PAM-N modulation penalty in pictures

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PAM-2 in pictures

- Probability of occurrence of each level = 1/N = 1/2
- Relative probability of error per symbol = 2.(N-1)/N = 1
- The error probability associated with a single noise tail on a particular signal level = \( P_i \cdot \frac{1}{2}.erfc\left(\frac{OMA/2}{\sigma_n \cdot \sqrt{2}}\right) \)
  - where \( P_i \) is the probability of occurrence of the \( i^{th} \) signal level
  - and \( \sigma_n \) is the RMS of the Gaussian noise
- Note: the symbol error ratio equals the bit error ratio for PAM-2
PAM-4 in pictures

N = 4 modulation levels

- Probability of occurrence of each level = 1/N = 1/4
- Relative probability of errors per symbol = 2.(N-1)/N = 3/2
  - The symbol error ratio increases!
  - For Gray coded PAM-4, 1 symbol error produces 1 bit error; but each symbol translates to log₂(N) = 2 bits, so the ratio of SER (symbol error ratio) to BER is: SER/BER = log₂(N) = 2
- Relative probability of errors per bit = 2.(N-1)/(N.log₂(N)) = 3/4
  - The bit error ratio decreases!

At time centre of eye

Modulation levels

Thresholds

N-1 = 3 thresholds

2.(N-1) = 6 noise tails causing errors

N-1 = 3 eyes
BER to power penalty

For a target BER of $2.4 \times 10^{-4}$:

- For ideal NRZ, the Q required is 3.492
- For ideal PAM-4, the Q required is 3.414
  - a negative Q penalty of $\sim 0.098$ dB
  - total modulation penalty for PAM-4 is 4.678 dB

  (including impact of OMA scaling, higher symbol error rate, lower bit error rate, for a given outer eye OMA and fixed receiver noise, at BER of $2.4 \times 10^{-4}$)
Back up
PAM-3 cartoon

N = 3 modulation levels

N-1 = 2 eyes

At time centre of eye

Modulation levels

Thresholds

noise N = 3 modulation levels

N-1 = 2 eyes N-1 = 2 thresholds 2.(N-1) = 2 noise tails causing errors

• Probability of occurrence of each level = 1/N = 1/3
• Relative probability of error per symbol = 2.(N-1)/N = 4/3
• Relative probability of errors per bit = 2.(N-1)/(N.log₂(N)) = 0.841
General PAM-N modulation penalty in words

Ideal Transmitter
• Negligible noise, negligible ISI, equally spaced modulation levels

Ideal Receiver
• Gaussian noise, perfect timing and slicing; Negligible added ISI/jitter
• Errors due to a noise tail crossing more than one threshold are ignored

PAM-N
• N levels with equal probability of occurrence; N-1 eyes; N-1 thresholds; \( \log_2 N \) bits/symbol
• 2.(N-1) noise tails causing symbol errors.
• 2.(N-1)/N times the number of symbol errors generated for PAM-N (compared to PAM-2) for same inner-eye OMA to noise ratio
• 2.(N-1)/(N.log_2 N) times the number of bit errors generated for PAM-N (compared to PAM-2) for same inner-eye OMA to noise ratio

For PAM-N, this allows a decrease in Q (cf. NRZ) to meet a given target BER
  • a negative power penalty, which increases in magnitude as target BER increases (slope of BER curve); \( \sim 0.098 \text{ dB for PAM4 at a BER of } 2.4 \times 10^{-4} \).
  • reduces the **PAM-4 modulation penalty to 4.678 dB** (compared to the OMA scaling penalty of 4.771 dB).
Simulation showing BER vs OMA for ideal Tx