

FEC performance on multi-part links

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Introduction

When analysing the performance of 400GbE FEC encoded links, two of the aspects that need consideration are:

- The interaction of burst errors due to decision feedback equalization (DFE) and dis-interleaving of bit-streams from higher rate lanes.
- Covering several sub-links with a single end-to-end FEC.

This contribution provides an analysis of the performance of RS(528,514) FEC for various interleaving schemes and also provides an analysis of the BER limit for a bursty sub-link to give a small penalty in a random sub-link for various interleaving schemes.

Reed-Solomon FEC with random errors

For a Reed-Solomon code RS(N,k), with symbol size of **m** and symbol correction capability **t**, if the input BER is:

$$BER_{in} = \frac{1}{2} \operatorname{erfc} \left(\sqrt{\frac{SNR}{2}} \right) \quad (1)$$

The input symbol error ratio (or the probability that an input symbol contains errors) SER_{in} is given by:

$$SER_{in} = 1 - (1 - BER_{in})^m \approx m \times BER_{in} \text{ (for small } BER_{in}) \quad (2)$$

The codeword error ratio (the probability of an uncorrectable FEC codeword) is:

$$CER = \sum_{i=t+1}^N \binom{N}{i} SER_{in}^i (1 - SER_{in})^{N-i} \quad (3)$$

The frame loss ratio (FLR) is then:

$$FLR = CER * (CER * 1 + (1 - CER) * (1 + MFC)/MFC) \quad (4)$$

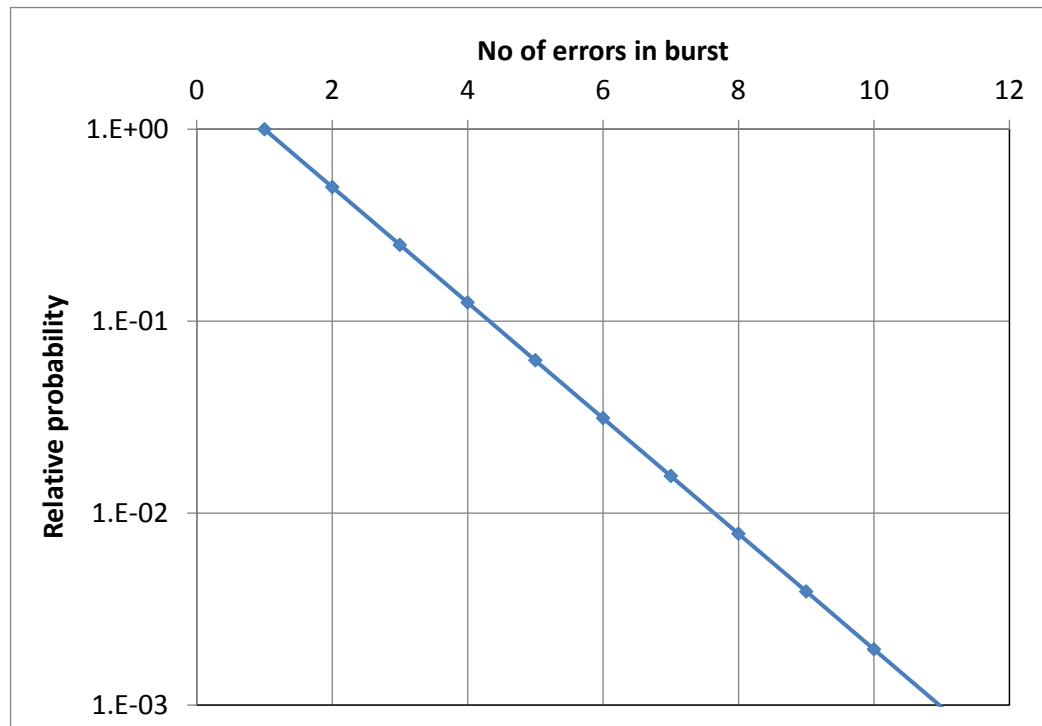
Where MFC is the number of MAC frames per codeword.

(This is an extension of slide 6 of [anslow_01a_1112_mmf](#) that still works for high CER)

Burst error model

The burst error probability is modelled in this contribution on the assumption that the probability of getting an error in the bit following an initial error (burst size of 2) is “a”, the probability of a burst of 3 errors is a^2 , the probability of a burst of 4 is a^3 , and so on. The worst case of $a=0.5$ is assumed throughout this contribution.

While $a=0.5$ may seem to be a pessimistic assumption, this simple model only applies to a 1-bit DFE. For a multi-bit DFE (particularly where the largest tap is not at 1 bit of delay) error burst patterns with several correct bits between errors are expected which makes the model used here not unreasonable.



RS(528,514) performance with bursts I

For the RS(528,514) code, $N=528$, $k=514$, $m=10$, $t=7$

Using similar analysis as slide 5 of [wang t 3bs 01 0514](#), the minimum burst size that can cause exactly 2 symbol errors is 2 bits, but this only occurs if the first error is the last bit of a symbol (i.e. a probability of 0.1).



If the burst size is 11 bits, then the probability that it will cause exactly 2 symbol errors is 1.



The maximum burst size that can cause exactly 2 symbol errors is 20 bits, but this only occurs if the first error is the first bit of a symbol (i.e. a probability of 0.1).



The probability of exactly two symbol errors (given an initial error) is:

$$P(2) = \sum_{i=1}^m \frac{i}{m} a^i (1-a) + \sum_{i=m+1}^{2m-1} \frac{2m-i}{m} a^i (1-a) = 0.0998 \text{ for } m=10, a=0.5 \quad (5)$$

RS(528,514) performance with bursts II

Similar analysis to that on the previous slide gives:

$P(1) = 0.9$, $P(2) = 0.0998$, $P(3) = 9.75\text{E-}5$, $P(4) = 9.52\text{E-}8$, etc.

These probabilities can then be used to calculate the overall probability of any combination of events that results in $t+1$ or more symbol errors:

$$\begin{aligned} \text{CER} = & \sum_{i=t+1}^N \binom{N}{i} \text{SER}_{\text{in}}^i (1 - \text{SER}_{\text{in}})^{N-i} + \binom{N}{1} \text{SER}_{\text{in}}^1 (1 - \text{SER}_{\text{in}})^{N-1} P(8) \\ & + \binom{N}{2} \text{SER}_{\text{in}}^2 (1 - \text{SER}_{\text{in}})^{N-2} \left(P(4)P(4) + \binom{2}{1} P(3)P(5) + \binom{2}{1} P(2)P(6) + \dots \right) \\ & + \binom{N}{3} \text{SER}_{\text{in}}^3 (1 - \text{SER}_{\text{in}})^{N-3} \left(\binom{3}{1} P(2)P(3)P(3) + \binom{3}{1} P(2)P(2)P(4) + \dots \right) \\ & + \dots \\ & + \binom{N}{7} \text{SER}_{\text{in}}^7 (1 - \text{SER}_{\text{in}})^{N-7} \left(\binom{7}{1} P(1)^6 P(2) + \binom{7}{2} P(1)^5 P(2)P(2) + \dots \right) \quad (7) \end{aligned}$$

