Further on a Path toward Incorporating Advanced Signal Processing in Electrical Channel Performance Assessment

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Outline

- Introduction
- Proposal Recap
- SNR, COM, and VEC
- Explanation of Steps (1 to 5)
- Recap of Steps
- Case Studies and Summary of Results
  - Native Noise
  - 1E-3
  - 1E-4
- Summary and Future Work
- Appendix
  A. Calculating SNR for L-PAM
  B. Error analysis of L-PAM 1-Tap DFE
  C. Error analysis of L-PAM 1+αD MLSE
  D. Analysis of the Conceptual Equivalent DFE
Introduction

• A method for incorporating performance effect of MLSE was proposed in shakiba_3df_01b_2211.pdf

• In response to the feedback, this is a first update, specifically:
  - Updated S3 channel models used in case studies
  - **Added 11 more cases (for a total of 22 cases)**
  - Used the same noise scaling approach for all channel cases (summarized in table on slide 7)
  - Corrected a minor typo in the table on slide 7
  - Updated case study results (slides 9-11) at two DER targets of 1E-3 and 1E-4 (with DFE)
  - **Added case study results for channel native noise levels (i.e. no noise scaling)**
  - **Provided more information and equations for the MLSE proposal**

• The equations are basis for incorporating the MLSE impact into the existing COM flow

• The equations can be easily and directly translated to Matlab for COM Matlab tool update

• More clarification and explanation will be provided based on the feedback
  - The received feedbacks and comments have been encouraging and are appreciated 😊
Proposal Recap

- The proposal outlined a method that enables incorporating MLSE impact in the COM flow
- As rates increase, we see this as a useful and maybe necessary update to COM for reflecting a more accurate and realistic representation of capabilities in the reference receiver
- RX FFE support is essential to considering more advanced detection techniques than DFE
- $1+\alpha D$ MLSE appears as a first natural alternative to a 1-tap DFE
- Proposal specified following steps:
  1) Use COM analysis to find the DFE tap, $\alpha$
  2) Use analysis to calculate SNR$_{DFE}$ and DER$_{DFE}$
  3) Use analysis to calculate DER$_{MLSE}$ at SNR$_{DFE}$
  4) Use analysis to calculate SNR$_{DFE,\text{equivalent}}$ for the same DFE that yields the same DER$_{MLSE}$
  5) Increase from SNR$_{DFE}$ to SNR$_{DFE,\text{equivalent}}$ is a much better estimate of COM improvement ($\Delta$COM) offered by the MLSE than $10\log_{10}(1+\alpha^2)$

![Diagram showing steps of the proposal with a line graph of Signal-to-Noise Ratio (SNR) vs. Decision Error Ratio (DER) with different steps labeled from 1 to 5.]
SNR, COM, and VEC

\[ COM = 20 \log_{10} \left( \frac{A_{signal}}{A_{noise}} \right) \]

\[ VEC = 20 \log_{10} \left( \frac{A_{signal}}{A_{eye}} \right) \]

\[ COM = -20 \log_{10} \left( 1 - 10^{-\frac{VEC}{20}} \right) \]

- COM and VEC are related to SNR

\[ SNR[dB] = 10 \log_{10} \left( \frac{1}{3} \frac{A_{peak}^2}{L-1 \sigma_{noise}^2} \right) \] (Appendix A)

\[ A_{peak} = (L-1)A_{signal} \]

\[ A_{noise} = k_{DER} \sigma_{noise} \]

\( k_{DER} \) is a multiplier factor that determines how many \( \sigma \)'s away from mean achieves target DER (a.k.a. Q factor for Gaussian noise)

- As a result COM can be expressed as

\[ COM = SNR[dB] - 10 \log_{10} \left( \frac{L^2-1}{3} k_{DER}^2 \right) \]

- Which suggests that COM is in fact a kind of SNR with a notion of DER directly built in it
SNR, COM, and VEC

\[ COM = SNR[dB] - 10 \log_{10} \left( \frac{L^2 - 1}{3} k_{DER}^2 \right) \]

