KEYSIGHT

# Codeword Error Rate TDECQ

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*Addressing comments #179, #180, #181 and #182 against IEEE P802.3dj D2.1*

# AGENDA

1. Introduction
2. Codeword Error Rate TDECQ
3. Experimental Data – Sampling Scope Architecture
4. Experimental Data – Real-Time Scope Architecture
5. Implementing CER TDECQ in the Draft – Suggested Remedy
6. Summary

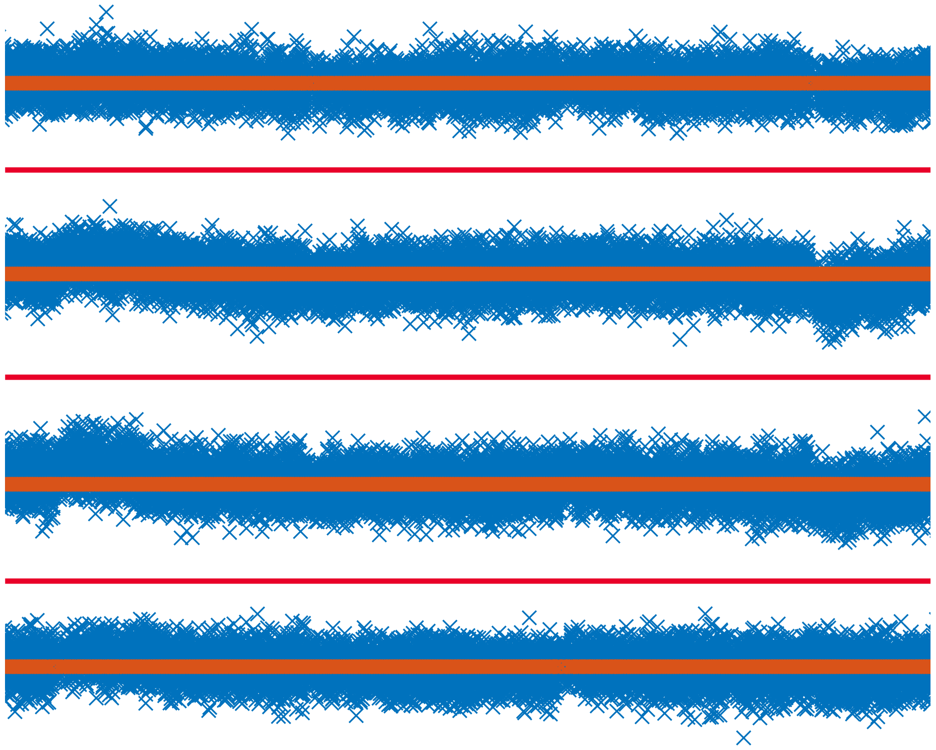
# Introduction

- The current definition of TDECQ is calculated at a theoretical pre-FEC target SER that assumes errors are uncorrelated and randomly distributed and was never intended to correlate to BER or BER floor.
- chayeb_3dj_01c_2507 proposed three different methods to calculate a CER TDECQ that correlates to link performance and leverages the existing test infrastructure.
- Straw poll TF-8 showed strong interest from the working group to see further refinement of the CER TDECQ methodology proposed in chayeb_3dj_01c_2507.
- This presentation provides additional test data and proposes adopting the rectangular (hyper-cubical) calculation method to estimate the codeword error ratio penalty, **CER TDECQ**.
- The new proposed metric, CER TDECQ, would be added as an **additional** transmitter metric for optical transmitters in clauses 180, 181, 182 and 183.
- This presentation addresses comments #179, #180, #181 and #182 against IEEE 802.3dj D2.1