Monte Carlo analysis

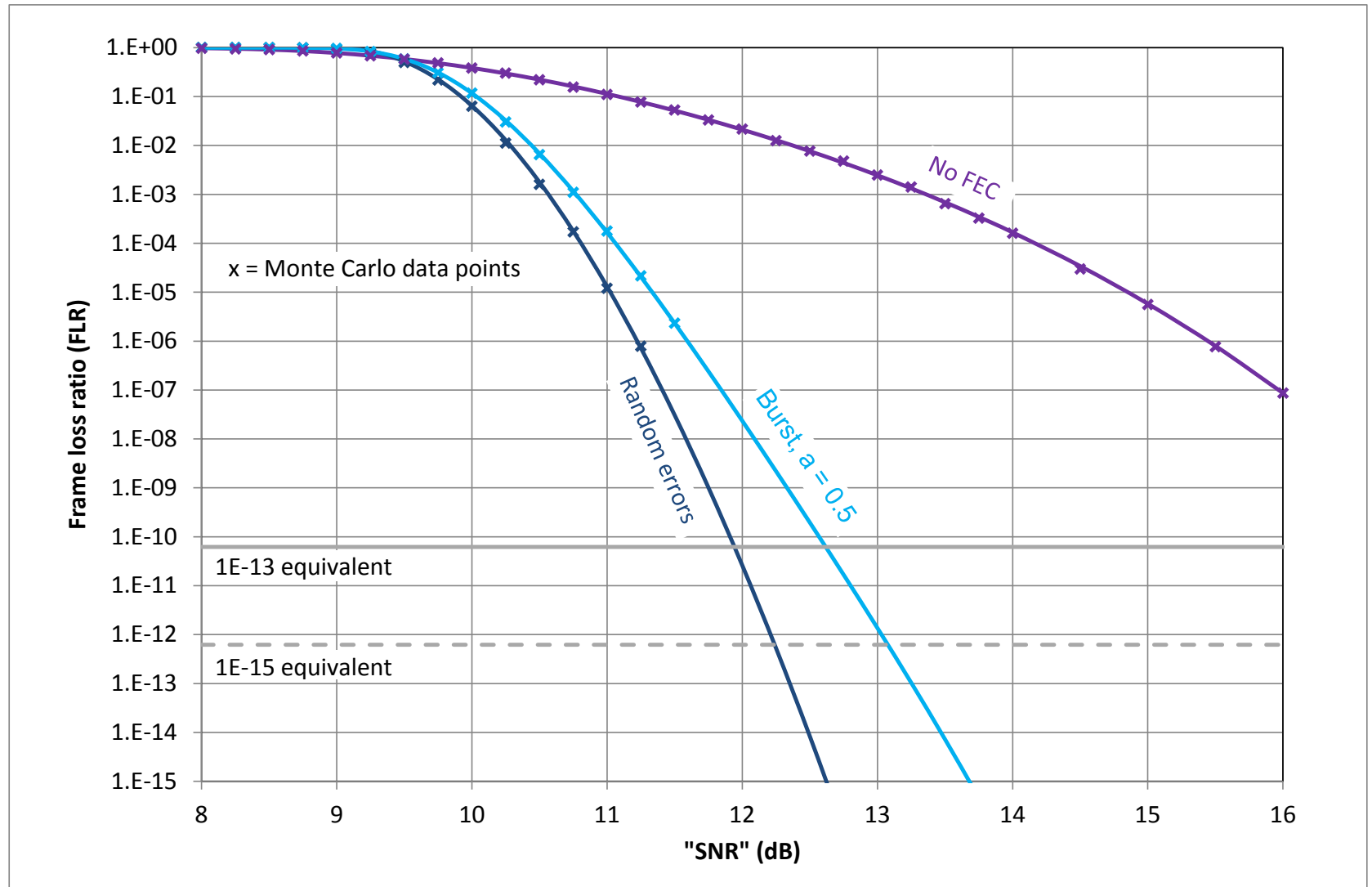
In an attempt to verify that the formulae on the previous slides give plausible results, a Monte Carlo based simulation was also performed using a random number generator to generate errors in a large number of codewords both with and without burst errors.

The results of plotting equations 1, 3, and 4 for random errors and equations 1, 5, and 4 for burst errors is shown on the next slide.

Also shown via “x” markers are the results of the Monte Carlo simulations which show good agreement with the formulae down to the probability where the simulations become too time consuming.

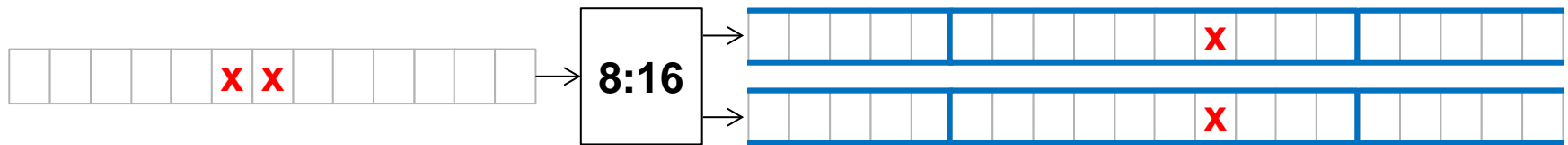
Note – the vertical axis for the plots is Frame Loss Ratio (FLR) for 64-octet MAC frames since uncorrectable codewords have to be discarded to provide an adequate Mean Time To False Packet Acceptance (MTTFPA).

RS(528,514) performance with bursts

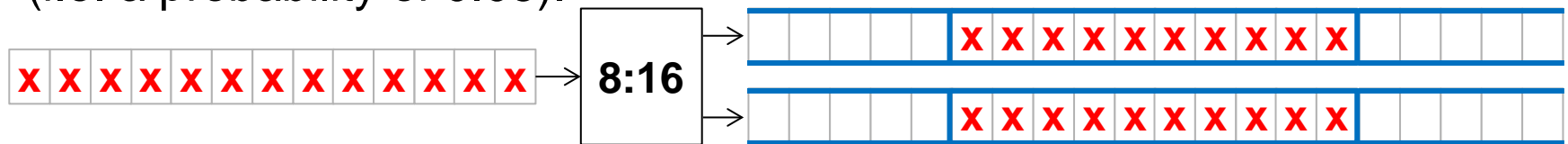


Bit interleaving – same FEC instance

For the case of bit interleaving two lanes from the same FEC instance, the minimum burst size that can cause exactly 2 symbol errors is again 2 bits, but here the probability of causing exactly 2 symbol errors is 1.



The maximum burst size that can cause exactly 2 symbol errors is 20 bits, but this only occurs if the first two errors are the first bits of a symbol (i.e. a probability of 0.05).



