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# Evaluation of FEC Performance with Symbol and Bit muxing Scenarios

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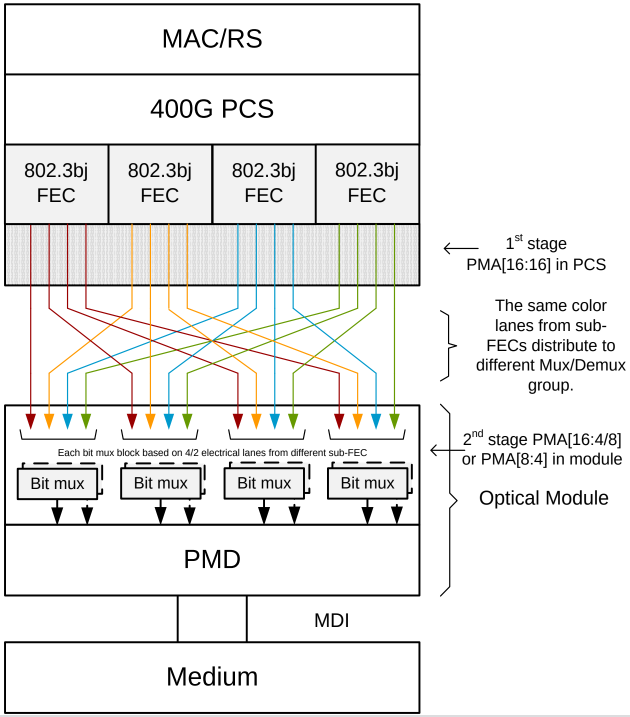
# Motivation

- When burst errors happen in electrical and optical lanes, interleaving data from parallel FEC instances in 400GbE maintains the FEC coding gain.
- The method of using FEC Orthogonal Multiplexing (FOM) in 400GbE was presented in the Indian Wells meeting in January 2014 (*wang_400_01a_0114.pdf*), which also enables protocol-agnostic optical modules with lower cost & increased broad market potential.




- This contribution includes the quantitative analysis of FOM.
- Four Muxing methods are illustrated: (use 4:1 muxing as example)
  - Orthogonal symbol mux
  - Orthogonal bit mux
  - Non-orthogonal symbol mux
  - Non-orthogonal bit mux
- Similar calculation applies to 2:1 Muxing

# Post FEC BER Calculation

- **Random Error Model**

$$SER_{pre} = 1 - (1 - BER_{pre})^{m}$$

$$P_{UE} = \sum_{i=t+1}^{n} \frac{i}{n} * \binom{n}{i} * SER_{pre}^{i} * \left(1 - SER_{pre}\right)^{n-i}$$

$$BER_{post} \approx P_{UE} / m$$

- **Burst Error Model (Gilbert model)**
  - Error propagation is modeled by probability calculation*

$$BER_{post} = \sum_{i=t+1}^{rll_{\max}} \sum_{all\ E} p(rll = i, E) * W(E) * BER_{pre} * (1 - BER_{pre})^{n - rll_{\max} - i}$$

Average weight of burst error

All the combinations of the error pattern

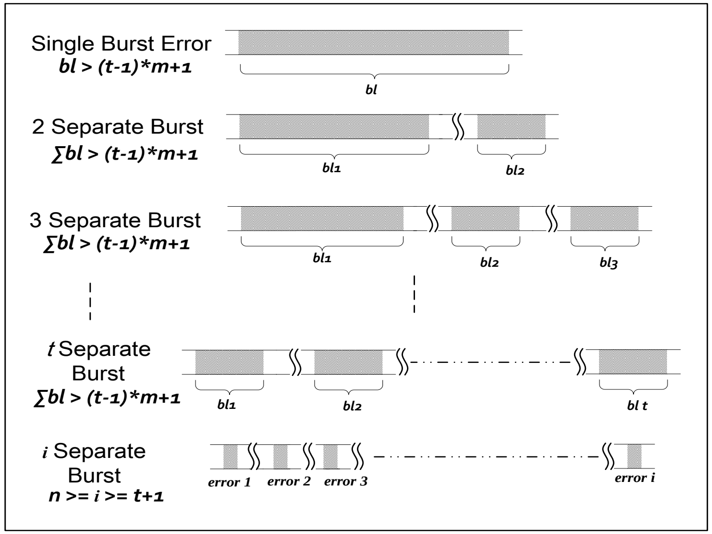
Burst error propagation length

Length of uncorrectable burst errors

* Cathy Ye Liu and Joe Caroselli, “Modeling and Mitigation of Error Propagation of Decision Feedback Equalization in High Speed Backplane Transceivers.” Proceedings of DesignCon 2006.

