Projecting BLER for AUIs

(in support of comment #406)

Adee Ran, Cisco

Problem statement

- Direct verification of the very low probabilities required for the AUI receiver error masks (Table 176C–6 and Table 176D–11) takes extremely long time
  - See ran_3dj_adhoc_01_260421 slides 10-11
  - The error masks can be found in the backup section
- The extrapolation method suggested informatively for the PMD clauses requires “bins with a count greater than 2”. This may be inapplicable for AUI receivers, which require very low error probabilities and often have 0 counts in most bins.

”Projection” mentioned in 174A.9.3

e) The total number of test blocks analyzed is determined according to Equation (174A–1). The value of test_block_total_count_*i* should be sufficiently large to reliably verify that the expected BLER is met, either by direct measurement or statistical projection. The projection should provide an accurate prediction of the value of $H_m^{(i)}(k)$ that would be observed over longer-term testing or at least provide an upper bound on the value.

The solution does not require advanced math; test engineers should be able to figure it out… ☹
But as a service to the industry, this presentation provides a possible recipe.

Outline

- Preview of the proposed informative extrapolation method:
  - Define the extrapolation based on bin n, where $n \geq 4$ and all bins $k \geq n$ have a count of zero.
  - Assume the count of bin n is 3 instead of 0.
  - Extrapolate using log-linear slope fitted from $H_m(k)$, $0 \leq k \leq n$.
- Illustration of the extrapolation with low BER and with borderline BER, assuming uncorrelated errors.
- Consideration of the effect of correlated errors.

Is extrapolation required at all? ⭐

- The note proposed in the comment is informative; the normative requirement is to meet either the full error mask or the BLER calculation with BER_{added}.
- For the AUIs, the BER_{added} is a much larger component than the errors expected on the receiver and creates H(16) close to the BLER (see backup slide). Convolution of the measured error histogram (with estimates of the bins that were not observed) can be used as the direct compliance method.
- The proposed extrapolation/projection is provided for users that prefer direct criteria without the convolution step.

Details of the extrapolation method

- Define n as the minimum value such that n≥4 and $H_m(k)=0$ for k≥n.
- Assign $H_m(n)=3/N$ (instead of 0), where N is the number of codewords observed during the test time
  - The minimum N required to project success can be found as shown on the right.
- From $H_m(k)$, 1≤k≤n, where $H_m(k)\neq 0$, find the best-fit log-linear slope A with intercept fixed at 1 (≈$H_m(0)$).