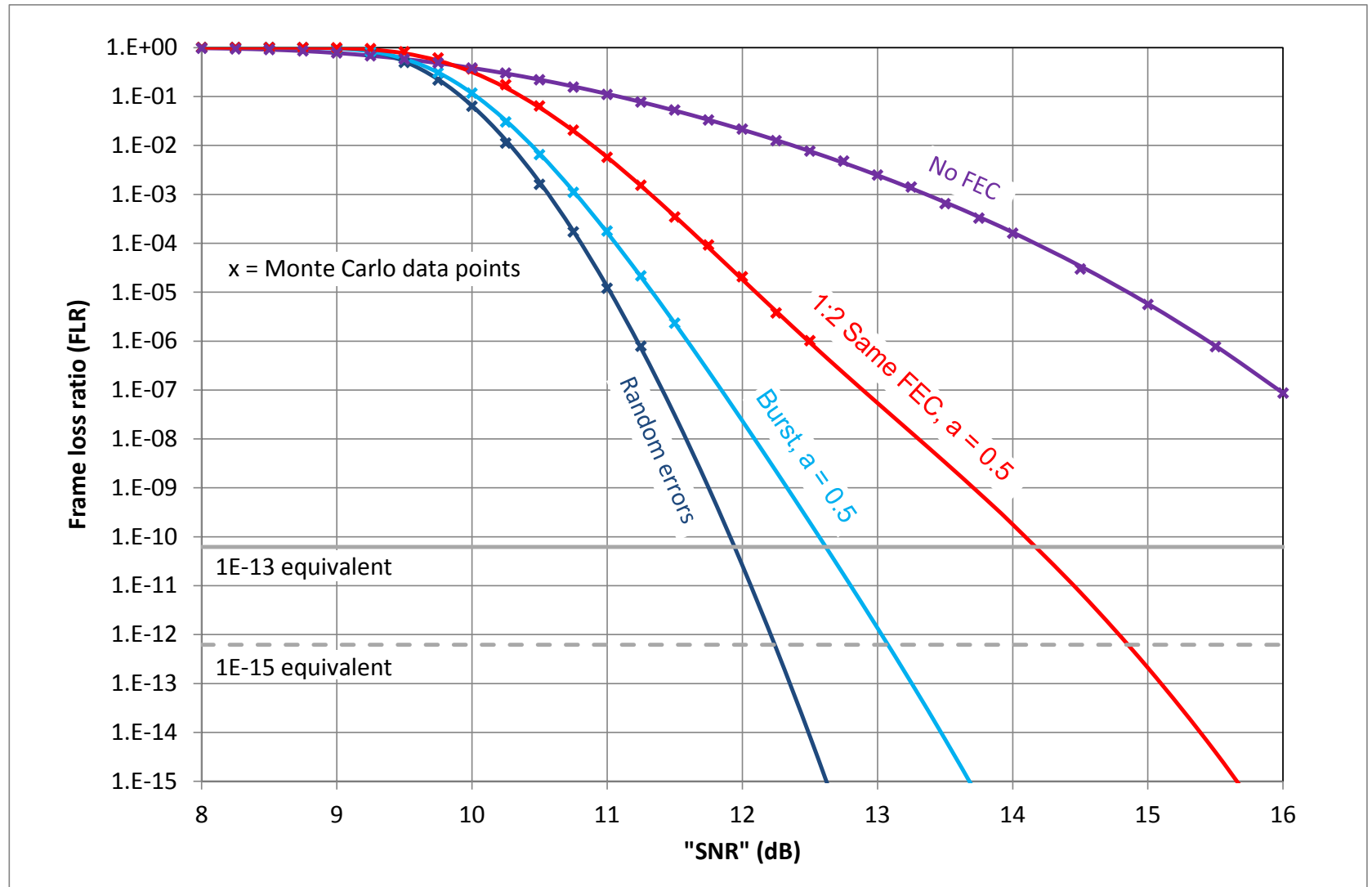
Note – this analysis assumes no skew between the two lanes as this is worse than assuming random alignment.

The probability of exactly two symbol errors (given an initial error) is:

$$P(2) = a(1-a) + \sum_{i=2}^{2m-1} \frac{2m-i}{2m} a^i (1-a) = 0.4625 \text{ for } m=10, a=0.5 \quad (8)$$

$P(1) = 0.5$, $P(2) = 0.4625$, $P(3) = 0.025$, $P(4) = 0.0125$, $P(5) = 2.4E-8$ etc.

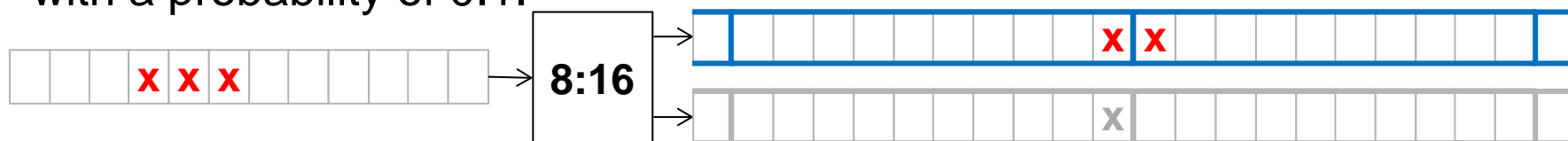
RS(528,514) with 1:2 interleaving and bursts



Bit interleaving – different FEC instance

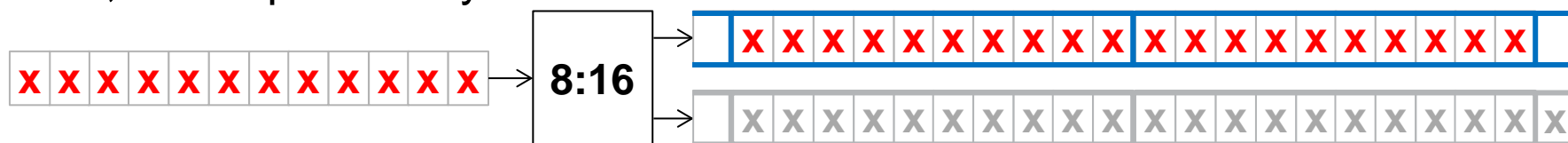
The case of bit interleaving two lanes from different FEC instances can be analysed by considering just the bits from one FEC instance with an input BER increased by a factor of $1 + a$.

The minimum burst size that can cause exactly 2 symbol errors is 3 bits, with a probability of 0.1.



If the burst size is 22 bits, then the probability that it will cause exactly 2 symbol errors is 1.

The maximum burst size that can cause exactly 2 symbol errors is 41 bits, with a probability of 0.1.

