TDECQ Considerations

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Associated with Comment No. 351 on 802.3dj D2.1

8 September 2025

Consider a grid of y_i values equally spaced. Limit attention to the left histogram.

TDECQ considers three thresholds, $\{T_1, T_2, T_3\}$, that are the boundaries of four *zones*, or decision regions, for the PAM4 signal.

Consider first T_1 and $y_i > T_1$.

$CF_1(y_i) \approx \text{Prob}(T_1 \leq y \leq y_i)$ (approximately, because we are using discrete histograms)

Suppose the noise n is given by $n = -(y_i - T_1)$ (Note: n is negative).

If y is above T_1 and below y_i, then $y + n$ will be below T_1, which results in a symbol error *if* y was in the correct zone to begin with.

TDECQ computes the probability of a making a symbol error by crossing from above T_1 to below T_1 as

$$\sum_{\{y_i | y_i > T_1\}} \text{Prob}(n = T_1 - y_i)\text{Prob}(T_1 \leq y \leq y_i)$$

$$= \sum_{\{y_i | y_i > T_1\}} \text{Prob}(n = T_1 - y_i)CF_1(y_i) \quad (1)$$

where n is Gaussian noise of the proper variance. The problem with this approach is when $y_i = y_{\text{max}} \equiv \max\{y_i\}$

In that case, $\text{Prob}(T_1 \leq y \leq y_{\text{max}})$ should not be weighted by $\text{Prob}(n = T_1 - y_{\text{max}})$

Instead it should be weighted by $\text{Prob}(n \leq T_1 - y_{\text{max}})$; otherwise we are ignoring the tail of the Gaussian distribution.

So the correct probability of crossing from above T1 to below T1 is given by

$$\sum_{\{y_i | T_1 < y_i < y_{\text{max}}\}} \text{Prob}(n = T_1 - y_i) CF_1(y_i) + \text{Prob}(n \leq T_1 - y_{\text{max}}) CF_1(y_{\text{max}}) \quad (2)$$

Similarly, the correct probability of crossing from below T1 to above T1 is given by

$$\sum_{\{y_i | y_{\text{min}} < y_i < T_1\}} \text{Prob}(n = T_1 - y_i) CF_1(y_i) + \text{Prob}(n \geq T_1 - y_{\text{min}}) CF_1(y_{\text{min}}) \quad (3)$$

(In this region, $CF_1(y_i) = \text{Prob}(y_i \leq y \leq T_1)$

Putting these together, assuming y was originally in the correct zone, the probability of making an error by crossing T_1 is given by

$$\sum_{\{y_i | y_{\text{min}} < y_i < y_{\text{max}}\}} \text{Prob}(n = T_1 - y_i) CF_1(y_i) + \text{Prob}(n \geq T_1 - y_{\text{min}}) CF_1(y_{\text{min}}) + \text{Prob}(n \leq T_1 - y_{\text{max}}) CF_1(y_{\text{max}}) \quad (4)$$

The second problem with the computation of symbol error probability in TDECQ is that it computes the probability of making an error by crossing $T1$, $T2$, and $T3$ independently, then adds those probabilities together to get a total probability. But this is double or triple counting in some cases, since a given y value has a certain probability of crossing one (or sometimes two, if an internal region) thresholds, and it does not make another symbol error if it crosses the next threshold.

All of these problems could be avoided by using the more conventional error probability conditioned on the value of y and then using the Q function to determine the probability of crossing a threshold out of the original zone.