Proposal for a causal transmission line model

IEEE P802.3bj Task Force

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Adam Healey, LSI Corporation

Charles Moore, Avago Technologies

Rich Mellitz, Intel Corporation

Adee Ran, Intel Corporation

Liav Ben-Artsi, Marvell Technology Group

Problem statement

- The transmission line models in IEEE P802.3bj/D3.1 are not causal
- Insertion loss deviation in the 1 mm section incorrectly modeled with a term proportional to f^2
- Expressions for cascading X sections to yield an X mm transmission line are inaccurate

A causal and stable transmission line model

- Complex propagation coefficient (units of f are GHz)

$$\gamma(f) = \begin{Bmatrix} \gamma_0 & f = 0 \\ \gamma_0 + \gamma_1\sqrt{f} + \gamma_2(f)f & f > 0 \end{Bmatrix}$$

$$\gamma_1 = a_1(1 + j)$$

$$\gamma_2(f) = a_2[1 - j(2/\pi)\log_e(f/1\ \mathrm{GHz})] + j2\pi\tau$$

- Constant reflection coefficient

$$\rho = \frac{Z_c - 2R_0}{Z_c + 2R_0}$$

A causal and stable transmission line model, continued

- Scattering parameters for a line of length d

$$s_{11}(f) = s_{22}(f) = \frac{\rho(1 - e^{-\gamma(f)2d})}{1 - \rho^2 e^{-\gamma(f)2d}}$$

$$s_{21}(f) = s_{12}(f) = \frac{(1 - \rho^2)e^{-\gamma(f)d}}{1 - \rho^2 e^{-\gamma(f)2d}}$$

Parameters of the model

- The model is a function of 5 real-valued parameters
- Coefficients of the complex propagation coefficient: γ_0, a_1, a_2, τ
- Differential characteristic impedance: Z_c
- Can be reduced to 4 parameters with some loss of generality (γ_0 = 0)

Values for the parameters

- 1 mm transmission line “section” calculated with commercial tools
- Calculated section is fit to the proposed model (see Appendix B)

Package transmission line

Table 93A−3—Transmission line model parameters

Parameter	Value	Units
γ_0	0	1/mm
a_1	1.734×10^{-3}	$ns^{1/2}$/mm
a_2	1.455×10^{-4}	ns/mm
τ	6.141×10^{-3}	ns/mm
Z_c	78.2	Ω

Host transmission line

Table 92−12—Transmission line model parameters

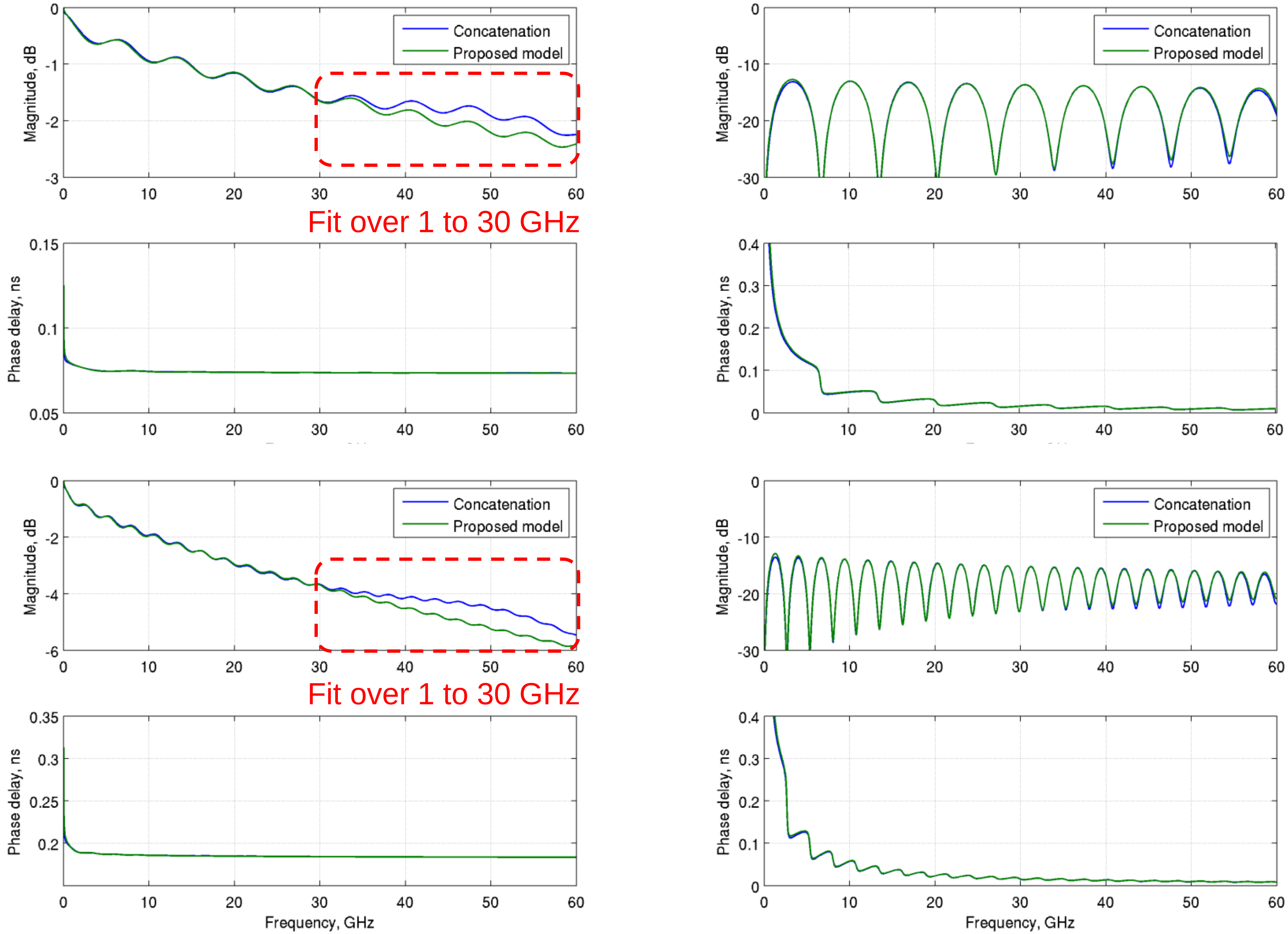
Parameter	Value	Units
γ_0	0	1/mm
a_1	4.114×10^{-4}	$ns^{1/2}$/mm
a_2	2.547×10^{-4}	ns/mm
τ	6.191×10^{-3}	ns/mm
Z_c	109.8	Ω

