Reliability Analysis of In-Band Signaling Message Fields for 200G proposal with inner code

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Outline

Recap of in-band signaling field : Size, Content, Bit-rate, Protection scheme

Existing methodology of MTTFPA calculation

- IEEE 802.3df task force, Beyond 800G study group

Analysis of MTTFPA vs probability of successful transmission for in-band signaling field for worst case BER scenarios

Strategies to improve reliability of transmission

- Various repetition codes with PV ("plurality voting") criterion
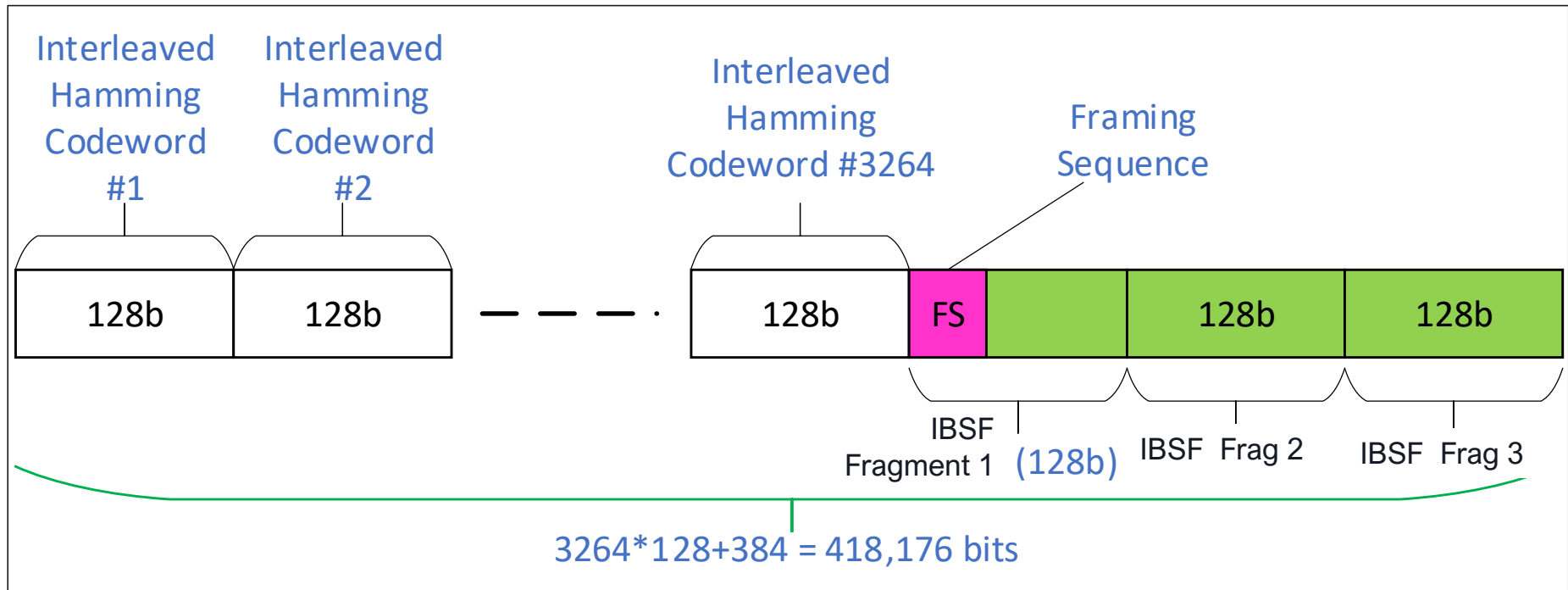
Proposal for baseline or default setting of repetitions and PV

- 15 repetitions of in-band message, 6/15 plurality voting at receiver

Summary

- Plurality voting scheme achieves both reliability of transmission and MTTFPA
  - BER=4.8E-3, HD Hamming, 15 repetitions, 6/15 PV criterion

Recap of In-Band Sync and Signaling Field (IBSF)

- Hitherto referenced as "Padding Field", the inserted 384-bit field is henceforth termed "In-Band Signaling Field" to signify its actual usage

Recap of In-Band Sync and Signaling Field (IBSF) – Cont'd

- **384 bits = 3 CW using 128, 120 code**
  - Payload bits = 360 (=45 B), parity = 24 bits

- **45 data bytes composed as follows**
  - 6 byte frame sync field (same as 200G/400G PCS AM, offers DC balance & hardware reuse):
    - 0x9A4A2665B5D9
  - Remaining 312 bits are scrambled with PRBS13, using generator polynomial $X^{13} + X^{12} + X^2 + X + 1$, seed reset to 0xCCC for start of each 312 bit instance):
    - 38 byte Message field – Start of scrambling with PRBS
      - 8 bit message index (8 bit counter 0 to 255)
      - 8 bit message type (see slides 4 & 5)
      - 36 bytes message content