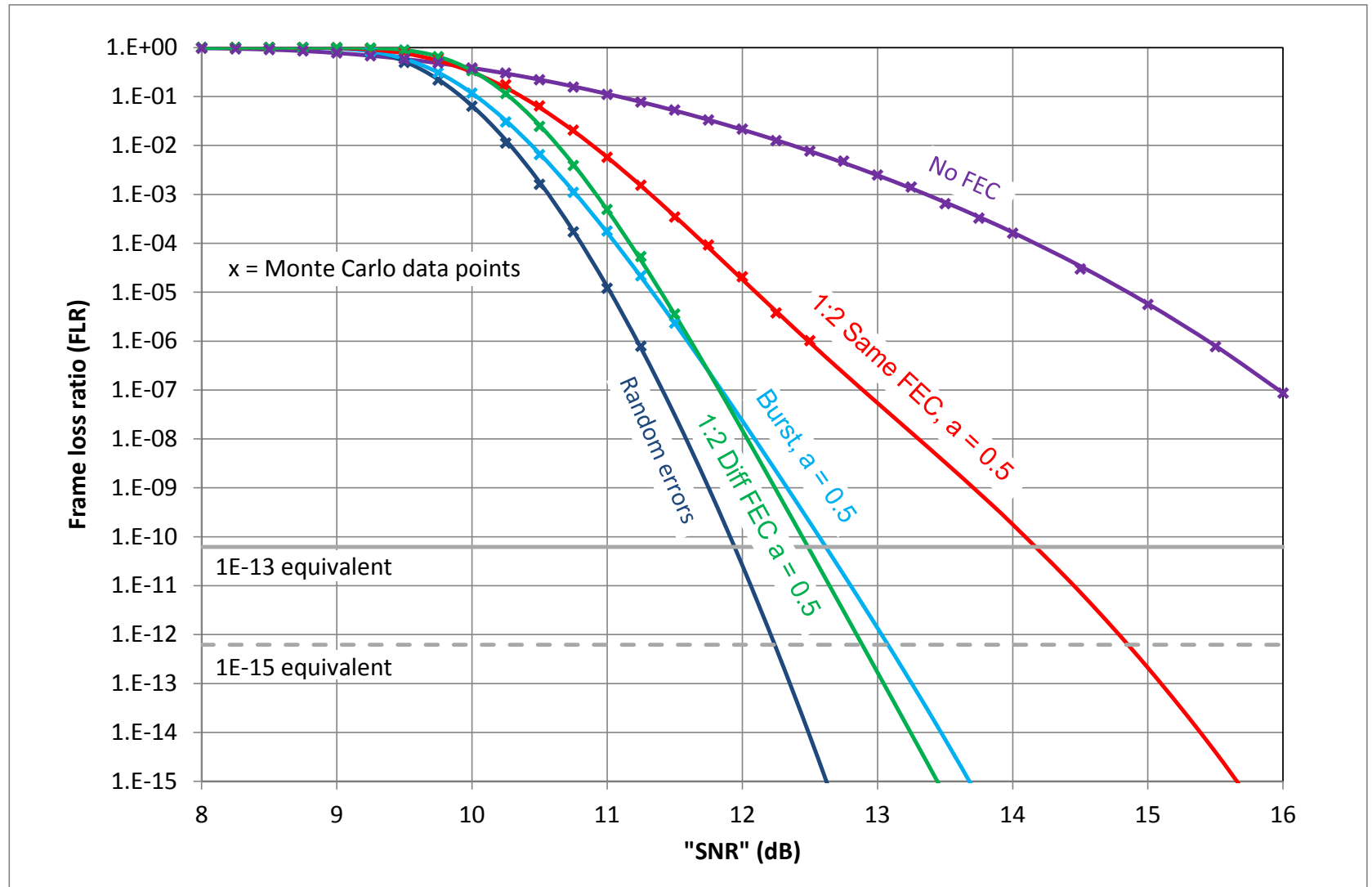


The probability of exactly two symbol errors (given an initial error) is:

$$P(2) = \sum_{i=2}^{2m+1} \frac{\text{floor}(i/2)}{m} a^i (1-a) + \sum_{i=2m+2}^{4m-1} \frac{2m - \text{floor}(i/2)}{m} a^i (1-a) = 0.0333 \quad (9)$$

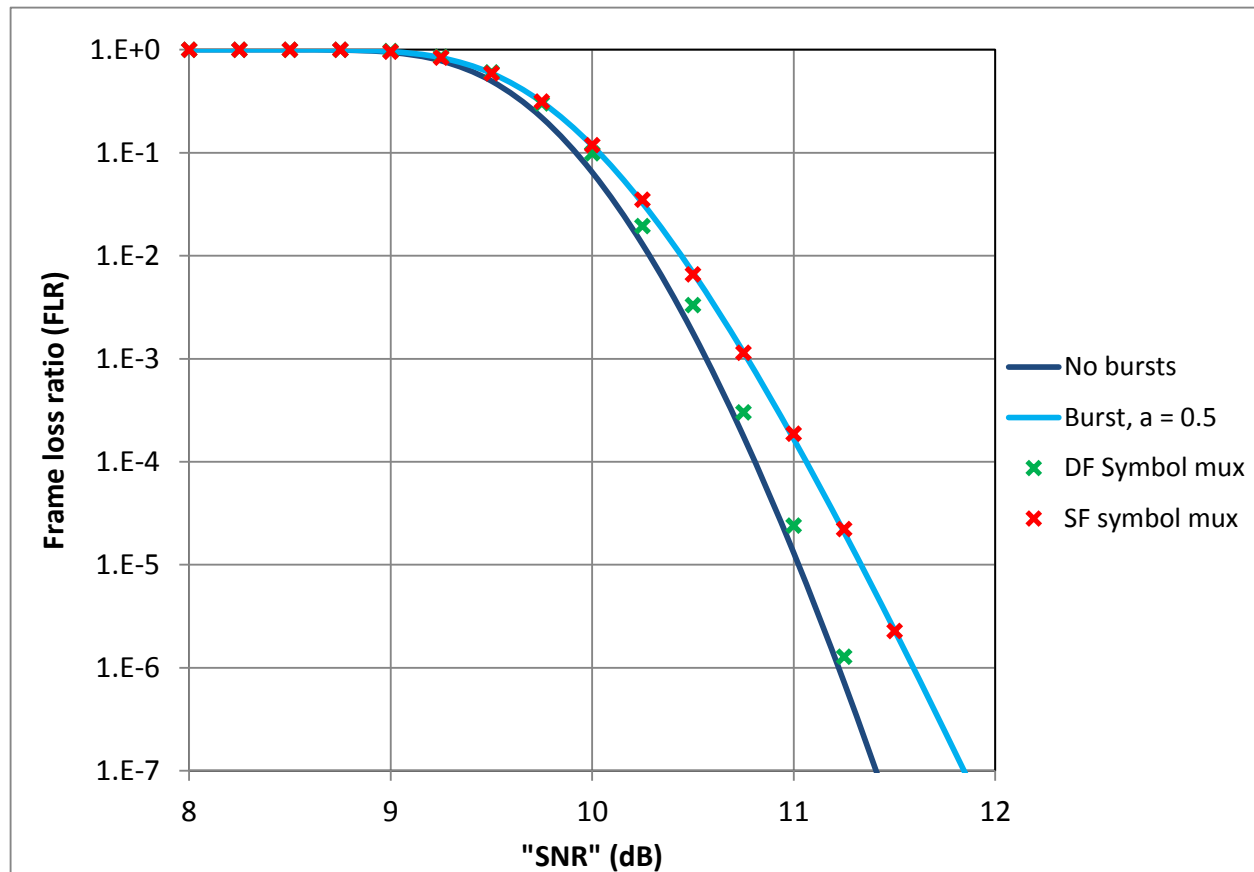
$P(1) = 0.9667$, $P(2) = 0.0333$, $P(3) = 3.2\text{E-}8$ etc.

RS(528,514) all curves



Symbol interleaving

Monte Carlo analysis of symbol interleaving shows that, as expected, 1:2 same FEC (SF) with bursts shows the same performance as for non-interleaved FEC with bursts and 1:2 different FEC (DF) with bursts has only slightly worse performance than for random errors.



Results for RS(528,514)

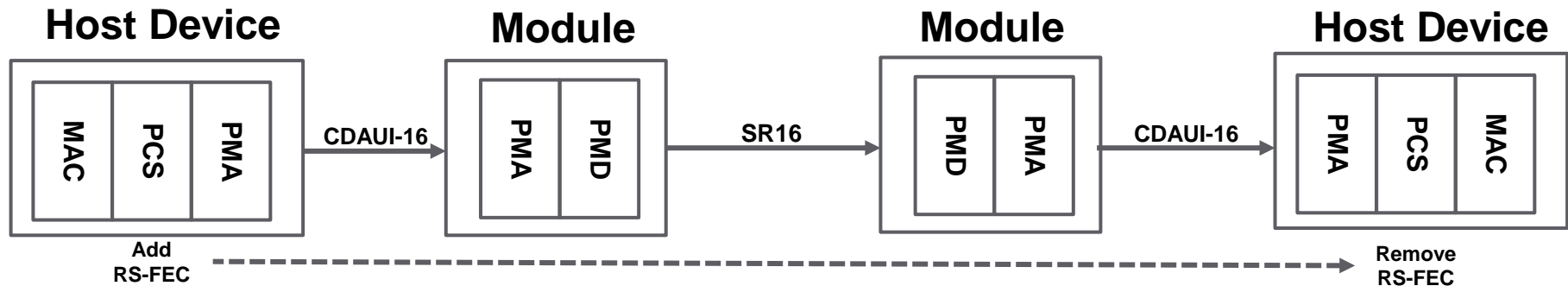
From the curves shown on the previous slide, the BERs (due to the “SNR”) at the input required to give FLRs equivalent to that of a BER of $1\text{E-}13$ and $1\text{E-}15$ are:

	For FLR = $6.2\text{E-}11$	For FLR = $6.2\text{E-}13$
No FEC	$1\text{E-}13$	$1\text{E-}15$
1:2 Same FEC, $a = 0.5$	$1.6\text{E-}7^*$	$1.6\text{E-}8^*$
Burst, $a = 0.5$	$9.6\text{E-}6^*$	$3.3\text{E-}6^*$
1:2 Different FEC, $a = 0.5$	$1.3\text{E-}5^*$	$5.2\text{E-}6^*$
Random errors	$3.8\text{E-}5$	$2.1\text{E-}5$

Note – these values are the BER derived from equation 1 and do not include the additional errors due to the bursts. To account for burst errors, the values marked with “*” must be multiplied by 2 when $a = 0.5$.

