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Dk & Df ALGEBRAIC MODEL v1.01

Let’s take a look…

WHAT YOU WILL SEE

LOOK & FEEL

NOTE: The Change Parameter window is a visual basic macro. If you save the file to your computer, be sure to select the Maco-Enabled file type.

Material/Design Characteristics	
~~Simple/Detailed~~	
~~Char. Imp. (Ohm)~~	
Length (inch)	40
Trace Width (mil)	8
Cu Thickness (mil)	0.67
Diel. Thickness (mil)	17

Freq	Dk	Df
1.00E+08	3.6	0.0092
1.00E+09	3.6	0.0092
2.00E+09	3.5	0.0115
5.00E+09	3.5	0.0115
1.00E+10	3.4	0.0125
2.00E+10	3.2	0.014

CHANGE PARAMETERS

Click to change the input parameters

Loss at 6GHz: 20.88 dB
Loss at 12.5GHz: 40.54 dB
Loss at 14GHz: 44.98 dB

DK fit to Second Order Equation	c2	3.64774E-23	Df fit to Second Order Equation	c2	-1.1521E-23
	c1	-2.02392E-11		c1	4.54439E-13
	b	3.591[illegible]		b	0.009448676

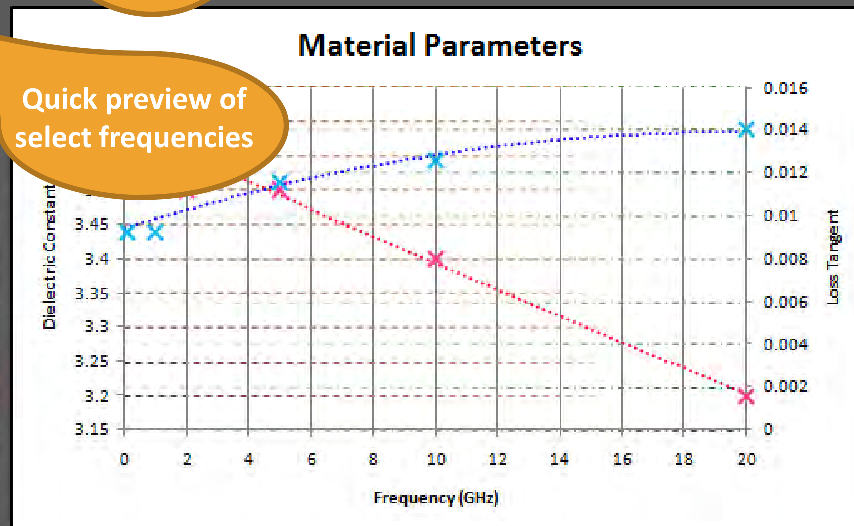
Fitted equation(s):
$Dk(f) = c2*f^2 + c1*f + b$

Quick preview of select frequencies

Frequency (GHz)	Zo	A_{total}
0.01	49.92196	0.378882
0.05	49.92759	0.89306
0.1	49.93462	1.311803
0.15	49.94166	1.652774
0.2	49.9487	1.953647
0.25	49.95574	2.229015
0.5	49.99096	3.404911
0.75	50.02623	4.414474
1	50.06154	5.341849
1.25	50.09689	6.219524
1.5	50.13229	7.064073
1.75	50.16772	7.885265
2	50.20321	8.6894
2.25	50.23873	9.480792
2.5	50.274[illegible]	10.26253
2.75	[illegible]	11.0369
3	[illegible]	11.80563
3.25	[illegible]	12.57005
3.5	[illegible]	13.33122
3.75	[illegible]	14.08995
4	[illegible]	14.84692
4.25	[illegible]	15.60264
4.5	[illegible]	16.35754
4.75	[illegible]	17.11196
5	50.63236	17.86617