- There are three ways to interpret this equation:
  1. If after a change in SNR same DER is targeted, \( k_{DER} \) remains constant and any increase (decrease) in SNR translates to the same increase (or decrease) in COM
     \[ \Delta COM = \Delta SNR[dB] \]
     e.g. this means that if a 3dB COM is targeted and if MLSE is used in the actual receiver with a verified 1.8dB SNR gain, COM target with the reference receiver can be lowered to 1.2dB.
  2. If after a change in SNR same COM is targeted, COM remains constant and any increase (or decrease) in SNR translates to an increase (or decrease) in \( k_{DER} \) and results in a decrease (or increase) in DER
  3. A change in SNR can be broken down to partially achieve both above as long as the equation holds

- Any change in SNR also translates to a change in VEC, but the change depends on VEC value
  \[ \Delta VEC = \Delta SNR[dB] - 20 \log_{10} \left( \left(10^{\Delta SNR[dB]/20} - 1\right)10^{VEC/20} + 1 \right) \]

- Note that if \( \Delta SNR > 0 \), \( \Delta VEC \) is negative, showing improvement in VEC
Explaination of Steps (Step 1)

1) Determining the optimum DFE tap is a standard practice in COM

\[ \alpha = DFE \text{ Tap} \]

- For optimization, it is possible, and maybe recommended to skip the DFE and optimize \( \alpha \) directly for best MLSE performance.
- This can be simply achieved by changing the signal energy in the COM-defined FOM to include energy of the post-cursor (MLSE treats the post-cursor as a part of the signal).
- The MLSE-based optimization has not been implemented here:
  - To provide a more direct and side-by-side comparison.
  - The additional performance improvement is not typically expected to be considerable.
  - To keep the existing COM flow untouched.
Explanation of Steps (Step 2)

2) Equation to calculate $\text{SNR}_{\text{DFE}}$ for L-PAM (Appendix A)

$$
\text{SNR}_{\text{DFE}} = \frac{1}{3} \frac{\text{main}^2}{\sigma_{\text{noise}}^2}
$$

Note that this can be written as

$$
\frac{\text{main}}{L-1} = \sigma_{\text{noise}} \sqrt{\frac{3}{L^2-1} \text{SNR}_{\text{DFE}}}
$$

$\text{main} =$ Main cursor of the pulse response at the RX FFE output
- Pulse amplitude = Peak value of the transmitted ± PAM signal swing
- Calculating main cursor is a standard practice in COM

$\sigma_{\text{noise}} =$ Standard deviation of the total noise (Xtalk, TX SNR, RX eta0, Jitter, and ISI) at the RX FFE output
- Calculated from the total noise PDF
- Calculating the total noise PDF is a standard practice in COM

Equation to calculate $\text{DER}_{\text{DFE}}$ for L-PAM (Appendix B)

$$
\text{DER}_{\text{DFE}} \approx \frac{2}{1 + CDF_{\text{noise}} \left(1 - 2\alpha \frac{\text{main}}{L-1}\right)} \left(1 - CDF_{\text{noise}} \left(\frac{\text{main}}{L-1}\right)\right)
$$

- The above equation includes effect of DFE error propagation
- Calculating the total noise CDF is a standard practice in COM
- Note that $\text{DER}_{\text{DFE}}$ is not needed for obtaining $\Delta \text{COM}$, but is still useful to calculate the decrease in error ratio
Explanation of Steps (Step 3)

3) Equation to calculate $\text{DER}_{\text{MLSE}}$ for L-PAM (Appendix C)

\[
\text{DER}_{\text{MLSE}} \approx 2 \sum_{j=1}^{\infty} j \left( \frac{L-1}{L} \right)^j \left( 1 - \text{CDF}_{\text{noise}} \left( \sqrt{1 + (j - 1)(1 - \alpha)^2 + \alpha^2 \frac{\text{main}}{L-1}} \right) \right)
\]

- The above equation includes effect of MLSE error propagation
- The summation is expected to include enough terms so that adding more terms doesn’t considerably change the result anymore
- Calculating the total noise CDF is a standard practice in COM
Explanation of Steps (Step 4)

