Further on a Path toward Incorporating Advanced Signal Processing in Electrical Channel Performance Assessment

Hossein Shakiba
Huawei Technologies
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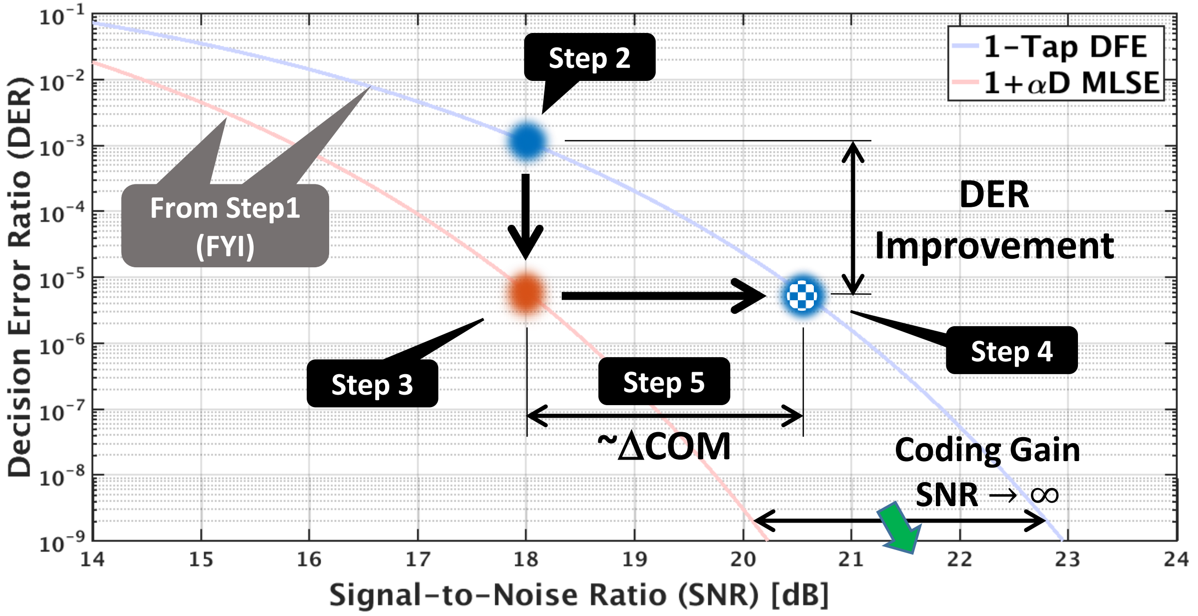
Introduction

- A method for incorporating performance effect of MLSE was proposed in shakiba_3df_01b_2211.pdf
- In response to the feedback, this is a first update, specifically:
  - Updated S3 channel models used in case studies
  - **Added 11 more cases (for a total of 22 cases)**
  - Used the same noise scaling approach for all channel cases (summarized in table on slide 7)
  - Corrected a minor typo in the table on slide 7
  - Updated case study results (slides 9-11) at two DER targets of 1E-3 and 1E-4 (with DFE)
  - **Added case study results for channel native noise levels (i.e. no noise scaling)**
  - **Provided more information and equations for the MLSE proposal**
- The equations are basis for incorporating the MLSE impact into the existing COM flow
- The equations can be easily and directly translated to Matlab for COM Matlab tool update
- More clarification and explanation will be provided based on the feedback
  - The received feedbacks and comments have been encouraging and are appreciated ☺

Proposal Recap

- The proposal outlined a method that enables incorporating MLSE impact in the COM flow
- As rates increase, we see this as a useful and maybe necessary update to COM for reflecting a more accurate and realistic representation of capabilities in the reference receiver
- RX FFE support is essential to considering more advanced detection techniques than DFE
- $1+\alpha D$ MLSE appears as a first natural alternative to a 1-tap DFE
- Proposal specified following steps:
  1) Use COM analysis to find the DFE tap, α
  2) Use analysis to calculate SNR_{DFE} and DER_{DFE}
  3) Use analysis to calculate DER_{MLSE} at SNR_{DFE}
  4) Use analysis to calculate $SNR_{DFE,equivalent}$ for the same DFE that yields the same DER_{MLSE}
  5) Increase from SNR_{DFE} to $SNR_{DFE,equivalent}$ is a much better estimate of COM improvement (ΔCOM) offered by the MLSE than $10\log_{10}(1+\alpha^2)$

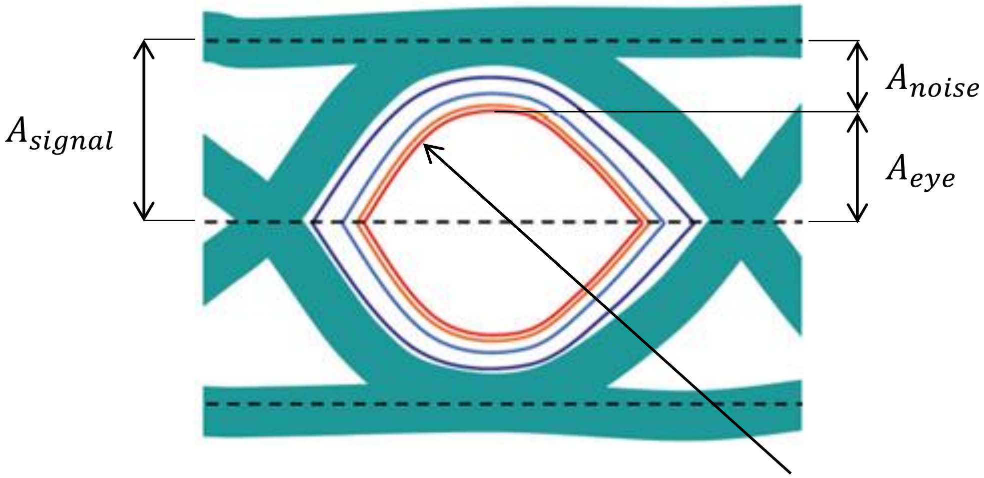


SNR, COM, and VEC

$$COM = 20\log_{10}\left(\frac{A_{signal}}{A_{noise}}\right)$$

$$VEC = 20\log_{10}\left(\frac{A_{signal}}{A_{eye}}\right)$$

$$\rightarrow \quad COM = -20\log_{10}\left(1 - 10^{-VEC/20}\right)$$



- COM and VEC are related to SNR

$$SNR[dB] = 10\log_{10}\left(\frac{1}{3}\frac{L+1}{L-1}\frac{{A_{peak}}^2}{{\sigma_{noise}}^2}\right)$$ (Appendix A)

$$A_{peak} = (L-1)A_{signal}$$

$$A_{noise} = k_{DER}\sigma_{noise}$$

← k_{DER} is a multiplier factor that determines how many σ's away from mean achieves target DER (a.k.a. Q factor for Gaussian noise)

- As a result COM can be expressed as

$$COM = SNR[dB] - 10\log_{10}\left(\frac{L^2-1}{3}{k_{DER}}^2\right)$$

- Which suggests that COM is in fact a kind of SNR with a notion of DER directly built in it

SNR, COM, and VEC

$$COM = SNR[dB] - 10\log_{10}\left(\frac{L^2-1}{3}{k_{DER}}^2\right)$$

- There are three ways to interpret this equation
  1) If after a change in SNR same DER is targeted, k_{DER} remains constant and any increase (decrease) in SNR translates to the same increase (or decrease) in COM

     $$\boxed{\Delta COM = \Delta SNR[dB]}$$

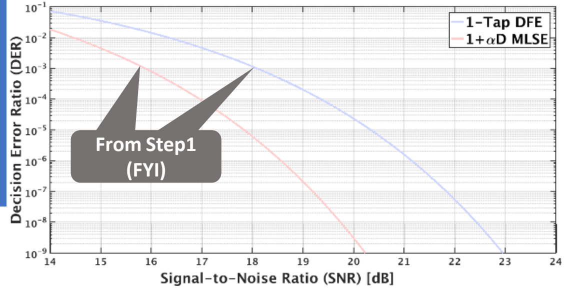
     e.g. this means that if a 3dB COM is targeted and if MLSE is used in the actual receiver with a verified 1.8dB SNR gain, COM target with the reference receiver can be lowered to 1.2dB.