      MTTFPA of message field is analyzed here
    - 1 byte CRC8 (calculated on previous 38 bytes) – polynomial is $X^8+X^5+X^4+1$

- **The 38-bytes message field (details to be specified) can be used to convey link and signal-related information, such as receiver state, channel pulse response, FEC stats, etc**

Terminology used

- **Successful transmission: Event where receiver decodes all the data of transmitted message correctly, and recognizes correct reception**
  - This is the most desired outcome for transmitted messages
- **Success Rate: Probability of successful transmission of message sequence**
- **Detected error: Event where receiver recognizes its inability to correctly receive and decode the received message**
  - May occur in an error-prone channel, but can be overcome with retransmissions
- **Undetected error: Event where receiver incorrectly believes the packet is decoded correctly, despite errors in reception and decoding**
  - MUST never occur in practice, and we try to drive its probability of occurrence to 0.
  - Egs. Undetected errors during decoding of a FEC protected transmission
- **MTTFPA : Mean elapsed time from when transmission of messages begins until the first occurrence of an undetected error**

Methodology for MTTFPA calculation

- A good reference in IEEE 802.3df when using KP4 FEC

Generated an easy-to-see formula based off this Eugene's contribution.
Note: This contribution is intended for "Stateless 64/66B encoding", but MTTFPA is part of the essential component to support this contribution.

https://www.ieee802.org/3/df/public/22_12/opsasnick_3df_01a_2212.pdf

MTTFPA Analysis and Updates for Stateless 64B/66B Coding

Eugene Opsasnick – Broadcom

802.3df December 2022

MTTFPA =

- At 800GE, an RS-FEC codeword arrives every 6.4ns
- Probability of undetected error packet from an undetected FEC error:
  - (FLR for FEC CWs with >15 FEC symbol errors) * Prob(FEC escape) * Prob(CRC32 escape)
  - 5.35E-9 * 1E-16 * 2.33E-10 = 1.24E-34

Calculated MTTFPA needs to be >
- Age of the universe ≈ 1.38E+10 years

- MTTFPA calculated based on **error** in transmission not detected in the PHY

Same methodology here

- Almost identical MTTFPA equation expressed in the same form
- MTTFPA reference: IEEE Beyond 400GE (pre-802.3df) when using KP4

https://www.ieee802.org/3/B400G/public/21_03/wang_b400g_01a_210315.pdf

BER objective for Beyond 400GbE

Xinyuan Wang, Xiang He, Hao Ren

HUAWEI

Calculating MTTFPA for BER objective (Cont'd)

- This leads to MTTFPA calculation:

$$BER_{out} = \sum_{i=t+1}^{n} \frac{i}{n} UCR_i \approx \frac{t+1}{n*m} * UCR$$

$$UMR \approx (2^m - 1)^{-(d-t-1)} \binom{n-d+t}{t}$$

$$MTTFPA > \frac{N * T_{bit}}{UCR * UMR * (1 + \frac{N}{k})} * 2^{32}$$

BER_{out}: Post-FEC BER.
UCR: Uncorrectable Codeword Ratio (Related to Pre-FEC BER).
UMR: UnMarked uncorrectable codeword Ratio.
T_{bit}: Bit time.
t: FEC error correction capability.
d: Number of erroneous symbols in a codeword.
n: FEC codeword size in symbols.
m: Galois Field index.
k: Number of message bits in a FEC codeword.
N: Ethernet MAC frame size in bits.