4) Equation to calculate $SNR_{DFE, equivalent}$ (Appendix D)

$$SNR_{DFE, equivalent} \approx \left( \frac{L-1}{\text{main}} CDF^{-1}_{\text{noise}} \left( 1 - \frac{1}{2} DER_{\text{MLSE}} \left( \frac{1}{L-1} + CDF_{\text{noise}} \left( 1 - 2\alpha \frac{\text{main}}{L-1} \right) \right) \right) \right)^2 SNR_{DFE}$$

$CDF^{-1}_{\text{noise}}$ = Inverse function of the total noise CDF

- Calculating the total noise CDF (hence inverse CDF) is a standard practice in COM
- The above equation includes effect of MLSE error propagation

Equation to calculate $\sigma_{\text{noise, equivalent}}$ (Appendix D)

$$\sigma_{\text{noise, equivalent}} \approx \frac{L-1}{\text{main}} CDF^{-1}_{\text{noise}} \left( 1 - \frac{1}{2} DER_{\text{MLSE}} \left( \frac{1}{L-1} + CDF_{\text{noise}} \left( 1 - 2\alpha \frac{\text{main}}{L-1} \right) \right) \right) \sigma_{\text{noise}}$$

- The above equation alternatively suggests that the noise PDF and CDF can be horizontally scaled by the given factor to obtain PDF and CDF of the equivalent noise
- Note that calculating $\sigma_{\text{noise, equivalent}}$ is not necessary and is only an alternative to calculating $SNR_{DFE, equivalent}$
Explanation of Steps (Step 5)

5) Equation to calculate increase in SNR

\[
\frac{SNR_{DFE, equivalent}}{SNR_{DFE}} \approx \left( \frac{L-1}{\text{main}} CDF^{-1}_{\text{noise}} \left( 1 - \frac{1}{2} DER_{\text{MLSE}} \left( \frac{1}{L-1} + CDF_{\text{noise}} \left( 1 - 2\alpha \right) \frac{\text{main}}{L-1} \right) \right) \right)^2
\]

Equation to calculate equivalent decrease in noise

\[
\frac{\sigma_{\text{noise}}}{\sigma_{\text{noise, equivalent}}} \approx \left( \frac{L-1}{\text{main}} CDF^{-1}_{\text{noise}} \left( 1 - \frac{1}{2} DER_{\text{MLSE}} \left( \frac{1}{L-1} + CDF_{\text{noise}} \left( 1 - 2\alpha \right) \frac{\text{main}}{L-1} \right) \right) \right)
\]

Equation to calculate \( \Delta COM \)

\[
\Delta COM \approx 10 \log_{10} \left( \frac{SNR_{DFE, equivalent}}{SNR_{DFE}} \right) = 20 \log_{10} \left( \frac{\sigma_{\text{noise}}}{\sigma_{\text{noise, equivalent}}} \right)
\]

Equation to calculate reduction in DER

\[
\text{Reduction in DER} \approx \frac{DER_{\text{MLSE}}}{DER_{DFE}}
\]
Recap (4-PAM, L = 4)

\[ SNR_{DFE} = \frac{1}{3} \frac{L + 1}{L - 1} \times \frac{\text{main}^2}{\sigma_{\text{noise}}^2} \leftrightarrow \frac{\text{main}}{L - 1} = \frac{3}{L^2 - 1} SNR_{DFE} \]

\[ DER_{DFE} \approx \frac{2}{L - 1 + CDF_{\text{noise}} \left( 1 - 2\alpha \frac{\text{main}}{L - 1} \right)} \left( 1 - CDF_{\text{noise}} \frac{\text{main}}{L - 1} \right) \]

\[ DER_{MLSE} \approx 2 \sum_{j=1}^{\infty} j \left( \frac{L - 1}{L} \right)^j \left( 1 - CDF_{\text{noise}} \left( \sqrt{1 + j - 1(1 - \alpha)^2 + \alpha^2 \frac{\text{main}}{L - 1}} \right) \right) \]

\[ SNR_{DFE,\text{equivalent}} \approx \left( \frac{L - 1}{\text{main}} \right)^{CDF_{\text{noise}}^{-1}} \left( 1 - \frac{1}{2} DER_{MLSE} \left( \frac{1}{L - 1} + CDF_{\text{noise}} \left( 1 - 2\alpha \frac{\text{main}}{L - 1} \right) \right) \right)^2 SNR_{DFE} \]

\[ \Delta COM \approx 10 \log_{10} \left( \frac{SNR_{DFE,\text{equivalent}}}{SNR_{DFE}} \right) \]
Case Studies