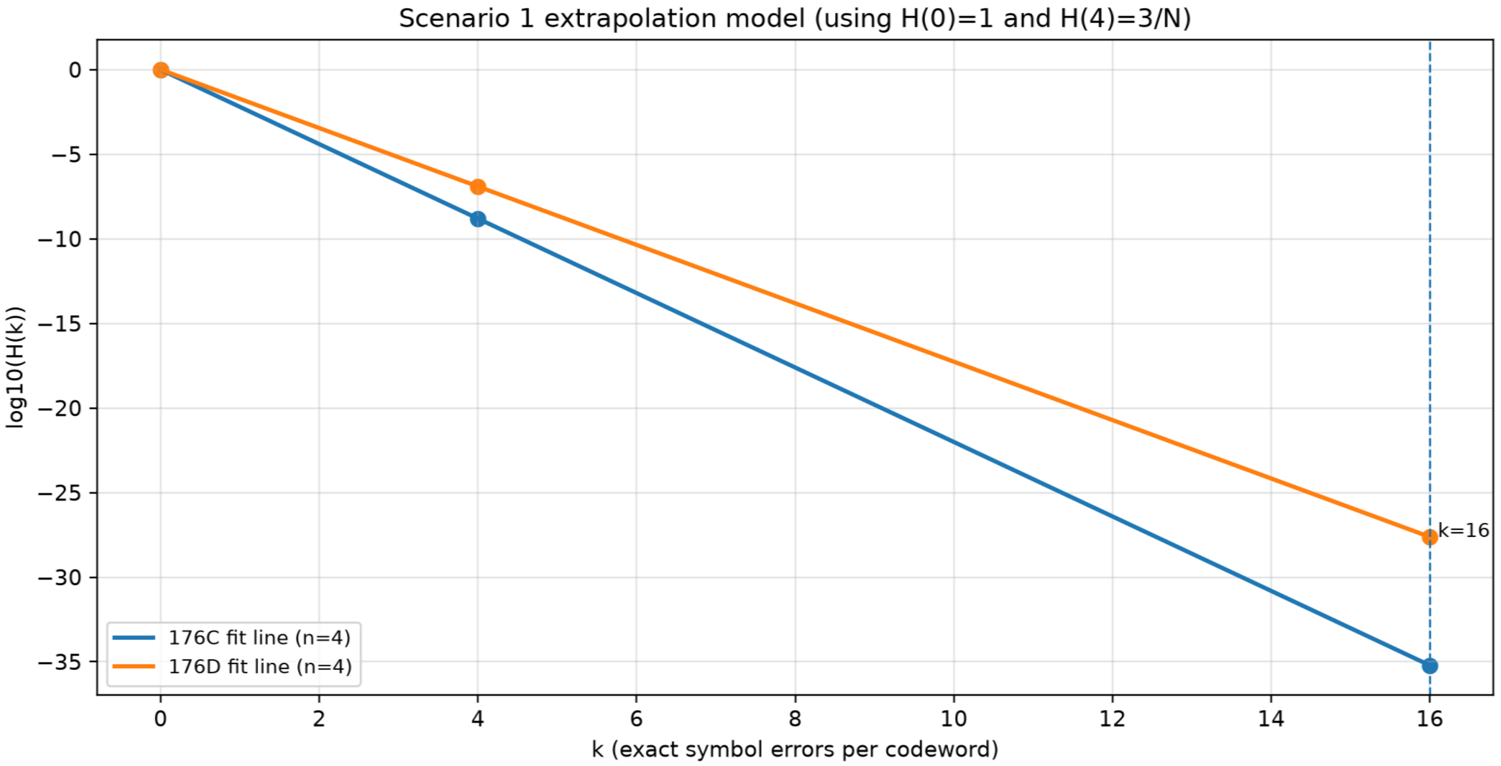
- Extrapolate using the slope A to $H_{ext}(16)$.
- If $H_{ext}(16)<H_{max}(16)$, the BLER requirement is likely to be met.

$$H(16) = \left(\frac{3}{N}\right)^{\frac{16}{n}}$$

$$N_{\min} = 3 \cdot H_{\max} (16)^{-\frac{n}{16}}$$

$$t_{test} = \frac{N_{\min}}{Rcw}$$

Graphical extrapolation concept



Left plot is in log10 scale.
Each mask connects H(0)=1 and H(16)=$H_{max}(16)$.
An “anchor” is set at $H_m(4)$=3/N, provided that in N codewords the “bin 4” count was 0.
The linear fit of the measured bins and the anchor should be below the mask to predict success.