What about multi-part links with FEC?

If the FEC bytes are added at the source PCS layer and then the correction is applied only at the destination PCS layer as in:



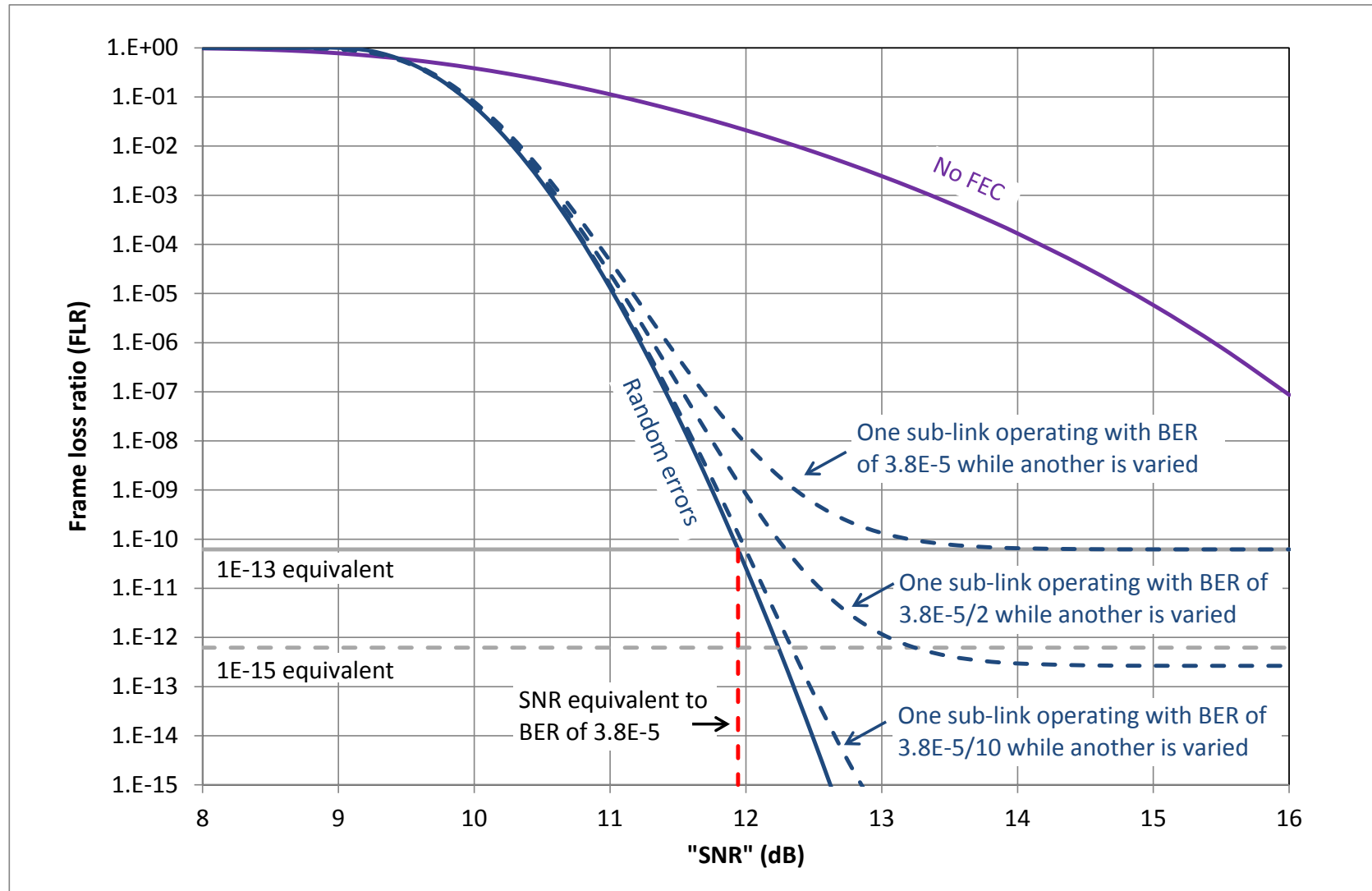
Source: [gustlin 3bs 02 0514](#)

Then the worst case input BER for the FEC decoder must be met by the concatenation of all of the sub-links.

In the case of CAUI-10 -> SR10 -> CAUI-10, the worst case BER for each of the sub-links is $1E-12$ which is the same as the end-to-end requirement. This situation is tolerated on the basis that it is unlikely that all three sub-links will be at the worst case BER at the same time and if two of them were, then the end-to-end BER would still be $2E-12$.

The situation for a pair of sub-links sharing the same RS(528,514) protection is shown on the next slide.

RS(528,514) multi-part (random)



Sub-links with bursts

The curves on the previous slide are all for random errors.

What if there are burst errors in the electrical sub-links but random errors in the optical part of the link?

The plot on the next slide models this situation (with non-interleaved bursts) for a BER floor that is reachable by a Monte Carlo simulation in a realistic time. This shows a good match between the analytical model and the Monte Carlo results.

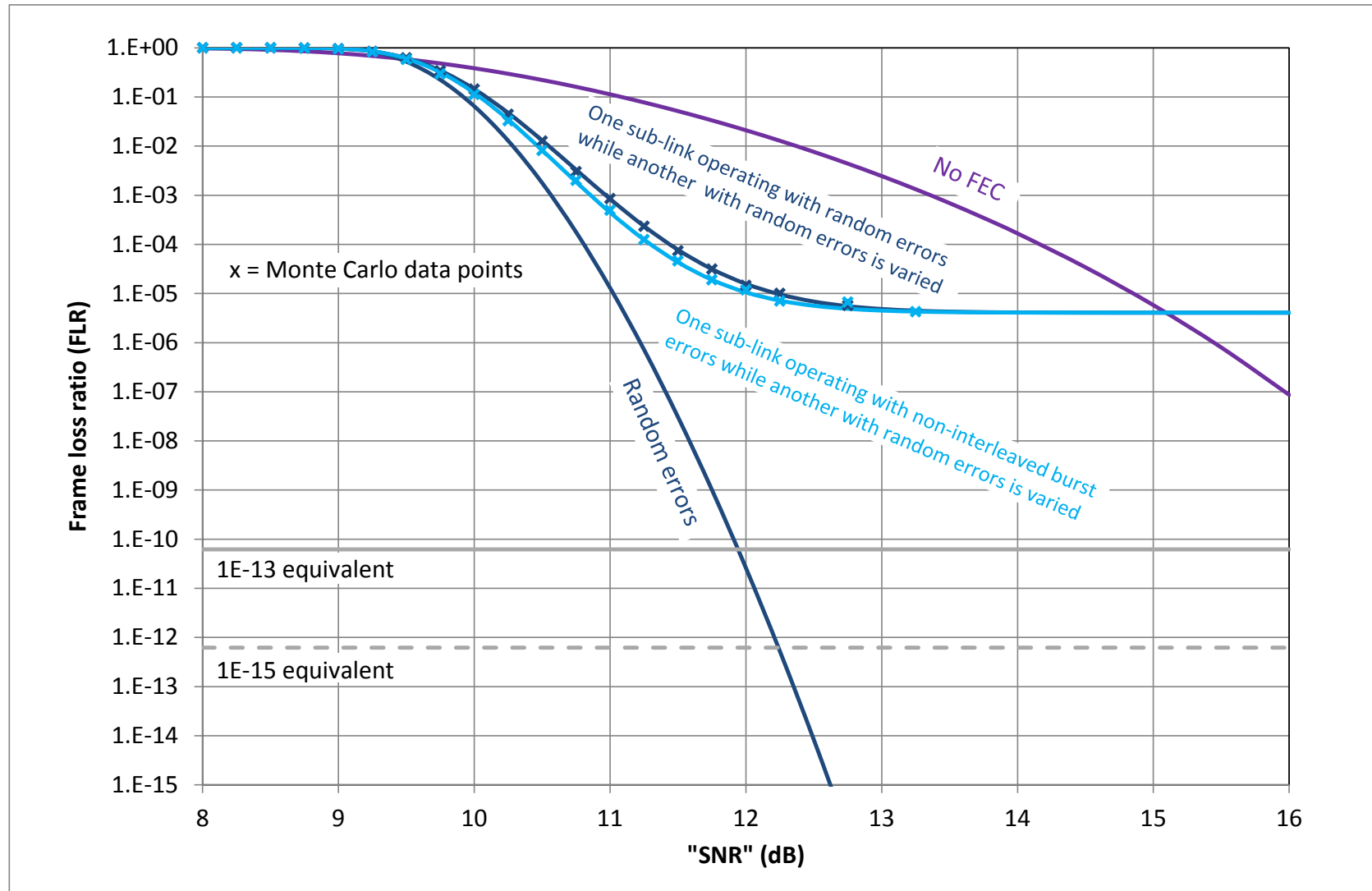
The same model was then used to predict the curves for a sub-link operating with non-interleaved bursts with a fixed BER together with additional random errors from another sub-link (i.e. similar to slide 16 but for a bursty fixed sub-link).