- Consider MLSE processing and application of the idea to few wireline examples.
## Link Parameters

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* To force more errors to facilitate time-domain simulation verifications
## Summary of the Case Study Results (Native Noise)

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<th>D\textsubscript{MLSE}</th>
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*Simulations do not include CDR; Jitter is applied using COM method; Maximum 1M symbols
### Summary of the Case Study Results (Native Noise)

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<th>Channel</th>
<th>Variant</th>
<th>DFE Tap = (\alpha)</th>
<th>Theoretical Coding Gain [dB]</th>
<th>SNR(_{\text{DFE}}) [dB]</th>
<th>DER(_{\text{DFE}})</th>
<th>SNR(_{\text{DFE, equivalent}}) [dB]</th>
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<th>(\Delta\text{SNR} = \Delta\text{COM}) [dB]</th>
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* Simulations do not include CDR; Jitter is applied using COM method; Maximum 1M symbols
Summary of the Case Study Results (Native Noise)

\[ \alpha = 0.7 \ 0.8 \ 0.9 \ 1 \]

S1 Channels
S2 Channels
S3 Channels
S4 CPC Channels
S4 NPC Channels
S4 PCB Channels

Calculated with Gaussian Noise

DER vs SNR [dB]
## Summary of the Case Study Results (1E-3)

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<th>Theoretical Coding Gain [dB]</th>
<th>SNR(_{\text{DFE}}) [dB]</th>
<th>DER(_{\text{DFE}})</th>
<th>DER(_{\text{MLSE}})</th>
<th>SNR(_{\text{DFE, equivalent}}) [dB]</th>
<th>Nosie Scaling Factor</th>
<th>(\Delta\text{SNR} = \Delta\text{COM}) [dB]</th>
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* Simulations do not include CDR; Jitter is applied using COM method; Maximum 1M symbols
## Summary of the Case Study Results (1E-3)

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<th>Channel</th>
<th>Variant</th>
<th>DFE Tap = α</th>
<th>Theoretical Coding Gain [dB]</th>
<th>SNR&lt;sub&gt;DFE&lt;/sub&gt; [dB]</th>
<th>DER&lt;sub&gt;DFE&lt;/sub&gt;</th>
<th>SNR&lt;sub&gt;DFE, equivalent&lt;/sub&gt; [dB]</th>
<th>Noisie Scaling Factor</th>
<th>ΔSNR for Simulation [dB]</th>
<th>DER Ratio [Order of Magnitude]</th>
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* Simulations do not include CDR; Jitter is applied using COM method; Maximum 1M symbols
Summary of the Case Study Results (1E-3)

DFE Tap = 0.7 0.8 0.9 1

- S1 Channels
- S2 Channels
- S3 Channels
- S4 CPC Channels
- S4 NPC Channels
- S4 PCB Channels

\[ \alpha = 0.7 0.8 0.9 1 \]
### Summary of the Case Study Results (1E-4)

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<th>Theoretical Coding Gain [dB]</th>
<th>SNR&lt;sub&gt;DFE&lt;/sub&gt; [dB]</th>
<th>DER&lt;sub&gt;DFE&lt;/sub&gt;</th>
<th>SNR&lt;sub&gt;MLSE&lt;/sub&gt;</th>
<th>Noise Scaling Factor Total</th>
<th>ΔSNR = ΔCOM [Order of Magnitude]</th>
<th>DER Ratio DFE, equivalent [dB]</th>
<th>DER Simulation Results *</th>
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*Simulations do not include CDR; Jitter is applied using COM method; Maximum 1M symbols
## Summary of the Case Study Results (1E-4)