# Codeword Error Rate TDECQ

## Unwrapping the Histogram – Calculating the probability of error for each sample

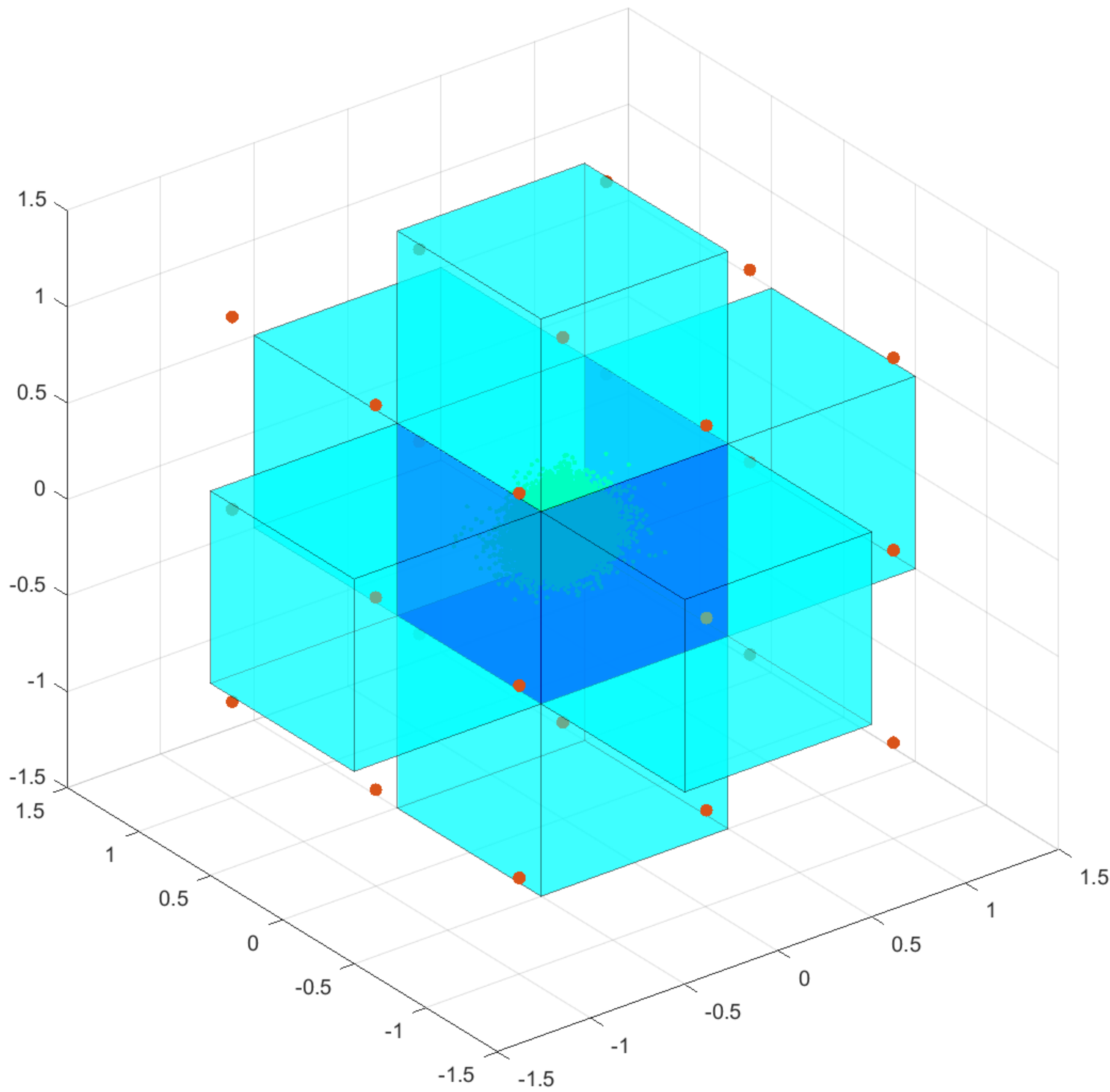
- An approach to calculate CER TDECQ by computing the probability of error per PAM4 symbol for each individual sample point.
- This approach unwraps the TDECQ histograms and uses an iterative approach to find the maximum intrinsic noise to achieve the target CER.
- The method proposed accurately predicts the codeword error rate for a given intrinsic noise value, however it is computationally expensive as it requires ~64K convolutions per sigma value.