# Post FEC BER Calculation(Cont'd)

- **Uncorrectable Burst Error Patterns for RSFEC(*n,k,t,m*)**
  - Single burst error (>*t* symbols)
  - Separate burst errors (∑***bl*** > t symbols)
    - *2, 3, … t* separate burst errors.
  - More than *t* separate burst errors




- **Post BER**
  - Assuming
    - Rllmax = 17bit, b=0.5*;
    - G(x) is the probability for having a burst error with length of x symbol.
    - M(x) is the probability for having a burst error equal to or longer than x symbol, i.e. $M(x)=\sum_{i=x}^{rll\max/m} G(i)$
  - $\mathrm{BER}_{\mathrm{Post}}$

$$BER_{post}=\binom{n}{1}*M(t+1)*W(E)+\binom{n}{2}*\sum_{i=1}^{t}G(i)*M(t-i)*W(E)+\binom{n}{3}*\sum_{i=1}^{t}\sum_{j=1}^{t-i}G(i)*G(j)*M(t-i)*W(E)+\ldots+\sum_{i=t+1}^{n}\binom{n}{i}*G(i)*W(E)$$

Probability of having more than *t+1* separate burst errors

Probability of having three burst error whose total length is longer than *t+1* symbols

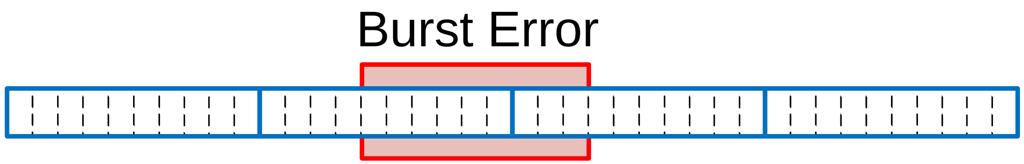
Probability of having two burst error whose total length is longer than *t+1* symbols

Probability of having a single burst error longer than *t+1* symbols

**refer to Cideciyan_02a_1111, liu_01_1105*

# Error Symbol Number and Probability

## - On Single RS(528,514)

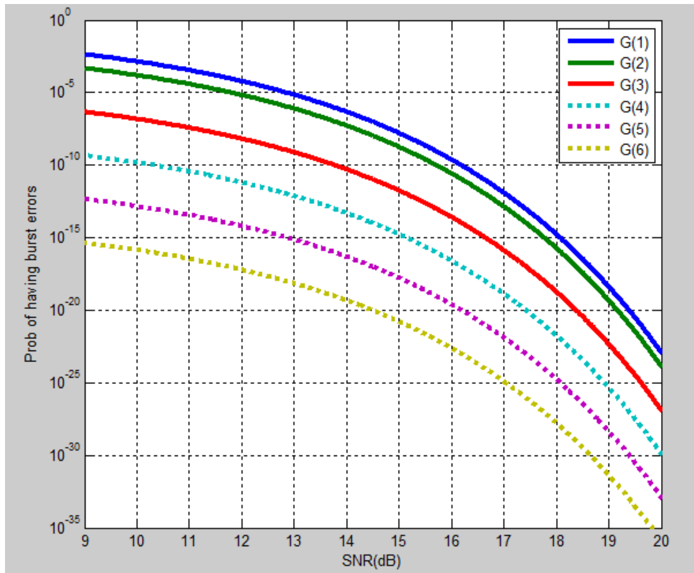



- **For a burst with length of *bl* bits, the number of error symbols and corresponding probability can be calculated as below,**

$$Error\ Symbol\ Number = \begin{cases} ceil(\frac{bl}{m})+1; & of\ \ prob_1 = \frac{|bl\%m-1|}{m} \\ ceil(\frac{bl}{m}); & of\ \ prob_2 = 1-prob_1 \end{cases}$$

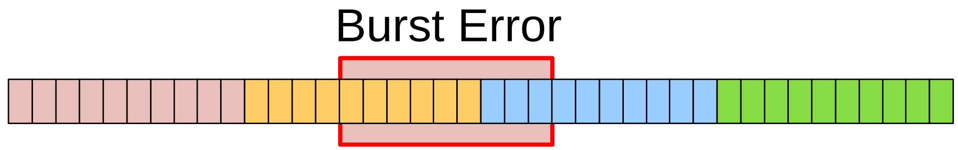
*Eq 1-1*

- **Offset of burst error in symbols affects the error symbol number.**
  - E.g., a 2 bit burst error may cause 2 error symbols by a probability of 10% , and may become 1 error symbol by 90%.
- **From *Eq1-1*, G(x) curves in figure shows**
  - G(1)>G(2)>G(3)>0, and G(4),…,G(7)=0 @Rllmax=17bit
  - G(1)>G(2)>G(3)>G(4)>G(5)>G(6), and G(7)=0 @Rllmax=50bit