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<th>$\text{SNR}_{\text{DFE, equivalent}}$ [dB]</th>
<th>Nosie Scaling Factor</th>
<th>$\Delta \text{SNR} = \Delta \text{COM}$ [dB]</th>
<th>DER Ratio [Order of Magnitude]</th>
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* Simulations do not include CDR; Jitter is applied using COM method; Maximum 1M symbols
Summary of the Case Study Results (1E-4)

DFE Tap = 0.7 0.8 0.9 1

S1 Channels
S2 Channels
S3 Channels
S4 CPC Channels
S4 NPC Channels
S4 PCB Channels

Variant 1
Variant 2
Variant 3
Variant 4

SNR [dB]

DER

α = 0.7 0.8 0.9 1
Summary

- The proposal for incorporating performance advantage of MLSE in COM, originally presented in November 2022 as shakiba_3df_01b_2211.pdf, was explained in further details.
- Validity of the proposal approach and its implementation method were demonstrated by analysis of several channels and cases.
- All the equations resulted from analysis of DFE and MLSE were presented.
- A summary of the DFE and MLSE analysis was provided (Appendix).
- The proposal boils down to quantifying the equivalent COM advantage of MLSE over DFE (ΔCOM).
- The ultimate equations to calculate ΔCOM were given.
- The equations are COM compatible and can be directly calculated from COM parameters.
- The proposal is extendable to higher order MLSE as well as other more advanced signal processing techniques.
Future Work

- Adding FFE support to COM is needed, but can be done independently.
- Analysis of error propagation in MLSE.
- Analysis of the effect of colored noise.
- Continued validation by running more cases.
- More time-domain simulations.
- Study of MLSE implementation and simplification.
- Discussion of margin (e.g., COM margin).
- Even though the cost of a higher order MLSE will be more concerning and likely not currently practical, its study will still be helpful in exploring the limit of performance.
Calculating SNR for L-PAM
SNR for L-PAM

- Assuming outer PAM levels of ±main and L equi-probable levels

\[
PAM\ Level\ Separation = \frac{2\text{main}}{L-1}
\]

\[
PAM\ Levels = -\text{main} + \frac{2\text{main}}{L-1}l, \text{ for } l = 0, \ldots, L - 1
\]

\[
\text{Signsl Power} = \frac{\text{main}^2}{L} \sum_{l=0}^{L-1} \left( -1 + \frac{l}{L-1} \right)^2 = \frac{1}{3} \frac{L+1}{L-1} \text{main}^2
\]

(Note that \( \frac{1}{L} \sum_{l=0}^{L-1} (PAM\ Levels) = 0 \))

\[
\text{SNR} = \frac{1}{3} \frac{L+1}{L-1} \frac{\text{main}^2}{\sigma_{\text{noise}}^2}
\]

- This is the equation used in step 2
Appendix B

Error analysis of L-PAM 1-Tap DFE
Error Analysis without Error Propagation

- Assuming outer PAM levels of ± main, Gaussian noise, and dominance of adjacent-level errors
- Symbol error probability without error propagation
  - 2L-2 tails extend to wrong decision sides

\[ \text{Symbol error probability without error propagation} \approx \frac{1}{L} (2L - 2) Q \left( \frac{\text{main}}{(L-1)\sigma_{\text{noise}}} \right) = 2 \frac{L-1}{L} Q \left( \frac{\text{main}}{(L-1)\sigma_{\text{noise}}} \right) \]

- With error propagation each error symbol extends to a burst of errors
- For the purpose of error calculation, error ratio multiplies by the average burst length

\[ \text{DER}_{DFE, \text{without error propagation}} \approx 2 \frac{L-1}{L} \overline{BL}_{DFE} Q \left( \frac{\text{main}}{(L-1)\sigma_{\text{noise}}} \right) \text{, where assuming exponential distribution for burst lengths} \overline{BL}_{DFE} \approx \frac{1}{1-\text{EPP}_{DFE}} \]
Error Analysis with Error Propagation