Package transmission line

s21 **s11**

d = 12 mm

d = 30 mm

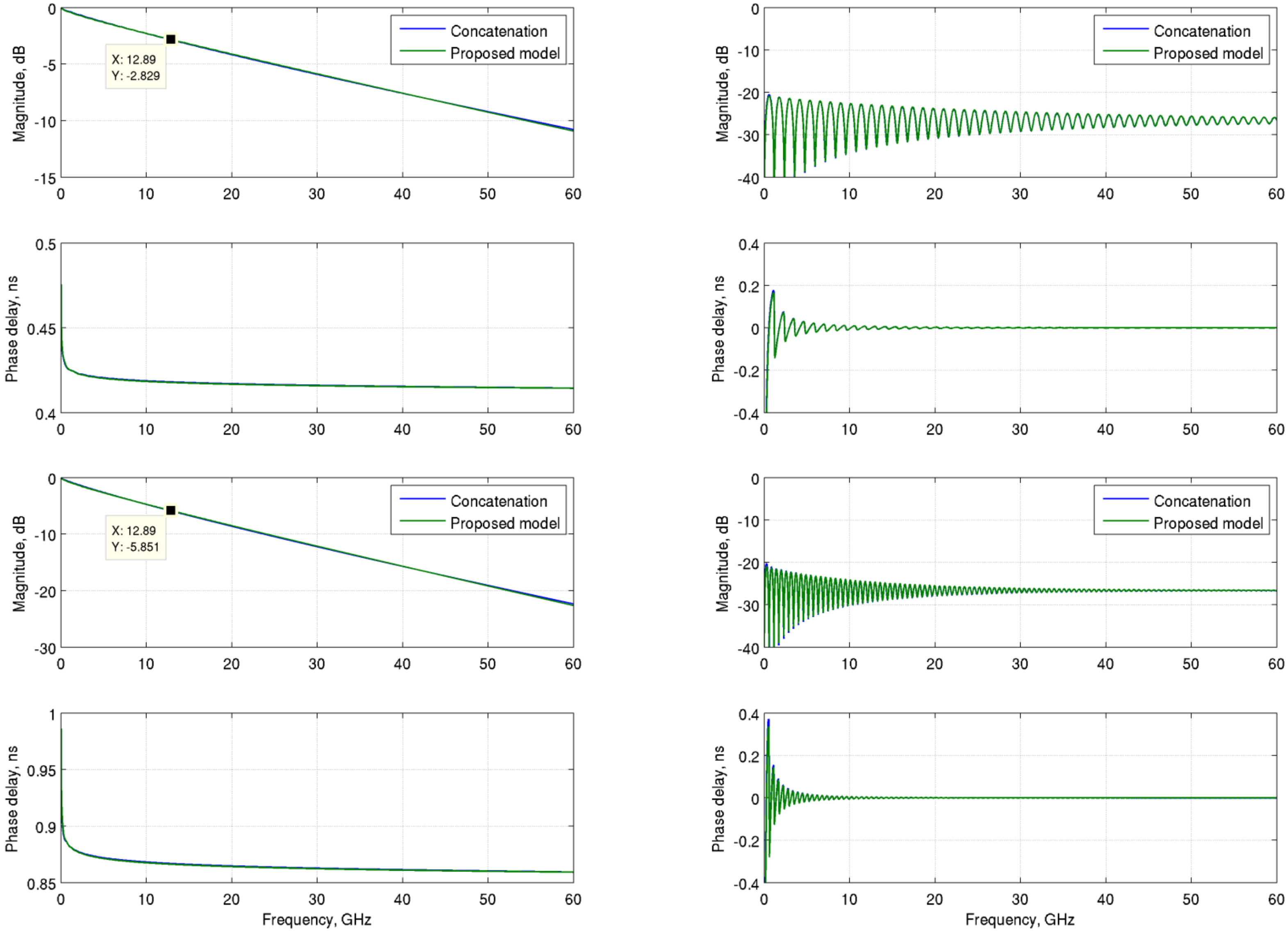


Host transmission line

s21 **s11**

d = 68 mm

d = 141 mm



Host transmission line length

- Insertion loss deviation in the 1 mm section