# Hyper-Cubicle TDECQ

## Three-Dimensional Visualization

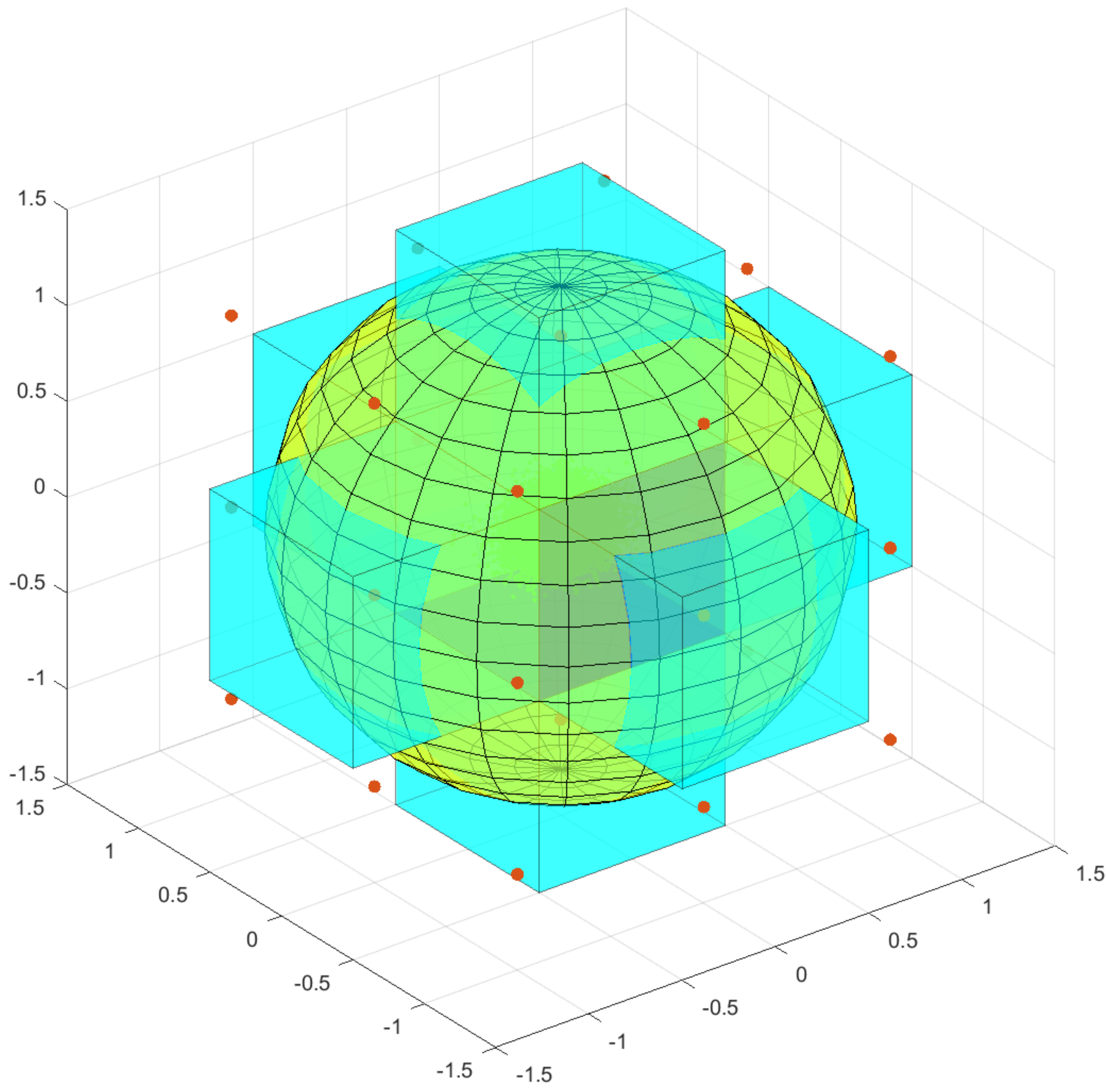
- A geometric approach to calculate the probability of k errors in a d-symbol codeword using residual vectors to compute the probability of errors.
- This approach uses an iterative approach to add a d-dimensional noise vector to the set of residual vectors defined by subtracting the nominal value of the PAM4 symbol from each individual sample point.
- This method is relatively fast and can accurately calculate the penalty for inner and outer FEC PMDs
- This presentation proposes adopting this method for calculating CER TDECQ