# Error Symbol Number and Probability

## - Orthogonal Symbol Mux 4:1



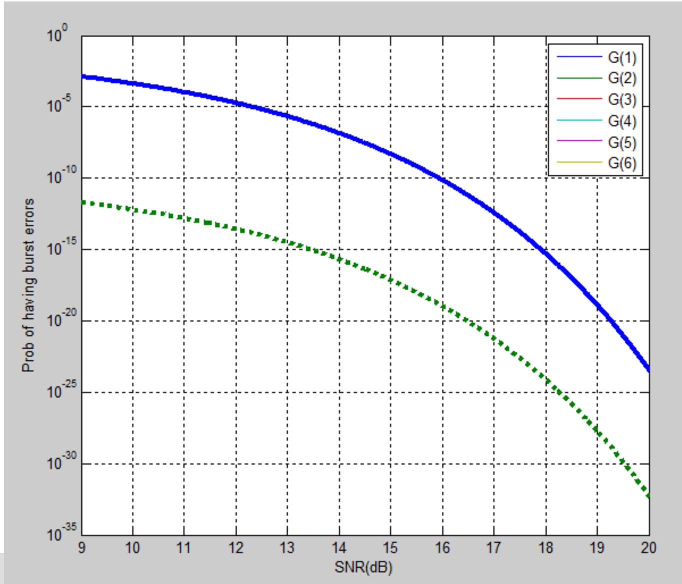

- **A burst with length of *bl* bits becomes x/x+1 symbols according to *Eq1-1*;**
- **The number of error symbols on each FEC lane can be calculated as**

$$Error\ Symbol\ \ Number = \begin{cases} ceil(\frac{x+1}{4}); & of\ \ prob_a = ((x\%4)/4) * prob_2 + (((x+1)\%4)/4) * prob_1; \\ floor(\frac{x}{4}); & of\ \ prob_b = 1 - prob_a; \end{cases}$$

*Eq 2-1*

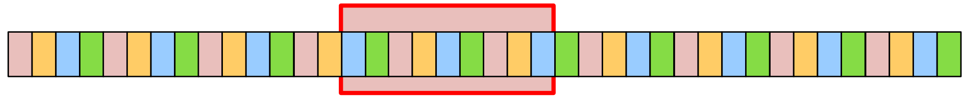
- For example, a 2bit burst become
  - Before 4:1 orthogonal symbol mux
    - 2 error symbol by 10% and 1 error symbols by 90%.
  - After
    - 1 error by 27.5% and 0 errors by 72.5%.

- **G(x) curves in figure shows**
  - G(1)> 0 and G(2),G(3),..,G(7)=0 @Rllmax=17bit;
  - G(1)> G(2)>0 and G(3),G(4)..,G(7)=0 @Rllmax=50bit;
  - Much less probability of having long burst in codeword.



# Error Symbol Number and Probability

## - Orthogonal Bit Mux 4:1

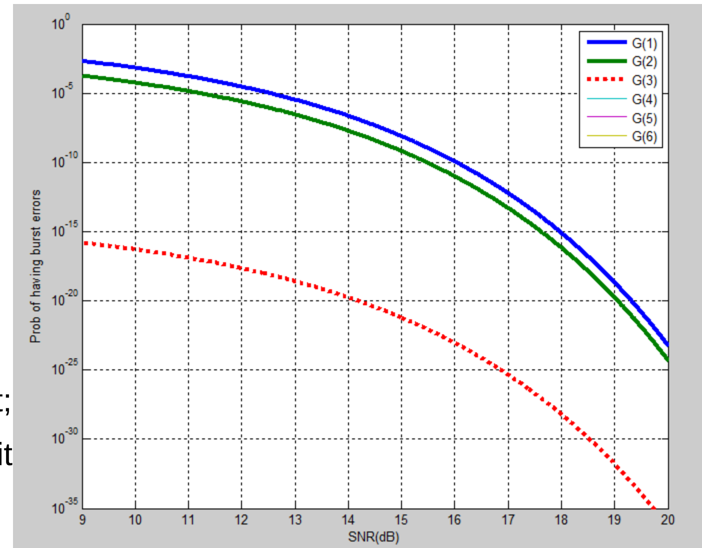



- **A burst with length of *bl* bits divides to shorter burst errors with length of floor(*bl*/4) or ceil(*bl*/4),**

$$Burst\ Length\,(bits) = \begin{cases} ceil(\frac{bl}{4}); & of\ \ prob_1 = (bl\%4)/4; \\ floor(\frac{bl}{4}); & of\ \ prob_2 = 1 - prob_1; \end{cases}$$