was incorrectly modeled with a term proportional to f^2

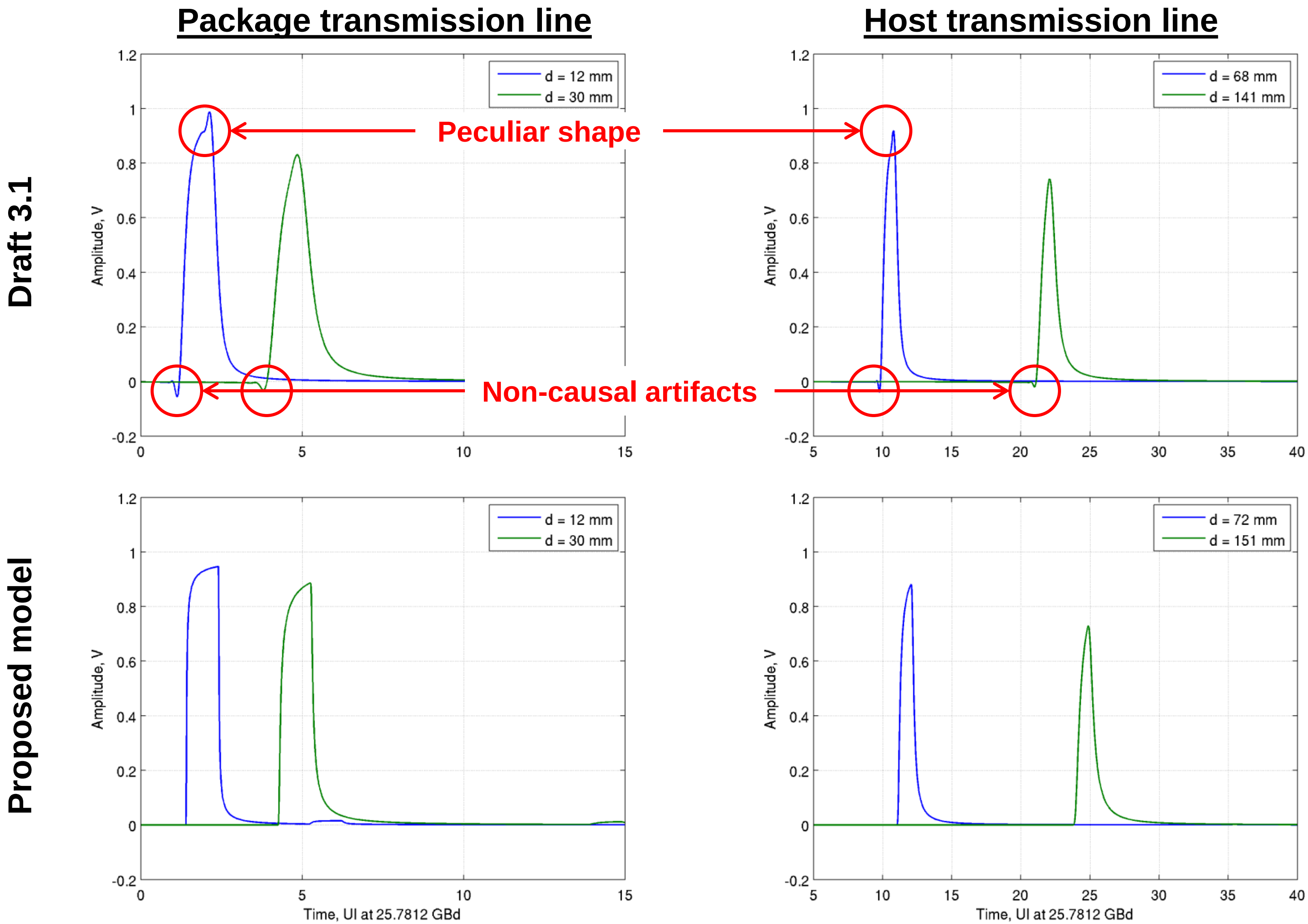
- With the insertion loss deviation correctly modeled, the f^2 term is no longer needed but this reduces the insertion loss at high frequencies