  2) If after a change in SNR same COM is targeted, COM remains constant and any increase (or decrease) in SNR translates to an increase (or decrease) in k_{DER} and results in a decrease (or increase) in DER
  3) A change in SNR can be broken down to partially achieve both above as long as the equation holds
- Any change in SNR also translates to a change in VEC, but the change depends on VEC value

$$\Delta VEC = \Delta SNR[dB] - 20\log_{10}\left(\left(10^{\Delta SNR[dB]/20} - 1\right)10^{VEC/20} + 1\right)$$

- Note that if $\Delta SNR > 0$, ΔVEC is negative, showing improvement in VEC

Explanation of Steps (Step 1)

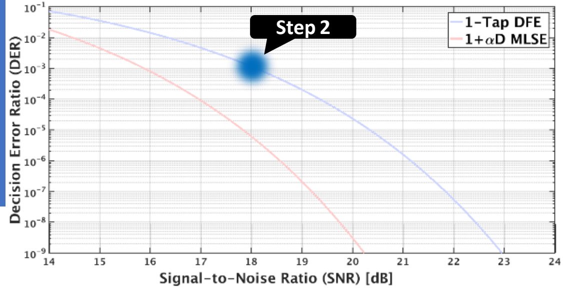


1) Determining the optimum DFE tap is a standard practice in COM

$$\boxed{\alpha = DFE\ Tap}$$

- For optimization, it is possible, and maybe recommended to skip the DFE and optimize α directly for best MLSE performance
- This can be simply achieved by changing the signal energy in the COM-defined FOM to include energy of the post-cursor (MLSE treats the post-curser as a part of the signal)
- The MLSE-based optimization has not been implemented here:
    - To provide a more direct and side-by-side comparison
    - The additional performance improvement is not typically expected to be considerable
    - To keep the existing COM flow untouched

Explanation of Steps (Step 2)



2) Equation to calculate SNR_{DFE} for L-PAM (Appendix A)

$$SNR_{DFE} = \frac{1}{3}\frac{L+1}{L-1}\frac{main^2}{\sigma_{noise}{}^2}$$ ← Note that this can be written as → $$\frac{main}{L-1} = \sigma_{noise}\sqrt{\frac{3}{L^2-1}SNR_{DFE}}$$

$main =$ Main cursor of the pulse response at the RX FFE output

- Pulse amplitude = Peak value of the transmitted ± PAM signal swing
- Calculating main cursor is a standard practice in COM

$\sigma_{noise} =$ Standard deviation of the total noise (Xtalk, TX SNR, RX eta0, Jitter, and ISI) at the RX FFE output

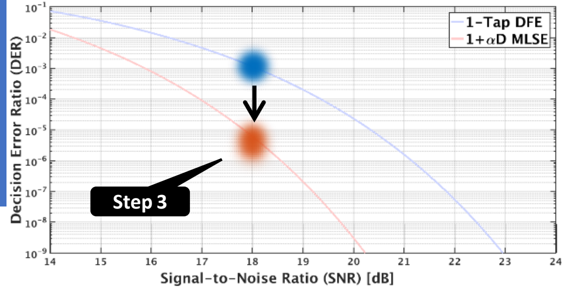
- Calculated from the total noise PDF
- Calculating the total noise PDF is a standard practice in COM

Equation to calculate DER_{DFE} for L-PAM (Appendix B)

$$DER_{DFE} \approx \frac{2}{\frac{1}{L-1}+CDF_{noise}\left((1-2\alpha)\frac{main}{L-1}\right)}\left(1 - CDF_{noise}\left(\frac{main}{L-1}\right)\right)$$

- The above equation includes effect of DFE error propagation
- Calculating the total noise CDF is a standard practice in COM
- Note that DER_{DFE} is not needed for obtaining ΔCOM, but is still useful to calculate the decrease in error ratio

Explanation of Steps (Step 3)

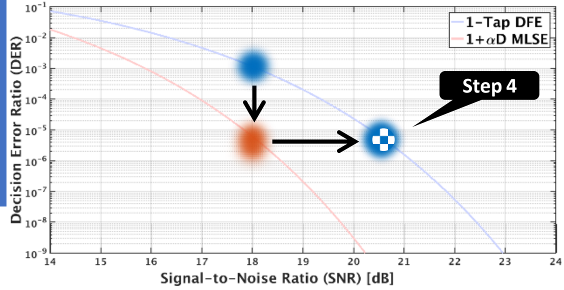


3) Equation to calculate DER_{MLSE} for L-PAM (Appendix C)

$$DER_{MLSE} \approx 2\sum_{j=1}^{\infty} j\left(\frac{L-1}{L}\right)^{j}\left(1 - CDF_{noise}\left(\sqrt{1+(j-1)(1-\alpha)^2+\alpha^2}\,\frac{main}{L-1}\right)\right)$$

- The above equation includes effect of MLSE error propagation
- The summation is expected to include enough terms so that adding more terms doesn't considerably change the result anymore
- Calculating the total noise CDF is a standard practice in COM

Explanation of Steps (Step 4)



4) Equation to calculate $\text{SNR}_{\text{DFE,equivalent}}$ (Appendix D)

$$SNR_{DFE,equivalent} \approx \left(\frac{L-1}{main} CDF_{noise}^{-1}\left(1-\frac{1}{2}DER_{MLSE}\left(\frac{1}{L-1}+CDF_{noise}\left((1-2\alpha)\frac{main}{L-1}\right)\right)\right)\right)^2 SNR_{DFE}$$

$CDF_{noise}^{-1} =$ Inverse function of the total noise CDF

- Calculating the total noise CDF (hence inverse CDF) is a standard practice in COM

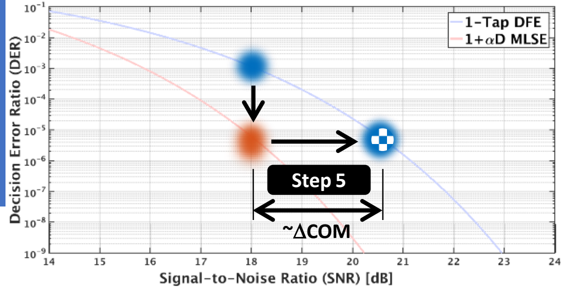
- The above equation includes effect of MLSE error propagation

Equation to calculate $\sigma_{\text{noise,equivalent}}$ (Appendix D)

$$\sigma_{noise,equivalent} \approx \frac{1}{\frac{L-1}{main} CDF_{noise}^{-1}\left(1-\frac{1}{2}DER_{MLSE}\left(\frac{1}{L-1}+CDF_{noise}\left((1-2\alpha)\frac{main}{L-1}\right)\right)\right)} \sigma_{noise}$$

- The above equation alternatively suggests that the noise PDF and CDF can be horizontally scaled by the given factor to obtain PDF and CDF of the equivalent noise
- Note that calculating $\sigma_{\text{noise,equivalent}}$ is not necessary and is only an alternative to calculating $SNR_{DFE,equivalent}$

Explanation of Steps (Step 5)



5) Equation to calculate increase in SNR

$$\frac{SNR_{DFE,equivalent}}{SNR_{DFE}} \approx \left(\frac{L-1}{main} CDF_{noise}^{-1}\left(1-\frac{1}{2} DER_{MLSE}\left(\frac{1}{L-1}+CDF_{noise}\left((1-2\alpha)\frac{main}{L-1}\right)\right)\right)\right)^2$$

Equation to calculate equivalent decrease in noise

$$\frac{\sigma_{noise}}{\sigma_{noise,equivalent}} \approx \frac{L-1}{main} CDF_{noise}^{-1}\left(1-\frac{1}{2} DER_{MLSE}\left(\frac{1}{L-1}+CDF_{noise}\left((1-2\alpha)\frac{main}{L-1}\right)\right)\right)$$

Equation to calculate ΔCOM

$$\boxed{\Delta COM \approx 10\log_{10}\left(\frac{SNR_{DFE,equivalent}}{SNR_{DFE}}\right)} = 20\log_{10}\left(\frac{\sigma_{noise}}{\sigma_{noise,equivalent}}\right)$$

Equation to calculate reduction in DER

$$Reduction\ in\ DER \approx \frac{DER_{MLSE}}{DER_{DFE}}$$

Recap (4-PAM, L = 4)

$$DER_{MLSE} \approx 2\sum_{l=1}^{\infty} l\left(\frac{L-1}{L}\right)^{l} Q\left(\frac{\sqrt{1+(i-1)(1-\alpha)^2+\alpha^2}}{(L-1)\sigma_{total\_noise}}\right)$$

$$DER_{DFE} \approx \frac{2}{\frac{L}{L-1} - Q\left(\frac{1-2\alpha}{(L-1)\sigma_{total\_noise}}\right)} Q\left(\frac{1}{(L-1)\sigma_{total\_noise}}\right)$$

from 1 (FYI)