The resulting plot is on slide 19

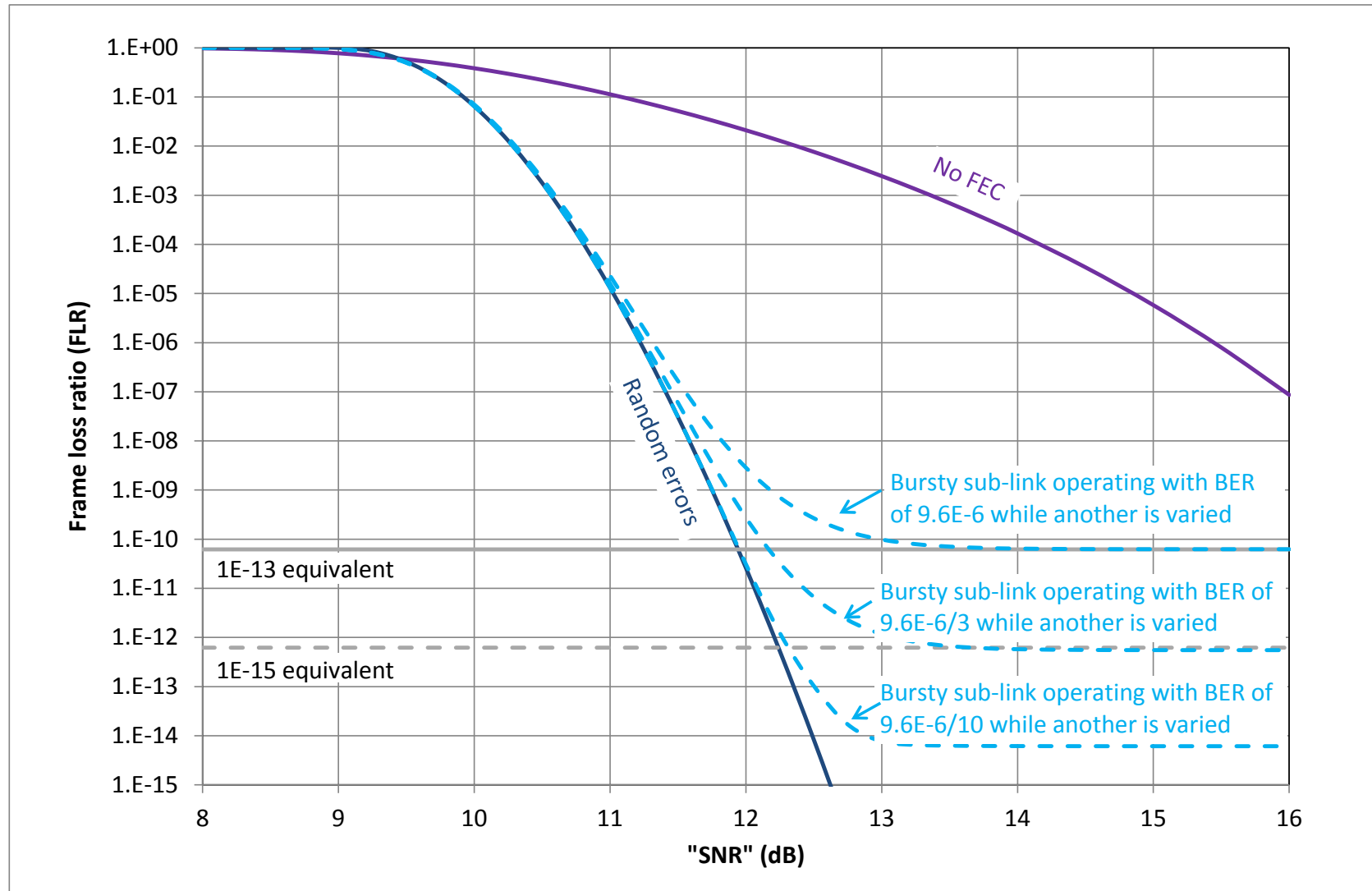
Slides 20 and 21 repeat this analysis for 1:2 same FEC (SF) bursts

Slides 22 and 23 repeat again for 1:2 different FEC (DF) bursts.