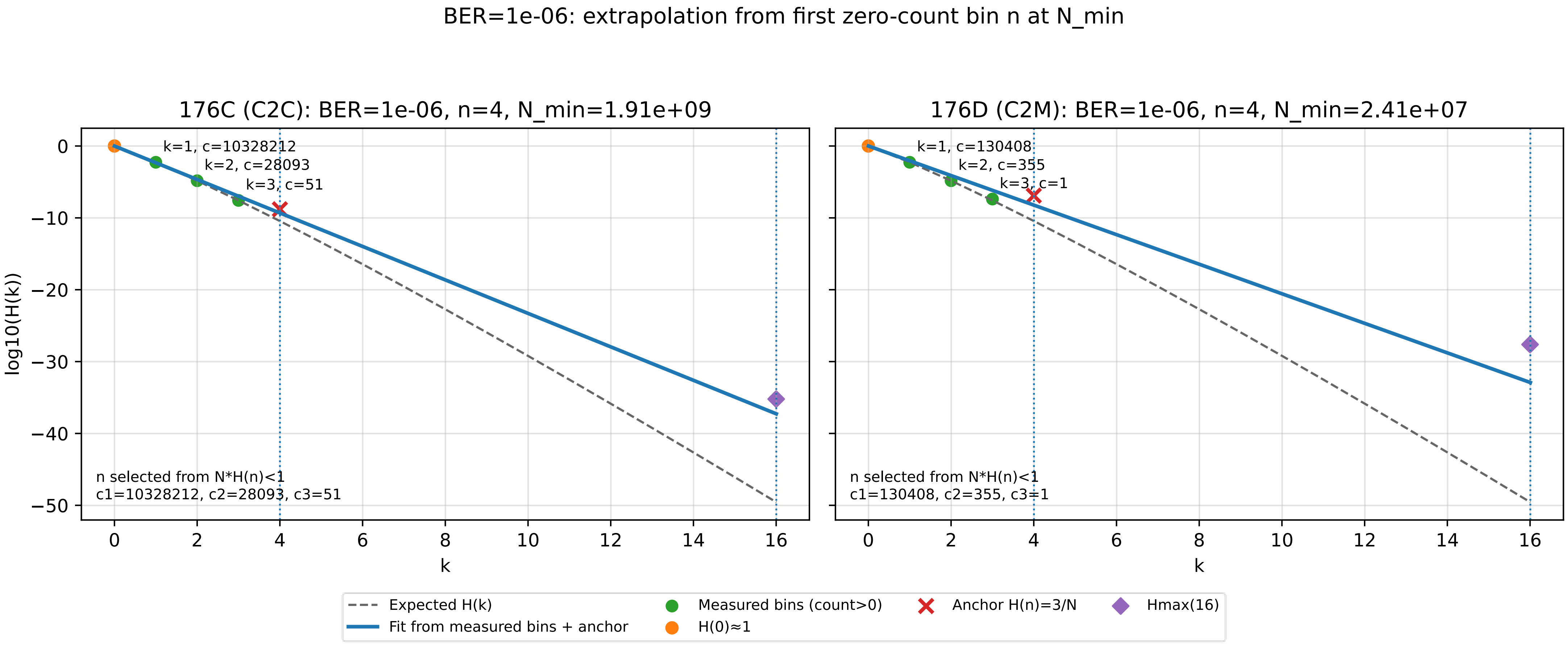
Example with BER=10^{-6}

Metric	176C (C2C)	176D (C2M)
$H_{max}(16)$ (p=1)	6.1×10^{-36}	2.4×10^{-28}
BER	1×10^{-6}	1×10^{-6}
λ (expected bit/symbol errors per codeword)	5.44×10^{-3}	5.44×10^{-3}
Anchor bin n (≥4)	4	4
N_{min}	1.9×10^{9}	2.4×10^{7}
Rcw (p=1)	39.0625×10^{6} cw/s	39.0625×10^{6} cw/s
Minimum test time (p=1)	48.9 s	0.6 s

For selected n=4, $N_{min} = 3 \cdot H_{max}(16)^{-\frac{4}{16}}$, and $N_{min} \cdot\ H(4) < 1$ is verified.
Low BER results in relatively small N_{min} for n=4, therefore extrapolation can be done after a short test time.

Illustration: extrapolation with BER=10^{-6}

(Assuming uncorrelated errors)



Example with $BER=BER_{max}$

Metric	176C (C2C)	176D (C2M)
$H_{max}(16)$ (p=1)	6.1×10^{-36}	2.4×10^{-28}
BER	8×10^{-6}	2.4×10^{-5}
λ (expected bit/symbol errors per codeword)	0.04352	0.13056
first zero-count bin n	18 (>16)	18 (>16)
N_{min}	1.2×10^{40}	3.5×10^{31}
Rcw (p=1)	39.0625×10^{6} cw/s	39.0625×10^{6} cw/s
Test time (p=1)	10^{25} years	2.9×10^{16} years

N_{min} and n were solved iteratively to satisfy the first expected zero-count bin criterion, $N_{min}*H(n)<1$.
The result shows that extrapolation does not help in this case.

Sensitivity to anchor n

Minimum bin n	4		5		6	
Specification	**176C (C2C)**	**176D (C2M)**	**176C (C2C)**	**176D (C2M)**	**176C (C2C)**	**176D (C2M)**
N_{min}	1.9×10^{9}	2.4×10^{7}	3.0×10^{11}	1.3×10^{9}	4.8×10^{13}	6.8×10^{10}
Minimum test time	48.9 s	0.6 s	2.2 h	32.9 s	0.5 months	29.1 min

Larger n requires much larger N for the BER_{max} values we use; n=4 is the maximum for a practical test time in C2C.
For C2M, n=5 can still be used with a reasonable test time.
Note that having nonzero count in bin 4 requires using a larger n and a longer test to predict success.

