significant.

An EPON operating at 10 Gbit/s, with high splitting ratios, was found to be a loss-limited system, or a dispersion-limited system, in the case were direct modulation is employed in the transmitter [15]. The nonlinearities that are present in such system do not degrade the system performance, as long as the launched power is kept below 11.1 dBm, which corresponds to the limit imposed by the SBS effect, in the worst case scenario analyzed. This limitation may be mitigated by increasing the source linewidth, or by providing a low frequency dither to the laser. The first option can be achieved through direct modulation of the source, since this causes the linewidth to broaden due to chirping. However, as referred above, the direct modulation will cause very high dispersion penalties, degrading the system performance. The second option seems to be the best active solution for minimizing the effect of SBS in a 10 Gbit/s EPON system. The third (passive) option is to use SBS enhanced fibers that can enable 3 dB higher launched power without SBS penalties and can be used together with dithering.

Appendix A: SBS theory for optical fibers

The starting point of our analysis is the equation for the acousto-optic interaction

$$\frac{\partial^2 \rho}{\partial t^2} - \Gamma \nabla^2 \frac{\partial \rho}{\partial t} - v_l^2(r)\nabla^2 \rho = -\frac{\gamma}{2}\nabla^2 E^2, \qquad \text{(A1)}$$

where ρ [kg/m^3] is the material density fluctuation around its mean value ρ_0, $\Gamma = \eta_{11}/\rho_0$ [m^2/s] is the damping factor, $v_l^2(r)=Y(r)/\rho(r)$ [m^2/s^2] is the squared longitudinal sound velocity that depends on the transverse radial coordinate r due to silica doping with GeO_2. This dependence creates guiding of acoustic waves in the optical fiber. $Y(r)$ [Pa] is the Young's modulus, $\gamma=n^4 \varepsilon_0 p_{12}$ [F/m] is the electrostriction constant, n is the glass refraction index, η_{11} [Pa s] and p_{12} [dimensionless] are the respective components of the viscosity and electrostriction tensors, and ε_0 is the vacuum permittivity. The electric field E in the right-hand side of (A1) is represented as a superposition of forward and backward propagating electro-magnetic waves

$$E(r,z,t) = \frac{f(r)}{2}\left[A_1(z,t)e^{i(\omega_1 t - \beta_1 z)} + A_2(z,t)e^{i(\omega_2 t + \beta_2 z)}\right] + c.c. \qquad \text{(A2)}$$

where $A_j(z,t)$, j=1,2, are the slowly varying envelopes of the optical field, ω_j and β_j are, respectively, frequencies and propagation constants of optical waves, and "c.c." stands for complex-conjugate. The dimensionless fundamental optical mode profile $f(r)$ satisfies the equation

$$\frac{\partial^2 f(r)}{\partial r^2} - \frac{1}{r^2}\frac{\partial f(r)}{\partial r} + \left[\frac{\omega^2 n^2(r)}{c^2} - \beta_j^{\,2}\right] f(r) = 0\,. \qquad \text{(A3)}$$

The solution to (A1) can be written in the form

$$\rho(z,t,r) = \frac{1}{2}\sum_{m=1}^{M} \rho_m(z,t)\xi_m(r)e^{i(\Omega t - qz)} + c.c. \qquad \text{(A4)}$$

where $\Omega=\omega_1-\omega_2$ is the acoustic frequency and $q=\beta_1+\beta_2 \approx 4\pi n/\lambda$ with λ being the input signal wavelength, M is the number of acoustic modes $\xi_m(r)$ which are solutions of the unperturbed (with zero right-hand side and Γ=0 equation (A1) and thus satisfy the eigenvalue equation

$$\frac{\partial^2 \xi_m(r)}{\partial r^2} - \frac{1}{r^2}\frac{\partial \xi_m(r)}{\partial r} + \left[\frac{\Omega_m^{\,2}}{v_l^{\,2}(r)} - q^2\right]\xi_m(r) = 0\,, \qquad \text{(A5)}$$

where we consider acoustic modes without axial variation since only those modes interact efficiently with the axially symmetric optical mode $f(r)$.

The nonlinear polarization induced by the acousto-optic interaction is $P_{NL} = \gamma\rho E/\rho_0$. Following the perturbative approach for the derivation of the nonlinear pulse propagation in an optical fiber with (A2) and (A4) substituted in the expression for the nonlinear polarization one can obtain the propagation equation for the optical powers $P_{1,2}$

$$\frac{dP_j}{dz} \pm \alpha P_j + \frac{g_B}{A_m^{ao}} \bar{\gamma}(\nu) P_1 P_2 = 0 . \tag{A6}$$

The upper sign applies to the forward-propagating optical wave (j=1), the lower sign applies to the backward-propagating (Stokes) wave (j=2), α, $g_B = 4\pi n^8 p_{12}^2/(\lambda^3\ \rho_0\ c\ w\ \nu_B)$, w, and ν_B are the loss and the Brillouin gain coefficients, the Brillouin gain FWHM, and the frequency shift, respectively; $\bar{\gamma}(\nu)$ is the normalized Lorentzian spectral shape

$$\bar{\gamma}(\nu) = \frac{(w/2)^2}{(\nu - \nu_1 + \nu_B)^2 + (w/2)^2}$$

and

$$A_m^{ao} = 2\pi \left[\frac{\int_0^\infty f^2(r) r dr}{\int_0^\infty \xi_m(r) f^2(r) r dr} \right]^2 \int_0^\infty \xi_m^2(r) r dr \tag{A7}$$

is the acousto-optic effective area which quantifies the peak value of the Brillouin gain.

To evaluate the SBST of the fiber one has to solve modal equations for acoustic (A5) and optical (A3) waves and calculate A_m^{ao} from (A7) for several lowest-order (i.e., having the smallest A_m^{ao}) acoustic modes. The relation for the longitudinal sound velocity used for numerical solution of (5) is

$$v_l(r) = 5944[1 - 0.078\Delta_{\%}(r)] \tag{A8}$$

where $\Delta_{\%}(r) = 100\lfloor n(r) - n_{SiO_2} \rfloor / n_{SiO_2}$.

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