Note:
1. The calculation is based on RS FEC.
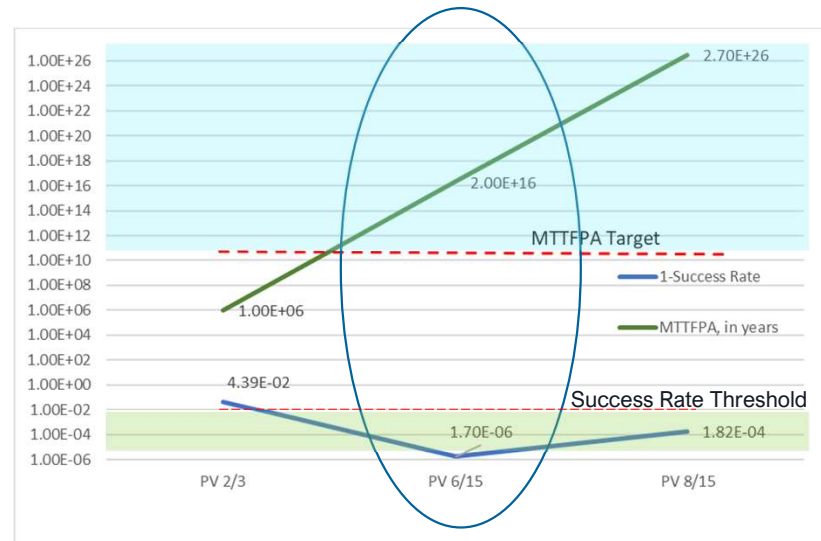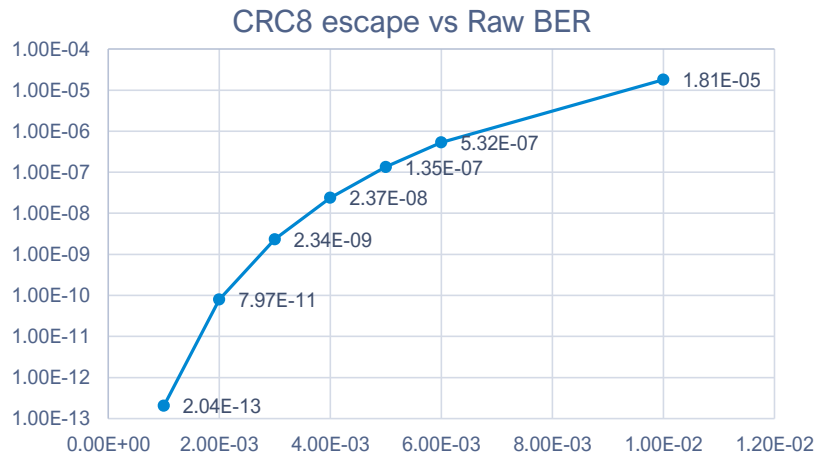2. For UMR calculation method please refer to cideciyan_01_0112.pdf.

- Again, MTTFPA is calculated based on **error** in transmission that is not detected; this may affect certain link configurations triggering off in-band messages

Calculate error in transmission that is not detected – for IBSF

- Adopt the existing criteria for in-band signaling field, with the following error protection:
  - Hamming(128,120) coding, CRC-8, and repetition coding of signaling field

- This analysis illustrates step-by-step:
  - 1. Given an input BER, calculate corresponding CER (codeword error rate)
  - 2. Using CER, calculate success rate when **repetition** of transmission is used
  - 3. From success rate, calculate the failure rate (i.e., error) in transmission
    - Highlight the importance of proper PV (plurality voting) criterion as used in repetition
  - 4. From failure rate, calculate "**false positive**" rate in repetition of transmission
    - For example, in 2/3 PV scheme, 2 padding codewords wrong in the same way is regarded as success, but is in fact a "false positive"
  - 5. From false positive rate (per padding codeword): (1) extend the calculation to 3 padding codewords in every 3264 codewords, (2) include "CRC-8 escape", and calculate the final MTTFPA. Lastly, compare with age of universe = 13.8 billion years (1.38E10 years)

Statistical Analysis

- Assume worst-case link BER = 4.8E-3
- Hamming(128,120)
  - Consider hard decision: able to correct 1 bit error in 128-bit codeword
- 128-bit codeword as 1 in-band signaling codeword (with 3 in-band signaling codewords sent every 3264 AUI payload codewords, or roughly 1.8 us)
- With repetition & PV, both Success Rate and MTTFPA targets can be met

CRC8 escape vs Raw BER

1.81E-05
5.32E-07
1.35E-07
2.37E-08
2.34E-09
7.97E-11
2.04E-13

2.70E+26
2.00E+16
MTTFPA Target
1-Success Rate
1.00E+06
MTTFPA, in years
4.39E-02
Success Rate Threshold
1.70E-06
1.82E-04
PV 2/3
PV 6/15
PV 8/15

Summary

- Using HD Hamming(128,120) coding of IBSF and applying CRC-8, as in
  https://www.ieee802.org/3/dj/public/23_01/23_0206/farhood_3dj_01a_230206.pdf
  - Repetition of in-band signaling messages
  - MTTFPA calculation methodology referencing IEEE 802.3df (Dec 2022)
    https://www.ieee802.org/3/df/public/22_12/opsasnick_3df_01a_2212.pdf
  - Extension of this methodology to Hamming, repetition, and PV criterion as shown here
- We have calculated:
  - MTTFPA of in-band signaling bits that meets AoU (age of universe) by using Hamming(128,120) encoding & CRC-8 protection with 15 repetitions in 6/15 PV criterion
  - All detailed calculations are made available in the appendix
- Baseline setting of 15 repetitions and 6 out 15 PV is recommended
- Full flexibility exists in selecting parameters:
  - Repetition can be autonomously set by transmitter, and updated with or without negotiation with receiver, based on prevailing BER.
  - PV scheme can be determined by receiver, based on BER.