Comparison of BER scenarios

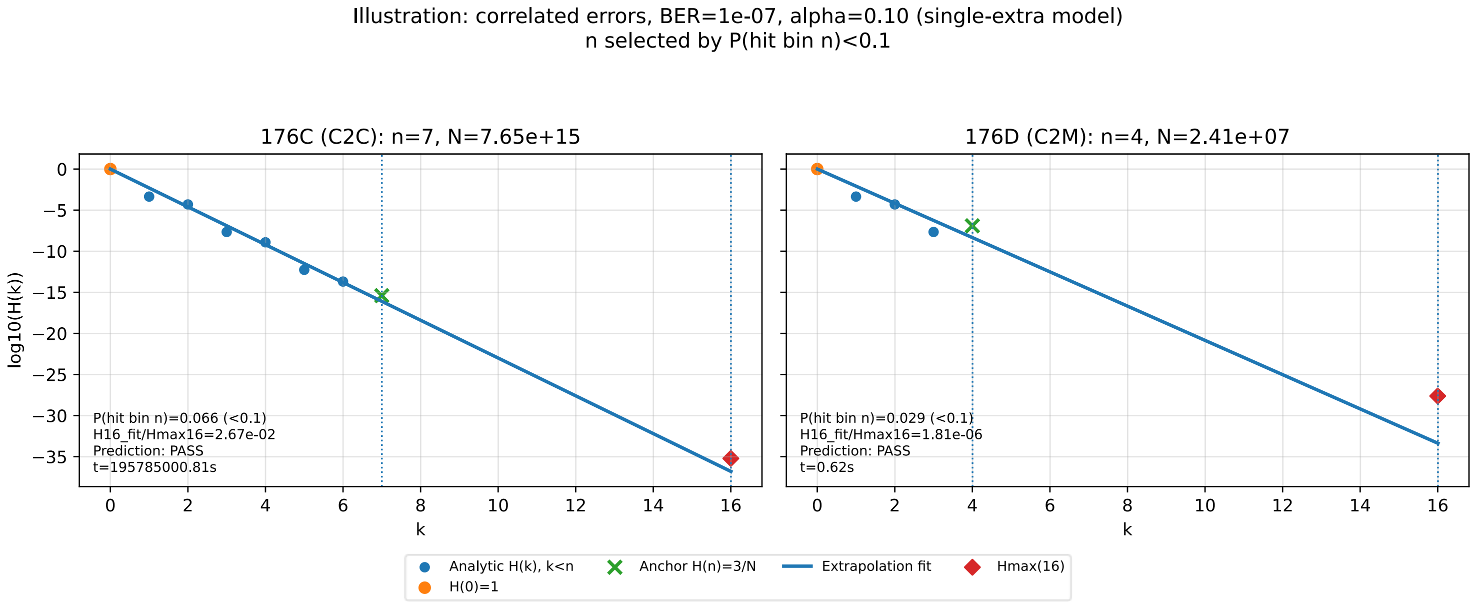
Observation	BER = 1×10^{-6}	BER = BER_{max}
Bin 4 event rate (counts per second)	$\sim 1.4 \times 10^{-3}$	176C: ~6 176D: ~415
First zero-count bin n with minimum N	Likely ≤4 for short runs	Likely >4
Implication for test duration	“pass” can be predicted after a very short test, based on extrapolation	“pass” can’t be predicted based on extrapolation (all extrapolation attempts predict failure)

Key point: the extrapolation starts at the first zero-count bin n, rather than a fixed bin.
As the BER of the DUT increases, higher values of N and n will be needed to make the extrapolation predict a “pass” (assuming the DUT actually meets the BLER requirement), and longer test time will be required.

What about correlated errors?

- Correlated errors will cause counts at some bins to be higher than the expected value based on the average BER.
  - If the count in bin 4 is nonzero, a larger n will be required.
- The slope based on the observed nonzero counts will be shallower, but may still pass (if the random BER is low enough).
- The next slide illustrates the case of average BER=10^{-7} with a "simplified precoding" model: a probability of $\alpha = 0.1$ to create one additional symbol error.

Illustration: correlated errors with BER=10^{-7}



For the Annex 176D case, using the proposed method (anchored at the first zero-count bin), the test time is sufficient for extrapolation from bin 4 to predict compliance with the BLER requirement.