- To compensate, changes are required to 92.10.7.1.1

- Change the length of the aggressor transmitter transmission line to 72 mm (from 68 mm)

- Change the length of the victim transmitter and receiver transmission line to 151 mm (from 141 mm)

Pulse response comparison



Recommendation

- Replace the package and host transmission line models in Draft 3.1 with the models proposed in this presentation
- Simpler models that provide more accurate representations of the transmission lines
- Causal by construction
- Adjustment to host transmission line length needed to maintain the 3 and 6.26 dB insertion loss targets
- The impact on the COM is shown in Appendix C.
- A companion document illustrating detailed changes to the draft will be provided

Appendix A

Derivation of the model

Complex propagation coefficient

- Model real part α as a polynomial in $\sqrt{|f|}$

$$\alpha(f) = a_0 + a_1\sqrt{|f|} + a_2|f|$$

- Derive complex propagation coefficient from α using the Hilbert transform

$$\gamma(f) = \alpha(f) + j\beta(f) = \mathcal{F}^{-1}[\mathcal{F}[\alpha(f)](\xi) \cdot 2H(\xi)](f)$$

- $H(x)$ is the Heaviside step function and $\mathcal{F}[f(x)](\xi)$ is the Fourier transform of $f(x)$ with frequency variable ξ

Complex propagation coefficient, continued

$$A(\xi) = \mathcal{F}[\alpha(f)](\xi) = a_0\delta(\xi) - \frac{a_1}{2|\xi|^{3/2}} - \frac{a_2\sqrt{2/\pi}}{|\xi|^2}$$

$$\gamma(f) = \mathcal{F}^{-1}[A(\xi)\cdot 2H(\xi)](f) = \gamma_0 + \gamma_1(f)\sqrt{|f|} + \gamma_2(f)|f|$$

$$\gamma_0 = a_0$$

$$\gamma_1(f) = a_1(1 + j\mathrm{sgn}(f))$$

$$\gamma_2(f) = a_2(1 + j\mathrm{sgn}(f)[2/\pi][\psi(1) + 1 - \log_e(f)])$$

- Note $\psi(x)$ is the digamma function and $-\psi(1)$ is the Euler-Mascheroni constant (0.57721566…)

- Also note $\lim_{x\to 0} x\log_e(x) = 0$

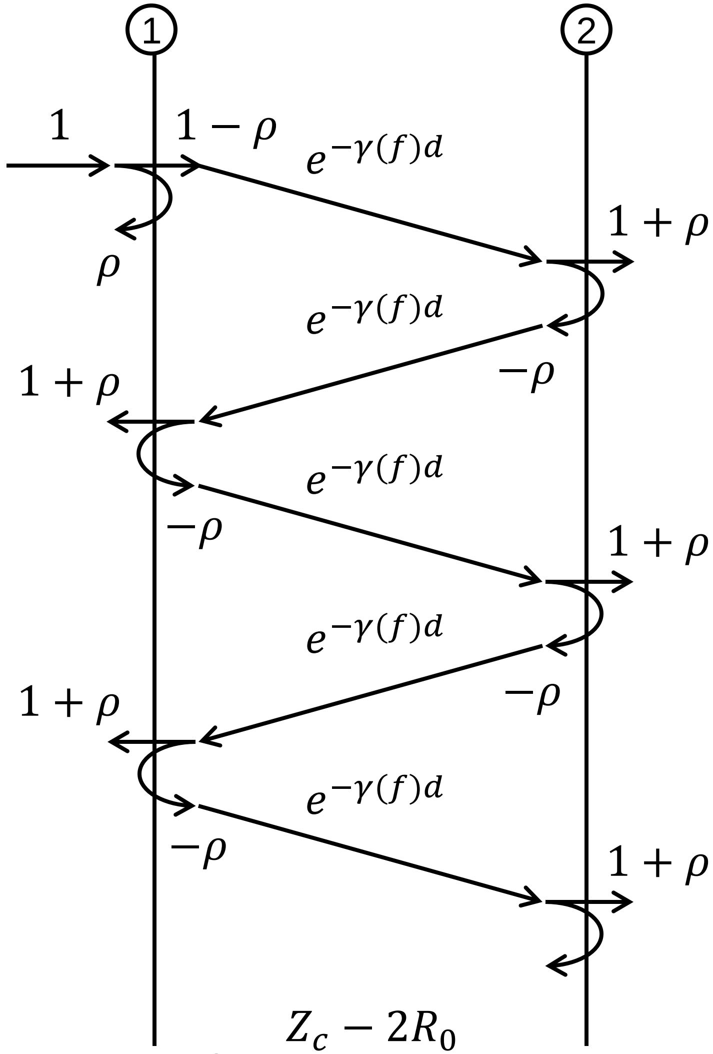
Complex propagation coefficient, continued