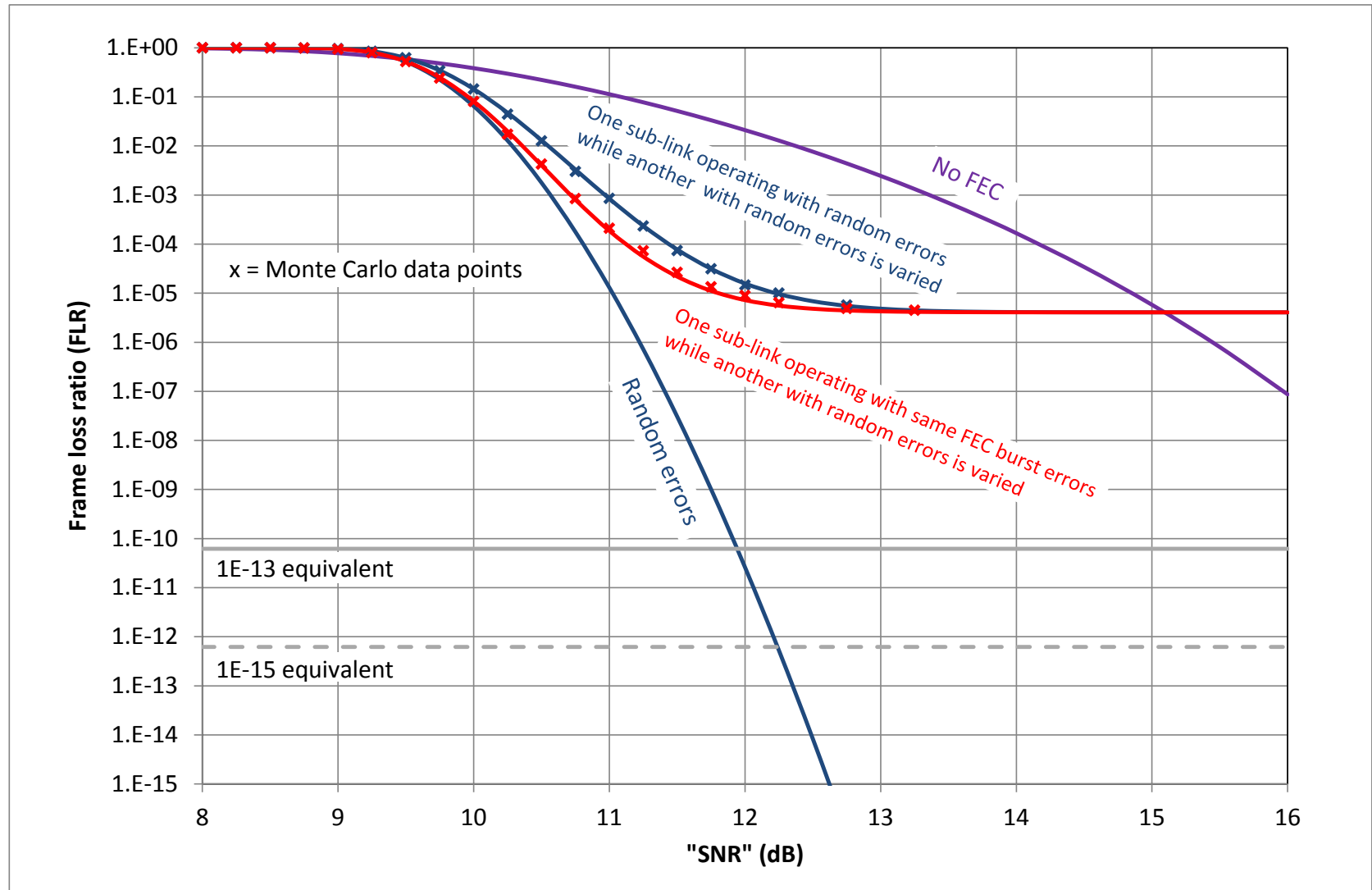
RS(528,514) multi-part (burst + random) I



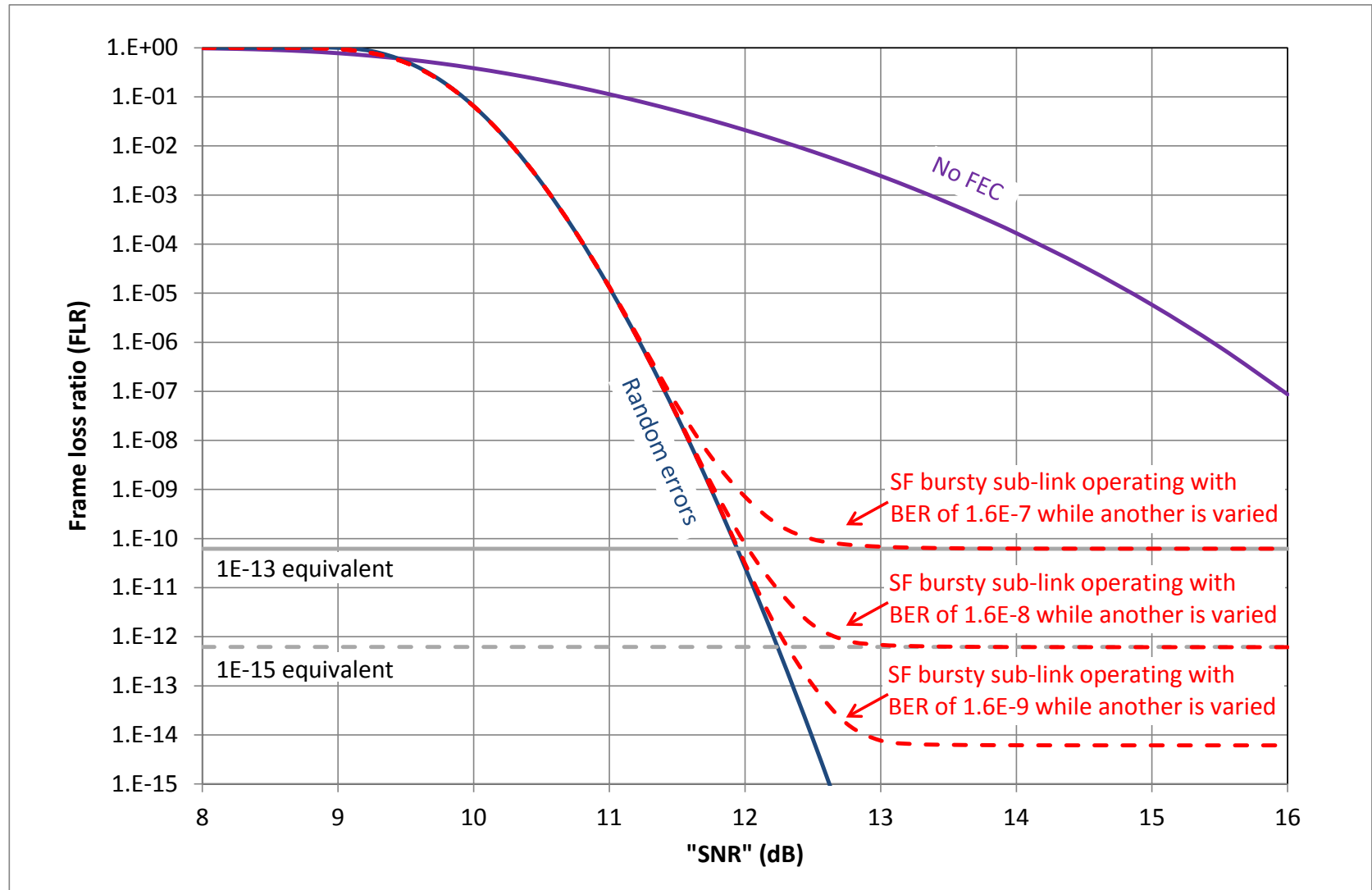
RS(528,514) multi-part (burst + random) II