# Hyper-Spherical TDECQ

## Reduction to a Singular Dimension

- Hyper-Spherical CER TDECQ is an approximation that reduces the d-dimensional residual vector into a singular hyper-sphere to simplify the CER TDECQ calculation
- Hyper-Spherical TDECQ is an efficient approximation that is computationally equivalent to the current SER TDECQ.
- Hyper-Spherical CER TDECQ can only be used to approximate the power penalty for inner FEC PMDs (does not work for outer FEC PMDs)



# Experimental Data – Sampling Scope Architecture

| | Ln0 | FIR1 | FIR2 | FIR3 | FIR4 | FIR5 | FIR6 | FIR7 | FIR8 | Default | FIR9 | FIR10 | FIR11 | FIR12 | FIR13 | FIR14 | FIR15 | FIR16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Data Rate | FIR1 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| | FIR2 | 19 | 14 | 9 | 4 | -1 | -6 | -11 | -16 | -21 | -26 | -31 | -36 | -41 | -42 | -42 | -42 | -42 |
| | FIR3 | 102 | 107 | 112 | 117 | 121 | 121 | 121 | 121 | 121 | 121 | 121 | 121 | 121 | 117 | 112 | 107 | 102 |
| | FIR4 | -42 | -42 | -42 | -42 | -41 | -36 | -31 | -26 | -21 | -16 | -11 | -6 | -1 | 4 | 9 | 14 | 19 |
| | FIR5 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| | Lvl0 | 0 2 -1 0 | 0 2 -1 0 | 0 2 -1 0 | 0 2 -1 0 | 0 2 -1 0 | 0 2 -1 0 | 0 2 -1 0 | 0 2 -1 0 | 0 2 -1 0 | 0 2 -1 0 | 0 2 -1 0 | 0 2 -1 0 | 0 2 -1 0 | 0 2 -1 0 | 0 2 -1 0 | 0 2 -1 0 | 0 2 -1 0 |
| | Sum | 168 | 168 | 168 | 168 | 168 | 168 | 168 | 168 | 168 | 168 | 168 | 168 | 168 | 168 | 168 | 168 | 168 |
| 212G | ER | 3.03 | 2.98 | 2.97 | 2.91 | 2.90 | 2.93 | 2.93 | 2.90 | 2.92 | 2.90 | 2.89 | 2.89 | 3.00 | 2.91 | 3.00 | 2.88 | 2.95 |
| | Ceq | 1.21 | 1.05 | 0.92 | 0.77 | 0.69 | 0.71 | 0.75 | 0.74 | 0.76 | 0.76 | 0.75 | 0.74 | 0.71 | 0.8 | 0.94 | 1.07 | 1.23 |
| | RLM | 0.996 | 0.994 | 0.99 | 0.983 | 0.978 | 0.978 | 0.986 | 0.99 | 0.989 | 0.988 | 0.993 | 0.992 | 0.991 | 0.991 | 0.993 | 0.993 | 0.988 |