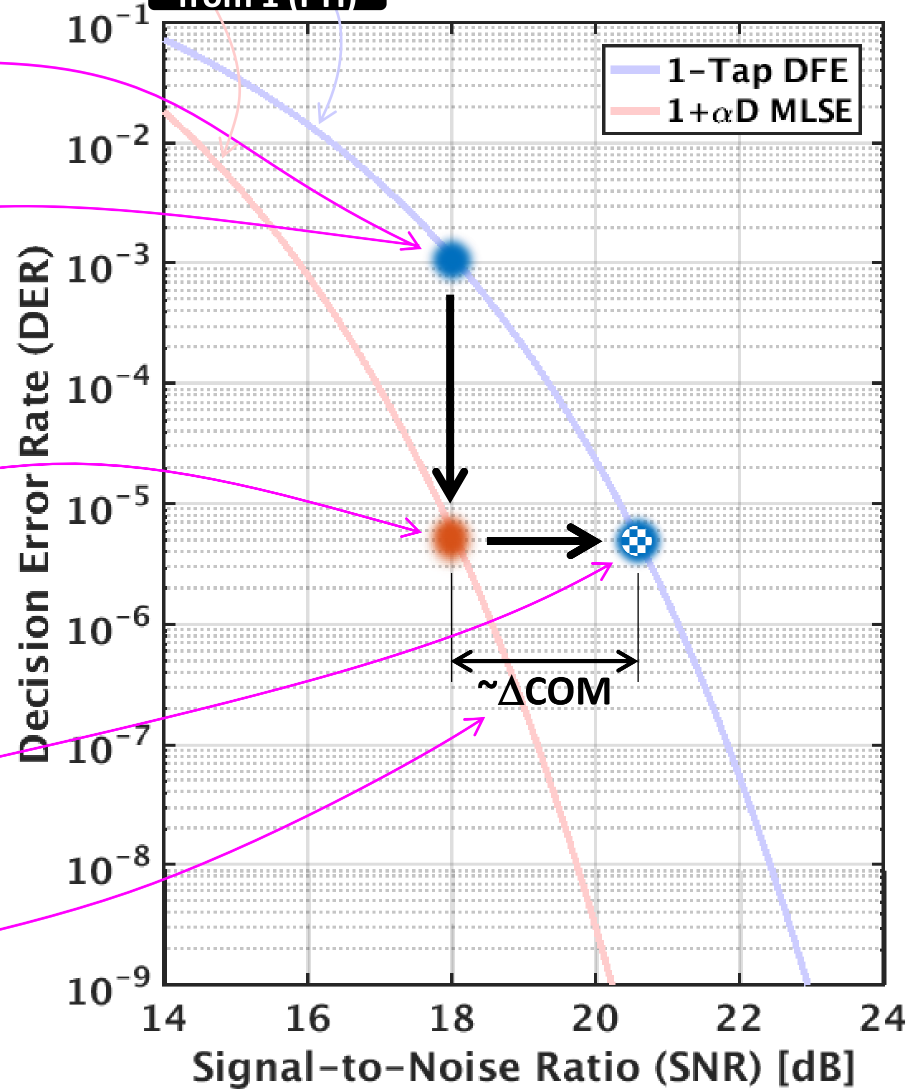
2
$$SNR_{DFE} = \frac{1}{3}\frac{L+1}{L-1} \times \frac{main^2}{\sigma_{noise}{}^2} \quad \leftrightarrow \quad \frac{main}{L-1} = \sigma_{noise}\sqrt{\frac{3}{L^2-1} SNR_{DFE}}$$

2
$$DER_{DFE} \approx \frac{2}{\frac{1}{L-1} + CDF_{noise}\left((1-2\alpha)\frac{main}{L-1}\right)}\left(1 - CDF_{noise}\left(\frac{main}{L-1}\right)\right)$$

3
$$DER_{MLSE} \approx 2\sum_{j=1}^{\infty} j\left(\frac{L-1}{L}\right)^{j}\left(1 - CDF_{noise}\left(\sqrt{1+(j-1)(1-\alpha)^2+\alpha^2}\,\frac{main}{L-1}\right)\right)$$

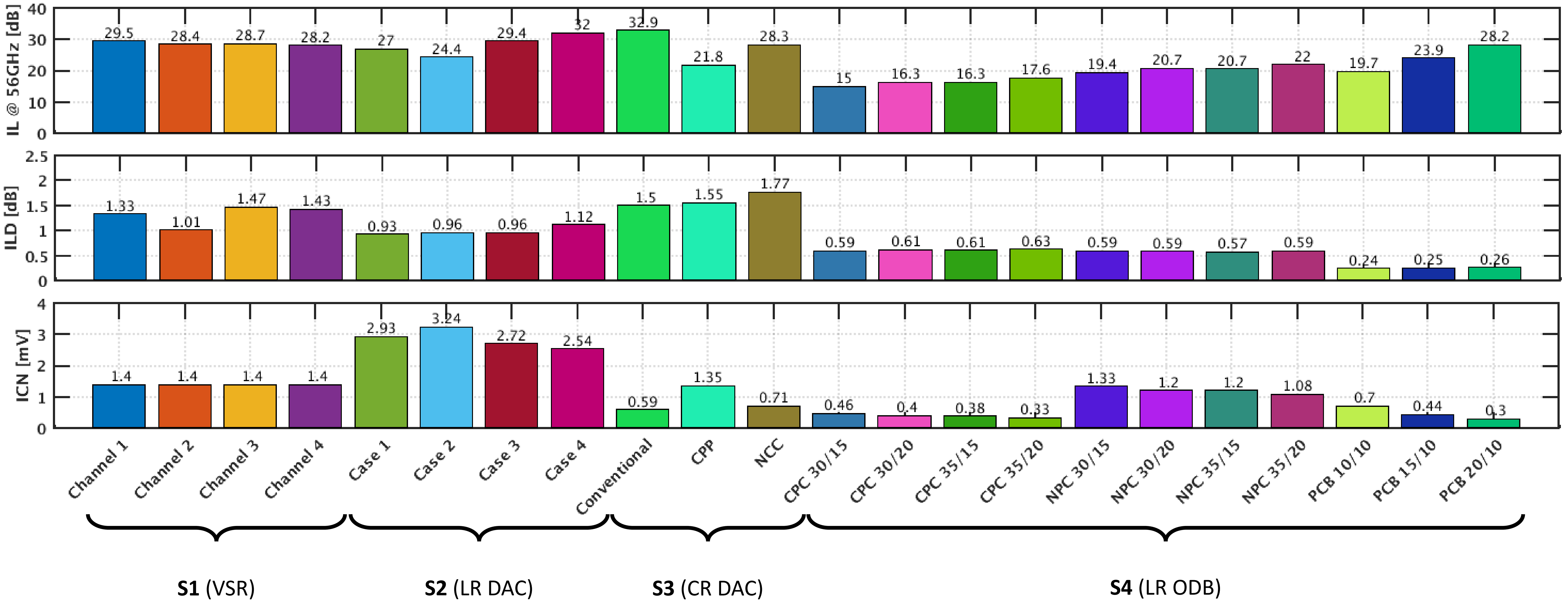
4
$$SNR_{DFE,equivalent} \approx \left(\frac{L-1}{main} CDF_{noise}^{-1}\left(1 - \frac{1}{2}DER_{MLSE}\left(\frac{1}{L-1} + CDF_{noise}\left((1-2\alpha)\frac{main}{L-1}\right)\right)\right)\right)^2 SNR_{DFE}$$

5
$$\Delta COM \approx 10\log_{10}\left(\frac{SNR_{DFE,equivalent}}{SNR_{DFE}}\right)$$



Case Studies

- Consider MLSE processing and application of the idea to few wireline examples



Link Parameters

Channel	Bit Rate [Gb/s]	Thru Swing [mV]	Fext Swing [mV]	Next Swing [mV]	TX FIR [Pre / Post]	Die C_d [fF] L_s [pH]	C_b [fF]	Package [mm] [Ω]	RX Filter BW	CTLE Pole/Zero Ratio	DFE [# of Taps]	RX FFE [Pre / Post]	TX SNR [dB]	RX Noise [V²/GHz]	Jitter Rand / DD [UI]	k_N *		
																Native	1E-3	1E-4
S1	224	413	413 x k_N	608 x k_N	3 / 1	40/90/110 130/150/140	Included In channel	Included In channel	0.75 x f_b	80/2.5/1	1	6 / 8	32.5 - $20\log_{10}(k_N)$	4.1E-8 x k_N^2	0.01 / 0.02 x k_N	1	2	1.675
																1	2.05	1.725
																1	1.9	1.575
																1	2	1.675
S2	224	442	442 x k_N	608 x k_N	3 / 1	40/90/110 130/150/140	30	30 92.5	0.75 x f_b	100/2.5/1	1	0 / 24	33 - $20\log_{10}(k_N)$	4.1E-8 x k_N^2	0.01 / 0.02 x k_N	1	1.85	1.6
																1	2	1.725
																1	1.7	1.425
																1	1.45	1.225
S3	224	413	413 x k_N	608 x k_N	3 / 1	40/90/110 130/150/140	30	30 92.5	0.75 x f_b	80/2.5/1	1	0 / 24	33 - $20\log_{10}(k_N)$	4.1E-8 x k_N^2	0.01 / 0.02 x k_N	1	0.9	0.735
																1	1.45	1.175
																1	0.95	0.775
S4	224	413	413 x k_N	608 x k_N	3 / 1	40/90/110 130/150/140	40	30 92.5	0.75 x f_b	80/2.5/1	1	0 / 24	33 - $20\log_{10}(k_N)$	4.1E-8 x k_N^2	0.01 / 0.02 x k_N	1	2.34	2
																1	2.25	1.9
																1	2.225	1.9
																1	2.1	1.775
																1	1.68	1.405
																1	1.525	1.275
																1	1.525	1.275
																1	1.41	1.175
																1	1.85	1.555
																1	1.46	1.235
																1	1.14	0.945

* To force more errors to facilitate time-domain simulation verifications

Summary of the Case Study Results (Native Noise)