Tab allows the user to input material and design specifics

UserInput | Meg6 | ImpvFR4 | KR Spec | AttenuationPlot



LOOK & FEEL

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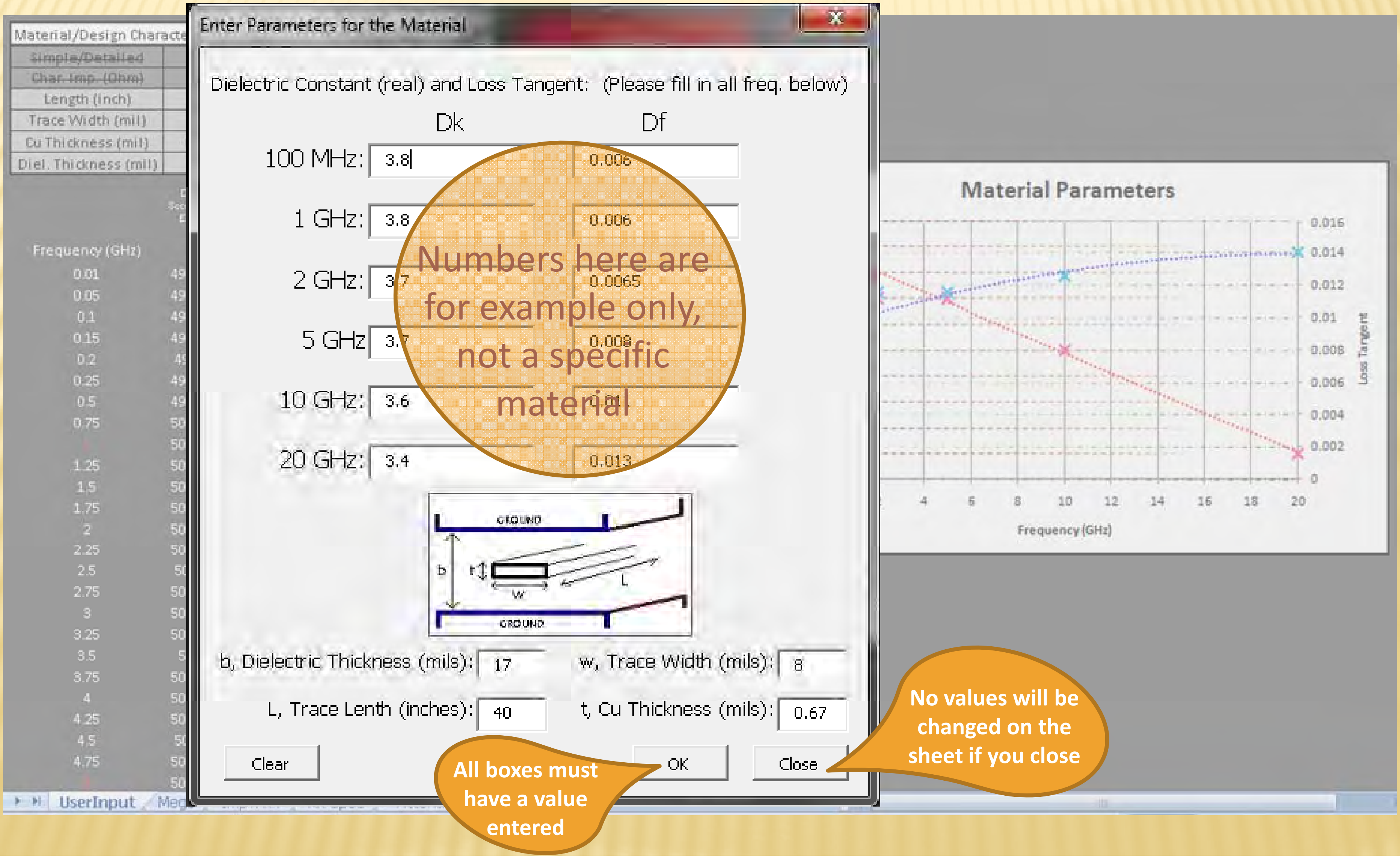
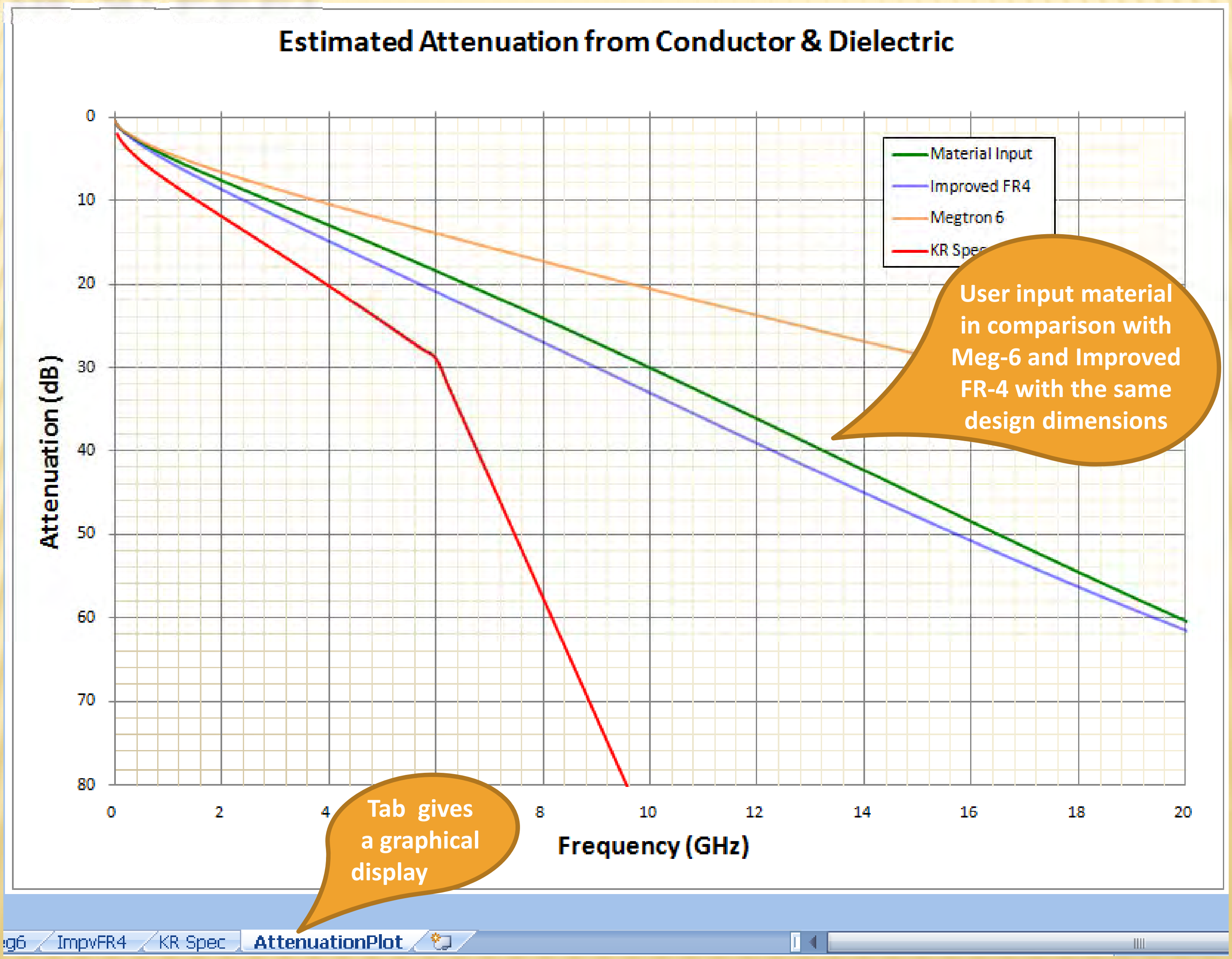