| | TDECQ (FFE) | 3.03 | 2.84 | 2.42 | 2.21 | 2.03 | 1.97 | 1.92 | 1.87 | 1.84 | 1.84 | 1.79 | 1.84 | 1.84 | 1.93 | 2.09 | 2.29 | 2.55 |
| | CER TDECQ (Outer FEC) | 3.19 | 2.92 | 2.76 | 2.15 | 2.02 | 2.05 | 1.88 | 1.89 | 1.8 | 1.89 | 1.78 | 1.82 | 1.95 | 1.93 | 2.1 | 2.24 | 2.54 |
| | TDECQ (+DFE) | 2.49 | 2.39 | 2.15 | 1.94 | 1.84 | 1.75 | 1.76 | 1.59 | 1.61 | 1.57 | 1.53 | 1.53 | 1.52 | 1.61 | 1.75 | 1.91 | 2.03 |
| | CER TDECQ (Outer FEC, +DFE) | 3.07 | 2.81 | 2.61 | 2.4 | 2.23 | 2.17 | 2.01 | 1.9 | 1.81 | 1.81 | 1.75 | 1.75 | 1.77 | 1.79 | 1.99 | 2.17 | 2.25 |
| 227G | ER | 3.07 | 2.97 | 3.00 | 2.96 | 2.98 | 2.97 | 2.93 | 2.92 | 2.91 | 2.91 | 2.93 | 2.92 | 2.97 | 2.94 | 2.99 | 2.92 | 2.86 |
| | Ceq | 1.55 | 1.39 | 1.22 | 1.1 | 0.99 | 1.03 | 1.04 | 1.05 | 1.06 | 1.08 | 1.06 | 1.04 | 1.04 | 1.11 | 1.25 | 1.37 | 1.54 |
| | RLM | 0.993 | 0.99 | 0.987 | 0.98 | 0.972 | 0.976 | 0.983 | 0.989 | 0.987 | 0.987 | 0.992 | 0.993 | 0.992 | 0.991 | 0.989 | 0.99 | 0.991 |
| | TDECQ (FFE) | 2.61 | 2.49 | 2.24 | 2.05 | 1.93 | 1.97 | 1.95 | 1.94 | 1.95 | 1.88 | 1.86 | 1.86 | 1.89 | 1.95 | 2.1 | 2.26 | 2.5 |
| | CER TDECQ (Inner FEC, FFE) | 2.63 | 2.48 | 2.23 | 2.04 | 1.93 | 1.96 | 1.95 | 1.93 | 1.93 | 1.87 | 1.84 | 1.85 | 1.89 | 1.94 | 2.09 | 2.27 | 2.51 |
| | TDECQ (+DFE) | 2.05 | 1.98 | 1.86 | 1.65 | 1.56 | 1.59 | 1.58 | 1.58 | 1.55 | 1.52 | 1.49 | 1.51 | 1.49 | 1.51 | 1.67 | 1.8 | 1.99 |
| | CER TDECQ (Inner FEC, +DFE) | 2.15 | 2.1 | 1.93 | 1.73 | 1.63 | 1.64 | 1.65 | 1.62 | 1.55 | 1.57 | 1.51 | 1.55 | 1.51 | 1.54 | 1.72 | 1.85 | 2.06 |
| | H. Sph. TDECQ (Inner FEC, FFE) | 2.57 | 2.45 | 2.2 | 2.01 | 1.91 | 1.9 | 1.92 | 1.92 | 1.88 | 1.86 | 1.78 | 1.78 | 1.84 | 1.94 | 2.08 | 2.21 | 2.49 |
| | H. Sph. TDECQ (Inner FEC, +DFE) | 0.78 | 0.85 | 0.82 | 0.69 | 0.68 | 0.72 | 0.76 | 0.76 | 0.62 | 0.7 | 0.6 | 0.62 | 0.53 | 0.5 | 0.55 | 0.57 | 0.71 |