Channel	Variant	DFE Tap $= \alpha$	Theoretical Coding Gain [dB]	SNR_{DFE} [dB]	DER_{DFE}	DER_{MLSE}	$SNR_{DFE,equivalent}$ [dB]	Nosie Scaling Factor		$\Delta SNR = \Delta COM$ [dB]	DER Ratio [Order of Magnitude]	DER Simulation Results *		
								Total	for Simulation			DFE	MLSE	$DFE_{Equivalent}$
S1	Channel 1	**0.8116**	2.1977	**22.4094**	**9.3440 E-9**	**3.2390 E-14**	**24.6754**	**0.7704**	0.5318	**2.2660**	5.4601	NA	NA	NA
	Channel 2	**0.7272**	1.8437	**22.8466**	**1.7362 E-9**	**1.6404 E-14**	**24.8377**	**0.7951**	0.6465	**1.9911**	5.0247	NA	NA	NA
	Channel 3	**0.7655**	2.0029	**21.9669**	**5.1313 E-8**	**1.6462 E-12**	**24.0787**	**0.7842**	0.5541	**2.1118**	4.4937	NA	NA	NA
	Channel 4	**0.7850**	2.0849	**22.4866**	**7.8439 E-9**	**5.1413 E-14**	**24.6685**	**0.7779**	0.5745	**2.1819**	5.1835	NA	NA	NA
S2	Case 1	**0.8599**	2.4042	**23.0054**	**1.0158 E-9**	**2.3304 E-16**	**25.1529**	**0.7810**	0.7324	**2.1475**	6.6394	NA	NA	NA
	Case 2	**0.8893**	2.5308	**23.8067**	**1.8718 E-11**	**6.6854 E-20**	**25.1540**	**0.8563**	0.8230	**1.3473**	8.4471	NA	NA	NA
	Case 3	**0.8702**	2.4481	**22.0362**	**4.7543 E-8**	**2.6394 E-13**	**24.4423**	**0.7580**	0.7004	**2.4060**	5.2556	NA	NA	NA
	Case 4	**0.8534**	2.3764	**20.8167**	**2.4914 E-6**	**3.8091 E-10**	**23.1275**	**0.7664**	0.7153	**2.3108**	3.8156	0	NA	NA
S3	Conventional	**0.9728**	2.8924	**17.3785**	**2.3461 E-3**	**2.5322 E-5**	**19.7544**	**0.7607**	0.6036	**2.3759**	1.9668	4.13 E-3	5.1 E-5	1.1 E-5
	CPP	**0.9999**	3.0100	**19.8950**	**1.3693 E-5**	**4.7054 E-10**	**22.6422**	**0.7289**	0.2504	**2.7472**	4.4639	1.9 E-5	NA	NA
	NCC	**0.9923**	2.9767	**17.7271**	**1.5176 E-3**	**9.9057 E-6**	**20.2105**	**0.7513**	0.5873	**2.4834**	2.1853	1.81 E-3	1.3 E-5	1.2 E-5

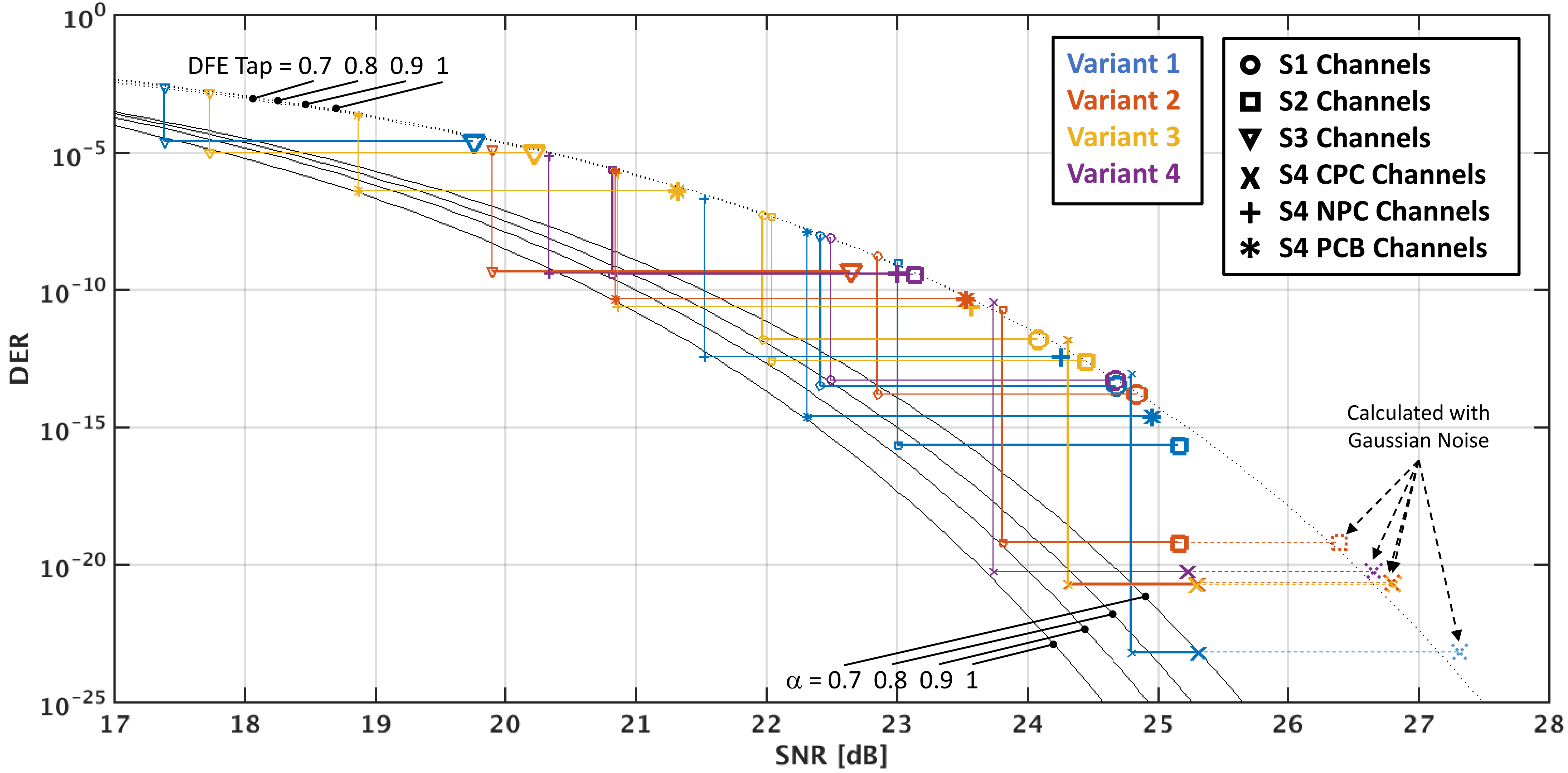
* Simulations do not include CDR; Jitter is applied using COM method; Maximum 1M symbols

Summary of the Case Study Results (Native Noise)

Channel	Variant	DFE Tap $= \alpha$	Theoretical Coding Gain [dB]	SNR_{DFE} [dB]	DER_{DFE}	DER_{MLSE}	$SNR_{DFE,equivalent}$ [dB]	Nosie Scaling Factor		$\Delta SNR = \Delta COM$ [dB]	DER Ratio [Order of Magnitude]	DER Simulation Results *		
								Total	for Simulation			DFE	MLSE	$DFE_{Equivalent}$
S4	CPC 30/15	**0.8389**	2.3141	**24.7915**	**8.7930 E-14**	**6.7051 E-24**	**25.3072**	**0.9424**	0.9211	**0.5157**	10.1177	NA	NA	NA
	CPC 30/20	**0.8361**	2.3021	**24.3010**	**1.6507 E-12**	**2.0676 E-21**	**25.2911**	**0.8923**	0.8508	**0.9901**	8.9022	NA	NA	NA
	CPC 35/15	**0.8388**	2.3136	**24.3061**	**1.5934 E-12**	**1.7804 E-21**	**25.2872**	**0.8932**	0.8519	**0.9812**	8.9518	NA	NA	NA
	CPC 35/20	**0.9843**	2.9419	**23.7363**	**3.5020 E-11**	**5.8234 E-21**	**25.2246**	**0.8425**	0.7736	**1.4883**	9.7791	NA	NA	NA
	NPC 30/15	**0.9819**	2.9315	**21.5206**	**2.2344 E-7**	**3.6820 E-13**	**24.2546**	**0.7300**	0.5384	**2.7340**	5.7831	NA	NA	NA
	NPC 30/20	**0.9847**	2.9430	**20.8551**	**1.8751 E-6**	**2.4601 E-11**	**23.5614**	**0.7323**	0.5527	**2.7063**	4.8821	0	NA	0
	NPC 35/15	**0.9850**	2.9452	**20.8528**	**1.8886 E-6**	**2.4803 E-11**	**23.5602**	**0.7322**	0.5523	**2.7073**	4.8816	0	NA	0
	NPC 35/20	**0.9837**	2.9379	**20.3274**	**7.9269 E-6**	**3.9988 E-10**	**23.0001**	**0.7351**	0.5559	**2.6727**	4.2972	1.2 E-5	NA	0
	PCB 10/10	**0.9906**	2.9693	**22.3082**	**1.3449 E-8**	**2.6110 E-15**	**24.9509**	**0.7377**	0.5560	**2.6427**	6.7119	NA	NA	NA
	PCB 15/10	**0.9815**	2.9300	**20.8384**	**2.2760 E-6**	**4.6077 E-11**	**23.5229**	**0.7341**	0.6260	**2.6845**	4.6937	5 E-6	NA	0
	PCB 20/10	**0.9542**	2.8113	**18.8703**	**2.2232 E-4**	**4.0606 E-7**	**21.3151**	**0.7547**	0.6565	**2.4448**	2.7384	3.81 E-4	NA	3 E-6