- Error propagation changes each distribution to a bimodal distribution
  - In $2(2L-2)$ cases error propagation is destructive and tails extend to wrong decision sides

$$
EPP_{DFE} = P(\text{error}|\text{previous error}) \approx \frac{1}{2L} (2L - 2) \left( Q \left( 1 - 2\alpha \frac{\text{main}}{(L-1)\sigma_{\text{noise}}} \right) + Q \left( 1 + 2\alpha \frac{\text{main}}{(L-1)\sigma_{\text{noise}}} \right) \right) \approx \frac{L-1}{L} Q \left( 1 - 2\alpha \frac{\text{main}}{(L-1)\sigma_{\text{noise}}} \right)
$$

$$
DER_{DFE} \approx 2 \frac{L-1}{L} \frac{1}{1-EPP_{DFE}} \frac{\text{main}}{(L-1)\sigma_{\text{noise}}} Q \left( \frac{2}{L-1} \frac{\text{main}}{(L-1)\sigma_{\text{noise}}} \right) Q \left( \frac{\text{main}}{(L-1)\sigma_{\text{noise}}} \right)
$$

4-PAM Example

For large $\alpha$ (e.g. $0.5 < \alpha < 1$)
Error Analysis with Error Propagation and Arbitrary Noise

- If noise is not Gaussian, change the Q function to 1 - CDF
- Note that since by definition of Q function its argument is normalized to standard deviation and the argument should now be de-normalized to $\sigma_{\text{noise}}$

$$DER_{DFE} \approx \frac{2}{\frac{1}{L-1} + CDF_{\text{noise}} \left(\frac{1 - 2\alpha}{L-1}\right)} \left(1 - CDF_{\text{noise}} \left(\frac{\text{main}}{L-1}\right)\right)$$

- The above expression includes the effect of error propagation
- This is the equation used in step 2
Appendix C

Error analysis of L-PAM 1+αD MLSE
Minimum Distant Error Events

- The obvious one (shortest event)
  \[ l \in \{0, \ldots, L - 1\} \]

- Not so obvious ones (longer events)
  \[ l \in \{0, \ldots, L - 1\} \]
Minimum Distant Error Events

- Assuming outer PAM levels of ±main, the Euclidean distance for the obvious short error event is \(2^{\text{main}} \frac{\sqrt{1 + \alpha^2}}{L-1}\).

- The Euclidean distance for the longer error events is \(2^{\text{main}} \frac{\sqrt{1 + (j-1)(1 - \alpha)^2 + \alpha^2}}{L-1}\) and the burst of errors it entails has a length of \(j (j \geq 1)\).

- Note that for \(j = 1\), this becomes the same as the short error event.

- Also note that as \(\alpha\) approaches 1 the Euclidean distance of all of the longer error events approaches the distance of the short error event.

- This is the error propagation mechanism in the MLSE and is maximized for \(\alpha = 1\).

- Combinational counting reveals that the fractional frequency of these error events is \(2^{\left(\frac{L-1}{L}\right)^j}\).
Error Analysis

Putting together fractional frequency, number of errors, and Euclidean distance for individual events and summation over all the events results in the following overall decision error ratio of the MLSE with Gaussian noise:

\[
DER_{MLSE} \approx 2 \sum_{j=1}^{\infty} j \left( \frac{L-1}{L} \right)^j Q \left( \sqrt{1 + (j - 1)(1 - \alpha)^2 + \alpha^2 \frac{main}{(L-1)\sigma_{noise}}} \right)
\]

which for arbitrary noise with a known CDF, and after de-normalization, becomes the following expression:

\[
DER_{MLSE} \approx 2 \sum_{j=1}^{\infty} j \left( \frac{L-1}{L} \right)^j \left( 1 - CDF_{noise} \left( \sqrt{1 + (j - 1)(1 - \alpha)^2 + \alpha^2 \frac{main}{L-1}} \right) \right)
\]

This is the equation used in step 3.
Analysis of the Conceptual Equivalent DFE
SNR of the ‘Equivalent’ DFE