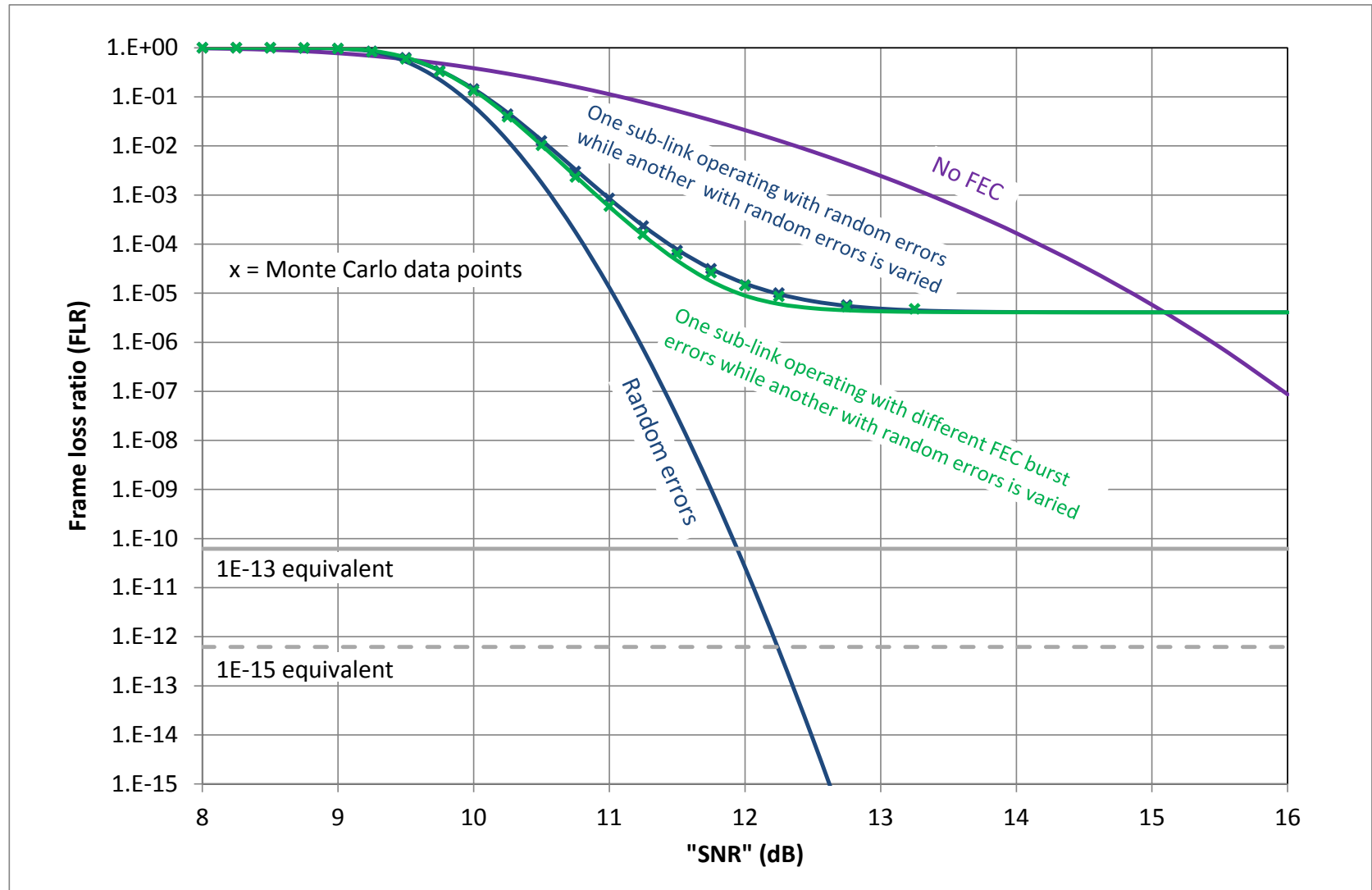
RS(528,514) multi-part (1:2 SF burst + random) I



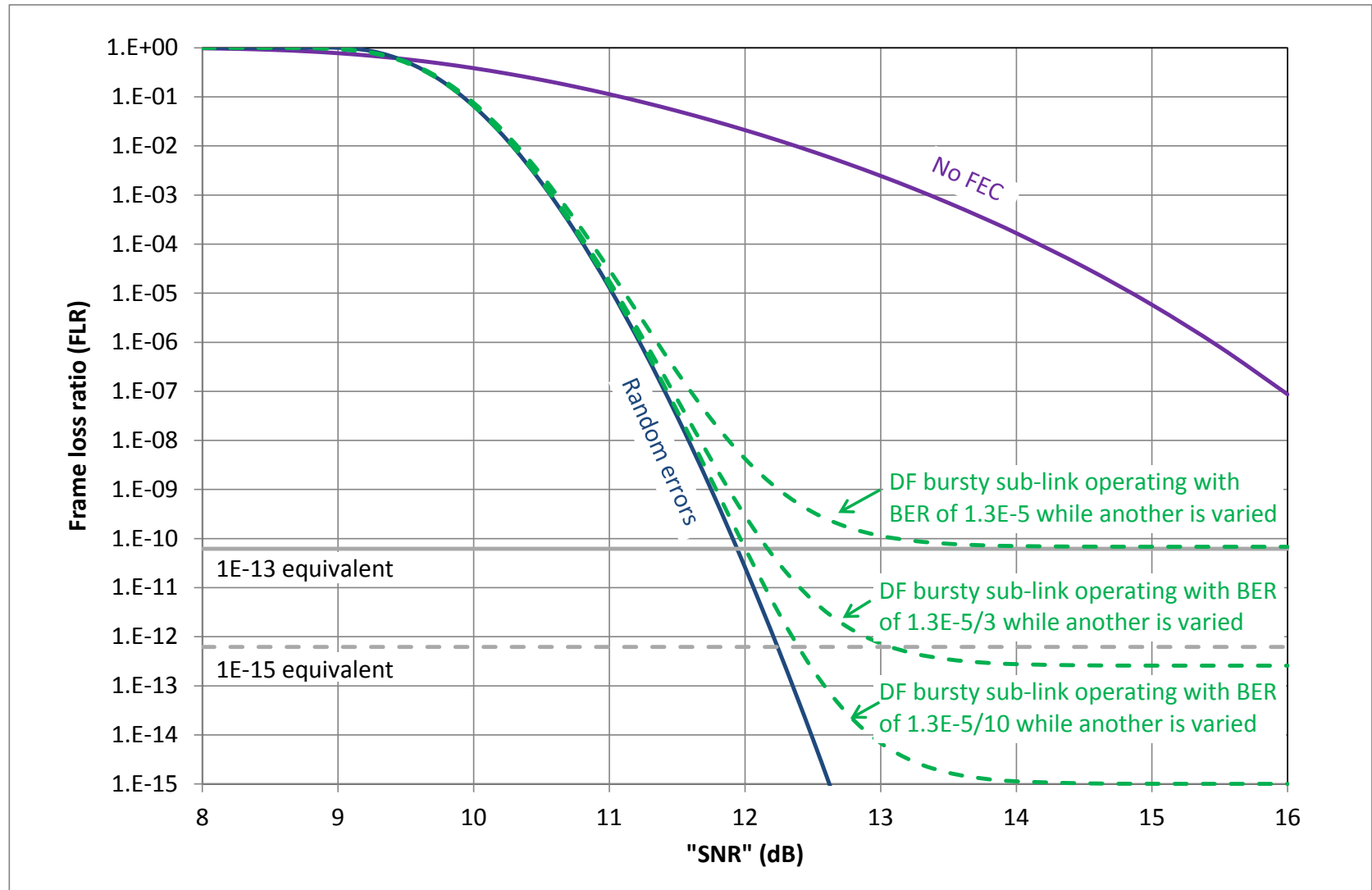
RS(528,514) multi-part (1:2 SF burst + random) II



RS(528,514) multi-part (1:2 DF burst + random) I



RS(528,514) multi-part (1:2 DF burst + random) II



Conclusions

This presentation contains curves predicting the performance of RS(528,514) FEC with various forms of interleaving supported by Monte Carlo analysis.

The performance curves of multi-part links containing one fixed bursty part and one random part has also been analysed.

The BERs (due to the “SNR”) of the bursty part for a penalty of 0.05 dB optical ($\text{BER} = 3.2\text{E-}5$ rather than $3.8\text{E-}5$ or $1.7\text{E-}5$ rather than $2.1\text{E-}5$) in the random part are:

	For FLR = $6.2\text{E-}11$	For FLR = $6.2\text{E-}13$
1:2 Same FEC, $a = 0.5$	$2.0\text{E-}8^*$	$2.3\text{E-}9^*$
Burst, $a = 0.5$	$1.5\text{E-}6^*$	$5.5\text{E-}7^*$
1:2 Different FEC, $a = 0.5$	$2.1\text{E-}6^*$	$8.8\text{E-}7^*$
Random errors	$6.7\text{E-}6$	$4.0\text{E-}6$

Note – these values do not include the additional errors due to the bursts. To account for burst errors, the values marked with “*” must be multiplied by 2 when $a = 0.5$.

Thanks!