LOOK & FEEL


Estimated Attenuation from Conductor & Dielectric
Attenuation (dB)
0
10
20
30
40
50
60
70
80
0
2
4
8
10
12
14
16
18
20
Frequency (GHz)
Material Input
Improved FR4
Megtron 6
KR Spec
User input material in comparison with Meg-6 and Improved FR-4 with the same design dimensions
Tab gives a graphical display
ImpvFR4
KR Spec
AttenuationPlot

Behind the Scenes…

EQUATIONS AND REFERENCES OF MODEL

FREQUENCY DEPENDENCE

- 6 input frequency points for Dk and Df
- Fit Dk and Df to second order equations
  - Coefficients shown on sheet
  - Graphical representation shown on sheet
- Note that frequency dependence fit is only approximated to 20G, therefore, loss approximations should only be considered to 20G
- Z_0 is calculated with Dk (or ε_r) at a given frequency; a similar technique is used in loss calculations

CHARACTERISTIC IMPEDANCE[2, EQU 4-5]

$$Z_0 = \frac{94.15}{\sqrt{\varepsilon_r}\left(\frac{\frac{w}{b}}{\frac{1-t}{b}} + \frac{c'_f}{0.0885\varepsilon_r}\right)} ohms$$

$$c'_f = \frac{0.0885\varepsilon_r}{\pi}\left\{\frac{2}{1-\frac{t}{b}}\log_e\left(\frac{1}{1-\frac{t}{b}}+1\right) - \left(\frac{1}{1-\frac{t}{b}}-1\right)\log_e\left(\frac{1}{\left(1-\frac{t}{b}\right)^2}-1\right)\right\}$$

ε_r = relative dielectric constant (at a given frequency)

b = plate spacing (mil)

w = trace width (mil)

t = trace thickness (mil)

c'_f = fringing capacitance (μf/cm) * assuming semi - infinite plate between two infinite ground planes, but good approximation for w/(b - t) $\geq$ 0.35

ATTENUATION IN LOSSY LINES

- Attenuation per length[1, EQN 9-54]:

$$\alpha_n = \sqrt{\frac{1}{2}\left[\sqrt{\left(R_L^2 + \omega^2 L_L^2\right)\left(G_L^2 + \omega^2 C_L^2\right)} - \omega^2 L_L C_L + R_L G_L\right]} \text{ nepers/len gth}$$