Appendix

Step 1: Use BER to calculate CER (for HD Hamming)

- Given input BER=4.8E-3
- A) Consider HD Hamming
  - Calculate CER (that is, Hamming codeword error, with 2b errors or more in a codeword)

Step 1. Calculate CER from BER

Hamming codeword error-free probability
BER=4.80E-3
nchoosek (128, 0) x BER^0 x (1 - BER)^128
nchoosek (128, 0) * 4.80E-3^0 * (1 - 4.80E-3)^128 = 5.401657210648095e-01

Hamming codeword error probability with 1 bit error (before HD Hamming)
BER=4.80E-3
nchoosek (128, 1) x BER^1 x (1 - BER)^127
nchoosek (128, 1) * 4.80E-3^1 * (1 - 4.80E-3)^127 = 3.334785158985319e-01

Hamming codeword error Probability with 2 bit errors
BER=4.80E-3
nchoosek (128, 2) x BER^2 x (1 - BER)^126
nchoosek (128, 2) * 4.80E-3^2 * (1 - 4.80E-3)^126 = 1.021344972325890e-01

CER of 1b HD Hamming correction
That is, to find Hamming codeword error Probability with 2 bit errors or more
BER=4.80E-3
1 - Codeword error-free probability - Codeword error probability with 1 bit error
= (1 - 5.401657210648095e-01 - 3.334785158985319e-01) = **1.263557630366586e-01**

- CER (with 2b errors or more) = **0.1263**

Step 1: Use BER to calculate CER (for SD Hamming)

- Given input BER=4.8E-3
- A) Consider SD Hamming
    - Calculate CER (that is, Hamming codeword error, with 4b errors or more in a codeword)
    - The calculation of CER for SD Hamming is carried out in a similar way, except that CER is calculated as:

CER after SD Hamming correction of 3 bit errors per codeword

That is, to find Hamming codeword error Probability with 4 bit errors or more
BER=4.80E-3
1 - Codeword error-free probability - Codeword error probability with 1 bit error
- Codeword error probability with 2 bit errors - Codeword error probability with 3 bit errors
= **3.53E-3**

- CER (with 4b errors or more) = **3.53E-3**

Step 2. General calculation of success rate in repetition of transmission

- As a simple example, there are 3 repetitions, at least 2 out of 3 repetitions are 'success', 1 'fail'
  - Plurality voting criterion is 2/3
    - Calculate probability (at least 2 out of 3 success)

Step 3. General calculation of failure rate in repetition of transmission

- Calculate failure rate = (1 – success rate)
  - That is, calculate probability that failed the **PV 2/3 success criterion**

- This is for conceptual illustration. See 15 repetitions later in the slides for practical usage.

Step 4. General calculation of "false positive" rate in repetition of transmission

- Definition of **false positive**: For example, in the "2/3 PV criterion", when two transmissions are wrong in the same way (i.e., identical bits but both are wrong).
    - E.g., in SD Hamming that can correct 3 bits. Since we care about the case when the transmission is wrong, e.g., having 4 error bits. Then, there are two in-band signaling codewords that have the same 4 wrong bits.
    - The decoder cannot detect these 4 error bits. But they are regarded as 'correct' since it satisfies the 2/3 PV criterion. It is a "false positive".

- PV accepts an incorrect codeword with this probability:
    - A. 2 codewords are wrong in the 3 repetitions
    - B. These 2 codewords have the exact same wrong bits
    - Probability of "false positive" in the 2 out of 3 wrong codewords = P(A) * P(B).
    - Probability of "false positive" in the 3 out of 3 wrong codewords << P(A) * P(B).
    - For this document, let's assume Prob. (false positive in 2/3 PV criterion) = P(A) * P(B).