212G only

| AOP (60s max) | OMA | FIR1 | FIR2 | FIR3 | FIR4 | FIR5 | FIR6 | FIR7 | FIR8 | Default | FIR9 | FIR10 | FIR11 | FIR12 | FIR13 | FIR14 | FIR15 | FIR16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1.8 | -0.08 | NL | NL | NL | 1.86E-08 | 2.80E-08 | 9.19E-09 | 3.24E-09 | 9.20E-10 | 2.68E-11 | 1.83E-11 | 4.67E-12 | 6.72E-12 | 6.37E-12 | 3.40E-12 | 2.85E-12 | 2.20E-12 | 4.95E-12 |
| 1 | -0.88 | NL | NL | NL | 1.97E-08 | 3.23E-08 | 1.27E-08 | 5.34E-09 | 1.77E-09 | 7.80E-11 | 3.31E-11 | 1.51E-11 | 1.24E-11 | 1.77E-11 | 6.29E-12 | 6.36E-12 | 5.93E-12 | 2.02E-11 |
| 0 | -1.88 | NL | NL | NL | 4.14E-08 | 4.94E-08 | 2.51E-08 | 1.20E-08 | 4.28E-09 | 3.12E-10 | 1.51E-10 | 9.43E-11 | 7.28E-11 | 8.73E-11 | 3.65E-11 | 3.80E-11 | 4.31E-11 | 1.11E-10 |
| -1 | -2.88 | NL | NL | NL | 1.23E-07 | 8.96E-08 | 4.51E-08 | 2.44E-08 | 1.08E-08 | 1.67E-09 | 8.80E-10 | 6.01E-10 | 5.80E-10 | 5.56E-10 | 3.51E-10 | 3.43E-10 | 3.65E-10 | 9.72E-10 |
| -2 | -3.88 | NL | NL | NL | 1.49E-06 | 4.91E-07 | 2.05E-07 | 1.10E-07 | 5.69E-08 | 1.70E-08 | 1.09E-08 | 9.18E-09 | 7.20E-09 | 6.97E-09 | 5.28E-09 | 5.17E-09 | 5.61E-09 | 1.31E-08 |
| -3 | -4.88 | | | | 5.53E-05 | 8.82E-06 | 2.18E-06 | 1.10E-06 | 5.55E-07 | 2.51E-07 | 1.90E-07 | 1.62E-07 | 1.21E-07 | 1.18E-07 | 1.09E-07 | 1.21E-07 | 1.31E-07 | 2.68E-07 |
| -4 | -5.88 | | | | | 4.10E-04 | 9.20E-05 | 2.32E-05 | 9.11E-06 | 5.37E-06 | 3.80E-06 | 3.01E-06 | 2.50E-06 | 2.33E-06 | 2.45E-06 | 2.78E-06 | 3.31E-06 | 7.49E-06 |
| -5 | -6.88 | | | | | | | | 3.20E-04 | 1.69E-04 | 9.63E-05 | 6.75E-05 | 5.43E-05 | 4.49E-05 | 6.98E-05 | 8.90E-05 | 1.06E-04 | 3.02E-04 |

Red: post-FEC seen during 60s gate time

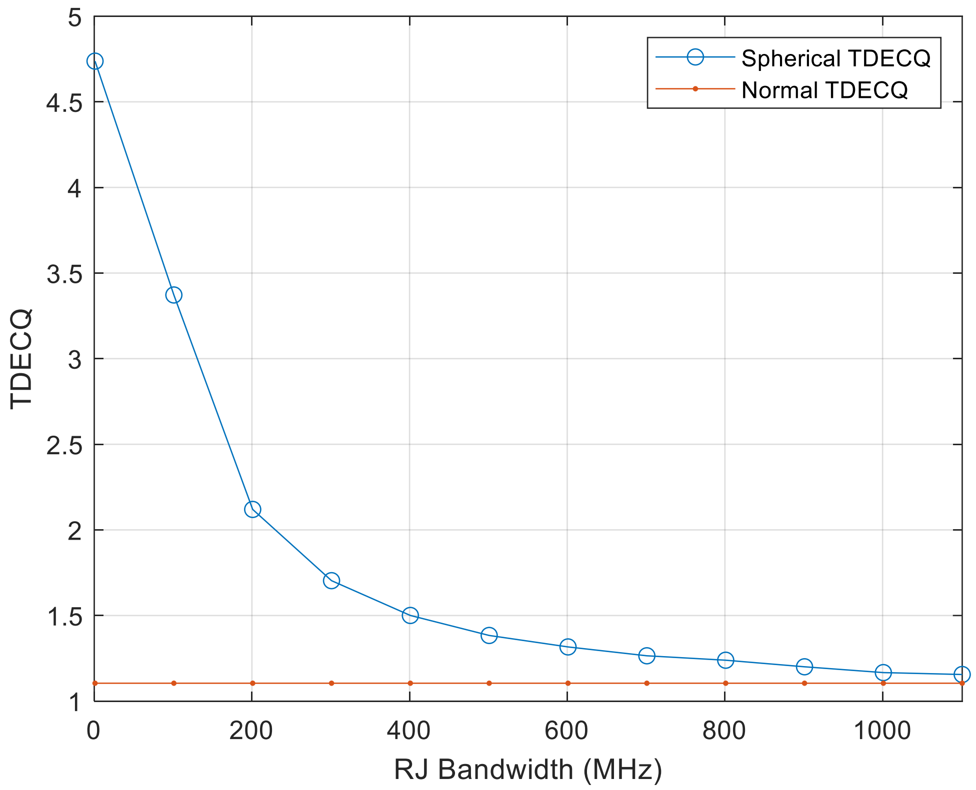
NL: CDR does not lock

# Experimental Data – Real Time Scope Architecture

## Bandlimited RJ - Simulated Test Case

- ran_3dj_02a_2407 and oif2024.449.02 demonstrated receivers with performance issue related to bandlimited random jitter
- Using a captured real signal and applying 400fs of bandlimited RJ in MATLAB
- Comparing standard TDECQ and Hyper-Spherical TDECQ, we can see that Hyper- Spherical TDECQ is sensitive to the lower frequency bandlimited RJ