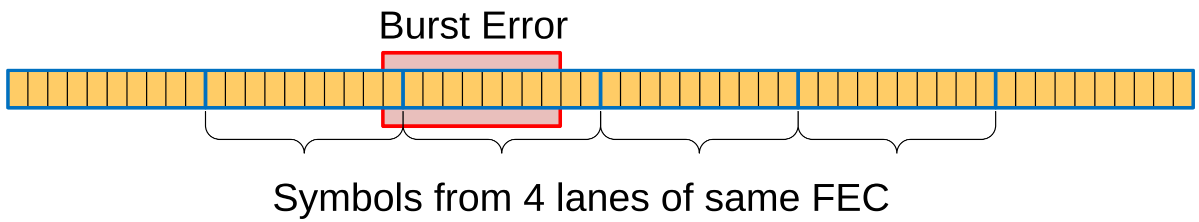
*Eq 3-1*

- **Then the number of error symbols caused by shorter burst on each FEC lane can be calculated by *Eq1-1*.**
- **G(x) curves in figure shows ,**
  - G(1)>G(2)>0, and G(3), G(4),G(5),G(6),G(7)=0;@Rllmax=17bit;
  - G(1)>G(2)>G(3)> 0, and G(4),G(5),G(6),G(7)=0;@Rllmax=50bit

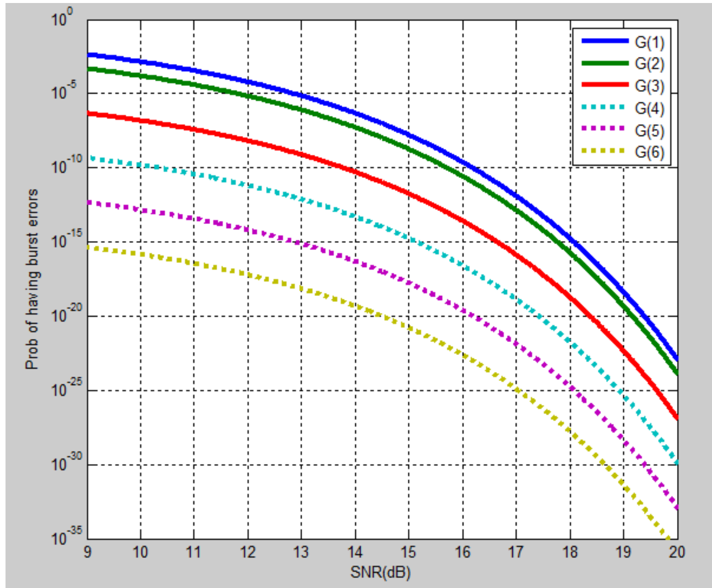


# Error Symbol Number and Probability

## - Non-Orthogonal Symbol Mux 4:1

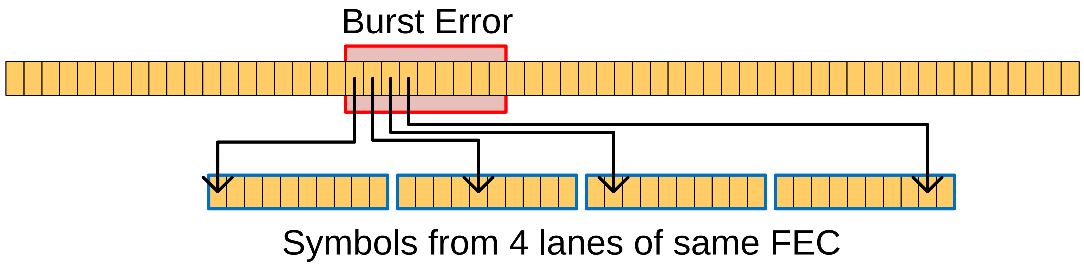



- **In worst case, symbols from 4 lanes belong to the same RS FEC codeword;**
- **The worst performance should be same as using one RS(528,514).**
- **A burst with length of *bl* bits becomes x/x+1 error symbols by *Eq1-1*;**
- **Add up error symbols from 4 lanes in one codeword.**
- **G(x) curves in figure shows,**
  - G(1)>G(2)>G(3)>0, and G(4),…,G(7)=0 @Rllmax=17bit;
  - G(1)>G(2)>G(3)>G(4)>G(5)>G(6)>0, and G(7)=0 @Rllmax=50bit;