Step 4. General calculation of "false positive" rate in repetition of transmission

- For PV 2/3 criterion
  - PV 2/3 criterion = For correct acceptance, 2 out of 3 codewords need to be the same
  - Not considering 3 codewords wrong here, probability is decreasing
  - P(A) = probability (any 2 codewords are wrong in 3 repetitions)
  - P(B) = probability (the 2 codewords have the same 2 bits of error in HD, or same 4 bits of error in SD)
  - "False positive" rate = P(A) x P(B)

- For PV 6/15 criterion
  - PV 6/15 criterion = For correct acceptance, 6 out of 15 codewords need to be the same
  - Not considering 7, …, 15 codewords are wrong here, probability is decreasing
  - P(A) = probability (any 6 codewords are wrong in 15 repetitions)
  - P(B) = probability (the 6 codewords have the same 2 bits of error in HD, or same 4 bits of error in SD)
  - "False positive" rate = P(A) x P(B)

Detailed example: Calculate failure rate in repetition of transmission using 15 repetitions (using 6/15 PV criterion)

If using 6/15 PV criterion, then 6 errors could lead to "false positive" if 6 codewords have exact same bit errors

- P(A)

P6
15 trials, **6 out of 15 trials errors**, 9 success
nchoosek (15, 6) x success probability^9 x failure probability^6
nchoosek (15, 6) * (1-CER)^9 * (CER)^6
nchoosek (15, 6) * (1-**1.263557630366586e-01**)^9 * (**1.263557630366586e-01**)^6
=6.039291019682385e-03

➡ Useful to part of the "false positive" rate calculation

P7
15 trials, **7 out of 15 trials errors**, 8 success
nchoosek (15, 7) x success probability^8 x failure probability^7
nchoosek (15, 7) * (1-CER)^8 * (CER)^7
nchoosek (15, 7) * (1-**1.263557630366586e-01**)^8 * (**1.263557630366586e-01**)^7
=1.123028726659370e-03

P8
15 trials, **8 out of 15 trials errors**, 7 success
nchoosek (15, 8) x success probability^7 x failure probability^8
nchoosek (15, 8) * (1-CER)^7 * (CER)^8
nchoosek (15, 8) * (1-**1.263557630366586e-01**)^7 * (**1.263557630366586e-01**)^8
=1.624244122096648e-04

P9
15 trials, **9 out of 15 trials errors**, 6 success
nchoosek (15, 9) x success probability^6 x failure probability^9
nchoosek (15, 9) * (1-CER)^6 * (CER)^9
nchoosek (15, 9) * (1-**1.263557630366586e-01**)^6 * (**1.263557630366586e-01**)^9
=1.827120846290191e-05

P10
15 trials, **10 out of 15 trials errors**, 5 success
nchoosek (15, 10) x success probability^5 x failure probability^10
nchoosek (15, 10) * (1-CER)^5 * (CER)^10
nchoosek (15, 10) * (1-**1.263557630366586e-01**)^5 * (**1.263557630366586e-01**)^10
=1.585546419872073e-06

Ignored in this slides due to
1) decreasing probability,
2) for illustration only
(although 6 codeword errors still exist)

Detailed example: Calculate "false positive" in repetition of transmission using 15 repetitions (using 6/15 PV criterion)

Calculate probability of "false positive" in the 6/15 PV criterion:
The definition of false positive here is: 6 OUT OF 15 TRIALS HAVE EXACTLY THE SAME ERROR, WHICH WILL BE MISTAKENLY REGARDED AS THE CORRECT MESSAGE IN 6/15 PV SCHEME NATURALLY.
(Ignore 7, 8, 9, 10, 11, 12, 13, 14, 15 codeword errors out of 15 repetitions, as probability will be decreasing quickly)

The probability that the 6/15 PV accepts an incorrect codeword would be: P(A) x P(B), where
P(A) = P6
P(B) = probability to have exact same 2 bit errors in each codeword in HD Hamming
P(B) = probability to have exact same 4 bit errors in each codeword in SD Hamming

For example, in HD Hamming, probability of same 2b errors in each codeword, for the 6 codewords, is:
$1 \times (1/128 \times 1/127)^5$
where 1 refers to the probability of the reference codeword, and $(1/128 \times 1/127)^5$ refers to the probability of same 2b errors for the rest of the 5 codewords, in this 6/15 PV criterion, to cause a "false positive"
$= 1 \times (6.151574803149606e\text{-}05)^5$

- P(B)

The total probability of "false positive" is therefore:
P(A) x P(B)
$= P6 \times (6.151574803149606e\text{-}05)^5$

To be absolutely accurate, the **"false positive" rate** calculation should have been
$(P6 * 6.151574803149606e\text{-}05^5) + (P7 * 6.151574803149606e\text{-}05^5)$ and so on. ^5 is due to identical error bits in 6 codewords, where 1st codeword is the reference = 1.