- Given transmission line length d, the construction of $\gamma(f)$ ensures that $e^{-\gamma(f)d}$ is minimum phase (causal and stable)
- Include propagation delay as the factor $e^{-j2\pi f\tau\prime}$
- Or equivalently…

$$\gamma_2(f) = a_2\left(1 - j\mathrm{sgn}(f)(2/\pi)\log_e(f)\right) + j\mathrm{sgn}(f)2\pi\tau$$

$$\tau = (\psi(1) + 1)/\pi^2 + \tau'$$

Scattering parameters



$$\rho = \frac{Z_c - 2R_0}{Z_c + 2R_0}$$

- Reflected signal at port 1

$$s_{11}(f) = \rho + (1-\rho^2)\,(-\rho)\,e^{-\gamma(f)2d} + (1-\rho^2)\,(-\rho)^3 e^{-\gamma(f)4d} + \cdots$$

$$s_{11}(f) = \rho + (1-\rho^2)(-\rho)e^{-\gamma(f)2d} \sum_{n=0}^{\infty} (-\rho)^{2n} e^{-\gamma(f)2nd}$$

- Transmitted signal at port 2

$$s_{21}(f) = (1-\rho^2)\,e^{-\gamma(f)d} + (1-\rho^2)\,(-\rho)^2 e^{-\gamma(f)3d} + \cdots$$

$$s_{21}(f) = (1-\rho^2)\,e^{-\gamma(f)d} \sum_{n=0}^{\infty} (-\rho)^{2n} e^{-\gamma(f)2nd}$$

Scattering parameters, continued

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \text{ for } |x| < 1$$

$$s_{11}(f) = \rho + \frac{(1-\rho^2)(-\rho)e^{-\gamma(f)2d}}{1-\rho^2 e^{-\gamma(f)2d}} = \frac{\rho\left(1-e^{-\gamma(f)2d}\right)}{1-\rho^2 e^{-\gamma(f)2d}}$$

$$s_{21}(f) = \frac{(1-\rho^2)\, e^{-\gamma(f)d}}{1-\rho^2 e^{-\gamma(f)2d}}$$

- Assuming the network is symmetric $s_{22(f)} = s_{11(f)}$ and $s_{12(f)} = s_{21(f)}$

Appendix B

Fitting the model

Step 1

- Convert differential mode scattering parameters to ABCD parameters

$$S = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix}$$

$$\Delta S = s_{11}s_{22} - s_{12}s_{21}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \frac{1}{2R_0 s_{21}} \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix}$$

$$A' = R_0(1 + s_{11} - s_{22} - \Delta S)$$

$$B' = R_0^2\,(1 + s_{11} + s_{22} + \Delta S)$$

$$C' = 1 - s_{11} - s_{22} + \Delta S$$

$$D' = R_0(1 - s_{11} + s_{22} - \Delta S)$$

Step 2

- Extract complex propagation coefficient and characteristic impedance as functions of frequency

- Given the ABCD parameters for a transmission line…

$$\begin{bmatrix} A(f) & B(f) \\ C(f) & D(f) \end{bmatrix} = \begin{bmatrix} \cosh(\gamma(f)d) & Z\,(f)\cdot\sinh(\gamma(f)d) \\ \sinh(\gamma(f)d)/Z(f) & \cosh(\gamma(f)d) \end{bmatrix}$$

- …$\gamma(f)$ and $Z_c(f)$ can be derived as follows

$$\gamma(f) = \cosh^{-1}(A(f))/d = \log_e(A(f) + \sqrt{A(f)+1}\sqrt{A(f)-1})/d$$

$$Z(f) = \sqrt{B(f)/C(f)}$$

Step 3

- Fit real part of the propagation coefficient $\alpha(f)$ to $\gamma_0 + a_1\sqrt{f} + a_2 f$
- Set γ_0 to $\alpha(0)$
- The values of a_1 and a_2 are chosen to minimize the mean squared error between $\alpha(f) - \gamma_0$ and $a_1\sqrt{f} + a_2 f$
- Derive τ from the imaginary part of the propagation coefficient $\beta(f)$