* Simulations do not include CDR; Jitter is applied using COM method; Maximum 1M symbols

Summary of the Case Study Results (Native Noise)



Summary of the Case Study Results (1E-3)

Channel	Variant	DFE Tap $= \alpha$	Theoretical Coding Gain [dB]	SNR_{DFE} [dB]	DER_{DFE}	DER_{MLSE}	$SNR_{DFE,equivalent}$ [dB]	Nosie Scaling Factor		$\Delta SNR = \Delta COM$ [dB]	DER Ratio [Order of Magnitude]	DER Simulation Results *		
								Total	for Simulation			DFE	MLSE	$DFE_{Equivalent}$
S1	Channel 1	**0.8121**	2.1997	**18.0966**	**1.0292 E-3**	**2.1588 E-5**	**20.1022**	**0.7938**	0.7465	**2.0055**	1.6783	1.04 E-3	2.2 E-5	2.5 E-5
	Channel 2	**0.7729**	2.0341	**18.0369**	**1.1216 E-3**	**3.0795 E-5**	**19.9627**	**0.8011**	0.7670	**1.9258**	1.5614	1.95 E-3	2.6 E-5	3.6 E-5
	Channel 3	**0.7778**	2.0546	**18.0641**	**1.0299 E-3**	**2.4942 E-5**	**20.0097**	**0.7993**	0.7467	**1.9456**	1.6159	5.5 E-4	1.4 E-5	3.3 E-5
	Channel 4	**0.7761**	2.0476	**18.0914**	**1.0044 E-3**	**2.4560 E-5**	**20.0336**	**0.7996**	0.7586	**1.9422**	1.6117	1.02 E-3	1.2 E-5	2.4 E-5
S2	Case 1	**0.9025**	2.5876	**18.1565**	**9.6653 E-4**	**1.2347 E-5**	**20.3399**	**0.7777**	0.7552	**2.1833**	1.8937	1.71 E-3	3.3 E-5	3.6 E-5
	Case 2	**0.8219**	2.2417	**18.1782**	**9.3605 E-4**	**1.7755 E-5**	**20.2069**	**0.7917**	0.7788	**2.0288**	1.7220	1.42 E-3	7.1 E-5	4.1 E-5
	Case 3	**0.9119**	2.6283	**17.9732**	**1.1996 E-3**	**1.6031 E-5**	**20.1700**	**0.7765**	0.7417	**2.1968**	1.8741	2.73 E-3	7.2 E-5	6.8 E-5
	Case 4	**0.9026**	2.5880	**18.0037**	**1.1001 E-3**	**1.3840 E-5**	**20.1903**	**0.7774**	0.7321	**2.1866**	1.9002	2.56 E-3	9 E-6	3.6 E-5
S3	Conventional	**0.9672**	2.8680	**17.9234**	**1.0536 E-3**	**5.9785 E-6**	**20.3362**	**0.7575**	0.6115	**2.4128**	2.2461	1.94 E-3	1.9 E-5	5 E-6
	CPP	**0.9938**	2.9833	**17.9546**	**1.0985 E-3**	**5.4707 E-6**	**20.4631**	**0.7492**	0.5976	**2.5085**	2.3028	1.06 E-3	1 E-6	5 E-6
	NCC	**0.9933**	2.9812	**18.0133**	**9.5421 E-4**	**3.8148 E-6**	**20.5386**	**0.7477**	0.5601	**2.5253**	2.3982	1.40 E-3	1.9 E-5	5 E-6

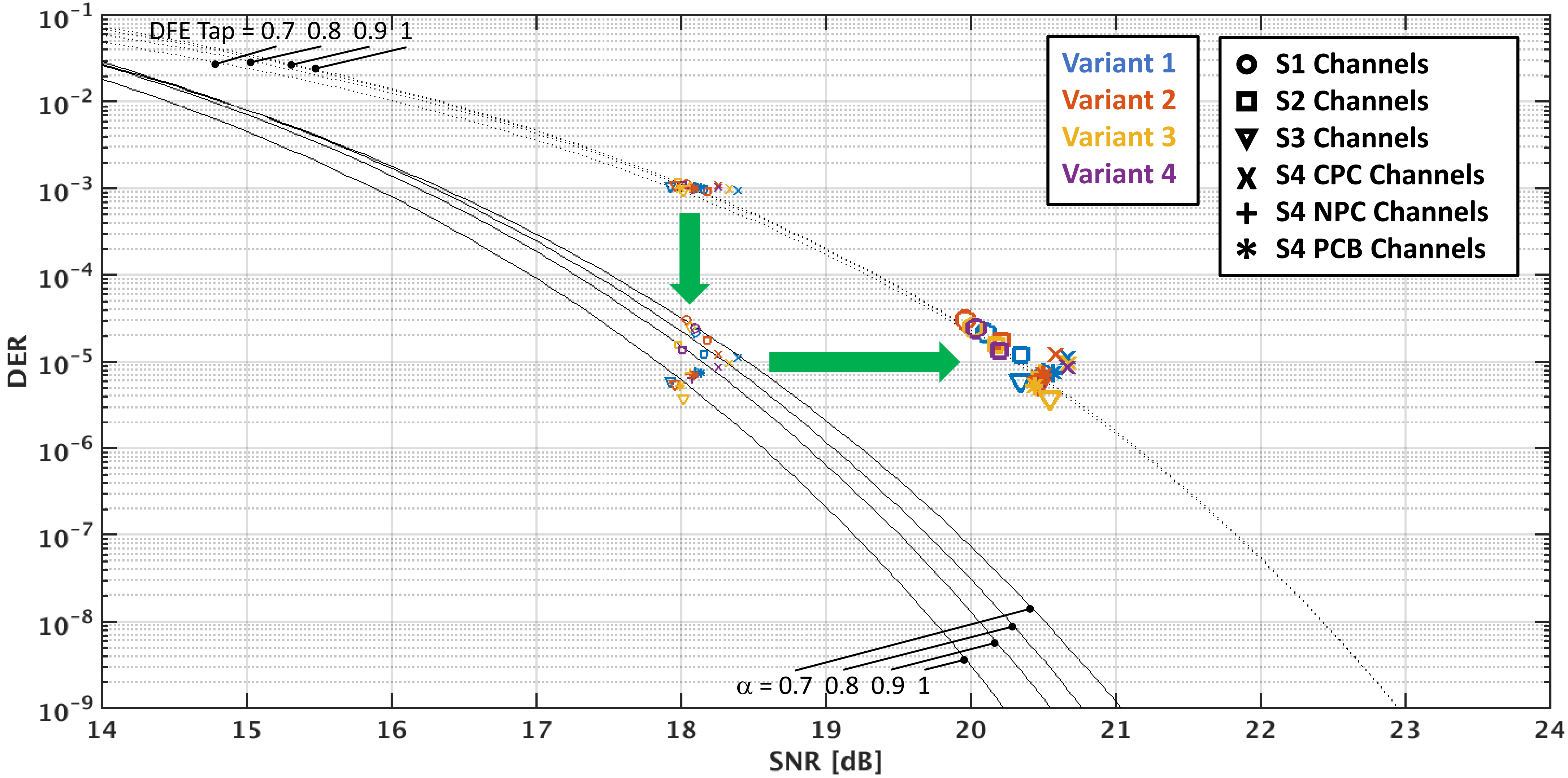
* Simulations do not include CDR; Jitter is applied using COM method; Maximum 1M symbols

Summary of the Case Study Results (1E-3)

Channel	Variant	DFE Tap = α	Theoretical Coding Gain [dB]	SNR_{DFE} [dB]	DER_{DFE}	DER_{MLSE}	$SNR_{DFE,equivalent}$ [dB]	Nosie Scaling Factor		ΔSNR = ΔCOM [dB]	DER Ratio [Order of Magnitude]	DER Simulation Results *		
								Total	for Simulation			DFE	MLSE	$DFE_{Equivalent}$
S4	CPC 30/15	**0.9421**	2.7590	**18.3817**	**9.6187 E-4**	**1.1310 E-5**	**20.6573**	**0.7695**	0.7467	**2.2756**	1.9297	6.4 E-4	NA	0
	CPC 30/20	**0.9598**	2.8358	**18.2483**	**1.0965 E-3**	**1.2076 E-5**	**20.5759**	**0.7649**	0.7421	**2.3276**	1.9581	6.8 E-4	NA	8 E-6
	CPC 35/15	**0.9602**	2.8376	**18.3260**	**9.7801 E-4**	**9.6982 E-6**	**20.6651**	**0.7639**	0.7402	**2.3391**	2.0037	9.4 E-4	NA	2.4 E-5
	CPC 35/20	**0.9783**	2.9160	**18.2488**	**1.0360 E-3**	**8.7069 E-6**	**20.6576**	**0.7578**	0.7332	**2.4088**	2.0755	1.12 E-3	NA	6 E-6
	NPC 30/15	**0.9739**	2.8969	**18.0914**	**1.0265 E-3**	**7.6231 E-6**	**20.4955**	**0.7582**	0.7034	**2.4041**	2.1292	1.29 E-3	NA	5 E-6
	NPC 30/20	**0.9779**	2.9144	**18.0586**	**1.0508 E-3**	**7.3813 E-6**	**20.4809**	**0.7566**	0.6932	**2.4224**	2.1534	9.3 E-4	NA	5 E-6
	NPC 35/15	**0.9782**	2.9157	**18.0532**	**1.0596 E-3**	**7.4717 E-6**	**20.4763**	**0.7566**	0.6927	**2.4231**	2.1517	1.95 E-3	NA	8 E-6
	NPC 35/20	**0.9784**	2.9166	**18.0672**	**1.0116 E-3**	**6.4992 E-6**	**20.4996**	**0.7557**	0.6786	**2.4324**	2.1922	2.38 E-3	NA	1 E-6
	PCB 10/10	**0.9822**	2.9329	**18.1274**	**1.0372 E-3**	**7.5393 E-6**	**20.5601**	**0.7557**	0.7116	**2.4327**	2.1385	1.06 E-3	NA	2 E-5
	PCB 15/10	**0.9742**	2.8982	**18.0862**	**1.0108 E-3**	**7.1265 E-6**	**20.4962**	**0.7577**	0.7130	**2.4100**	2.1518	1.72 E-3	NA	4 E-6
	PCB 20/10	**0.9768**	2.9096	**17.9897**	**1.0075 E-3**	**5.2994 E-6**	**20.4366**	**0.7545**	0.6426	**2.4470**	2.2790	1.40 E-3	NA	0