Therefore, the **probability that the PV accepts an incorrect codeword** would be:
- P(A) x P(B)

P(A) x P(B)
$= P6 \times (6.151574803149606e\text{-}05)^5$
$= 6.039291019682385e\text{-}03 * 6.151574803149606e\text{-}05^5$
$= \mathbf{5.320071936675846e\text{-}24}$

Step 5. **Example 1**: Calculate MTTFPA for SD Hamming, but only use 2/3 PV criterion with 3 repetitions

Conditions in Example 1:
SD Hamming can correct up to 3 bits, but repetition 3x, using 2/3 PV criterion

- Smallest packet size = 3 in-band signaling codewords = 3x128 bits = 384 bits
  - 3 in-band signaling codewords are sent in every 3264 codewords
  - On average, there is 1 in-band signaling codeword every 1088 codewords
  - Since all the analysis before is based on 1 in-band signaling codeword=128 bits, the MTTFPA time length calculated from the calculated probability will be extended by a factor of 1088
- Probability of CRC-8 escape = 1.35e-7
  - Ref: PaddingErrProbCalcs Excel from Sridhar
    - Provides a plot of CRC escape vs. raw input BER
- MTTFPA = Packet size duration / SD Hamming false positive escape with only 2/3 PV 3 repetitions / CRC-8 escape
  - (128 / 226e9) * 1088 / (1.4549e-13) / (1.35e-7)sec ~1e6 years
  - Can provide details of calculation for SD Hamming escape (with 2/3 PV using 3 repetitions), which is similar in calculation on pages 12, 13
- MTTFPA not OK although with SD Hamming but only 3 repetitions with 2/3 PV criterion

Step 5. **Example 2**: Calculate MTTFPA for HD Hamming, but has 8/15 PV criterion with 15 repetitions

Conditions in Example 2:
HD Hamming can correct up to 1 bit, but repetition 15x, using 8/15 PV criterion

- Smallest packet size = 384 bits (same as Example 1)
  - 3 in-band signaling codewords are sent in every 3264 codewords
  - On average, there is 1 in-band signaling codeword every 1088 codewords
  - Since all the analysis before is based on 1 in-band signaling codeword=128 bits, the MTTFPA time length calculated from the calculated probability will be extended by a factor of 1088

- Probability of CRC-8 escape = 1.35e-07 (same as Example 1)
  - Ref: PaddingErrProbCalcs Excel from Sridhar
    - Provides a plot of CRC escape vs. raw input BER

- MTTFPA = Packet size duration / HD Hamming false positive escape with 8/15 MV 15 repetitions / CRC-8 escape
  - (128 / 226e9) * 1088 / (5.4145e-34) / (1.35e-7)sec ~2.7e26 years
  - Can provide details of calculation for HD Hamming escape (with 8/15 MV using 15 repetitions), which is similar in calculation on pages 12, 13

- MTTFPA is OK with HD Hamming using 15 repetitions with 8/15 PV criterion

Step 5. **Example 3**: Calculate MTTFPA for HD Hamming, but has 6/15 PV criterion with 15 repetitions (a detailed example)

First, calculate MTTFPA of **one** 128b in-band signaling codeword message:
(**three** 128b in-band signaling codeword message in every 3264 Hamming codewords will be extended, see below)

Bit rate = 226Gbps
MTTFPA (for one 128b codeword message with 15 repetitions) = 128 bit time / Probability of "false positive" of 15 repetitions of HD Hamming (128,120) codeword that correct up to 1b error / CRC-8 escape rate
Note: CRC-8 escape is considered at the end of this calculation

For 226Gbps
MTTFPA for one codeword message with 15 repetitions using 6/15 PV criterion
= 128 / 226e9 / 5.320071936675846e-24 (calculated and shown before)
= 1.064594028346573e+14 sec

This can be easily extended to 800GBASE-R or 1.6TBASE-R
By dividing MTTFPA by 4 and 8, respectively. This will not affect the big picture result.

Consider 3 in-band signaling codewords, rather than just 1 in-band signaling codeword:
3 in-band signaling codewords are inserted every 3264 codewords, on average, therefore, we have 1 in-band signaling codeword every 3264/3 codewords. That is, 1 in-band signaling codeword every 1088 codewords. This will lengthen the MTTFPA time as calculated above by 1088x.

For 226Gbps,
MTTFPA (for three 128b codeword message with 15 repetitions, inserted every 3264 codewords) = 1.064594028346573e+14 sec * 1088 = 1.158278302841071e+17 sec
= 3.672876404239825e+09 years

Consider CRC-8 escape added for in-band signaling:
For 226Gbps,
MTTFPA (for three 128b codeword message with 15 repetitions, with 1 codeword inserted every 3264 codewords, now consider CRC-8 escape)
= 3.672876404239825e+09 / 1.35E-7 = 2.720649188325796e+16 years
>1.38e+10 years (13.8 billion years AoU)
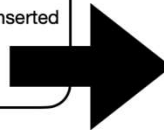
HD Hamming(128,120) using Sridhar's in-band signaling scheme. With repetitions (from Vasu/Will) here using 15 repetitions with 6/15 PV criterion, can satisfy AoU thus meeting MTTFPA criterion.