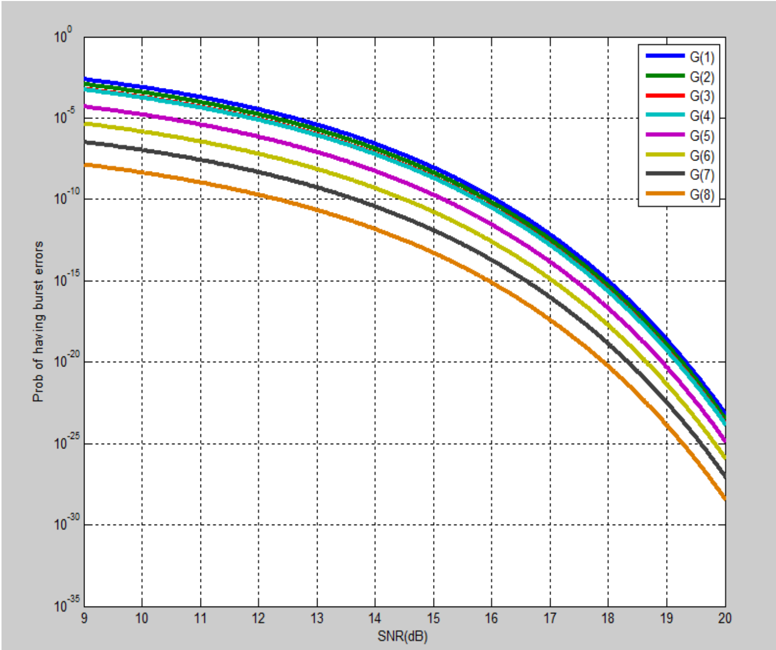


# Error Symbol Number and Probability

## - Non-Orthogonal Bit Mux 4:1

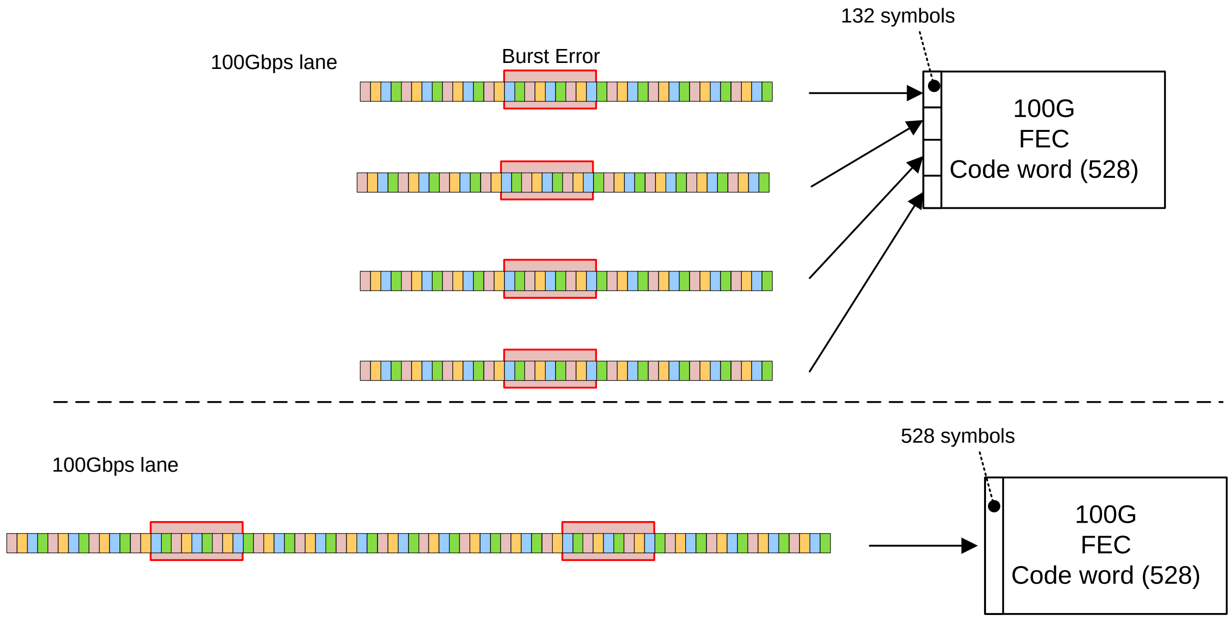



- **A burst with length of *bl* bits becomes shorter bursts on each lane by *Eq3-1*;**
- **Use *Eq1-1* to get number of error symbols on each lane;**
- **Add up errors on all 4 lanes in one codeword.**
- **G(x) curves in figure shows,**
  - G(1)>G(2)>…>G(7)>G(8)>0 @Rllmax=17bit;
  - G(1)>G(2)>…>G(7)>G(8)>0 @Rllmax=50bit;
  - Larger probability than in other muxing methods.