* Simulations do not include CDR; Jitter is applied using COM method; Maximum 1M symbols

Summary of the Case Study Results (1E-3)



Summary of the Case Study Results (1E-4)

Channel	Variant	DFE Tap $= \alpha$	Theoretical Coding Gain [dB]	SNR_{DFE} [dB]	DER_{DFE}	DER_{MLSE}	$SNR_{DFE,equivalent}$ [dB]	Nosie Scaling Factor		$\Delta SNR = \Delta COM$ [dB]	DER Ratio [Order of Magnitude]	DER Simulation Results *		
								Total	for Simulation			DFE	MLSE	$DFE_{Equivalent}$
S1	Channel 1	**0.8121**	2.1999	**19.3358**	**1.1358 E-4**	**4.8160 E-7**	**21.4596**	**0.7831**	0.7105	**2.1238**	2.3726	6.5 E-5	NA	NA
	Channel 2	**0.7677**	2.0122	**19.2972**	**1.2762 E-4**	**8.1949 E-7**	**21.3249**	**0.7918**	0.7413	**2.0277**	2.1924	9.2 E-5	NA	NA
	Channel 3	**0.7746**	2.0413	**19.3574**	**1.0349 E-4**	**5.1857 E-7**	**21.4090**	**0.7896**	0.7089	**2.0516**	2.3001	1.24 E-4	NA	NA
	Channel 4	**0.7778**	2.0548	**19.3624**	**1.0649 E-4**	**5.4976 E-7**	**21.4184**	**0.7892**	0.7267	**2.0559**	2.2871	1.58 E-4	NA	NA
S2	Case 1	**0.8710**	2.4519	**19.3156**	**1.2101 E-4**	**3.6782 E-7**	**21.5521**	**0.7730**	0.7497	**2.2365**	2.5172	2.71 E-4	NA	NA
	Case 2	**0.8382**	2.3112	**19.3738**	**1.1817 E-4**	**4.4691 E-7**	**21.5464**	**0.7787**	0.7607	**2.1727**	2.4223	2.23 E-4	NA	NA
	Case 3	**0.8883**	2.5262	**19.2597**	**1.2679 E-4**	**3.3241 E-7**	**21.5347**	**0.7696**	0.7341	**2.2751**	2.5814	4.01 E-4	NA	NA
	Case 4	**0.8803**	2.4919	**19.2705**	**1.1275 E-4**	**2.6353 E-7**	**21.5379**	**0.7702**	0.7243	**2.2674**	2.6313	2.4 E-4	NA	NA
S3	Conventional	**0.9704**	2.8818	**19.0707**	**1.0416 E-4**	**4.6160 E-8**	**21.6348**	**0.7444**	0.4914	**2.5641**	3.3535	1.93 E-4	NA	NA
	CPP	**0.9978**	3.0006	**19.1129**	**1.1263 E-4**	**4.6876 E-8**	**21.7741**	**0.7361**	0.4644	**2.6613**	3.3807	9.3 E-5	NA	NA
	NCC	**0.9966**	2.9957	**19.0863**	**1.1472 E-4**	**4.2448 E-8**	**21.7498**	**0.7359**	0.3967	**2.6636**	3.4318	1.89 E-4	NA	NA

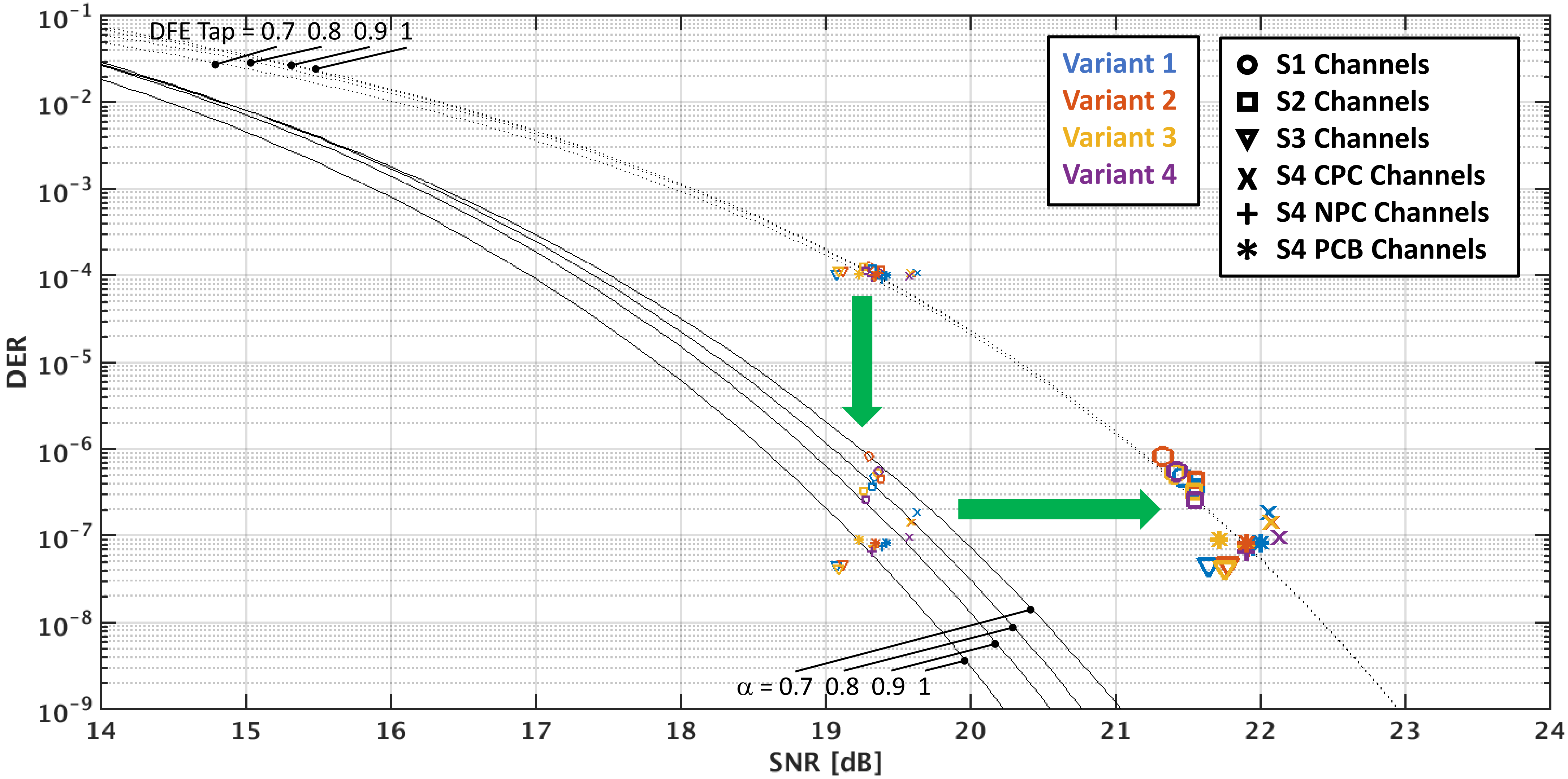
* Simulations do not include CDR; Jitter is applied using COM method; Maximum 1M symbols

Summary of the Case Study Results (1E-4)