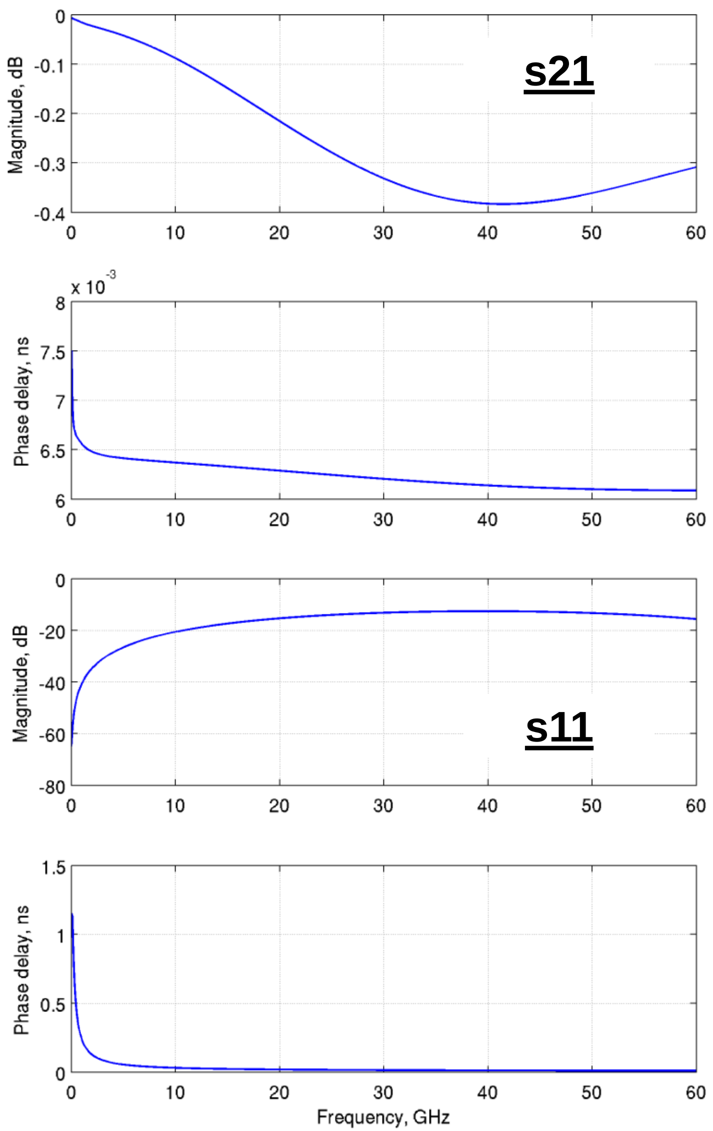
$$\tau = \frac{\beta(f_{\text{max}})}{2\pi f_{\text{max}}} - \frac{a_1/(2\pi)}{\sqrt{f_{\text{max}}}} + \frac{a_2}{\pi^2}\log_e\left(\frac{f_{\text{max}}}{1\text{ GHz}}\right)$$

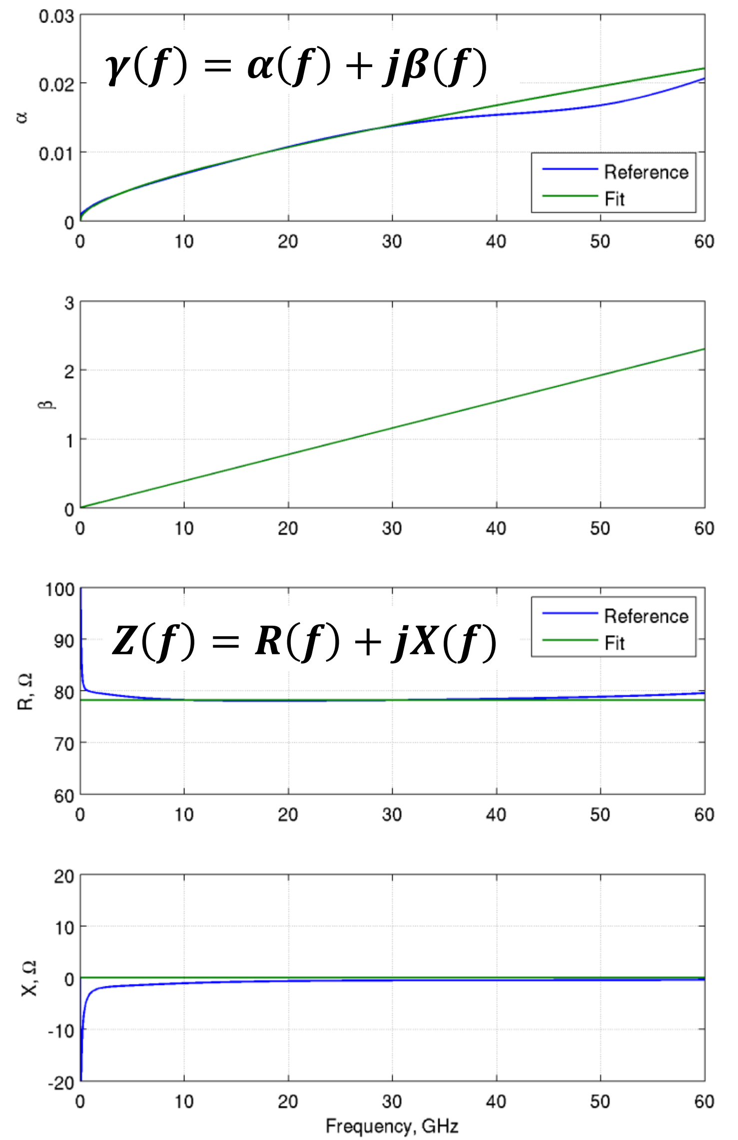
- Note f_{max} is the highest frequency included in the fit