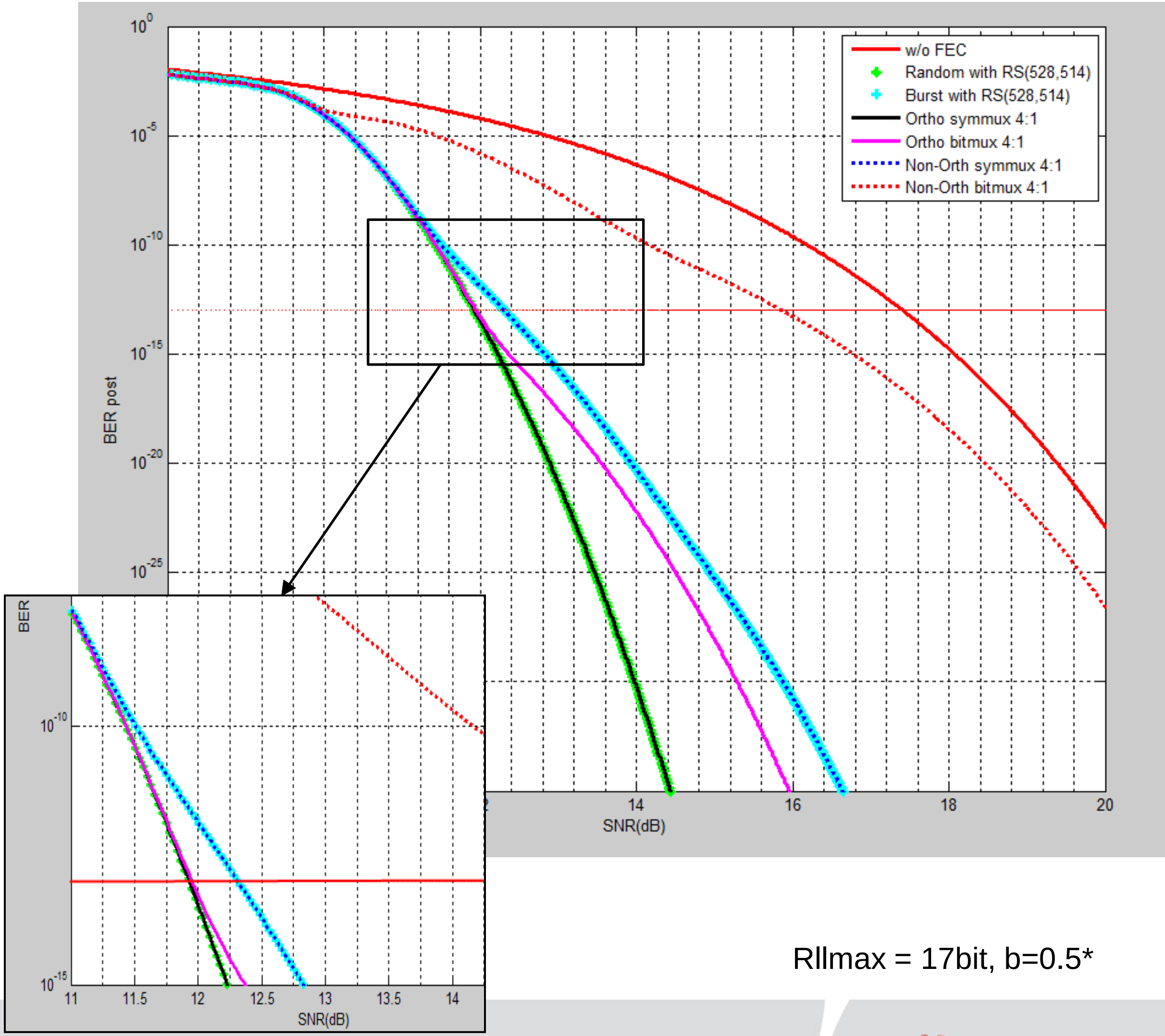
# Consideration on Errors From Multi-lane

- **While every lane has same error probability, errors from a single lane or from multiple lanes can be considered identically.**
  - Use $C_n^i = \binom{n}{i}$ to present the chances of having *i* errors in a codeword.