Channel	Variant	DFE Tap = α	Theoretical Coding Gain [dB]	SNR_{DFE} [dB]	DER_{DFE}	DER_{MLSE}	$SNR_{DFE,equivalent}$ [dB]	Nosie Scaling Factor		ΔSNR = ΔCOM [dB]	DER Ratio [Order of Magnitude]	DER Simulation Results *		
								Total	for Simulation			DFE	MLSE	$DFE_{Equivalent}$
S4	CPC 30/15	**0.9437**	2.7659	**19.6268**	**1.0728 E-4**	**1.8526 E-7**	**22.0459**	**0.7569**	0.7238	**2.4190**	2.7627	1.09 E-4	NA	NA
	CPC 30/20	**0.9618**	2.8445	**19.5849**	**1.0578 E-4**	**1.4261 E-7**	**22.0673**	**0.7514**	0.7177	**2.4824**	2.8703	7.5 E-5	NA	NA
	CPC 35/15	**0.9621**	2.8456	**19.5808**	**1.0668 E-4**	**1.4455 E-7**	**22.0637**	**0.7514**	0.7174	**2.4829**	2.8680	7.0 E-5	NA	NA
	CPC 35/20	**0.9803**	2.9249	**19.5680**	**1.0012 E-4**	**9.6782 E-8**	**22.1262**	**0.7449**	0.7080	**2.5582**	3.0147	1.01 E-4	NA	NA
	NPC 30/15	**0.9775**	2.9125	**19.3856**	**9.4633 E-5**	**7.5737 E-8**	**21.9409**	**0.7451**	0.6602	**2.5553**	3.0967	1.19 E-4	NA	NA
	NPC 30/20	**0.9814**	2.9297	**19.3345**	**1.0004 E-4**	**7.5113 E-8**	**21.9067**	**0.7437**	0.6453	**2.5722**	3.1245	1.54 E-4	NA	NA
	NPC 35/15	**0.9817**	2.9309	**19.3297**	**1.0142 E-4**	**7.6746 E-8**	**21.9008**	**0.7436**	0.6446	**2.5729**	3.1211	1.24 E-4	NA	NA
	NPC 35/20	**0.9816**	2.9306	**19.3192**	**9.8216 E-5**	**6.6343 E-8**	**21.8977**	**0.7431**	0.6248	**2.5785**	3.1704	1.94 E-4	NA	NA
	PCB 10/10	**0.9855**	2.9474	**19.4163**	**1.0046 E-4**	**8.2442 E-8**	**21.9985**	**0.7428**	0.6760	**2.5823**	3.0859	1.14 E-4	NA	NA
	PCB 15/10	**0.9780**	2.9148	**19.3419**	**1.0110 E-4**	**8.2962 E-8**	**21.8978**	**0.7451**	0.6780	**2.5559**	3.0859	1.38 E-4	NA	NA
	PCB 20/10	**0.9555**	2.8170	**19.2298**	**1.0486 E-4**	**9.2064 E-8**	**21.7147**	**0.7512**	0.6387	**2.4849**	3.0565	1.77 E-4	NA	NA

* Simulations do not include CDR; Jitter is applied using COM method; Maximum 1M symbols

Summary of the Case Study Results (1E-4)



Summary

- The proposal for incorporating performance advantage of MLSE in COM, originally presented in November 2022 as shakiba_3df_01b_2211.pdf, was explained in further details
- Validity of the proposal approach and its implementation method were demonstrated by analysis of several channels and cases
- All the equations resulted from analysis of DFE and MLSE were presented
- A summary of the DFE and MLSE analysis was provided (Appendix)
- The proposal boils down to quantifying the equivalent COM advantage of MLSE over DFE (ΔCOM)
- The ultimate equations to calculate ΔCOM were given
- The equations are COM compatible and can be directly calculated from COM parameters
- The proposal is extendable to higher order MLSE as well as other more advanced signal processing techniques

Future Work

- Adding FFE support to COM is needed, but can be done independently
- Analysis of error propagation in MLSE
- Analysis of the effect of colored noise
- Continued validation by running more cases
- More time-domain simulations
- Study of MLSE Implementation and simplification
- Discussion of margin (e.g. COM margin)
- Even though the cost of a higher order MLSE will be more concerning and likely not currently practical, its study will still be helpful in exploring the limit of performance

Appendix A

Calculating SNR for L-PAM

SNR for L-PAM

- Assuming outer PAM levels of ±main and L equi-probable levels

$$PAM\ Level\ Separation = \frac{2main}{L-1}$$

$$PAM\ Levels = -main + \frac{2main}{L-1} l \quad , for\ l = 0, \cdots, L-1$$

$$Signsl\ Power = \frac{main^2}{L} \sum_{l=0}^{L-1} \left(-1 + \frac{l}{L-1}\right)^2 = \frac{1}{3} \frac{L+1}{L-1} main^2$$

(Note that $\frac{1}{L}\sum_{l=0}^{L-1}(PAM\ Levels) = 0$)

$$SNR = \frac{1}{3} \frac{L+1}{L-1} \frac{main^2}{\sigma_{noise}{}^2}$$

- This is the equation used in step 2

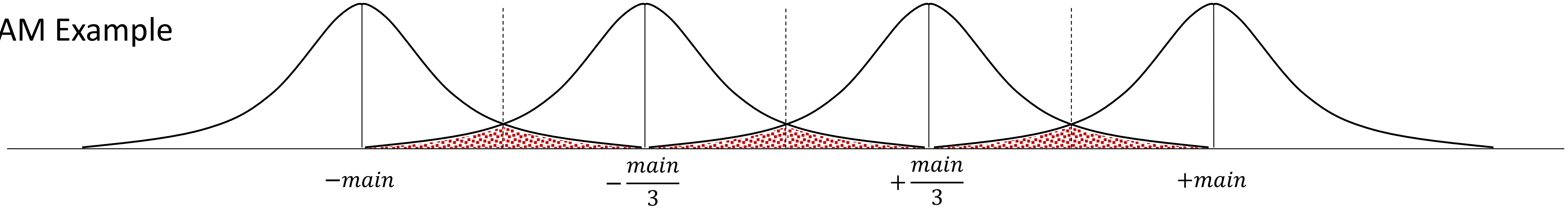
Appendix B

Error analysis of L-PAM 1-Tap DFE

Error Analysis without Error Propagation

- Assuming outer PAM levels of ±main, Gaussian noise, and dominance of adjacent-level errors
- Symbol error probability without error propagation
  - 2L-2 tails extend to wrong decision sides

4-PAM Example



$$DER_{DFE,without\ error\ propagation} \approx \frac{1}{L}(2L-2)Q\left(\frac{main}{(L-1)\sigma_{noise}}\right) = 2\frac{L-1}{L}Q\left(\frac{main}{(L-1)\sigma_{noise}}\right)$$

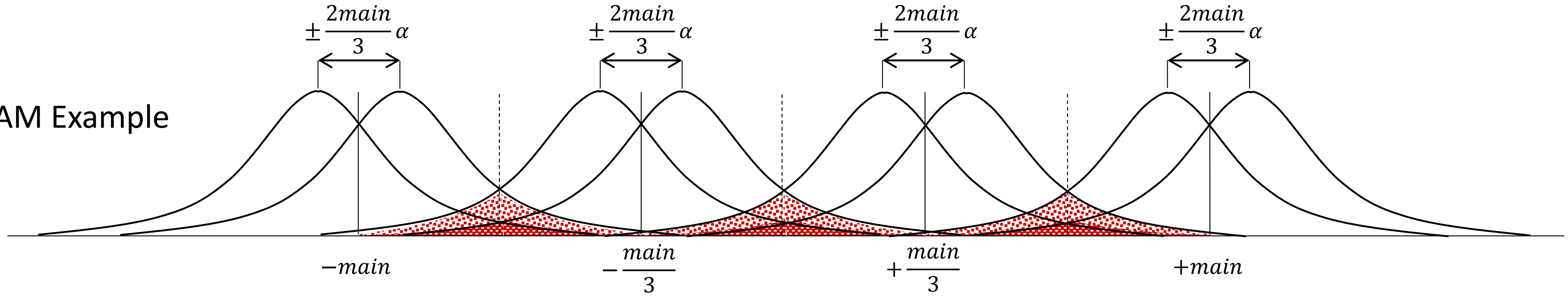
- With error propagation each error symbol extends to a burst of errors
- For the purpose of error calculation, error ratio multiplies by the average burst length

$$DER_{DFE} \approx 2\frac{L-1}{L}\overline{BL}_{DFE}Q\left(\frac{main}{(L-1)\sigma_{noise}}\right)$$, where assuming exponential distribution for burst lengths $\overline{BL}_{DFE} \approx \frac{1}{1-EPP_{DFE}}$

Error Analysis with Error Propagation

- Error propagation changes each distribution to a bimodal distribution
  - In 2(2L-2) cases error propagation is destructive and tails extend to wrong decision sides

4-PAM Example



for large α (e.g. 0.5 < α < 1)

$$EPP_{DFE} = P(error|previous\ error) \approx \frac{1}{2L}(2L-2)\left(Q\left((1-2\alpha)\frac{main}{(L-1)\sigma_{noise}}\right) + Q\left((1+2\alpha)\frac{main}{(L-1)\sigma_{noise}}\right)\right) \approx \frac{L-1}{L}Q\left((1-2\alpha)\frac{main}{(L-1)\sigma_{noise}}\right)$$

$$DER_{DFE} \approx 2\frac{L-1}{L}\frac{1}{1-EPP_{DFE}}Q\left(\frac{main}{(L-1)\sigma_{noise}}\right) = \frac{2}{\frac{L}{L-1}-Q\left((1-2\alpha)\frac{main}{(L-1)\sigma_{noise}}\right)}Q\left(\frac{main}{(L-1)\sigma_{noise}}\right)$$