Step 4

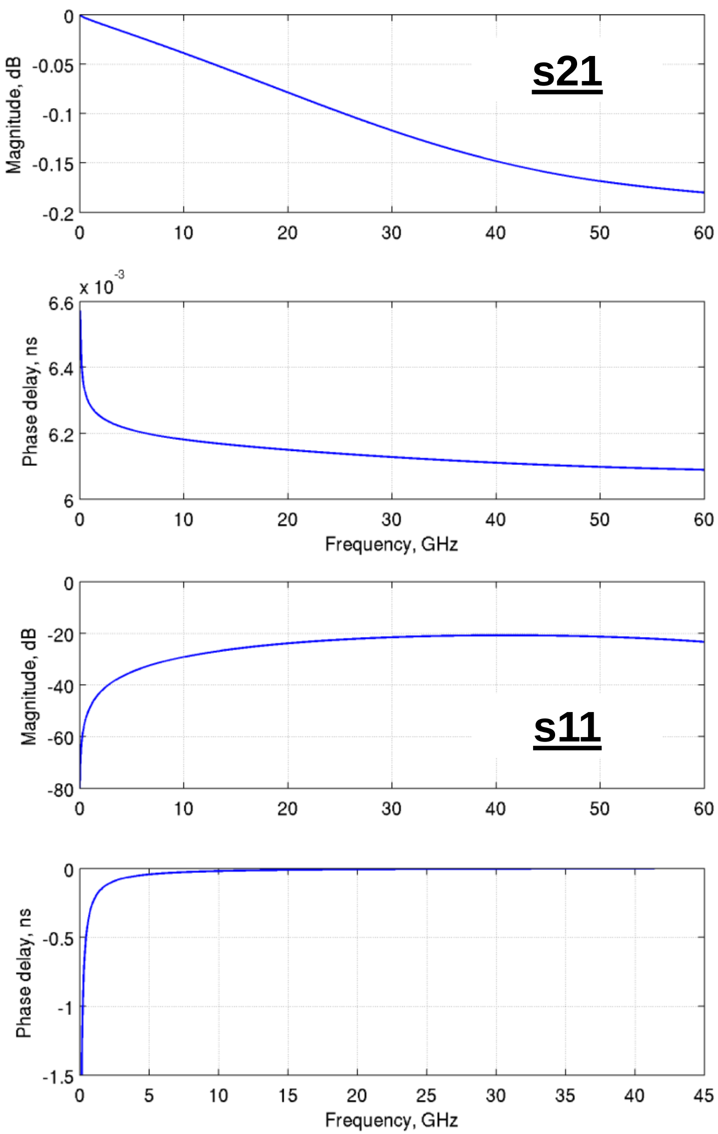
- Choose Z_c to be $|Z(f_{\mathrm{max}})|$