# 4:1 Mux Post FEC BER vs. $SNR_{NRZ}$

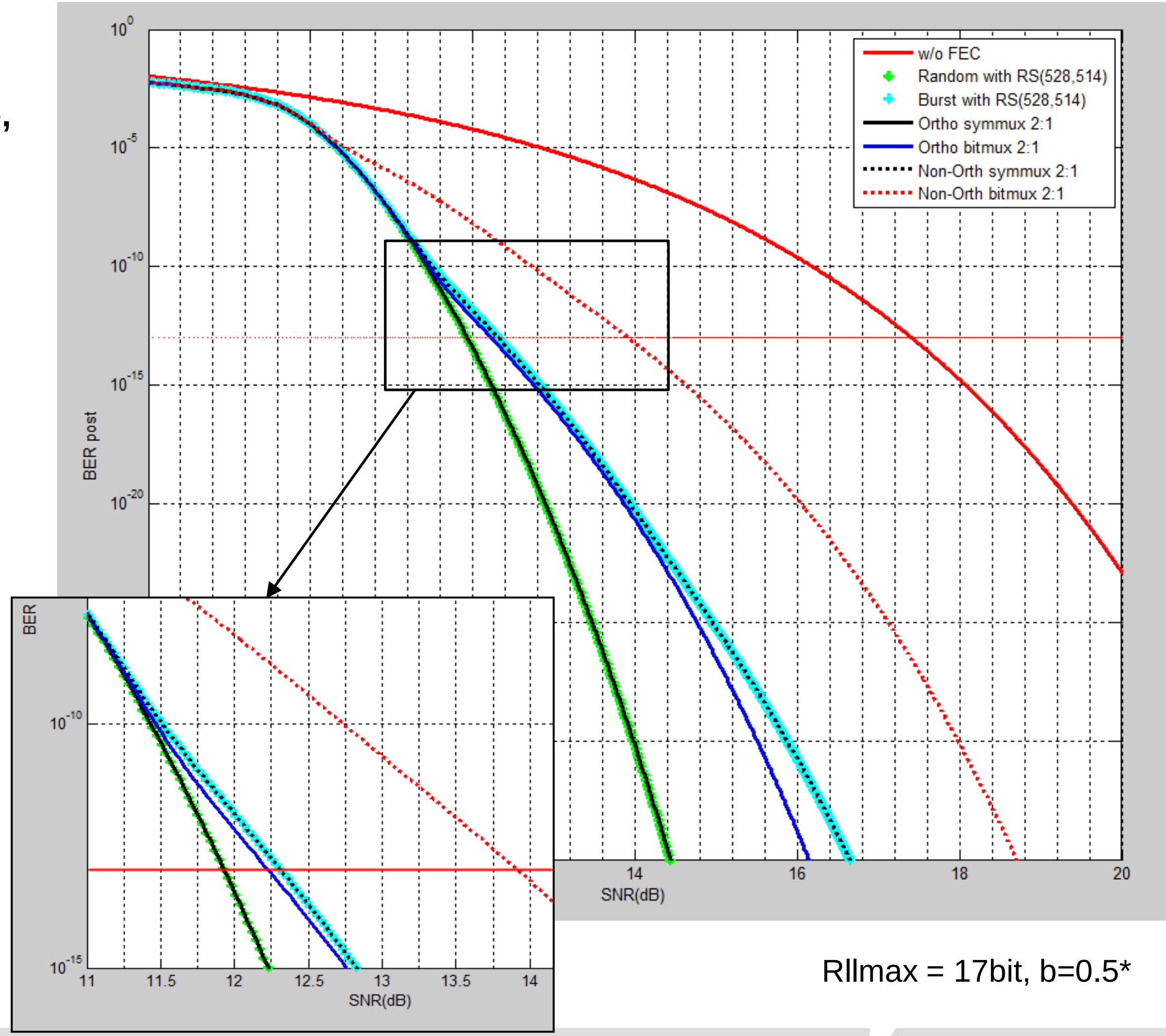
- **Orthogonal symbol mux has excellent performance, with the same coding gain as single RS(528,514) random error curve.**
- **Orthogonal bitmux also has better coding gain than 802.3bj CR4/KR4 burst error curve.**
- **Non-orthogonal bitmux is not acceptable for links with burst errors.**
- **By orthogonal bitmux, to obtain $BER_{post}$ = 1e-13; $BER_{in}$ should be 3.4e-5;**