Error Analysis with Error Propagation and Arbitrary Noise

- If noise in not Gaussian change the Q function to 1-CDF
- Note that since by definition of Q function its argument is normalized to standard deviation and the argument should now be de-normalized to σ_{noise}

$$DER_{DFE} \approx \frac{2}{\frac{1}{L-1}+CDF_{noise}\left((1-2\alpha)\frac{main}{L-1}\right)}\left(1-CDF_{noise}\left(\frac{main}{L-1}\right)\right)$$

- The above expression includes the effect of error propagation
- This is the equation used in step 2

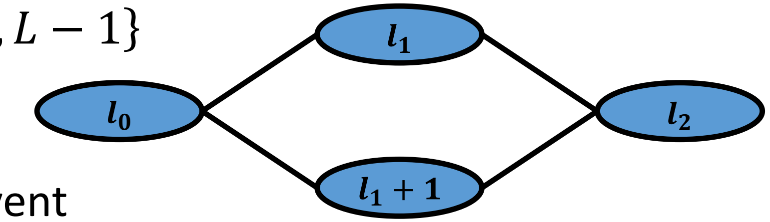
Appendix C

Error analysis of L-PAM 1+αD MLSE

Minimum Distant Error Events

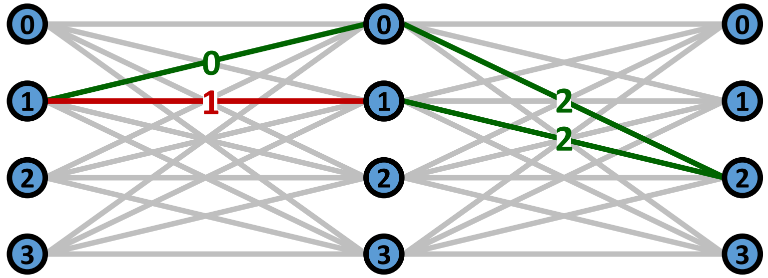
- The obvious one (shortest event)

$l \in \{0, \cdots, L-1\}$



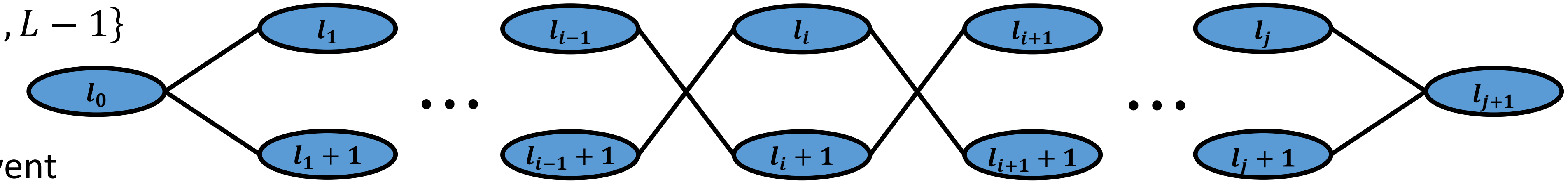
1-Error Event

4-PAM Example
1-Error Event



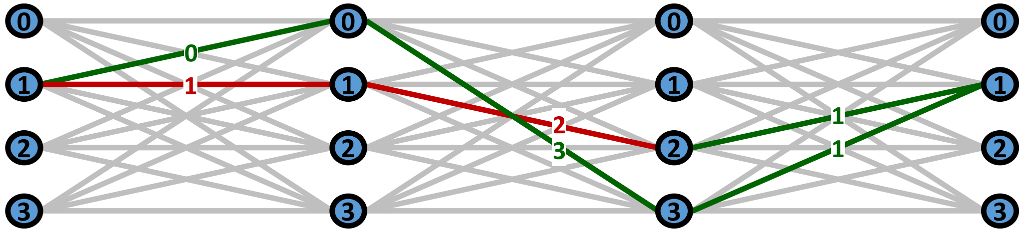
- Not so obvious ones (longer events)

$l \in \{0, \cdots, L-1\}$



j-Error Event

4-PAM Example
2-Error Event



Minimum Distant Error Events

- Assuming outer PAM levels of ±main, the Euclidean distance for the obvious short error event is $2\frac{main}{L-1}\sqrt{1+\alpha^2}$
- The Euclidean distance for the longer error events is $2\frac{main}{L-1}\sqrt{1+(j-1)(1-\alpha)^2+\alpha^2}$ and the burst of errors it entails has a length of j $(j \geq 1)$
- Note that for $j=1$, this becomes the same as the short error event
- Also note that as α approaches 1 the Euclidean distance of all of the longer error events approaches the distance of the short error event
- This is the error propagation mechanism in the MLSE and is maximized for $\alpha=1$
- Combinational counting reveals that the fractional frequency of these error events is $2\left(\frac{L-1}{L}\right)^j$

Error Analysis

- Putting together fractional frequency, number of errors, and Euclidean distance for individual events and summation over all the events results in the following overall decision error ratio of the MLSE with Gaussian noise

$$DER_{MLSE} \approx 2\sum_{j=1}^{\infty} j\left(\frac{L-1}{L}\right)^{j} Q\left(\sqrt{1+(j-1)(1-\alpha)^2+\alpha^2}\,\frac{main}{(L-1)\sigma_{noise}}\right)$$

which for arbitrary noise with a known CDF, and after de-normalization, becomes the following expression

$$DER_{MLSE} \approx 2\sum_{j=1}^{\infty} j\left(\frac{L-1}{L}\right)^{j} \left(1-CDF_{noise}\left(\sqrt{1+(j-1)(1-\alpha)^2+\alpha^2}\,\frac{main}{L-1}\right)\right)$$

- This is the equation used in step 3

Appendix D

Analysis of the Conceptual Equivalent DFE

SNR of the 'Equivalent' DFE

- How much does SNR need to increase so that a conceptual equivalent DFE performs as well as the MLSE?

$$DER_{DFE,equivalent} = DER_{MLSE}$$

$$\frac{2}{\frac{L}{L-1} - Q\left((1-2\alpha)\sqrt{\frac{3}{L^2-1}SNR_{DFE,equivalent}}\right)} Q\left(\sqrt{\frac{3}{L^2-1}SNR_{DFE,equivalent}}\right) = DER_{MLSE}$$

- Solving this equation requires iterations, but noticing that the Q function in the denominator of left hand side is a weak function of its argument, particularly $SNR_{DFE,equivalent}$ (which only changes from SNR_{DFE} by as much as a factor of 2), $SNR_{DFE,equivalent}$ can be replaced with SNR_{DFE} to avoid iterations with negligible accuracy penalty, yielding

$$SNR_{DFE,equivalent} = SNR_{DFE}\left(\frac{(L-1)\sigma_{noise}}{main} Q^{-1}\left(\frac{1}{2} DER_{MLSE}\left(\frac{L}{L-1} - Q\left((1-2\alpha)\frac{main}{(L-1)\sigma_{noise}}\right)\right)\right)\right)^2$$

Noise of the ‘Equivalent‘ DFE

- How much does noise need to decrease to give the same increase in SNR so that the conceptual equivalent DFE performs as well as the MLSE?

$$SNR_{DFE} = \frac{1}{3}\frac{L+1}{L-1}\frac{main^2}{\sigma_{noise}{}^2} \qquad SNR_{DFE,equivalent} = \frac{1}{3}\frac{L+1}{L-1}\frac{main^2}{\sigma_{noise,equivalent}{}^2}$$

$$\sigma_{noise,equivalent} = \frac{1}{\frac{L-1}{main}Q^{-1}\left(\frac{1}{2}DER_{MLSE}\left(\frac{L}{L-1}-Q\left((1-2\alpha)\frac{main}{(L-1)\sigma_{noise}}\right)\right)\right)}$$

‘Equivalent‘ SNR and Noise with Arbitrary Noise

- Change the Q function to 1-CDF, de-normalized, and solve

$$\frac{2}{\frac{1}{L-1}+CDF_{noise}\left((1-2\alpha)\sigma_{noise}\sqrt{\frac{3}{L^2-1}SNR_{DFE,equivalent}}\right)}\left(1-CDF\left(\sigma_{noise}\sqrt{\frac{3}{L^2-1}SNR_{DFE,equivalent}}\right)\right)=DER_{MLSE}$$

- Similarly, replace $SNR_{DFE,equivalent}$ in the denominator with SNR_{DFE} to avoid iterations to yield

$$SNR_{DFE,equivalent}=\left(\frac{L-1}{main}CDF_{noise}^{-1}\left(1-\frac{1}{2}DER_{MLSE}\left(\frac{1}{L-1}+CDF_{noise}\left((1-2\alpha)\frac{main}{L-1}\right)\right)\right)\right)^2 SNR_{DFE}$$

- Equivalently, this increase in SNR can be expressed as a decrease in noise

$$\sigma_{noise,equivalent}=\frac{1}{\frac{L-1}{main}CDF_{noise}^{-1}\left(1-\frac{1}{2}DER_{MLSE}\left(\frac{1}{L-1}+CDF_{noise}\left((1-2\alpha)\frac{main}{L-1}\right)\right)\right)}\sigma_{noise}$$

- Last two equations are the equations used in step 4