- NOTE: On a Sampling Oscilloscope, the spectrum of the RJ will be aliased, and will appear uncorrelated.



# Implementing CER TDECQ in the Draft – Suggested Remedy

- This presentation proposes adding CER TDECQ as an additional transmitter interoperability metric and using the rectangular (hyper-cubical) method for calculating the penalty.
- The new penalty metric will be simply called *Codeword Error Ratio Transmitter Dispersion Eye Close Quaternary* or *CER TDECQ* and should be implemented in clauses 180, 181, 182 and 183.
- CER TDECQ is measured using an SSPRQ pattern and is calculated using the equalized waveform and all optimized parameters from the TDECQ measurement.
- CER TDECQ will be measured using the same reference receiver and reference equalizer defined in draft 2.1 for TDECQ. The proposed limits for CER TDECQ are the same as the existing TDECQ limits.
- The text below is suggested to implement CER TDECQ in IEEE 802.3dj

# CER TDECQ

$\text{TDECQ}_{\text{CER}}$ is calculated using the equalized waveform and all optimized parameters from the TDECQ measurement.

$CER_L(\sigma) = 1 - G_L(\sigma)$ and $CER_R(\sigma) = 1 - G_R(\sigma)$ are the codeword error ratios given a noise rms value, $\sigma$, associated with the left and right histograms. With $G_L(\sigma)$ and $G_R(\sigma)$ the rate of correctable codeword.

$CER(\sigma)$ is the larger of $CER_L(\sigma)\ and\ CER_R(\sigma)$ and it must be less than $CER_{target}$. An iterative process is used to find $\sigma_g$, the maximum $\sigma$ that does not exceed the codeword error rate target.

$$\sigma_g = \underset{CER(\sigma) \leq CER_{target}}{\arg\max} \quad CER(\sigma)$$

The RMS noise, R, that can be added by a receiver is given by

$$R = \sqrt{C_{eq}^{-2}\sigma_g^2 + \sigma_s^2}$$

## CER TDECQ – *Continued*

$TDECQ_{CER}$ is given by

$$TDECQ_{CER} = 10\log_{10}\left(\frac{\sigma_{ref}}{R}\right)$$

$\sigma_{ref}$ is the RMS noise that can be added by a receiver and achieve the target error rate if the input signal is ideal, given by

$$\sigma_{ref} = \frac{OMA_{Outer}}{6Q_t}$$

Where $Q_t$ is consistent with the target error rate and can be calculated as:

$$Q_t = \sqrt{2}\, erfc^{-1}\left(\frac{4}{3}SER_{target}\right)$$

## Generating $G_L(\sigma)$ $and$ $G_R(\sigma)$

To generate the correctable codeword rate $G_X(\sigma)$ for either the left or right histogram start by generating a sequence of error probabilities for each symbol of the equalized waveform assuming an Additive White Gaussian Noise, with rms, $\sigma$, is added to the waveform.

The probability of error for the *nth* symbol is calculated by first taking the sample point closest to the center of the target histogram while being within the limits of the histogram. The amplitude of that sample is $y_n$.