Step 5. **Example 3**: Calculate MTTFPA for HD Hamming, but has 6/15 MV criterion with 15 repetitions

Conditions in Example 3:
HD Hamming can correct up to 1 bit, but repetition 15x, using 6/15 PV criterion

- Smallest packet size = 384 bits (same as Example 1)
  - 3 in-band signaling codewords are sent in every 3264 codewords
  - On average, there is 1 in-band signaling codeword every 1088 codewords
  - Since all the analysis before is based on 1 in-band signaling codeword=128 bits, the MTTFPA time length calculated from the calculated probability will be extended by a factor of 1088

- Probability of CRC-8 escape = 1.35e-7 (same as Example 1)
  - Ref: PaddingErrProbCalcs Excel from Sridhar
    - Provides a plot of CRC escape vs. raw input BER

- MTTFPA = Packet size duration / HD Hamming false positive escape with 6/15 PV 15 repetitions / CRC-8 escape
  - (128 / 226e9) * 1088 / (5.3200e-24) / (1.35e-7)sec ~2e16 years
  - Details of calculation for HD Hamming escape (with 6/15 PV using 15 repetitions), have been provided on pages 12, 13, 16

- MTTFPA is OK with HD Hamming using 15 repetitions with 6/15 PV criterion

Codeword success rate for 15, 14, 13, …, 6 correct codewords

P0

15 trials, 0 out of 15 trials errors, 15 success
nchoosek (15, 15) x success probability^14 x (failure probability)^1
nchoosek (15, 15) * (1-CER)^15 * (CER)^0
nchoosek (15, 15) * (1-**1.263557630366586e-01**)^15 * (**1.263557630366586e-01**)^0
1.318315159684798e-01

P1

15 trials, 1 out of 15 trials errors, 14 success
nchoosek (15, 14) x success probability^14 x (failure probability)^1
nchoosek (15, 14) * (1-CER)^14 * (CER)^1
nchoosek (15, 14) * (1-**1.263557630366586e-01**)^14 * (**1.263557630366586e-01**)^1
2.860032337140398e-01

P2

15 trials, 2 out of 15 trials errors, 13 success
nchoosek (15, 13) x success probability^13 x (failure probability)^2
nchoosek (15, 13) * (1-CER)^13 * (CER)^2
nchoosek (15, 13) * (1-**1.263557630366586e-01**)^13 * (**1.263557630366586e-01**)^2
2.895539020179443e-01

P3

15 trials, 3 out of 15 trials errors, 12 success
nchoosek (15, 12) x success probability^12 x (failure probability)^3
nchoosek (15, 12) * (1-CER)^12 * (CER)^3
nchoosek (15, 12) * (1-**1.263557630366586e-01**)^12 * (**1.263557630366586e-01**)^3
1.814729744911440e-01

P4

15 trials, 4 out of 15 trials errors, 11 success
nchoosek (15, 11) x success probability^11 x (failure probability)^4
nchoosek (15, 11) * (1-CER)^11 * (CER)^4
nchoosek (15, 11) * (1-**1.263557630366586e-01**)^11 * (**1.263557630366586e-01**)^4
7.873968095547366e-02

P5

15 trials, 5 out of 15 trials errors, 10 success
nchoosek (15, 10) x success probability^10 x (failure probability)^5
nchoosek (15, 10) * (1-CER)^10 * (CER)^5
nchoosek (15, 10) * (1-**1.263557630366586e-01**)^10 * (**1.263557630366586e-01**)^5
2.505398250727623e-02

P6

15 trials, 6 out of 15 trials errors, 9 success
nchoosek (15, 9) x success probability^9 x (failure probability)^6
nchoosek (15, 9) * (1-CER)^9 * (CER)^6
nchoosek (15, 9) * (1-**1.263557630366586e-01**)^9 * (**1.263557630366586e-01**)^6
6.039291019682385e-03

P7

15 trials, 7 out of 15 trials errors, 8 success
nchoosek (15, 8) x success probability^8 x (failure probability)^7
nchoosek (15, 8) * (1-CER)^8 * (CER)^7
nchoosek (15, 8) * (1-**1.263557630366586e-01**)^8 * (**1.263557630366586e-01**)^7
1.123028726659370e-03