However, in the Annex 176C case, the test time is longer and some bin 4 events are expected. The error statistics in this scenario are still within the BLER requirement (borderline), but the projection requires an impractical test time (~6 years!), and the first usable anchor is n=7. Any shorter test will predict failure.

Proposal

- Add the following NOTE after the text in 176C.6.4.4 and in 176D.8.11:

  NOTE—In order to predict whether a receiver meets the BLER requirement in a short test time, extrapolation of the measured histogram (see 174A.9.3) to $H_m(16)$ can be performed. One way of doing this is using a linear fit of the slope of $\log_{10}(H_m(k))$ for $k = 0$ to n, where n is at least 4 and $H_m(k)$=0 for all $k \geq n$. For the linear fit, $H_m(n)$ is set to $3/N$ where N is the total number of test blocks, and the intercept $H_m(0)$ is set to 1. The fit is performed only for values of k that have $H_m(k)$>0.

Backup

Error masks

Table 176C–6—Receiver error mask

Test symbol errors per test block, k (see 174A.9.5)	Probability $H_{max}(k)$			
	$p = 1$	$p = 2$	$p = 4$	$p = 8$
1	4.2×10^{-2}	2.1×10^{-2}	1.1×10^{-2}	5.4×10^{-3}
2	9.1×10^{-4}	2.3×10^{-4}	5.8×10^{-5}	1.5×10^{-5}
3	1.3×10^{-5}	1.7×10^{-6}	2.1×10^{-7}	2.6×10^{-8}
4	1.4×10^{-7}	8.9×10^{-9}	5.5×10^{-10}	3.3×10^{-11}
5	1.2×10^{-9}	3.8×10^{-11}	1.2×10^{-12}	3.4×10^{-14}
6	8.8×10^{-12}	1.4×10^{-13}	2.0×10^{-15}	2.9×10^{-17}
7	5.4×10^{-14}	4.2×10^{-16}	3.0×10^{-18}	2.0×10^{-20}
8	2.9×10^{-16}	1.1×10^{-18}	3.9×10^{-21}	1.2×10^{-23}
9	1.4×10^{-18}	2.6×10^{-21}	4.4×10^{-24}	6.6×10^{-27}
10	5.9×10^{-21}	5.4×10^{-24}	4.5×10^{-27}	3.1×10^{-30}
11	2.3×10^{-23}	1.0×10^{-26}	4.1×10^{-30}	1.3×10^{-33}
12	8.2×10^{-26}	1.8×10^{-29}	3.4×10^{-33}	5.0×10^{-37}
13	2.7×10^{-28}	2.9×10^{-32}	2.6×10^{-36}	1.7×10^{-40}
14	8.1×10^{-31}	4.3×10^{-35}	1.9×10^{-39}	5.4×10^{-44}
15	2.3×10^{-33}	5.9×10^{-38}	1.2×10^{-42}	1.6×10^{-47}
16	6.1×10^{-36}	7.5×10^{-41}	7.3×10^{-46}	4.1×10^{-51}

Table 176D–11—Receiver error mask

Test symbol errors per test block, k (see 174A.9.5)	$H_{max}(k)$			
	$p = 1$	$p = 2$	$p = 4$	$p = 8$
1	1.1×10^{-1}	6.1×10^{-2}	3.2×10^{-2}	1.6×10^{-2}
2	7.5×10^{-3}	2.0×10^{-3}	5.1×10^{-4}	1.3×10^{-4}
3	3.2×10^{-4}	4.3×10^{-5}	5.5×10^{-6}	6.8×10^{-7}
4	1.1×10^{-5}	6.9×10^{-7}	4.4×10^{-8}	2.7×10^{-9}
5	2.7×10^{-7}	8.9×10^{-9}	2.8×10^{-10}	8.2×10^{-12}
6	5.9×10^{-9}	9.5×10^{-11}	1.5×10^{-12}	2.1×10^{-14}
7	1.1×10^{-10}	8.7×10^{-13}	6.5×10^{-15}	4.4×10^{-17}
8	1.7×10^{-12}	6.9×10^{-15}	2.5×10^{-17}	8.0×10^{-20}
9	2.5×10^{-14}	4.9×10^{-17}	8.6×10^{-20}	1.3×10^{-22}
10	3.2×10^{-16}	3.1×10^{-19}	2.6×10^{-22}	1.8×10^{-25}
11	3.7×10^{-18}	1.8×10^{-21}	7.2×10^{-25}	2.3×10^{-28}
12	4.0×10^{-20}	9.2×10^{-24}	1.8×10^{-27}	2.6×10^{-31}
13	3.9×10^{-22}	4.4×10^{-26}	4.1×10^{-30}	2.7×10^{-34}
14	3.6×10^{-24}	2.0×10^{-28}	8.7×10^{-33}	2.6×10^{-37}
15	3.0×10^{-26}	8.1×10^{-31}	1.7×10^{-35}	2.2×10^{-40}
16	2.4×10^{-28}	3.1×10^{-33}	3.1×10^{-38}	1.8×10^{-43}