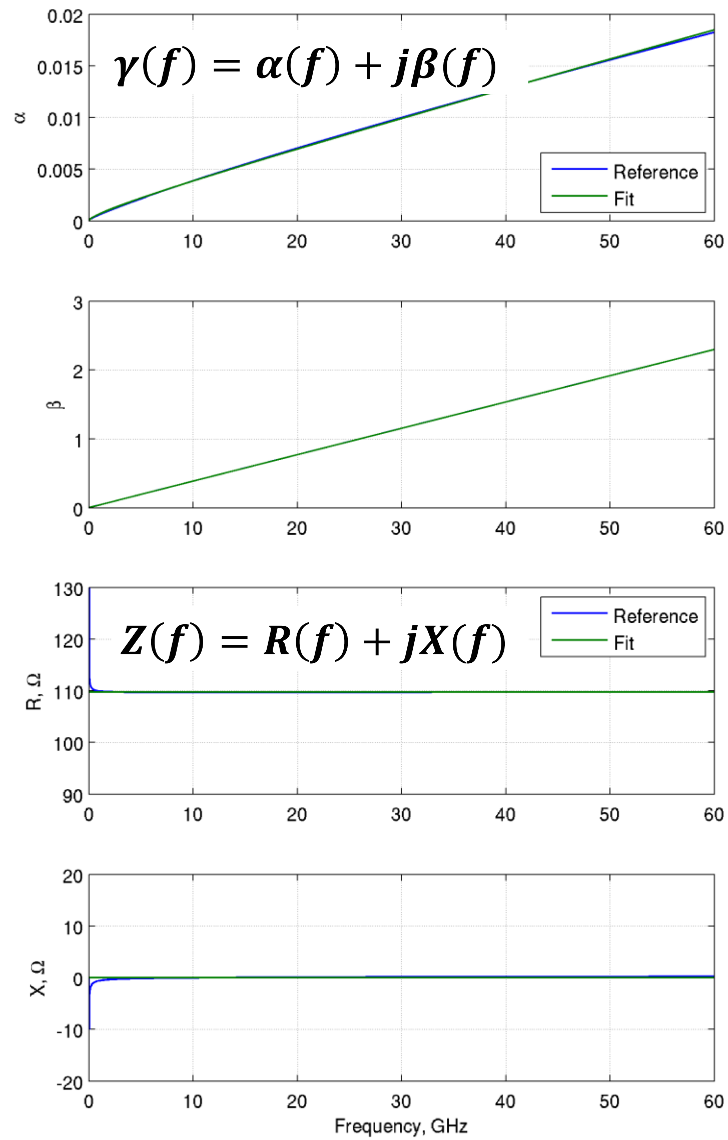
Example: 1 mm package transmission line





Example: 1 mm host transmission line

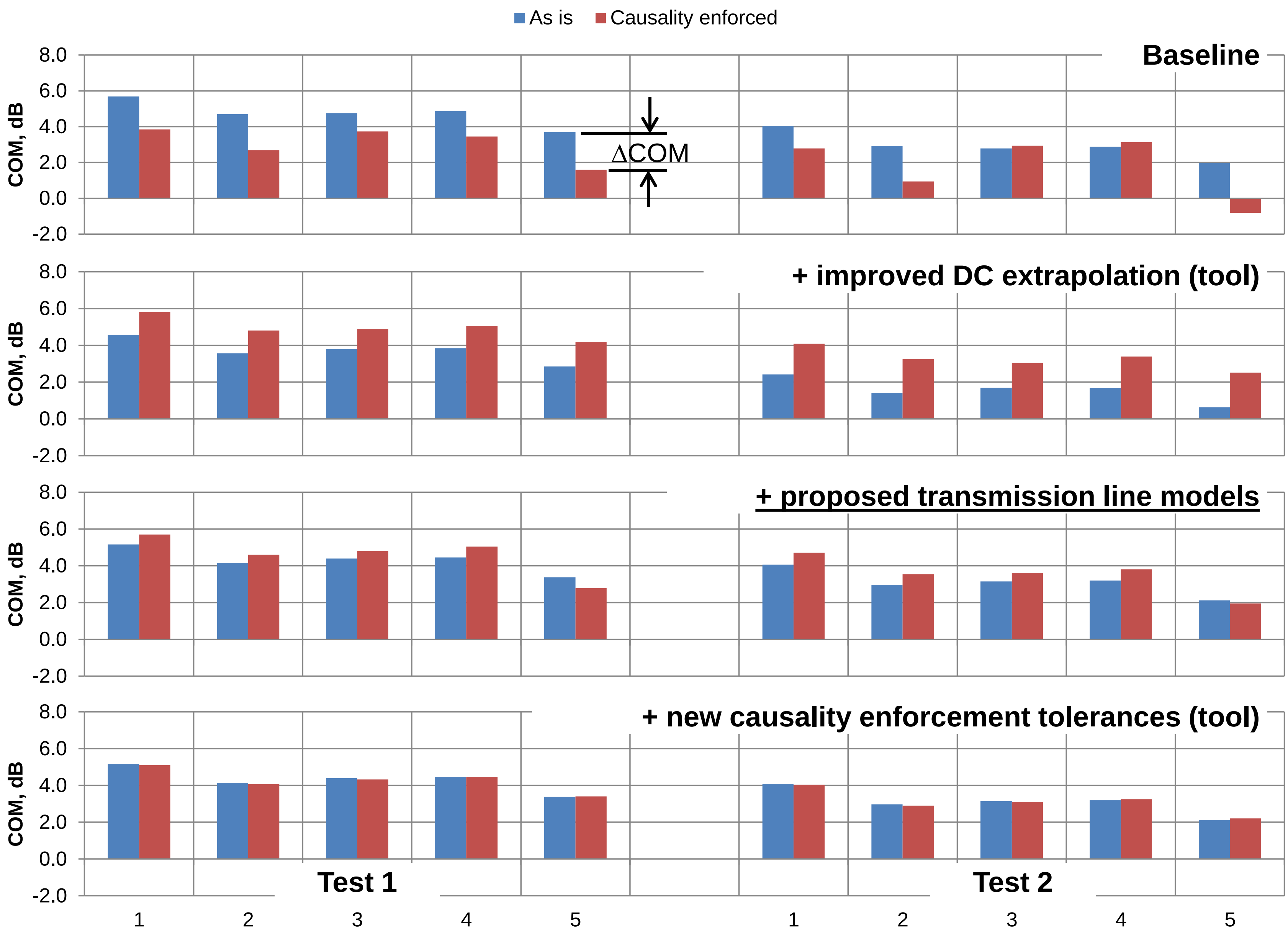




Appendix C

Regression testing

100GBASE-CR4 channels



ΔCOM ≤ 2.79 dB

ΔCOM ≤ 1.88 dB

ΔCOM ≤ 0.65 dB

ΔCOM ≤ 0.08 dB

100GBASE-KR4 channels