- How much does SNR need to increase so that a conceptual equivalent DFE performs as well as the MLSE?

\[
DER_{DFE, equivalent} = DER_{MLSE}
\]

\[
\frac{L}{L-1}Q\left(\frac{2}{(1-2\alpha)\sqrt{3\text{SNR}_{DFE,equivalent}}}\right)Q\left(\frac{\text{SNR}_{DFE,equivalent}}{\sqrt{L^2-1}}\right) = DER_{MLSE}
\]

- Solving this equation requires iterations, but noticing that the Q function in the denominator of left hand side is a weak function of its argument, particularly \(\text{SNR}_{DFE,equivalent}\), which only changes from \(\text{SNR}_{DFE}\) by as much as a factor of 2, \(\text{SNR}_{DFE,equivalent}\) can be replaced with \(\text{SNR}_{DFE}\) to avoid iterations with negligible accuracy penalty, yielding

\[
\text{SNR}_{DFE,equivalent} = \text{SNR}_{DFE}\left(\frac{(L-1)\sigma_{\text{noise main}}}{Q^{-1}}\left(\frac{1}{2} \cdot DER_{MLSE} \left(\frac{L}{L-1} - Q\left(1 - 2\alpha \frac{\sigma_{\text{noise main}}}{(L-1)\sigma_{\text{noise}}}\right)\right)\right)^2\right)
\]
Noise of the ‘Equivalent’ DFE

• How much does noise need to decrease to give the same increase in SNR so that the conceptual equivalent DFE performs as well as the MLSE?

\[
SNR_{DFE} = \frac{1}{3} \frac{L+1}{L-1} \frac{\text{main}^2}{\sigma_{noise}^2}
\]

\[
SNR_{DFE, equivalent} = \frac{1}{3} \frac{L+1}{L-1} \frac{\text{main}^2}{\sigma_{noise, equivalent}^2}
\]

\[
\sigma_{noise, equivalent} = \frac{1}{L-1} \frac{Q^{-1}\left(\frac{1}{2} \text{DER}_{MLSE} \left(\frac{L}{L-1} Q \left(1-2\alpha \frac{\text{main}}{(L-1)\sigma_{noise}}\right)\right)\right)}{\text{main}^2}
\]
'Equivalent' SNR and Noise with Arbitrary Noise

- Change the Q function to $1$-CDF, de-normalized, and solve

$$\frac{2}{\frac{1}{L-1} + \text{CDF}_{\text{noise}} \left( (1-2\alpha)\sigma_{\text{noise}} \sqrt{\frac{3}{L^2-1}} \text{SNR}_{DFE,\text{equivalent}} \right)} \left( 1 - \text{CDF} \left( \sigma_{\text{noise}} \sqrt{\frac{3}{L^2-1}} \text{SNR}_{DFE,\text{equivalent}} \right) \right) = \text{DER}_{\text{MLSE}}$$

- Similarly, replace $\text{SNR}_{DFE,\text{equivalent}}$ in the denominator with $\text{SNR}_{DFE}$ to avoid iterations to yield

$$\text{SNR}_{DFE,\text{equivalent}} = \left( \frac{L-1}{\text{main}} \text{CDF}_{\text{noise}}^{-1} \left( 1 - \frac{1}{2} \text{DER}_{\text{MLSE}} \left( \frac{1}{L-1} + \text{CDF}_{\text{noise}} \left( (1-2\alpha)\frac{\text{main}}{L-1} \right) \right) \right) \right)^2 \text{SNR}_{DFE}$$

- Equivalently, this increase in SNR can be expressed as a decrease in noise

$$\sigma_{\text{noise, equivalent}} = \frac{L-1}{\text{main}} \text{CDF}_{\text{noise}}^{-1} \left( 1 - \frac{1}{2} \text{DER}_{\text{MLSE}} \left( \frac{1}{L-1} + \text{CDF}_{\text{noise}} \left( (1-2\alpha)\frac{\text{main}}{L-1} \right) \right) \right) \sigma_{\text{noise}}$$

- Last two equations are the equations used in step 4