- Using a low-loss approximation[1, EQN 9-55]:

$$\alpha_n = \frac{1}{2}\left(\frac{R_L}{Z_0} + G_L Z_0\right) \text{ nepers/len gth}$$

- But we don't typically discuss in nepers…[1, EQN 9-57]

$$10^{\frac{\alpha_{dB}}{20}} = e^{\alpha_n} \qquad \therefore \alpha_{dB} = 20\log_{10} e \times \alpha_n$$

$$\text{for ease of notation}: Y_{n\to dB} = 20\log_{10} e$$

CONDUCTOR LOSS (per inch)

$$\alpha_{cond} = \left(\mathrm{Y}_{\mathrm{n}\to\mathrm{dB}}\right)\times\frac{\mathrm{R_L}}{\mathrm{Z_0}} \quad \begin{cases} \alpha_{\mathrm{cond}} = \text{attenuation of amplitude due to conductor loss, in dB/length}^{[1,\text{ EQN 9-59}]} \\ \mathrm{Y}_{\mathrm{n}\to\mathrm{dB}} = \text{converstion from nepers to dB} \\ \mathrm{R_L} = \text{resistance per length of conductor} \\ \mathrm{Z_0} = \text{characteristic impedance} \end{cases}$$

- Skin effect is accounted for in resistance[3, EQN 7]:

$$R = \frac{l}{w}\sqrt{\pi\mu\rho f} = l\times R_L \quad \begin{cases} l = \text{lenth of trace (inch)} \\ w = \text{width of trace (inch)} \\ \mu = \text{permeability of Cu} \approx 4\pi\times10^{-7}\times1 \\ \rho = \text{resistivity of Cu} = \dfrac{1}{\sigma} = \dfrac{1}{5.69\times10^{7}} \end{cases}$$

- Conductor loss per inch as entered in the model:

$$\boxed{\alpha_{cond} = \frac{1}{2}\times\left(20\log_{10} e\right)\times\left(\sqrt{\frac{\pi\left(4\pi\times10^{-7}\right)}{5.69\times10^{7}}}\,\frac{\sqrt{f_{\text{in Hz}}}}{w_{\text{in mils}}}\right)\times\frac{1}{Z_0}}$$

DIELECTRIC LOSS (per inch)

$$\alpha_{diel} = \left(\mathrm{Y}_{\mathrm{n}\to\mathrm{dB}}\right) \times \mathrm{G}_L Z_0 \quad \begin{cases} \alpha_{\mathrm{cond}} = \text{attenuation of amplitude due to dielectric loss, in dB/length}^{[1,\text{ EQN 9-60}]} \\ \mathrm{Y}_{\mathrm{n}\to\mathrm{dB}} = \text{converstion from nepers to dB} \\ \mathrm{G}_{\mathrm{L}} = \text{shunt conductance per length from dielectric} \\ \mathrm{Z}_0 = \text{characteristic impedance} \end{cases}$$

- As developed by Bogatin…

$$G_L = \omega \tan(\delta) C_L \qquad Z_0 = \frac{\sqrt{\varepsilon_r}}{cC_L} \quad \begin{cases} G_L \text{ equation}^{[1,\text{ EQN 9-19, EQN 9-60}]} \\ Z_0 \text{ equation}^{[1,\text{ EQN 9-67}]} \text{ is used to cancel the capacitance value,} \\ \qquad \text{the } \mathrm{Z}_0 \text{ value for a given frequency is NOT used} \\ c = \text{speed of light m/s} \quad \therefore \text{converstion m} \to \text{in. is needed} \end{cases}$$

- Dielectric loss per inch as entered in the model:

$$\boxed{\alpha_{diel} = \frac{1}{2} \times \left(20 \log_{10} e\right) \times \left(2\pi f \times D_f\right) \times \frac{\sqrt{\varepsilon_r}}{299792458 * 39.37}}$$

REFERENCES

[1] E. Bogatin, *Signal Integrity – Simplified*. Pearson Education, Inc., 2004. ISBN 0-13-066946-6

[2] S. B. Cohn, "Problems in Strop Transmission Lines," *IRE Trans. Microwave Theory and Techniques*, Vol. MTT-3, March, 1955, pp 119-126.

[3] H. A. Wheeler, "Formulas for the Skin Effect," *Proceedings of IRE*, Vol 30, Septempber, 1942, pp 412-424.

TRACKING THE CHANGES

Version	Change
1.01	9/26/2011 – Initial release – second order Dk & Df approximation, track user input channel along with Meg-6 & Improved FR-4 for given length/width/thickness, 3 materials compared to KR limit line.