Expected bin counts in 1 minute (p=1)

	Good margin case		176C BER_{max}		176D BER_{max}	
	BER=1×10^{-6}		BER=8×10^{-6}		BER=2.4×10^{-5}	
	$H(k)$	$E[k, 60s]$	$H(k)$	$E[k, 60s]$	$H(k)$	$E[k, 60s]$
k=1	5.41×10^{-3}	1.3×10^{7}	4.17×10^{-2}	1×10^{8}	1.15×10^{-1}	3×10^{8}
k=2	1.47×10^{-5}	3.5×10^{4}	9.07×10^{-4}	2×10^{6}	7.48×10^{-3}	2×10^{7}
k=3	2.67×10^{-8}	6.3×10^{1}	1.32×10^{-5}	3×10^{4}	3.26×10^{-4}	8×10^{5}
k=4	3.63×10^{-11}	8.5×10^{-2}	1.43×10^{-7}	3×10^{2}	1.06×10^{-5}	2×10^{4}

Expected 1-minute count for bin k: $E[k, 60s] = 60 \times Rcw \times H(k)$

Detailed calculation for Annex 176D, assuming $BER = 1 \times 10^{-6}$

Inputs

- bits/codeword = 5440
- Bit rate = 212×10^9 bits/s
- Codeword rate = 39,062,500 codewords/s
- $Hmax(16) = 2.4 \times 10^{-28}$
- $BER = 1 \times 10^{-6}$
- $\lambda = 5440 \cdot \mathrm{BER} = 5.44 \times 10^{-3}$

H(16) extrapolation from anchor n=4:

$$H(16) = \left(\frac{3}{N}\right)^{\frac{16}{4}} = \left(\frac{3}{N}\right)^{4}$$

Set H(16)=H_{max}(16):

$$\left(\frac{3}{N}\right)^{4} = 2.4 \times 10^{-28}$$

$N = 3 \cdot (2.4 \times 10^{-28})^{-\frac{1}{4}} \approx 2.4 \times 10^{7}$ codewords

Consistency checks

Expected bin-4 probability at BER=10^{-6}:

$$H4_{expected} = e^{-\lambda} \cdot \frac{\lambda^4}{4!} \approx 3.6 \times 10^{-11}$$

Expected bin-4 count at N:

$$N \cdot H4_{expected} = 8.75 \times 10^{-4} < 1$$

So first expected zero-count bin is n=4 (valid).

Anchor used in fit:

$$H(4) = \frac{3}{N} \approx 1.2 \times 10^{-7}$$

Projected endpoint:

$$\left(\frac{3}{N}\right)^{4} = 2.4 \times 10^{-28} = H_{\max}(16)$$

Sufficient test time:

$$t = \frac{N}{Rcw} \approx \frac{2.4 \times 10^{7}}{3.9 \times 10^{7}} \approx 0.617\ s$$

Alternative proposal

- Comments 200 and 201 suggest adding the following note:

In order to predict whether a receiver meets the BLER requirement in a short test time it is acceptable to have zero counts in a 10 second test for any bin which has a probability less than 10^{-11}.

- As shown on slide 15, the H_{max} values less than 10^{-11} (for p=1) are k≥6 (176C) and k≥8 (176D).
- Having 0 counts in these bins in a 10-second test is ”too easy” – with BER_{max}, the expected count in these bins is <<1:
  - For 176C (BER_{max}=8×10^{-6}): E[6, 10s]≈ 3.5×10^{-3}
  - For 176D (BER_{max}=2.4×10^{-5}): E[8, 10s]≈7.2×10^{-4}

Bin distribution due to BER_{added}