RIlmax = 17bit, b=0.5*

# 2:1 Mux Post FEC BER vs. $SNR_{NRZ}$

- **Orthogonal symbol mux has excellent performance, with the same code gain as single RS(528,514) random error curve.**
- **Orthogonal bitmux also has similar code gain as 802.3bj CR4/KR4 burst error curve.**
- **Non-orthogonal bitmux is not acceptable for links with burst errors.**
- **By orthogonal bitmux, to obtain $BER_{post}$ = 1e-13; $BER_{in}$ should be 1.9e-5;**



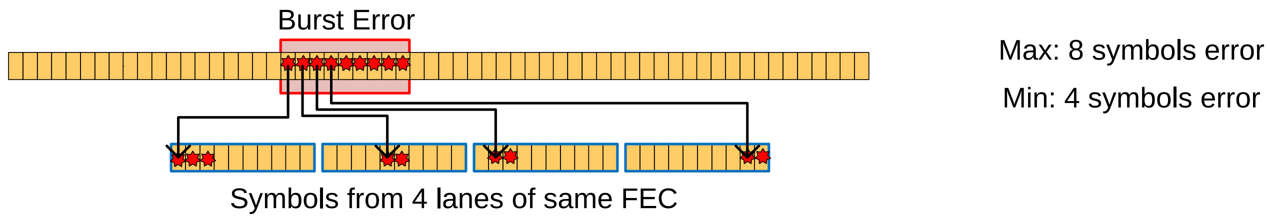

Rllmax = 17bit, b=0.5*

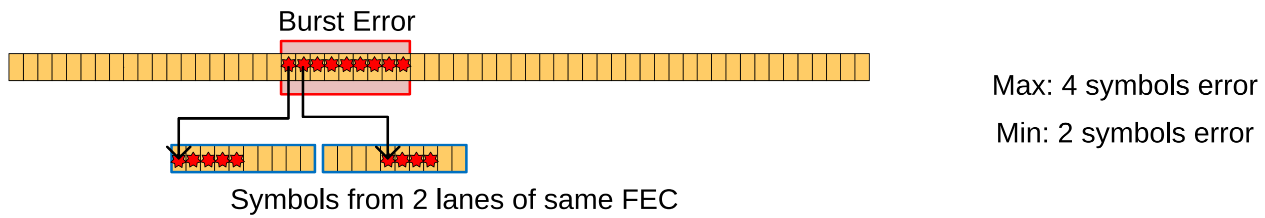
# Why orthogonal bit mux can obtain better performance than symbol mux based 802.3bj CR4/KR4?

- **When a burst error happens on bit mux links, it will divide into shorter bursts on each FEC. Thus on each FEC data stream, the probability of having a burst of given length is much smaller than using no mux at all.**
  - For instance, a 16 bit long burst will become 4 bit burst on each FEC lane in 4:1 mux scenario; the probability of having 16bit burst is BER*b^(16-1), while the chance of having 4bit burst is BER*b^(4-1); So on each FEC data lane, The probability of having same size burst error has been reduced by 1000 times. (~b^12 = 0.00024).
- **On page 7, with orthogonal bitmux 4:1, only G(1),G(2) > 0@assuming Rllmax =17bit; which means that maximally 1 or 2 symbols will be broken when a up-to-17bit burst error happens.**
- **By Comparison, the CR4/KR4 performance on page 5 with Rllmax=17bit, shows the value of G(1),G(2), G(3) > 0, which means when a up-to-17bit burst occurs, 1 or 2 or 3 symbols may be contaminated. Also note that G(x) value on this page is greater than orthogonal bitmux 4:1. That's why FEC performance of orthogonal bitmux 4:1 is better than CR4/KR4.**

# Why non-orthogonal bit mux 4:1 is worse than non-orthogonal bit mux 2:1?

- **By common sense, mux 4:1 would have better FEC performance than mux 2:1, the result of orthogonal bitmux conforms with this conclusion.**
- **However with non-orthogonal bitmux, 2:1 mux outperform the 4:1 mux.**
- **Why ?**




- **Only orthogonal Mux/Demux will reduce the error probability on each FEC;**

# Summary

- These slides present a method to evaluate how different symbol and bit mux arrangements affect the FEC code gain against burst error in link.
- Orthogonal symbol and bit mux have better performance than 100GBASE-CR4/KR4.
- By FOM bitmux 2:1, requirement of $BER_{in}$ = 1.9e-5 to satisfy $BER_{post}$ = 1e-13 is proposed.
- FOM symbol mux 2:1 will help get better $BER_{post.}$
- Further analysis on RS(544,514) will be undertaken.

# Thank you