P8

15 trials, 8 out of 15 trials errors, 7 success
nchoosek (15, 7) x success probability^7 x (failure probability)^8
nchoosek (15, 7) * (1-CER)^7 * (CER)^8
nchoosek (15, 7) * (1-**1.263557630366586e-01**)^7 * (**1.263557630366586e-01**)^8
1.624244122096648e-04

P9 = 6 success

15 trials, 9 out of 15 trials errors, 6 success
nchoosek (15, 6) x success probability^6 x (failure probability)^9
nchoosek (15, 6) * (1-CER)^6 * (CER)^9
nchoosek (15, 6) * (1-**1.263557630366586e-01**)^6 * (**1.263557630366586e-01**)^9
1.827120846290191e-05

Success rate in the PV repetition scheme (two cases)

Table 1 Calculate success rate PV 6/15

	Success rate
6 successes	1.82712085E-05
7 successes	1.62424412E-04
8 successes	1.12302873E-03
9 successes	6.03929102E-03
10 successes	2.50539825E-02
11 successes	7.87396810E-02
12 successes	1.81472974E-01
13 successes	2.89553902E-01
14 successes	2.86003234E-01
15 successes	1.31831516E-01
# of success >6	**9.99998305E-01**

Considered success of transmission is when 6 codewords out of 15 are identical. In this case, then success rate in PV 6/15 repetition scheme has five 9's.

Table 2 Calculate success rate PV 8/15

	Success rate
8 successes	1.12302873E-03
9 successes	6.03929102E-03
10 successes	2.50539825E-02
11 successes	7.87396810E-02
12 successes	1.81472974E-01
13 successes	2.89553902E-01
14 successes	2.86003234E-01
15 successes	1.31831516E-01
# of success >8	**9.99817609E-01**

Considered success of transmission is when 8 codewords out of 15 are identical. In this case, then success rate in PV 8/15 repetition scheme has at least three 9's.

- Depending on the definition of PV acceptance criterion, the lower the codeword # (i.e., 6) considered as acceptance, the higher the success rate, since there are more scenarios considered as "success" in the in-band messaging transmission

- It therefore is a balance with False Positive rate. Conversely, the lower the codeword # (i.e., 6 here), the higher the False Positive rate

- But since the False Positive rate for 6/15 is already low enough to meet MTTFPA, the balance to 6/15 is a good way comparing to 8/15, since "success rate" having five 9's is achieved here

- Since 6 codeword success is already >3 in-band messaging codeword, it should have included the 3-codeword messaging scenario

Success rate in the PV repetition scheme (PV 2/3 repetition)

3 trials, 0 out of 3 trials error, 3 success
nchoosek (3, 0) x success probability^3 x (failure probability)^0
nchoosek (3, 0) * (1-CER)^3 * (CER)^0
nchoosek (3, 0) * (1-**1.263557630366586e-01**)^3 * (**1.263557630366586e-01**)^0
6.668126792783610e-01

3 trials, 1 out of 3 trials error, 2 success
nchoosek (3, 1) x success probability^2 x (failure probability)^1
nchoosek (3, 1) * (1-CER)^2 * (CER)^1
nchoosek (3, 1) * (1-**1.263557630366586e-01**)^2 * (**1.263557630366586e-01**)^1
2.893247205026939e-01

3 trials, 2 out of 3 trials errors, 1 success
nchoosek (3, 2) x success probability^1 x (failure probability)^2
nchoosek (3, 2) * (1-CER)^1 * (CER)^2
nchoosek (3, 2) * (1-**1.263557630366586e-01**)^1 * (**1.263557630366586e-01**)^2
4.184523204955321e-02

3 trials, 3 out of 3 trials errors, 0 success
nchoosek (3, 3) x success probability^0 x (failure probability)^3
nchoosek (3, 3) * (1-CER)^0 * (CER)^3
nchoosek (3, 3) * (1-**1.263557630366586e-01**)^0 * (**1.263557630366586e-01**)^3
2.017368169391816e-03

Table 1 Calculate success rate PV 2/3

	Success rate
2 successes	2.89324721E-01
3 successes	6.66812679E-01
# of success >2	**9.56137400E-01**

Table 2 Calculate success rate PV 3/3

	Success rate
3 successes	6.66812679E-01
# of success >2	**6.66812679E-01**

- This is the case with only PV 2/3 where maximum # of codeword transmission is 3. That also covers the case with 3 in-band messaging codeword
- The success rate here is obviously lower than the 15 repetition cases (in the last slide) – less repetition, less reliable in transmission