The probability that the *nth* symbol is in error, can be calculated as:

$$P_{err,n}(\sigma) = \frac{1}{2}\begin{cases} erfc\left(\frac{P_{th_1} - y_n}{\sqrt{2}\sigma}\right) & l_n = 0 \\ erfc\left(\frac{y_n - P_{th_1}}{\sqrt{2}\sigma}\right) + erfc\left(\frac{P_{th_2} - y_n}{\sqrt{2}\sigma}\right) & l_n = 1 \\ erfc\left(\frac{y_n - P_{th_2}}{\sqrt{2}\sigma}\right) + erfc\left(\frac{P_{th_3} - y_n}{\sqrt{2}\sigma}\right) & l_n = 2 \\ erfc\left(\frac{y_n - P_{th_3}}{\sqrt{2}\sigma}\right) & l_n = 3 \end{cases}$$

# Generating $G_L(\sigma)$ *and* $G_R(\sigma)$ – *Continued*

Where $erfc(\cdot)$ is the complimentary error function and $l_n$ is the symbol value of the *nth* symbol.

Let $m$ be the number of PAM4 symbols per FEC codeword symbol. The probability that an FEC codeword symbol has an error is:

$$P_{FEC,n}(\sigma) = 1 - \prod_{i=0}^{m-1}\left(1 - P_{err,n+i}(\sigma)\right)$$

Group FEC symbol error probabilities into codeword groups. Let $d$ be the number of FEC symbols per codeword and $r$ the stride of the symbol interleaving. Then the array of symbol errors probabilities associated with the *nth* codeword is

$$C_n(\sigma) = \left[P_{FEC,n}(\sigma), P_{FEC,n+r}(\sigma), P_{FEC,n+2r}(\sigma), \dots, P_{FEC,n+(d-1)r}(\sigma)\right].$$

## Generating $G_L(\sigma)$ and $G_R(\sigma)$ – Continued

The PMF for the number of symbol errors in the *nth* symbol is

$$p_n(e,\sigma) = \begin{cases} 1 - P_{FEC,n}(\sigma) & e = 0 \\ P_{FEC,n}(\sigma) & e = 1 \\ 0 & else \end{cases}$$

where $e$ is the number of errors. The PMF of number of errors in the *kth* codeword is calculated by convolving the symbol error PMFs.

$$C_n(e,\sigma) = p_n(e,\sigma) * p_{n+r}(e,\sigma) * p_{n+2r}(e,\sigma) * \ldots * p_{n+(d-1)r}(e,\sigma)$$

## Generating $G_L(\sigma)$ and $G_R(\sigma)$ – Continued

Assuming that a codeword can only be corrected if it has at most $k$ errors, the probability that the *nth* codeword can be decoded correctly is

$$G_n(\sigma) = \sum_{i=0}^{k} C_n(i,\sigma)\,.$$

and the overall correctable codeword rate is

$$G(\sigma) = \frac{1}{N}\sum_{n=0}^{N-1} G_n(\sigma)$$

where N is the pattern length. Note, this method generates overlapping codewords. Because this is estimating codeword error rate on an unencoded test pattern and there is no start symbol, this improves the consistency of the measurement.

## Calculating $CER_{target}$ from $SER_{target}$

Let $m$ be the number of PAM4 symbols per FEC symbol, $d$ the number of FEC symbols per codeword, and $k$ the maximum number of FEC symbol errors that can be corrected in a codeword. The target codeword error rate is calculated from the target PAM4 symbol error rate as:

$$p = SER_{target}^{m}$$

$$CER_{target} = 1 - \sum_{i=0}^{k} \binom{d}{i} p^{i} (1-p)^{k-i}$$

# Summary

- The current definition of TDECQ is calculated at a theoretical pre-FEC target SER that assumes errors are uncorrelated and randomly distributed and was never intended to correlate to BER or BER floor.
- chayeb_3dj_01c_2507 proposed three different methods to calculate a CER TDECQ that correlates to link performance and leverages the existing test infrastructure.
- This presentation proposes adding CER TDECQ as an **additional** transmitter interoperability metric and using the rectangular (hyper-cubical) method for calculating the penalty.
- The proposed CER TDECQ is relatively fast to calculate and will provide an additional tool that captures correlated errors synchronous to the pattern which are not captured by the current definition of TDECQ.
- CER TDECQ is measured using an SSPRQ pattern and is calculated using the equalized waveform and all optimized parameters from the TDECQ measurement (including the reference receiver and reference equalizer).
- This presentation provides a suggested remedy for implementing CER TDECQ in IEEE 802.3dj which would be applicable to clauses 180, 181, 182 and 183.

# Thank you